Lecture IV: Search and Matching

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Overview

- So far application to growth and savings (related)
- We have seen that the recursive formulation is very general
- Now look at two canonical applications to the labor market

This lecture: Search and Matching

The wage search model: McCall (1970)

The search and matching model: Diamond (1982), Mortensen (1982) and Pissarides (1990)

The wage-posting model: Burdett and Mortensen (1998)

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Key ingredients

- Unemployed worker who is looking for a job
- Draw a job offer with exogenous probability λ
- Offer is a wage draw from exogenous wage offer distribution $F(.)$: "take-it-or-leave-it"
- Linear utility; preferences given by

$$\sum_{t=0}^{\infty} \beta^t y_t$$

y_t: income this period

How should we think about λ and $F(.)$?

How should we think about λ?

- Probability to get a job offer in each period
- Microfoundation: imperfect info in two-sided market
    - Many job-seekers
    - Limited number of vacant jobs
    - Takes time to get an offer
- We use a **matching function** to formalize this idea. More on this soon.

How should we think about $F(.)$?

- Perfectly competitive labor markets:
  - Single wage for similar workers: paid their marginal product
  - Implicitly: costless to find alternative employer
- Monopsonistic competition:
  - Infinite cost to find alternative employer
  - Workers paid their reservation wage
- Microfoundation for $F(.)$: somewhere in between perfect competition and monopsonistic competition, **search frictions**

In the data: Residual wage dispersion in some "homogenous" labor market

Residual wage dispersion

- Typical measure: Residual in regression

$$\ln w_{it} = x_{it}'\beta + \alpha_i + \psi_{j(i,t)} + \varepsilon_{it}$$

  with

$$\begin{aligned} x_{it} &: \text{covariates (age polynomial, etc.)} \\ \alpha_i &: \text{worker fixed-effect} \\ \psi_{j(i,t)} &: \text{firm fixed effect (firm } j \text{ of worker } i \text{ in period } t\text{)} \end{aligned}$$

- Known as **two-way fixed-effect model** (sometimes error component model)

- Share of explained variance in this regression is 70-80%

The McCall search model (1970)

- Income is

$$y_t = \begin{cases} w & \text{if worker employed} \\ b & \text{if worker unemployed } (b > 0) \end{cases}$$

- Recursive formulation: $V(w)$ is value for worker with offer w in hand

$$V(w) = \max_{\text{accept,reject}} \left\{ \frac{w}{1-\beta},\ b + \beta \cdot \lambda \int_0^{\overline{w}} V(w') dF(w') \right\}$$

- Note: Jobs assumed to last forever!

Solution concept: Reservation wage

$$V(w) = \max_{\text{accept,reject}} \left\{ \frac{w}{1-\beta},\ b + \beta \cdot \lambda \int_0^{\overline{w}} V(w')dF(w') \right\}$$

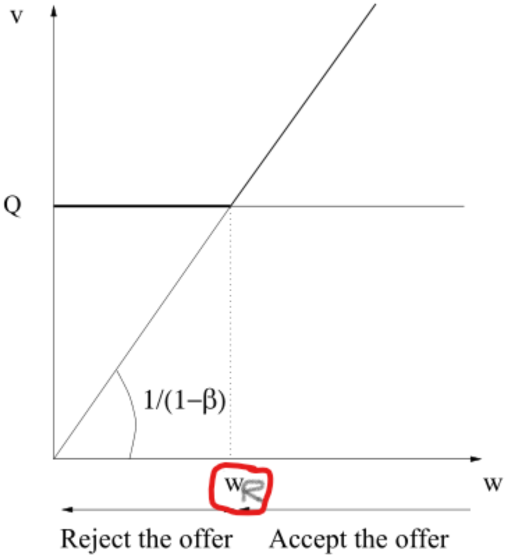
- Left-hand side is increasing in w
- Right-hand side is constant

$$Q := b + \beta \cdot \lambda \int_0^{\overline{w}} V(w')dF(w')$$

So there must exist a reservation wage $w_R > 0$ such that

$$\frac{w_R}{1-\beta} = Q = b + \beta \cdot \lambda \int_0^{\overline{w}} V(w')dF(w')$$

