Lecture III: The Lifecycle Model

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ECS506: PhD Macroeconomics I

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Overview

- Until now, we have mostly maintained the assumption of an infinite planning horizon
- Relaxing this assumption allows to introduce a notion of age in this framework
- The savings problem now yields a wealth profile for wealth and consumption that can be taken to the data

This lecture: The lifecycle model

The age profile of consumption, income, and wealth

The lifecycle model

Solving the lifecycle model numerically

Extensions to the lifecycle model

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Consumption and income age profile

Figure 2.2: Age profile of income and consumption

Note: Data are drawn from the pooled 1980–2010 Interview Surveys of the Consumer Expenditure Survey (CEX).

Wealth age profile

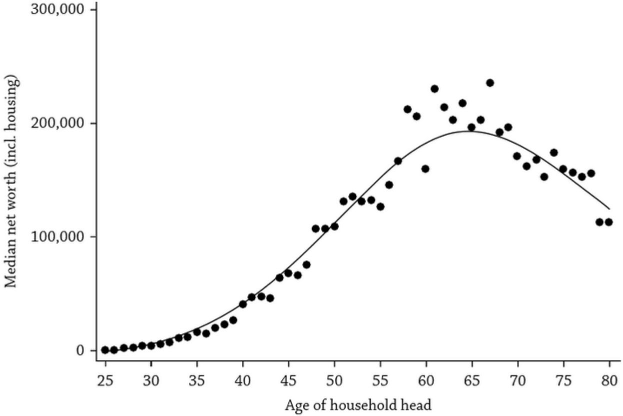

Figure 2.3: The cross-sectional profile of wealth

Note: Data are drawn from the pooled 1983–2007 Survey of Consumer Finances (SCF).

Measurement issues

1. Measuring consumption

   - Surveys: Allows to breakdown consumption in various spending categories (durable vs non-durable), but potential measurement issues

   - Alternative: From good income AND wealth data using the accounting identity (see Eika et al. 2020 for Norway)

$$c_{it} + \sum_k p_{kt} A_{ikt} = (y_{it} - \tau_{it} + \sum_k r_{kt} A_{ikt-1}) + \sum_k p_{kt} A_{ikt-1}$$

2. Econometrics: age vs year vs cohort effects

3. Definitions: household (children, divorce), head of household (arbitrary), etc.

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The finite horizon savings problem

- A household chooses consumption and assets to

$$
\begin{aligned}
\max_{\{c_t, a_{t+1}\}} \; & \sum_{t=0}^{T} \beta^t u(c_t) \\
\text{s.t.} \quad & c_t + a_{t+1} \leq (1+r)a_t + y_t, \;\; t = 0, 1, \ldots, T \\
& c_t \geq 0 \\
& a_0 \text{ given} \\
& a_{T+1} = 0
\end{aligned}
$$

- No income uncertainty: path for y_t is known
- Does not let anything behind $a_{T+1} = 0$, a form of borrowing constraint

The Euler equation again

- For $t < T$, we can attach a Lagrange multiplier to the period-by-period budget constraint (BC) and again find

$$u'(c_t) = (1+r)\beta u'(c_{t+1}), \quad t < T.$$

- Defining the **discount rate** ρ as $\beta := (1+\rho)^{-1}$ and taking logs

$$\Delta \ln u'(c_{t+1}) = \ln\left(\frac{1+\rho}{1+r}\right)$$

The Euler equation: Discount rate and interest rate

- Starting from the log-Euler equation

$$\Delta \ln u'(c_{t+1}) = \ln\left(\frac{1+\rho}{1+r}\right)$$

- A first-order Taylor expansion around c_t of $\ln u'(c_{t+1})$ gives

$$\Delta \ln u'(c_{t+1}) \approxeq \frac{u''(c_t)}{u'(c_t)} \cdot (c_{t+1} - c_t) \approxeq \rho - r$$

$$\Rightarrow \quad \frac{c_{t+1} - c_t}{c_t} \approxeq -\frac{u'(c_t)}{c_t u''(c_t)} \cdot (r - \rho)$$

The Euler equation: Discount rate and interest rate

1. Consumption response to $r, \ \rho$. Since

$$\frac{c_{t+1} - c_t}{c_t} \approxeq -\frac{u'(c_t)}{c_t u''(c_t)} \cdot (r - \rho)$$

   optimal consumption increases (decreases) with r (ρ) along the equilibrium path.

