The Default Point

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Abstract

We posit equations for equity value S and asset value A, connected by a common Wiener process. We derive a second-order time-independent ODE for S as a function of A, and set two boundary conditions: S is zero if and only if A is equal to some default point value DP, and when A is large changes in A fully accrue to shareholders. We obtain an analytical solution $S(A, \sigma_A)$, where σ_A is the asset volatility. The equation, however, cannot be solved for A, so that given S we have to solve for A and σ_A simultaneously. This is done in a loop, starting with an initial volatility, calculating asset values (and hedge ratios and put options) for the past 3/5 years using that volatility for all periods, using asset values and hedge ratios to delever equity returns over the past 3/5 years, and obtaining a new volatility, until initial and final volatilities are the same. The result of the loop is an estimate of asset volatility. We then use the volatility to calculate the current asset value, HR, and PO (somewhat repetitively), and calculate DD (with a different LongTermDebtCoupon as defore, now using the put option).

With a different (left) boundary condition we get a new asset value equation (solution to the differential equation), and therefore have to resolve everyting. The value of equity in default

Our discussion on 7/3/2019...

Contents

1 Equity in default

The value of equity is bounded below by zero. Even in a world where equity would theoretically be zero when a firm defaults, any noise would lead to a positive average observed equity value in default.

2 VK model solution

The equation for asset value is

$$dA = (\mu_A A - a)dt + \sigma_A A\, dW\,, \tag{1}$$

where $a = rD^* + c + q + p$ with r the risk-free rate, D^* current liabilities, c annual coupon payments on long-term liabilities, q annual common dividend, and p annual preferred dividend.[1] The market capitalization of a firm S can be viewed as a function of A, since equity is a perpetual down-and-out call option on the firm's assets with strike price equal to the book value of liabilities. The equation for equity value is

$$dS = (\mu_S S - q)dt + \sigma_S S\, dW, \tag{2}$$

where the Wiener process W is the same for assets and equity, that is in equations (1) and (2). Note that A and S are Ito processes, but neither follows a geometric Brownian motion. Applying Ito's lemma to $S(A)$ yields

$$dS = \left(\frac{\partial S}{\partial A}(\mu_A A - a) + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} + \frac{\partial S}{\partial t}\right)dt + \frac{\partial S}{\partial A}\sigma_A A\, dW \tag{3}$$

Matching terms in the equations for equity (2) and (3) and gives

$$\frac{\partial S}{\partial t} + (\mu_A A - a)\frac{\partial S}{\partial A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} = \mu_S S - q \tag{4}$$

and

$$\frac{\partial S}{\partial A} = \frac{\sigma_S}{\sigma_A}\frac{S}{A}. \tag{5}$$

The no-arbitrage condition is

$$\frac{\mu_A - r}{\sigma_A} = \frac{\mu_S - r}{\sigma_S}, \qquad \text{or} \qquad \mu_A = \frac{\sigma_A}{\sigma_S}(\mu_S - r) + r. \tag{6}$$

Plugging the latter expression into equation (4), and using (5) twice, we get

$$\begin{aligned}
\mu_S S - q &= \frac{\partial S}{\partial t} + \left[\left(\frac{\sigma_A}{\sigma_S}(\mu_S - r) + r\right)A - a\right]\frac{\sigma_S}{\sigma_A}\frac{S}{A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} \\
&= \frac{\partial S}{\partial t} + \frac{\sigma_A}{\sigma_S}(\mu_S - r)A\frac{\sigma_S}{\sigma_A}\frac{S}{A} + rA\frac{\sigma_S}{\sigma_A}\frac{S}{A} - a\frac{\sigma_S}{\sigma_A}\frac{S}{A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} \\
&= \frac{\partial S}{\partial t} + \mu_S S - rS + (rA - a)\frac{\partial S}{\partial A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2}
\end{aligned}$$

This is the second-order ODE

$$\frac{\partial S}{\partial t} + (rA - a)\frac{\partial S}{\partial A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} - rS + q = 0$$