The value function in the McCall model



Reservation wage: Characterization

- For simplicity, let's now assume $\lambda = 1$
- The value function is given by

$$V(w) = \begin{cases} \frac{w_R}{1-\beta} = b + \beta \int_0^{\overline{w}} V(w') dF(w') & \text{if} \quad w < w_R \\ \frac{w}{1-\beta} & \text{if} \quad w \geq w_R \end{cases}$$

- The reservation wage must satisfy

$$\begin{aligned} \frac{w_R}{1-\beta} &= b + \beta \int_0^{\overline{w}} V(w') dF(w') \\ &= b + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w') + \beta \int_{w_R}^{\overline{w}} \frac{w'}{1-\beta} dF(w') \end{aligned}$$

Reservation wage: Characterization

Manipulating this last expression

$$\frac{w_R}{1-\beta} = b + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w') + \beta \int_{w_R}^{\overline{w}} \frac{w'}{1-\beta} dF(w')$$

we obtain

$$w_R = \underbrace{b}_{\text{income in unemp.}} + \beta \underbrace{\int_{w_R}^{\overline{w}} \frac{w' - w_R}{1-\beta} dF(w')}_{\text{expected gains from waiting}}$$

Homework: check this.

Reservation wage: Characterization

From

$$w_R = b + \beta \int_{w_R}^{\overline{w}} \frac{w' - w_R}{1-\beta} dF(w'),$$

we also get

$$\frac{dw_r}{db} \underbrace{\left(1 + \frac{\beta}{1-\beta} \int_{w_R}^{\overline{w}} dF(w')\right)}_{>0} = 1 \qquad \text{(use Leibniz rule)}$$

$$\Rightarrow \quad \frac{dw_r}{db} > 0.$$

So, not surprisingly, the reservation wage increases with b.

Do workers ever want to quit?

- Intuitively no: they face the same problem as before if they quit

- The value to take an offer w and quit after t periods is

$$
\begin{aligned}
& w\frac{1-\beta^t}{1-\beta} + \beta^t \left(b + \beta \int V(w')dF(w') \right) \\
= \; & w\frac{1-\beta^t}{1-\beta} + \beta^t \frac{w_R}{1-\beta} \\
= \; & \frac{w}{1-\beta} - \beta^t \frac{w - w_R}{1-\beta} < \frac{w}{1-\beta}
\end{aligned}
$$

- So it is never optimal to take an offer and then quit provided the offer was above the reservation wage

Numerical solution to the McCall model

VFI will still work here:

1. Start from a grid for wages $\{w_i\}_{i=1}^{N}$ and define a stopping rule $\varepsilon > 0$
2. Make an initial guess for the value function at each point in the wage grid $V_0(w_i)$
3. At iteration k, update the value function as

$$V_{k+1}(w_i) = \max\left\{\frac{w_i}{1-\beta}, b + \beta \sum_j \Pr(w = w_j) V_k(w_j)\right\}$$

4. If $||V_{k+1}(w) - V_k(w)|| < \varepsilon$ stop. Else set $k = k + 1$, go back to 3.

A better algorithm?

- Note: the reservation wage fully summarizes workers' choices
- An alternative solution strategy would use the expression we derived for the reservation wage

$$w_R = b + \beta \int_{w_R}^{\overline{w}} \frac{w' - w_R}{1-\beta} dF(w')$$

- We could define the function

$$h(x) := x - b - \beta \int_{x}^{\overline{w}} \frac{w' - x}{1-\beta} dF(w')$$

  and directly look for the "root" w_R such that $h(w_R) = 0$.

- See PS4

This lecture: Search and Matching

The wage search model: McCall (1970)

The search and matching model: Diamond (1982), Mortensen (1982) and Pissarides (1990)

The wage-posting model: Burdett and Mortensen (1998)

The matching function

- λ: chance to draw an offer. This probability can be made endogenous by way of a **matching function**