2. Elasticity of Intertemporal Substitution (EIS)

$$EIS := \frac{d\left(c_{t+1} - c_t\right)/c_t}{dr} = -\frac{u'(c_t)}{c_t u''(c_t)}$$

   since the LHS is just (approximately) $d \ln(x/y)/d \ln(p_x/p_y)$ for this case

EIS with CRRA utility

- With constant relative risk aversion (CRRA) utility $u(c) := (1-\sigma)^{-1}(c^{1-\sigma}-1)$

$$\text{relative risk aversion} := -c \cdot \frac{u''(c)}{u'(c)} = \sigma = \frac{1}{EIS}$$

- Plugging into our local approximation gives

$$\frac{c_{t+1}-c_t}{c_t} \approxeq \frac{1}{\sigma} \cdot (r-\rho)$$

- So a more risk-averse agent (larger σ) responds less to a change in r, because of the curvature of utility

- σ and ρ are difficult to separately identify from data on consumption (or wealth) alone with these preferences

The Modigliani and Brumberg (1954) model

- Landmark paper—what people have in mind when they talk about the lifecycle model
- Analytically tractable: get closed form solutions
- We make two additional assumptions:
  1. No discounting by agents/markets: $r = \rho = 0$
  2. Specific income path

Assumption 1: $r = \rho = 0$

- Summing the budget constraint across the agent's lifetime with $r = \rho = 0$ gives

$$\begin{aligned} y_t - c_t &= a_{t+1} - a_t \\ \Rightarrow \sum_{t=0}^{T} y_t - c_t &= \sum_{t=0}^{T} a_{t+1} - a_t = a_{T+1} - a_0 = -a_0 \\ \Rightarrow \sum_{t=0}^{T} c_t &= a_0 + \sum_{t=0}^{T} y_t \end{aligned}$$

- We have already seen that with $r = \rho = 0$, the Euler equation gives

$$u'(c_t) = \frac{1+r}{1+\rho} u'(c_{t+1}) \Rightarrow u'(c_t) = u'(c_{t+1}) \Rightarrow c_t = c \quad \forall t$$

Assumption 2: Income path

- We assume the following income profile

$$y_t = \begin{cases} y & \text{if } t < N \text{ (working life)} \\ 0 & \text{if } t \geq N \text{ (retirement)} \end{cases}$$

- From the lifetime budget constraint, this gives the following consumption function

$$c = \frac{N}{T+1} y + \frac{1}{T+1} a_0$$

from which we can compute the MPCs

$$\text{MPC}(y) = \frac{N}{T+1} \geq \frac{1}{T+1} = \text{MPC}(a_0)$$

Savings and wealth in Modigliani and Brumberg (1954)

- Saving/borrowing in each period follows directly from the identity $s_t + c_t := y_t$. With $a_0 = 0$

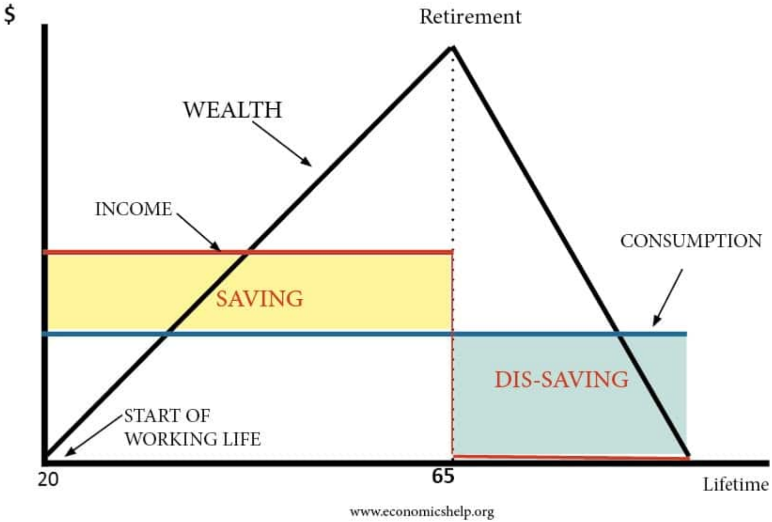
$$s_t = \begin{cases} y - c = y\left(1 - \frac{N}{T+1}\right) & \text{if } t < N \text{ (working life)} \\ 0 - c = -y\frac{N}{T+1} & \text{if } t \geq N \text{ (retirement)} \end{cases}$$

  The agent saves during her working life and dis-save during retirement. There is no borrowing.