The boundary conditions are

$$S = 0 \Leftrightarrow A = DP \tag{7}$$

[1]Equation (1) does not imply maturities for current liabilities, long-term liabilities, and preferred stock. It assumes the levels of liabilities and dividends, and of the risk-free rate, stay the same, from year to year.

and

$$\frac{\partial S}{\partial A} = 1 - \gamma \text{ as } A \to \infty \tag{8}$$

Since neither coefficients nor boundary conditions depend on t, the partial with respect to t drops out, and the equation to be solved becomes

$$\boxed{(rA - a)\frac{\partial S}{\partial A} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 S}{\partial A^2} - rS + q = 0,} \tag{9}$$

or, in compact notation,

$$S_{AA}\frac{1}{\alpha}A^2 + S_A(A - A^*) - S + S^* = 0, \tag{10}$$

where

$$\alpha = \frac{2r}{\sigma_A^2}, \quad A^* = \frac{a}{r}, \quad S^* = \frac{q}{r}.$$

With $P^* = \frac{p}{r}$ and $B^* = \frac{c}{r}$, it also holds that $A^* = D^* + B^* + P^* + S^*$.

The particular solution to equation (10) (that does not fulfill the boundary conditions) is the constant function $S(A) = S^*$. The homogenous equation to (10) is

$$S_{AA}\frac{1}{\alpha}A^2 + S_A(A - A^*) - S = 0 \tag{11}$$

Two independent solutions for (11), again disregarding the boundary conditions, are

$$S(A) = A - A^* \tag{12}$$

and[2]

$$S(A) = F\left(\alpha, \frac{A}{A^*}\right) \tag{14}$$

For both (12) and (14), adding S^* yields solutions to the inhomogeneous ODE (10). Now we need to satisfy the boundary conditions. We set up the general solution as

$$S(A) = C_1(A - A^*) + C_2 F\left(\alpha, \frac{A}{A^*}\right) + S^*, \tag{15}$$

with constants C_1 and C_2. The first boudary condition (7) yields

$$0 = C_1(DP - A^*) + C_2 F\left(\alpha, \frac{DP}{A^*}\right) + S^*. \tag{16}$$

For the second boundary condition, note that $F(\alpha, \infty) = 0$, so that

$$\left.\frac{\partial F(\alpha, \frac{A}{A^*})}{\partial A}\right|_{A=\infty} = 0,$$

and we have

$$1 - \gamma = C_1. \tag{17}$$

Plugging this into (16) we obtain

$$C_2 = -\frac{(1-\gamma)(DP - A^*) + S^*}{F\left(\alpha, \frac{DP}{A^*}\right)}. \tag{18}$$

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[2]Check that (14) is a solution.

$$S(A) = F\left(\alpha, \frac{A}{A^*}\right) \equiv \left(1 - \frac{A}{A^*}\right)\Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right) + \Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)$$

$$\begin{aligned}
S_A &= -\frac{1}{A^*}\Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right) + \left(1 - \frac{A}{A^*}\right)\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)\frac{-\alpha A^*}{A^2} + \Gamma''\left(\alpha+1, \frac{\alpha A^*}{A}\right)\frac{-\alpha A^*}{A^2} \\
&= -\frac{\Gamma(\alpha+1, \frac{\alpha A^*}{A})}{A^*} - \frac{\alpha A^*}{A^2}\left[\left(1 - \frac{A}{A^*}\right)\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right) + \Gamma''\left(\alpha+1, \frac{\alpha A^*}{A}\right)\right] \\
&= -\frac{\Gamma(\alpha+1, \frac{\alpha A^*}{A})}{A^*} - \frac{\alpha A^*}{A^2}\left[\left(1 - \frac{A}{A^*}\right)\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right) + \left(\frac{\alpha A}{\alpha A^*} - 1\right)\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)\right] \\
&= -\frac{\Gamma(\alpha+1, \frac{\alpha A^*}{A})}{A^*}
\end{aligned} \tag{13}$$

$$S_{AA} = -\frac{1}{A^*}\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)\frac{-\alpha A^*}{A^2} = \frac{\alpha}{A^2}\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)$$