- Number of "matches"

$$\text{jobs created} := m(u, v)$$

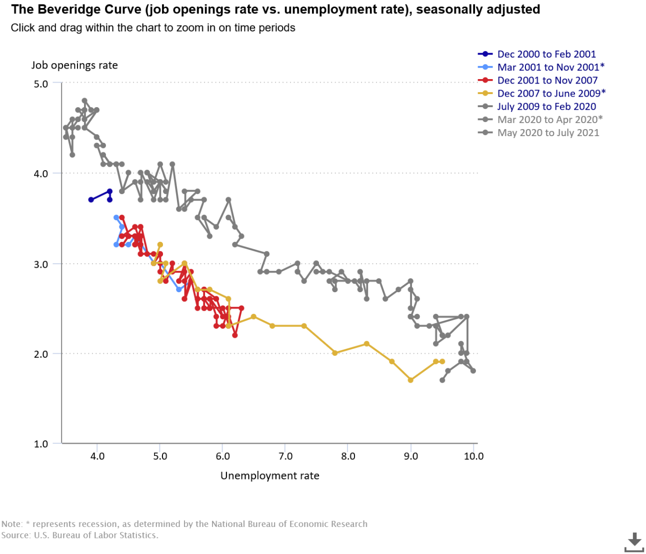
  with

$$u := \text{job seekers}$$
$$v := \text{vacancies}$$

- It takes time for job seekers and employers to meet

- Labor markets don't clear: both job seekers and vacant jobs

Empirical counterpart: Beveridge curve



The matching function

The standard assumptions on the function $m(.,.)$ are:

1. Increasing in both arguments
2. Concave in both arguments
3. Homogeneous of degree one:

$$m(\kappa \cdot u, \kappa \cdot v) := \kappa \cdot m(u, v)$$

for some $\kappa \in \mathbb{R}$

You can check that the Cobb-Douglas matching function $m(u, v) := Au^{\alpha} v^{1-\alpha}$ satisfies these assumptions

The matching function

- The probability a vacancy get filled

$$\frac{m(u,v)}{v} \quad \underbrace{= m\left(\frac{u}{v}, 1\right)}_{m \text{ homog. of degree } 1} \quad := q\left(\frac{v}{u}\right) := q(\theta)$$

where we define the **tightness ratio** $\theta := v/u$

- The probability an unemployed worker finds a job

$$\frac{m(u,v)}{u} = m\left(1, \frac{v}{u}\right) := \lambda(\theta) = \theta q(\theta)$$

Note: In steady-state θ is constant, so implicitly $\lambda(\theta) = \lambda$ in our general framework

Search and matching model: aggregate flows

- We normalize the measure of workers to 1: u_t is share unemployed (or unemployment rate) $1 - u_t$ is share employed

- $\delta \in (0, 1)$ is the exogenous probability to separate from a job into unemployment

- So in steady state ($u_t = u$), the inflow and outflow to and from unemployment must be equal

$$\delta \cdot (1 - u) = \lambda(\theta) \cdot u \quad \Rightarrow \quad u = \frac{\delta}{\delta + \lambda(\theta)}.$$

We need more structure to pin down θ!

DMP model: Firm side

- **Assumption:** All firms and workers are identical
- Linear production technology. We focus on jobs, with productivity y paying wage w
- Cost of opening a vacancy is c
- Denote J the value of a filled job and V the value of a vacancy

$$J = y - w + \beta\Big[\delta V + (1-\delta)J\Big]$$
$$V = -c + \beta\Big[q(\theta)J + (1-q(\theta))V\Big]$$

Note: The firm isn't making a choice here

Free entry condition

- Free-entry of firms: Firms will enter as long as the value of posting a vacancy is non-negative

- So in equilibrium $V = 0$, which gives

$$\beta q(\theta) J = c \;\Rightarrow\; J = \frac{c}{\beta q(\theta)}$$

- Plugging this back in the expression for J,

$$w = y - \frac{c}{q(\theta)}(\rho + \delta)$$

  where $\beta := (1+\rho)^{-1}$

$\Rightarrow$ First expression for w as a function of θ

DMP model: Worker side

- Again workers are all identical
- Earn wage w when employed and have exogenous home production b when unemployed
- Denote E and U the value, respectively, for an employed and unemployed worker

$$E = w + \beta\Big[\delta U + (1-\delta)E\Big]$$
$$U = b + \beta\Big[\lambda(\theta)E + (1-\lambda(\theta))U\Big]$$

We need another assumption on wage determination to pin down θ and w

Nash bargaining

- **Nash-bargaining** is a bargaining protocol to determine how to split the surplus created by a job

$$S := \underbrace{J}_{\text{firm part of surplus}} + \underbrace{E - U}_{\text{worker part of surplus}}$$