- Wealth follows simply from keeping track of the stock of savings over the agent's life span

$$a_t = \begin{cases} ty\left(1 - \frac{N}{T+1}\right) & \text{if } t \leq N \\ Ny\left(1 - \frac{N}{T+1}\right) - (t-N)y\frac{N}{T+1} & \text{if } t > N \end{cases}$$

Modigliani and Brumberg (1954) in one picture



Actually not that bad

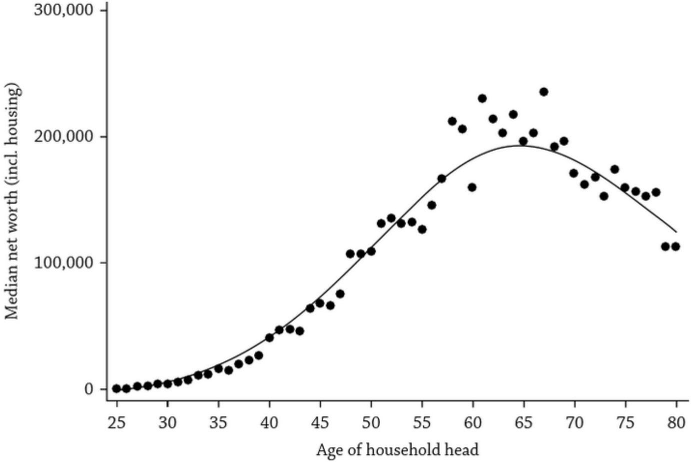


Figure 2.3: The cross-sectional profile of wealth

Note: Data are drawn from the pooled 1983–2007 Survey of Consumer Finances (SCF).

This lecture: The lifecycle model

Beyond Modigliani and Brumberg (1954)

- We got a lifecycle profile for wealth that makes sense in a simple setting
- Many of the assumptions/results in Modigliani and Brumberg (1954) are not in line with the data: constant income, constant consumption
- Given what we know about the savings problem, at the very least we want to introduce income shocks and a borrowing limit

A more general lifecycle model

- A household chooses consumption and assets to

$$
\begin{aligned}
\max_{\{c_t, a_{t+1}\}} \ & \mathbb{E}_0 \sum_{t=0}^{T} \beta^t u(c_t) \\
\text{s.t.} \quad & c_t + a_{t+1} \leq (1+r)a_t + y_t, \ \ t = 0, 1, \ldots, T \\
& c_t \geq 0 \\
& a_0 \text{ given} \\
& a_{T+1} = 0 \\
& a_{t+1} \geq \underline{a}
\end{aligned}
$$

- We can get the Euler equation. But we need the optimal consumption and savings rule to simulate the model

- There is no closed-form solution in this case

The lifecycle model in recursive form

- Define $V_t(a, y)$ the present discounted utility of an agent with assets a, current income y, in period t (or age t)

- For $t < T$, the problem in recursive form is given by

$$V_{t}(a, y) = \max_{c_t, a_{t+1}} u(c_t) + \beta \mathbb{E}_t V_{t+1}(a_{t+1}, y_{t+1})$$
$$\text{s.t.} \quad c_t + a_{t+1} \leq (1+r)a_t + y_t,$$
$$a_{t+1} \geq \underline{a}$$

- t is a state variable here: this is a finite-horizon problem

Terminal condition and numerical solution

- At age T, the terminal condition $a_{T+1} = 0$ gives

$$c_T = a_T(1+r) + y_T$$
$$\Rightarrow V_T(a_T, y_T) = u(a_T(1+r) + y_T).$$

- Numerically we solve for the value function at each t starting from the known function in period T and moving backward

$$V_{T-1}(a_{T-1}, y_{T-1}) = \max_{c_{T-1}, a_T} u(c_{T-1}) + \beta \mathbb{E}_{T-1} V_T(a_T, y_T)$$
$$\text{s.t.} \quad c_{T-1} + a_T \leq (1+r)a_{T-1} + y_{T-1},$$
$$a_T \geq \underline{a}$$

and so on at $T-2, T-3, \ldots$

- See PS3

A digression about calibration/estimation

- We have only emphasized solving the model conditional on some parameter values so far
- For instance, with CRRA utility, we need to assume a value for the relative risk aversion parameter, σ, in

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

- How should we go about pinning down this value?