Using these, (11) is confirmed:

$$\begin{aligned}
&\frac{\alpha}{A^2}\Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)\frac{1}{\alpha}A^2 - \frac{\Gamma(\alpha+1, \frac{\alpha A^*}{A})}{A^*}(A - A^*) \\
&\qquad = \left(1 - \frac{A}{A^*}\right)\Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right) + \Gamma'\left(\alpha+1, \frac{\alpha A^*}{A}\right)
\end{aligned}$$

The solution thus is

$$S(A) = S^* + (1-\gamma)(A - A^*) + \Big((1-\gamma)(A^* - DP) - S^*\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})} \tag{19}$$

Rearranging, we get

$$S = S^* + (1-\gamma)(A - A^*) + \Big((1-\gamma)(A^* - DP) - S^*\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$\frac{S - S^*}{1-\gamma} = (A - A^*) + \Big((A^* - DP) - \frac{S^*}{1-\gamma}\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$A = \frac{S - S^*}{1-\gamma} + A^* - \Big((A^* - DP) - \frac{S^*}{1-\gamma}\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$\boxed{A = \frac{S}{1-\gamma} + DP + \Big(\underbrace{A^* - DP - \frac{S^*}{1-\gamma}}_{\text{DefaultableDebt}}\Big)\Big(1 - \underbrace{\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}}_{\text{PutOption}}\Big)} \tag{20}$$

Equation (20) is the asset value equation. It represents a decomposition of asset value into three components: 1) the diluted equity value, the default point, and the market value of defaultable debt. PutOption is the price of a put option (with strike price equal to the face value?) for every dollar of liability.

2.1 4 values

$$S = S^*(1 - \tfrac{FA}{FDstar} + (1-\gamma)(A - A^*) + (1-\gamma)(A^* - DP)\tfrac{FA}{FDstar}$$

2.2 DD derivation

Apply Ito's lemma with $f(A) = \log(A)$ to the simplified asset value equation $dA = \mu_A A dt + \sigma_A A\, dW$

Ito's lemma: if $dX = \mu dt + \sigma dB$, and f smooth, then $df(X) = (\mu f'(X) + \frac{1}{2}\sigma^2 f''(X))dt + \sigma f'(X)dB = f'(X)dX + \frac{1}{2}\sigma^2 f''(X)dt$

$$d\log(A) = \frac{1}{A}(\mu_A A dt + \sigma_A A\, dW) + \frac{1}{2}\sigma_A^2 A^2 \frac{-1}{A^2}dt = \sigma_A dW + (\mu_A - \frac{\sigma_A^2}{2})dt$$

Integrate to get

$$\log(A) = \log(A_0) + (\mu_A - \frac{\sigma_A^2}{2})t + \sigma_A\sqrt{t}W$$

2.3 Implementation

In the code (Emp-Vol2_delta.R), the implementation proceeds in the following steps

- financial statements are pre-processed (PreProcess)
  - In this step, LTDC is defined as r-f rate + 50bps times long-term liabilities, and Bstar is the present value of this (LTCD/r-f rate). Sstar and Pstar are the present values of common and

preferred dividends, resp. Astar = Dstar + Sstar + Pstar + Bstar. Also, the debt duration theta is calculated.

- asset returns are calculated (ARet)
  - This step contains the loop that starts with an initial guess for empirical volatility, asset values are calculated (using the asset value function/equation),

$$A = \frac{S}{1-\gamma} + DP + \left(A^* - DP - \frac{S^*}{1-\gamma}\right)\left(1 - \frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}\right),$$

then asset returns are calculated with the asset return equation,

$$\mu_A = \frac{S}{A\frac{\partial S}{\partial A}}\mu_S + \frac{A - S - D^*}{A}\theta\mu_B r_f$$

In the code, $\mu_B r_f$ is called "bonddelta". Mktfs_Prod_Recalc.R: br.rate_t0 - br.rate_t1 as bonddelta, br = "input_bond_return". These rates are in percent, 7.334819794 - 7.250360012.