- Under Nash-bargaining, profits are split according to

$$\max_{J, E-U} (E - U)^{\phi} J^{1-\phi} \quad \text{s.t.} \quad S = J + E - U$$

- You can check the solutions to this problem are

$$E - U = \phi S \quad \text{and} \quad J = (1 - \phi) S$$

The parameter ϕ is called the **Nash bargaining weight**

Solving for w and θ

- After some (somewhat tedious) algebra we can obtain a second equation for wages

$$w = b + \phi(y - b + \theta c).$$

- Recall that we already found

$$w = y - \frac{c}{q(\theta)}(\rho + \delta)$$

- Combining these two expressions, θ is implicitly defined by

$$y - b = c \cdot \frac{\rho + \delta + \phi\lambda(\theta)}{(1 - \phi)q(\theta)}$$

And we're done: we have a solution for θ as a function of the model's parameter

Defining the equilibrium

- This is the first General Equilibrium model we've seen in the course: θ arises from the supply of vacancy from firms and the demand for jobs from unemployed workers
- Vast majority of macro papers include a formal **definition of equilibrium**
- "An equilibrium is [define some objects], such that [list some conditions]"
- For the DMP model, such a definition can be very succinct: "An equilibrium is a tightness $\theta > 0$, such that

$$y - b = c \cdot \frac{\rho + \delta + \phi\lambda(\theta)}{(1-\phi)q(\theta)}$$

holds."

Planner's problem

- In the model we have laid out so far, agents make decisions that are optimal for them individually
- For example, implicitly firms keep opening vacancies until the value is driven to zero ($V = 0$)
- The "planner" makes choices for all agents in the same environment, but to maximize outcomes for the whole economy
- This yields a **social optimum**, which can be used to benchmark the **decentralized equilibrium** we found above

The planner's problem in the DMP model

The planner chooses $\{u_{t+1}, v_t\}_{t=0}^{\infty}$ to

$$\max_{\{u_{t+1}, v_t\}_{t=0}^{\infty}} \sum_t \beta^t \Big[y(1-u_t) + bu_t - cv_t \Big]$$

$$\text{s.t} \quad u_{t+1} = \delta(1-u_t) + \left[1 - \lambda\left(\frac{v_t}{u_t}\right)\right] u_t$$

$$u_0 \text{ given}$$

Remarks:

1. The planner operates in the same environment: matching frictions, etc.

2. Only quantities $\{u_{t+1}, v_t\}_{t=0}^{\infty}$ are chosen — no need to specify wage determination (how to split output)

The planner's problem in the DMP model

The planner chooses $\{u_{t+1}, v_t\}_{t=0}^{\infty}$ to

$$\max_{\{u_{t+1},v_t\}_{t=0}^{\infty}} \sum_t \beta^t \Big[y(1-u_t) + bu_t - cv_t \Big]$$

$$\text{s.t} \quad u_{t+1} = \delta(1-u_t) + \left[1 - \lambda\left(\frac{v_t}{u_t}\right)\right] u_t \qquad (\mu_t \geq 0)$$

$$u_0 \text{ given}$$

with associated FOCs

$$\begin{aligned}
[v_t]: \quad & 0 = -\beta^t c + \mu_t \lambda'\left(\frac{v_t}{u_t}\right) \\
[u_{t+1}]: \quad & 0 = \beta^{t+1}(-y+b) + \mu_t \\
& \quad - \mu_{t+1}\Big[-\delta + \lambda'(\theta_{t+1})\theta_{t+1} + (1-\lambda(\theta_{t+1}))\Big]
\end{aligned}$$

Hosios condition

- Combining the FOCs to eliminate μ_t and evaluating at the steady-state gives

$$y - b = \frac{c}{\lambda'(\theta)}\Big[\rho + \delta + \lambda(\theta) - \lambda'(\theta)\theta\Big].$$

- This is close to the condition we got in the decentralized equilibrium:

$$y - b = \frac{c}{(1-\phi)q(\theta)}\Big[\rho + \delta + \phi\lambda(\theta)\Big].$$

- So the $\theta := v/u$ ratio the planner would choose is related to that arising in the decentralized equilibrium