A digression about calibration/estimation

- We know σ is related to how individuals shift consumption over time
- Our model generates a wealth profile, so we could pick σ to match, say, median wealth by age
- This is known as the Method of Simulated Moments (MSM)
- Possible to retrieve standard errors
- There is clearly a need for a fast solution method—you need to solve and simulate for many different σs

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Extensions to the lifecycle model

Many potential extensions

Framework that can be extended in many different directions to bring it in line with data:

1. Lifetime uncertainty: T is stochastic

2. Bequest motives: What if the agent cares about what's left at T?

3. Distinction between liquid and illiquid wealth

4. Additional components of utility: consumption habits, durable consumption, work vs leisure, home production, etc.

5. Non-standard preferences: mental accounting, hyperbolic discounting, etc.?

Two recent examples

1. **More micro.** De Nardi, French, and Jones (2011): lifetime uncertainty, bequests, medical expenses

2. **More macro.** Kaplan and Violante (2014): distinction between liquid and illiquid wealth

De Nardi, French, and Jones (2011)

- Why do elderly keep such large amount of wealth until very late in life?
- Data on single, retired elderly individuals in the US
- Competing explanations:

  1. Bequest motive
  2. Health status and medical expenditures (it's US data!)
  3. Uncertainty about time of death

Some details

1. Bequest motive "warm glow"

$$\phi(e) = \theta \frac{(e+k)^{1-\nu}}{\nu}$$

   e is wealth net of taxes ("estate")

2. Health status $h = \{\text{good health, bad health}\}$ and medical expenditures

$$\begin{aligned} \ln m_t &= m(g,h,I,t) + \sigma(g,h,I,t) \cdot \psi_t \\ \psi_t &= \zeta_t + \xi_t, \quad \xi_t \sim \mathcal{N}(0, \sigma_\xi) \\ \zeta_t &= \rho_m \zeta_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon) \end{aligned}$$

3. Survival probability $s(g,h,I,t)$

Note: Survival, health status transitions, and medical expenditures taken from the data

What are the states?

- Everything needed to solve the agent's problem next period!
- Variables with some "persistence": assets, persistent part of medical expenditures ζ_t
- Here rewrite the problem in terms of **cash on hand**

$$x_t = a_t + y_n(ra_t + y_t, \tau) - m_t$$

- Age, gender, health status

Problem in recursive form and estimation

- Putting all pieces together

$$V_t(x_t, \zeta_t, h_t, g, I) = \max_{c_t, x_{t+1}} u(c_t, h_t) + \beta s_{g,h,I,t} \mathbb{E}_t V_{t+1}(x_{t+1}, \zeta_{t+1}, h_{t+1}, g, I) + \beta(1 - s_{g,h,I,t})\phi(x_t - c_t)$$

  subject to budget constraint, etc.

- This yields decision rules from which they can simulate the model

- Target median wealth by cohort and permanent income quintile (the I in the state space)

The determinants of wealth in old-age

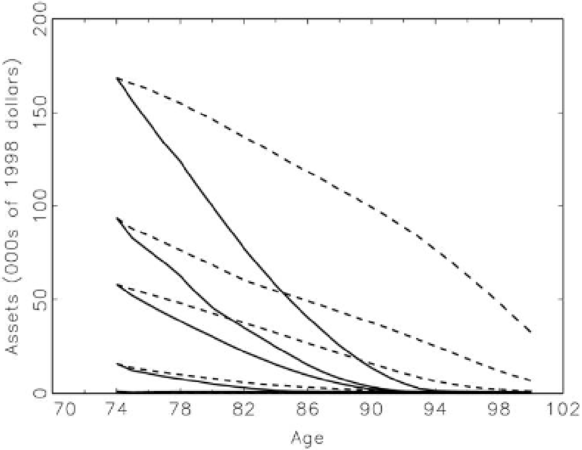


FIG. 9.—Median assets by cohort and permanent income quintile: baseline model (dashed lines) and model with no medical expenses (solid lines).