From the asset returns a new value for empirical volatility is calculated. ARet returns (only) asset returns.

- with the asset returns, empirical volatility is calculated.

in DD.R

- calculate combined volatility (line 134)
- some of the definitions that were done during the preprocessing are repeated
  - LTDC = (r-f rate + 50bps) × long-term liabilities
  - Bstar = p.v. of LTDC, Sstar = p.v. of common dividend, Pstar = p.v. of preferred dividend
  - Astar = Dstar + Sstar + Pstar + Bstar
- call asset value function to give AVL, HR, PO
- LTDC is redefined: LTDC = (r-f rate + POiS) times long-term liabilities, where POiS as in equation (21), and POiS is capped at 3 percent.

  PO is the price per dollar of face value. We have the that a perpetual coupon discounted at rf plus the POiS should have the same value as the present value of the coupon discounted at the r-f rate and discounted with the risk premium of the put option,

$$\frac{c}{r_f + POiS} = \frac{c}{r_f}(1 - PO).$$

  Solving for POiS:

$$r_f + POiS = \frac{r_f}{1 - PO}.$$

$$POiS = r_f\left(\frac{1}{1 - PO} - 1\right) \tag{21}$$

- cash leakage = r-f rate × Dstar + LTDC + common dividend + preferred dividend
- DD

$$\text{DD} = \frac{\log\left(\frac{A}{DP(1+r_f)+\text{cash leakage}}\right) + Drift - \frac{csg^2}{2} + 0.15\,(1-w_{fin})}{csg} \tag{22}$$

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$a = r_f CL + (r_f + 50bps)LTL + \text{comdiv} + \text{prefdiv}$
$r_f + \text{POiS}$
$A^* = \frac{a}{r_f}$

$$A^* = \frac{a}{r_f}$$

$$A = \underbrace{\frac{S}{1-\gamma}}_{\substack{\text{Diluted}\\ \text{equity value}}} + \underbrace{DP}_{\substack{\text{Safe}\\ \text{liabilities}}} + \underbrace{\left(A^* - DP - \frac{S^*}{1-\gamma}\right)}_{\substack{\text{Defaultable}\\ \text{debt...}}} \underbrace{\left(1 - \frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}\right)}_{\substack{\text{... discounted}\\ \text{with Put Option}}}$$

2.4 Asset value function

For given volatility, that is, α, and equity value S we solve the asset value equation (20) for A. We do this by finding a zero of the function f defined as

$$f(A) = A - \underbrace{\left(A^* + \frac{S-S^*}{1-\gamma}\right)}_{\text{CallPlusStrike}} + \underbrace{\left(A^* - DP - \frac{S^*}{1-\gamma}\right)}_{\text{DefaultableDebt}} \frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})} \tag{23}$$

Taking the derivative, and using equation (13), we obtain

$$f'(A) = 1 - \frac{\left(A^* - DP - \frac{S^*}{1-\gamma}\right)}{F\left(\alpha, \frac{DP}{A^*}\right)} \frac{\Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right)}{A^*} = 1 - \frac{\text{DefaultableDebt}}{A^*\,FD^*}\,\Gamma A \tag{24}$$

where $\Gamma A \equiv \Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right)$ and $FD^* \equiv F\left(\alpha, \frac{DP}{A^*}\right)$.

By taking the derivative of (19), again using equation (13), we obtain an expression for the hedge ratio

$$\begin{aligned}\frac{\partial S}{\partial A} &= 1 - \gamma - \frac{\left((1-\gamma)(A^* - DP) - S^*\right)\Gamma\left(\alpha+1, \frac{\alpha A^*}{A}\right)}{A^* F\left(\alpha, \frac{DP}{A^*}\right)} \\ &= (1-\gamma)\left(\underbrace{1 - \frac{\text{DefaultableDebt}\ \Gamma A}{A^*\,FD^*}}_{\text{BiasedHR}}\right)\end{aligned} \tag{25}$$

It holds that $f'(A) = \text{BiasedHR}$.