- Get stronger result with Cobb-Douglas matching function

Hosios condition

- With $m(u,v) = Au^{\alpha}v^{1-\alpha},\ A > 0,\ \alpha \in (0,1)$, you can check that:

$$\lambda'(\theta)\theta = (1-\alpha)\lambda(\theta); \quad \lambda'(\theta) = (1-\alpha)q(\theta).$$

- Substituting in the planner's solution, we get

$$\begin{aligned} \text{planner}: \quad & y - b = \frac{c}{(1-\alpha)q(\theta)}\left[\rho + \delta + \alpha\lambda(\theta)\right] \\ \text{decentralized}: \quad & y - b = \frac{c}{(1-\phi)q(\theta)}\left[\rho + \delta + \phi\lambda(\theta)\right] \end{aligned}$$

- They are the same if we "set" ϕ such that $\alpha = \phi$

- This is known as the **Hosios condition**

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The wage-posting model

- Combine ingredients from two previous models:
  - Wage offer distribution $F(.)$
  - Flows in-and-out of unemployment (UE and EU transitions)
- In addition, we allow workers to **search on the job**: There are EE transitions (sometimes "JJ" or "J2J")
- Framework where worker mobility (UE, EU, and EE flows) and wage distribution jointly determined

Most papers in this literature are written in continuous time, so we introduce this notation first

DMP model in continuous time

- We have seen that the value of an employed worker is given by

$$E = w + \frac{1}{1+\rho}\Big[\delta U + (1-\delta)E\Big].$$

- Let's rewrite the same expression in some interval of time Δ small enough that $1 - \Delta\delta \leq 1$

$$E = \Delta w + \frac{1}{1+\Delta\rho}\Big[\Delta\delta U + (1-\Delta\delta)E\Big].$$

- Rearranging and letting Δ go to zero, we get

$$\rho E = w + \delta(U - E).$$

DMP model in continuous time

- So in continuous time, workers' value functions in DMP are given by

$$
\begin{aligned}
\rho E &= w + \delta(U - E) \\
\rho U &= b + \lambda(E - U)
\end{aligned}
$$

- Formally δ and λ are parameters of their respective **Poisson process**

- Equations for "value of an asset yielding income w (b) and changing to state U (E) at **Poisson rate** δ (λ)."

See References slide for some additional pointers on these definitions.

Wage-posting model: Worker side

- Workers are all identical
- Unemployed (employed) workers sample job offers at Poisson rate λ_0 (λ_1)
- Lose their job at Poisson rate δ
- Offer is still a constant wage w until the worker leaves, voluntarily (λ_1-shock) or unvoluntarily (δ-shock)
- Offer is still a draw from a wage offer distribution $F(.)$ with support $[\underline{w}, \overline{w}]$

Assumption: There is a single wage offer distribution from which both employed and unemployed workers draw

Employed worker value function

- Employed workers only change jobs if it gives them higher lifetime value
- Formally, let $V(w)$ be the lifetime value of a job paying w

$$\rho V(w) = w + \delta\Big[U - V(w)\Big] \\ + \lambda_1 \left[\int_{\underline{w}}^{\overline{w}} \max\left\{V(x), V(w)\right\} dF(x) - V(w)\right].$$

- It can be shown that $V(.)$ is **strictly increasing** in w

Employed worker value function

- So we can rewrite the lifetime value of a job paying w as

$$[\rho + \delta + \lambda_1 \overline{F}(w)] V(w) = w + \delta U + \lambda_1 \int_{w}^{\overline{w}} V(x) dF(x)$$

  with $\overline{F} = 1 - F$.

- Workers then **"climb the wage ladder"** as a result of their reservation strategy

Homework: Check you can obtain the last expression as the limit as $\Delta \to 0$ of the worker's value function in discrete time.

Unemployed worker value function

- Lifetime value of unemployment U is given by

$$\rho U = b + \lambda_0 \left[\int_{\underline{w}}^{\overline{w}} \max\{V(x), U\} dF(x) - U \right]$$

- Since $V(.)$ is increasing, we can define the **reservation wage** w_R as $V(w_R) = U$,

$$[\rho + \lambda_0] U = b + \lambda_0 \int_{w_R}^{\overline{w}} V(x) dF(x)$$

- The lowest posted wage (lower support of $F(.)$) is $\underline{w} = \max\{w_R, w_{\min}\}$ where $w_{\min}$ reflects institutional constraints (e.g. minimum wage)