Kaplan and Violante (2014)

- US households spend a large share of tax rebates on non-durable consumption: 25% in the next quarter
- Single asset model does poorly at replicating this fact: only constrained households respond
- What is needed to bring model in line with empirical evidence?
- Propose a model with two assets: liquid and illiquid (a **portfolio choice** model)

Portfolio choice

- Notation (potentially confusing) for asset type

$$m_t := \text{liquid asset (cash, saving accounts, stocks)}$$
$$a_t := \text{illiquid asset (housing, retirement accounts)}$$

- Return higher on illiquid assets than liquid assets
- a_t enters utility as housing services

$$c_t^{\psi} \cdot s_t^{1-\psi} \text{ with } s_t = h_t + \zeta a_t$$

- Transaction cost κ to adjust balance of illiquid assets

Agent's problem (simplified)

- I focus on the (simplified) budget constraints, which give the key insight

- With no adjustment:

$$c_t + h_t + m_{t+1}/R_m = y_t + m_t$$
$$a_{t+1}/R_a = a_t$$

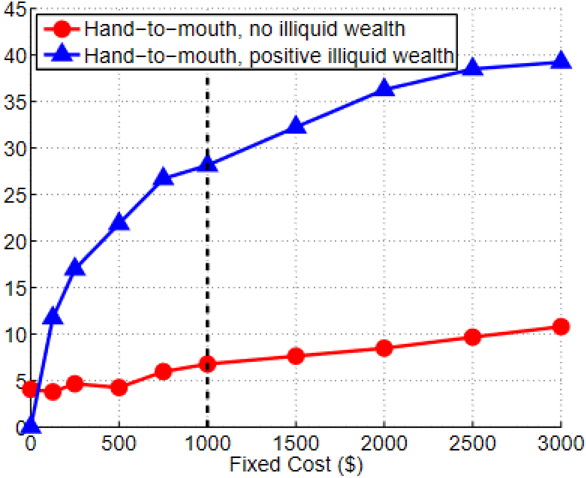
- With adjustment:

$$c_t + h_t + m_{t+1}/R_m + a_{t+1}/R_a = y_t + m_t + a_t - \kappa$$

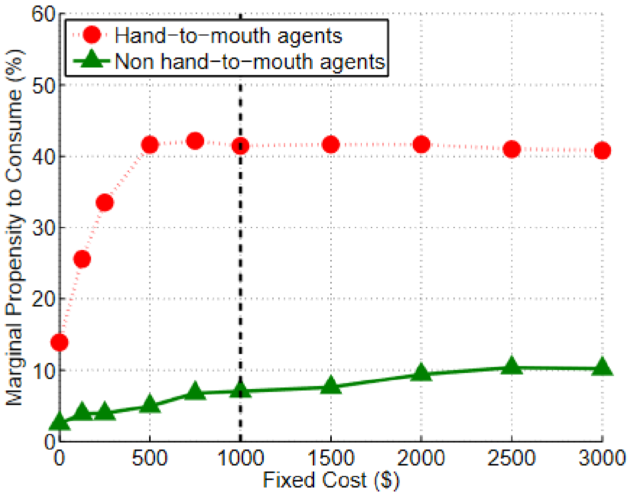
  $\kappa = 0 \Rightarrow$ similar to one asset model

- The states are a_t (illiquid), m_t (liquid), y_t (income with some shocks)

Hand-to-mouth by adjustment cost



MPC to tax rebate by adjustment cost



Literature

Lifecycle model

EoC, Chapter 1, 2, and 7

Calibration and Simulation

Fatih Guvenen's slides on optimization: `https://fatihguvenen.com/teaching/econ8185-phd-computation-empirics/`

"Simulation-Based Econometric Methods" by Christian Gouriéroux and Alain Monfort

Literature

Recent examples

De Nardi, Mariacristina, Eric French, and John B. Jones. "Why do the elderly save? The role of medical expenses." Journal of political economy 118.1 (2010): 39-75.

Kaplan, Greg, and Giovanni L. Violante. "A model of the consumption response to fiscal stimulus payments." Econometrica 82.4 (2014): 1199-1239.