To find a zero of f, we use Newton's method, iterating the asset value as

$$A_1 = A_0 + \frac{f(A_0)}{f'(A_0)}.$$

For the implementation, FD^* is defined differently depending on α as

$$FD^* = F\left(\alpha, \frac{DP}{A^*}\right) = \left(1 - \frac{DP}{A^*}\right)\Gamma D^* + \underbrace{e^{-\log\left(\Gamma(\alpha+1)\right) + \alpha \log\left(\frac{\alpha A^*}{DP}\right) - \frac{\alpha A^*}{DP}}}_{\Gamma'(\alpha+1, \frac{\alpha A^*}{DP})}, \quad \text{for } \alpha < 70,$$

and

$$FD^* = \left(1 - \frac{DP}{A^*}\right)\Gamma D^* + \underbrace{\frac{\Gamma(\alpha+1, 1.00001\frac{\alpha A^*}{DP}) - \Gamma D^*}{0.00001\frac{\alpha A^*}{DP}}}_{\text{Approximation of } \Gamma'}, \quad \text{for } \alpha \geq 70,$$

with

$$\Gamma D^* = \Gamma\left(\alpha + 1, \frac{\alpha A^*}{DP}\right)$$

If

$$f'(A_0) = \left(1 - \frac{\text{DefaultableDebt } \Gamma A_0}{A^* \, FD^*)}\right)$$

very small for a given iteration process, we regard it as "converged" – implication? Otherwise, we calculate

$$f(A_0) = A_0 - \text{CallPlusStrike} + \frac{\text{DefaultableDebt } FA_0}{FD^*}, \tag{26}$$

where $FA_0 = (1 - \frac{A_0}{A^*})\Gamma A_0 + \Gamma' A_0$, with ΓA_0 as in (24), and $\Gamma' A_0$ defined dependent on α as

$$\Gamma' A_0 = \frac{1}{\Gamma(\alpha+1)}\left(\frac{\alpha A^*}{A_0}\right)^{\alpha} e^{-\frac{\alpha A^*}{A_0}}, \quad \text{for } \alpha < 70,$$

$$\Gamma' A_0 = \frac{\Gamma\big(\alpha+1, 1.00001\,\frac{\alpha A^*}{A_0}\big)}{0.00001\,\frac{\alpha A^*}{A_0}}, \quad \text{for } \alpha \geq 70.$$

We calculate the biased hedge ratio as

$$f'(A_0) = 1 - \frac{\text{DefaultableDebt } \Gamma A_0}{A^* \, FD^*} \tag{27}$$

Now the correction term $\frac{f(A_0)}{f'(A_0)}$ can be calculated using (26) and (27). We update A_1 as $A_0 - \frac{f(A_0)}{f'(A_0)}$ and call the iteration process "converged" if

$$\frac{f(A_0)}{f'(A_0)} < 0.00001\,A^*.$$

Intuition? The asset value function returns three values: The final asset value (A_1), the (unbiased) hedge ratio as

$$\text{HR} = (1-\gamma)f'(A_0), \tag{28}$$

and the put option

$$\text{PO} = \frac{FA_0}{FD^*}. \tag{29}$$

2.5 Asset return calculation

The asset return equation is

$$\mu_A = \frac{S}{A\frac{\partial S}{\partial A}}\mu_S + \frac{A - S - D^*}{A}\theta\mu_B r_f \tag{30}$$

So to calculate an asset return we need an estimate of the asset value and of the hedge ratio.

2.6 Calculate EDF

3 Meeting 14/3/2019

The logic of the model is that when asset value falls to the default point, default happens. We can define and calculate the default point directly from the data, but asset value is not directly observable. In order to estimate it we set up a boundary condition that states that asset value is equal to the default point if and only if equity value is zero. Consequently, the causal assumption of the model is that default occurs when equity value is zero (at which point by construction asset value will be equal to the default point).

Given this setup, to what extent can we think of asset value as an autonomous process?