Flow-Balance equations

- We're in steady-state: **inflows equal outflows**
- Unemployment flow-balance equation is the same as before

$$\lambda_0 \cdot N \cdot u = \delta \cdot N \cdot (1 - u) \;\Rightarrow\; u = \frac{\delta}{\delta + \lambda_0}$$

N: "number" of workers
u: unemployment rate

Note: Unemployed workers accept all offers given $\underline{w} \geq w_R$

Flow-balance equations

- We're in steady-state: **inflows equal outflows**
- Define $G(.)$ the wage distribution in the population of employed workers
- Employed workers with current wage $\leq w$ have flow-balance equation

$$\left[\delta + \lambda_1 \overline{F}(w)\right] \cdot N \cdot (1-u) \cdot G(w) = \lambda_0 \cdot F(w) \cdot N \cdot u$$

- Using the expression for steady-state unemployment rate, $G(.)$ and $F(.)$ are related as follows:

$$G(w) = \frac{\delta F(w)}{\delta + \lambda_1 \overline{F}(w)} \;\Leftrightarrow\; F(w) = \frac{(\delta + \lambda_1) G(w)}{\delta + \lambda_1 G(w)}$$

The F/G relationship

You can check the F/G relationship can also be written

$$\frac{F(w) - G(w)}{\overline{F}(w)G(w)} = \frac{\lambda_1}{\delta} := \kappa_1.$$

So:

1. So $G(.)$ first-order stochastically dominates $F(.)$ ($G \leq F$)
2. This depends on how fast **employed workers** sample offers relative to the rate at which they fall back into unemployment

Question: What happens as $\lambda_1 \to 0$?

Wage-posting: Firm side

We can anchor the wage offer distribution in the optimal recruitment behavior of employers:

- Assume **wage-posting**: Offers are "take-it-or-leave-it"
- Assume **equal treatment** of workers (rules out firm response to outside offers)
- Firms operate a constant-returns to scale production technology with labor as only input
- Recall that all workers are equally skilled

Size of a firm

- Define a "firm" as a collection of jobs paying w, average "firm size" $l(w)$ is given by

$$l(w) := \frac{\text{workers in type-}w\text{ firms}}{\text{type-}w\text{ firms}} = \frac{(1-u)N}{M} \cdot \frac{g(w)}{\gamma(w)}$$

  with M the measure of firms in the market and $\gamma(.) = \Gamma'(.)$ the density of w in the population of firms

- Differentiating the expression for $G(w)$ above, we get

$$l(w) = \frac{(1-u)N}{M} \cdot \frac{1+\kappa_1}{\left[1+\kappa_1\overline{F}(w)\right]^2} \cdot \frac{f(w)}{\gamma(w)}.$$

Firm profits

- Let's assume **equal sampling weights**: $\gamma(w) = f(w)$ [you can relax this by assuming firms post vacancies]

- The average size of a firm becomes

$$l(w) = \frac{(1-u)N}{M} \cdot \frac{1+\kappa_1}{\left[1+\kappa_1 \overline{F}(w)\right]^2}$$

so $l(.)$ **is increasing in** w.

- Flow profits at a firm of productivity p are given by

$$\pi(p,w) := (p-w) \cdot l(w) \propto \frac{(p-w)(1+\kappa_1)}{\left[1+\kappa_1 \overline{F}(w)\right]^2}$$

"$\propto$" means "proportional to"

Wage posting equilibrium: Heterogeneous firms

- Assume that firms have **heterogeneous productivities** p
- Their marginal productivities differ
- When receiving an offer, workers sample the type of (p) of a firm from an exogenous distribution $F_p(.)$ with support $[\underline{p}, \overline{p}]$
- So workers are not inferring firm productivity (p) from the wage [that's another model]
- We focus on **pure strategy equilibria**, where all type-p firms post a single wage $w(p)$

Characterization of equilibrium

Result: $w(.)$ is increasing in p

Proof: Consider two firms $p' > p$ whose respective optimal wage are w' and w. By optimality of the wage,

$$(p' - \overset{(1)}{w'})l(w') > (p' - \overset{(2)}{w})l(w) > (p - \overset{(3)}{w})l(w) > (p - \overset{(4)}{w'})l(w')$$

So $(1) - (4) > (2) - (3)$ and therefore

$$(p' - p)l(w') > (p' - p)l(w) \Leftrightarrow l(w') > l(w) \Leftrightarrow w' > w$$

since $l(.)$ is increasing in w.