In reality, default usually occurs when there is still sizeable equity value in the firm. Default is triggered by someone taking an action (e.g. file for bankruptcy protection), and they may not wait until equity is worth zero.

One way to have the model reflect this aspect of the data is to set up the boundary condition so that assets equal the default point when equity is not zero, but instead some positive value (possibly expressed relative to the default point). This preserves the meaning of the default point as the boundary of asset value at which default occurs.

Another idea is to increase the default point in DD by a fraction of asset value, while keeping the estimation of asset value and asset volatility as is. The idea is that at the new default point $DP + \rho A$, equity value is positive. However, default would not necessarily occur (or rather, be predicted to occur) at the new default point – we would have firms with asset value below the new default point. What this setup instead effectively amounts to is an ad-hoc adjustment to DD that penalizes low-volatility firms (which are a perceived weakness of the model). (Negative DDs are possible and occur in the present setup.)

In order to be consistent and ensure default occurs at the default point, we need to incorporate the point at which the firm defaults into the boundary condition.

A natural idea then is to use the new default point in the original boundary condition. This would not change the central assumption that default occurs when the equity value is zero. Given that all of short-term liabilities is already included into the default point, increasing the default point would increase the weight of long-term liabilities. Intuitively, this will penalize (relative to the current model) firms with more long-term liabilities, since for those firms the default point would be affected more, and thus they would be predicted to default quicker. (This interpretation assumes that the asset value is given, whereas in practice its estimation will be affected by the default point definition...) But I do not quite see through the details of this channel... Also, what role does volatility play here?

Finally, the first idea could be modified to state that the equity level at which default occurs is also included in asset value at default, that is, in the default point.

Idea: Database of defaults that contains a measure of the market value of firms at the time of default that is independent of our model. What is this price? "Price at which a distressed firm was purchased on or about the time of default". This could be the equity value (price paid for the shares of the target) or the enterprise value (accounting definition: equity value + financial debts - cash).

The price paid for a firm in distress is expected to be the asset value Use this information to estimate the value of assets when default is triggered.

Predict the purchase price with: total liabilities (perhaps split into CL and LTL, maybe are priced differently), market cap.

$$\frac{\text{Default point}}{\text{Book assets}} = \frac{\text{Current liabilities} + 0.5 * \text{Long-term debt}}{\text{Book assets}} + \frac{0.03}{\text{Book assets}}$$

then boundary condition is different, and everything changes.

Default is trigger when asset value is less ¡(CL + 0.5*LTD) /(book assets) + .03/bo * Equity

$$A < CL + 0.5 * LTL + 0.03 * S$$

$$Marketvalueofadistressedfirm = \alpha + \beta_1 * CL + \beta_2 * LTL + \beta_3 * BookAssets$$

$$Marketvalueofadistressedfirm = CL + 0.5 * LTL + 0.03 * BookAssets$$

or

$$Marketvalueofadistressedfirm = CL + 0.5 * LTL + 0.03 * (NetWorth + CL + LTL)$$

Equity = 0.03 of Assets when Asset Value = Default Point,

Lets take E(equity/default) point at default, and then

default occurs when Asset Value = CL + 0.5*LTD, and equity is 0.

Taking literally, what happes is CL paid in full, LTD paid in half, equity holders get zero when Asset value hits the default point. But if we said, default happens when Asset value falls by default point, but equity holders get 3Trigger is equity value = 0.03*(CL + 0.5*LTD), asset value is equal to (CL + 0.5*LTD)*(1.03) (

How much is equity worth at default relative to something? How much is debt worth at default? – from LC How much does it cost to buy a distressed company that would otherwise be bankrupt?

– adjustment to the default point – depends on asset volatility ———simulation approach

In the Zephyr database deal value is described simply as "the consideration paid for the actual stake acquired". When not available, this value is estimated, meaning that it is taken from a news source, often as an estimated or approximated value, or that it is calculated as number of shares times offer price (or closing price until the offer price is available). Furthermore, deal value can be an equity value (essentially meaning the price paid for the shares, that is, number of shares times offer price) or an enterprise value (equity value for 100% + debt, where debt is based on the assumption involved in the deal).