Quod Erat Demonstrandum.

Characterization of equilibrium

Result: $w(.)$ is increasing in p

Key implication: $\forall p \in [\underline{p}, \overline{p}],\; F(w(p)) = F_p(p)$

To sum up:

1. There is a one-to-one increasing mapping from productivity p to wages w
2. The productivity rank of a firm $F_p(p)$ is the same as their posted wage rank $F(w(p))$

The equilibrium wage

- The equilibrium level of profits at type-p firm is

$$\pi(p, w(p)) = \frac{(p - w(p))(1+\kappa_1)}{\left[1+\kappa_1 \overline{F}(w(p))\right]^2} = \frac{(p - w(p))(1+\kappa_1)}{\left[1+\kappa_1 \overline{F}_p(p)\right]^2}$$

- The Envelope theorem gives

$$\frac{d\pi}{dp}(p, w(p)) = \frac{\partial \pi}{\partial p}(p, w(p)) = \frac{1+\kappa_1}{1+\kappa_1 \overline{F}_p(p)}$$

- Integrating and substituting $\pi(p, w(p))$ back in, we get:

$$w(p) = p - \left[1+\kappa_1 \overline{F}_p(p)\right]^2 \left\{ \int_{\underline{p}}^{p} \frac{dx}{\left[1+\kappa_1 \overline{F}_p(x)\right]^2} + \underbrace{\frac{\pi(\underline{p}, w(\underline{p}))}{1+\kappa_1}}_{=0 \text{ if free entry}} \right\}$$

Wage distribution in the model and data

- The equilibrium wage is given by

$$w(p) = p - \left[1 + \kappa_1 \overline{F}_p(p)\right]^2 \left\{ \int_{\underline{p}}^{p} \frac{dx}{\left[1 + \kappa_1 \overline{F}_p(x)\right]^2} \right\}$$

- So given values for $\kappa_1 := \lambda_1/\delta$ and a distribution $F_p(.)$ on some interval $[\underline{p}, \overline{p}]$, we can compute the wage distribution implied by the model

Wage distribution in the model and data

- We can also do the opposite: Start from the observed wage distribution and find $F_p(.)$ to rationalize it

- From the FOC for wages in the firm's maximization problem,

$$\frac{\partial \pi}{\partial w}(p, w) = 0$$

we get, using the F/G relationship,

$$p(w) = w + \frac{1 + \kappa_1 \overline{F}(w)}{2\kappa_1 f(w)} = w + \frac{1 + \kappa_1 G(w)}{2\kappa_1 g(w)}$$

- **Note**: The model implies a very, very long right tail in p because the observed distribution of wages has a thin right tail

$$g(w) \xrightarrow[w \to \overline{w}]{} 0 \;\Rightarrow\; p(w) - w = \frac{1 + \kappa_1 G(w)}{2\kappa_1 g(w)} \xrightarrow[w \to \overline{w}]{} \infty$$

Some recent examples based on BM98

Many, many papers have built on this framework. Here are two recent examples:

1. Meghir, C., Narita, R., & Robin, J. M. (2015). Wages and informality in developing countries. American Economic Review, 105(4), 1509-46. [Formal and informal sector: how does increasing the cost of informality affects labor markets?]

2. Lise, J. (2013). On-the-job search and precautionary savings. Review of economic studies, 80(3), 1086-1113. [What if workers can accumulate savings in this framework?]

Wrapping up

Some key concepts:

- Reservation wage, offer distribution, offer arrival rate
- Matching function, tightness
- Definition of Equilibrium, Planner's problem, Hosios condition
- Continuous time notation, Job ladder

References

Wage search model

RMT, Chapter 6

Search and matching model

RMT, Chapter 28

References

Job ladder model

`Fabien Postel-Vinay's lecture slides`: Lectures 2 and 3

Burdett, Kenneth, and Dale T. Mortensen. "Wage differentials, employer size, and unemployment." International Economic Review (1998): 257-273.

A very clear and short intro to the continuous time notation in the labor context can be found in *Labor Economics* (2nd Edition) by Cahuc et al. (2014), Appendix 4.