They say deal value is "the consideration paid for the actual stake acquired", and that it can be stated as an equity value or an enterprise value.

For equity value they use deal value if it is stated as an equity value, or if no info is available how deal value was stated, and if number of shares and price per share is available, it is calculated as the product.

Enterprise value is said to mean 100% of shares + publicly disclosed debt. If deal value is stated as enterprise value, it is used here. If don't know how deal value is stated, and have details of debt assumption in deal, use equity value for 100% + debt.

Zephyr also gives an modeled enterprise value, equal to equity value + financials debts – cash.

The fact that both equity value and enterprise value are used for deal value does not mean that these numbers are directly comparable. An equity value will be the price paid for the shares, whereas an enterprise value will be

3.1 Factiva

Summary: *Frequent formulation: ”enterprise value paid”. Is the price paid for the assets. EV = equity + net debt. Net debt can be total debt minus cash. ”acquire for a total enterprise value x with an additional y representing net cash in the business upon completion”. Enterprise value not uncommonly less than equity value. ”private equity value of the remaining assets is below the public market enterprise value, after creditors are paid off, equity value could be half of the current public market cap”.*

The proposed acquisition represented an enterprise value of around $480 million, of which $115-120 million was the purchase price in cash and $380 million the project debt to be assumed.

The enterprise value for the acquisition is Rs 1662 crore, which includes Rs 946 crore for the equity and Rs 716 crore of debt.

In July, Michelin paid an enterprise-value $1.7 billion for Canadian off-road tire maker Camso.

The enterprise value paid for NewVoiceMedia represents approximately 3.8x projected 2019 revenue.

In May 2017, the company sold its microwave Microwave Telecoms business to Infinite Electronics Inc., paid an enterprise value of GBP85 million ($118.16 million).

definitive agreement to acquire Blue Buffalo Pet Products, Inc. for $40/per share or approximately $8.0 billion enterprise value, to be paid in cash.

Swiss investment bank UBS also said the transaction was “strategically attractive” for BASF, although adding that the high multiple paid for the assets (the enterprise value (EV) paid for BASF came in at 15 times (x) earnings before interest, taxes, depreciation and amortisation, EBITDA) would cause the pay-back to “take time”.

each shareholder of Urban Communications will receive cash consideration of CAD0.07 for each share held, which together with anticipated debt being assumed at closing results in an enterprise value being paid of CAD15.6 million on a fully-diluted in-the-money basis.

Nestlé paid for Galderma for an enterprise value of 3.1 billion euros via 21.1 million L’Oréal shares. For the remaining 27.3 million shares, L’Oréal paid 3.4 billion euros.

The British firm has paid an enterprise value of EUR78 million ($81 million) for the asset.

The price paid equates to an enterprise value of EUR1.025 billion.

The firm’s analysts, however, said they ”do not think Dominion is overpaying,” with its deal value at 9.5x enterprise value/2016 consensus EBITDA, which is a ”slight discount” to the 10.5x enterprise value/EBITDA that Southern paid for AGL.

IDP will acquire Hotcourses for a total enterprise value of GBP30.1 million with an additional GBP4.9 million representing net cash in the business upon completion.

But he cautioned: ”The bad news, however, is when we lay out our itemized list of sale-able assets and estimated liquidation values, we believe the private equity value of the remaining assets (about $12 billion) is well below the public market enterprise value. After the creditors [are] paid off, the equity value could be as little as half of the current public market cap.”

F2i is EGP's existing partner in EF Solare Italia, which has an enterprise value of EUR1.3 billion, of which EUR430 million is equity value and EUR900 million third party debt.

The deal represented W148.4 billion in equity value and W200.8 billion in enterprise value.

The deal announced on December 14 represents an equity value of US$26bn in a transaction with an enterprise value of US$32bn.

The London-based investment firm will buy all Trade Me shares for NZ$6.45 per share, representing an implied equity value of NZ$2.56 billion and an enterprise value of NZ$2.74 billion.

The transaction values Nutrisystem at an enterprise value of $1.3 billion and an equity value of $1.4 billion, or approximately $47.00 per share.

The new debt would effectively cut the value of the company's equity based on the idea that the enterprise value (equity value plus net debt) shouldn't change. If you take out a $250,000 mortgage on a house valued at $1 million with no debt on it, you haven't created any value because the equity in your home has fallen to $750,000.

Pursuant to the merger agreement, CECO will acquire all of the outstanding shares of PMFG common stock for cash and stock valued at $6.85 per share (approximately $150 million equity value or $130 million enterprise value)

Sweco has acquired all shares in Vectura from the Swedish Government for a consideration consisting of a purchase price in cash amounting to 927 MSEK (equity value). The final value of the acquisition (enterprise value) amounts to approximately 900 MSEK.

At closing, the purchase consideration was valued at $107.19 per Covance share, consisting of $75.76 in cash and 0.2686 LabCorp shares for each Covance share, or an equity value of approximately $6.2 billion and an enterprise value of approximately $5.7 billion.

4 Augment the default point with term proportional to asset value

Keep the estimation of asset value and asset volatility as is, but change the calculation of DD. Replace DP by $DP + \rho\, A$, where ρ is a constant. The intuition is that empirically asset value is not equal to the default point in default. The parameter ρ models residual equity value in default.

Model DD:

$$\mathrm{DD} = \frac{\log(A) - \log\Big(DP(1+r_f) + \text{cash leakage}\Big) + Drift - \dfrac{csg^2}{2} + 0.15\,(1-w_{fin})}{csg}$$

Alternative DD:

$$\mathrm{DD} = \frac{\log(A) - \log\Big((DP+\rho\, A)(1+r_f) + \text{cash leakage}\Big) + Drift - \dfrac{csg^2}{2} + 0.15\,(1-w_{fin})}{csg}$$

The change affects low-volatility firms proportionally more. Example calculation:[3]

Asset value	Default point	CSG	Model DD	Alternative DD ($\rho = 0.02$)
5.00	4.77	0.10	1.83	1.63
5.00	4.00	0.20	1.83	1.71

Table 1: Corporate, $r_f = 0.03$, cash leakage = 0.2, $Drift = 0.05$

Changing the DD list this is something else than changing the boundary condition to $S = 0 \Leftrightarrow A = DP + 0.02\, A$, or $S = 0 \Leftrightarrow A = \frac{DP}{0.98}$, which would not change anything. (Everything would have $\frac{DP}{0.98}$, and in the ad-hoc DD this would also have to appear.)

4.1 Data preparation

Download from output_edf_monthly, from 199900. Test, can replicate EDF.

Public default data. Take out covenant defaults. Merge into EDF data with yearmon of default closest strictly after EDF yearmon. (edf_date is late in EDF yearmon, or early in next yearmon.) In a few cases, this results in defdate on or before edf_date. Can calculate time to default in days or in (yearmon) months.

Define two default dummies, 1 month and 1 year.

Vary ρ between 0 (Model DD) and 20%. Calculate DD1r's and EDFs for each ρ

Figure 1: Accurary ratios.

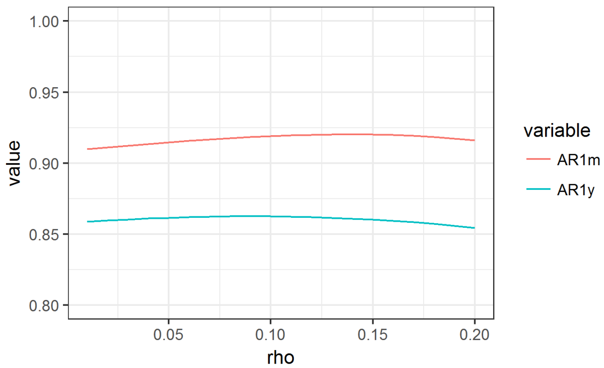
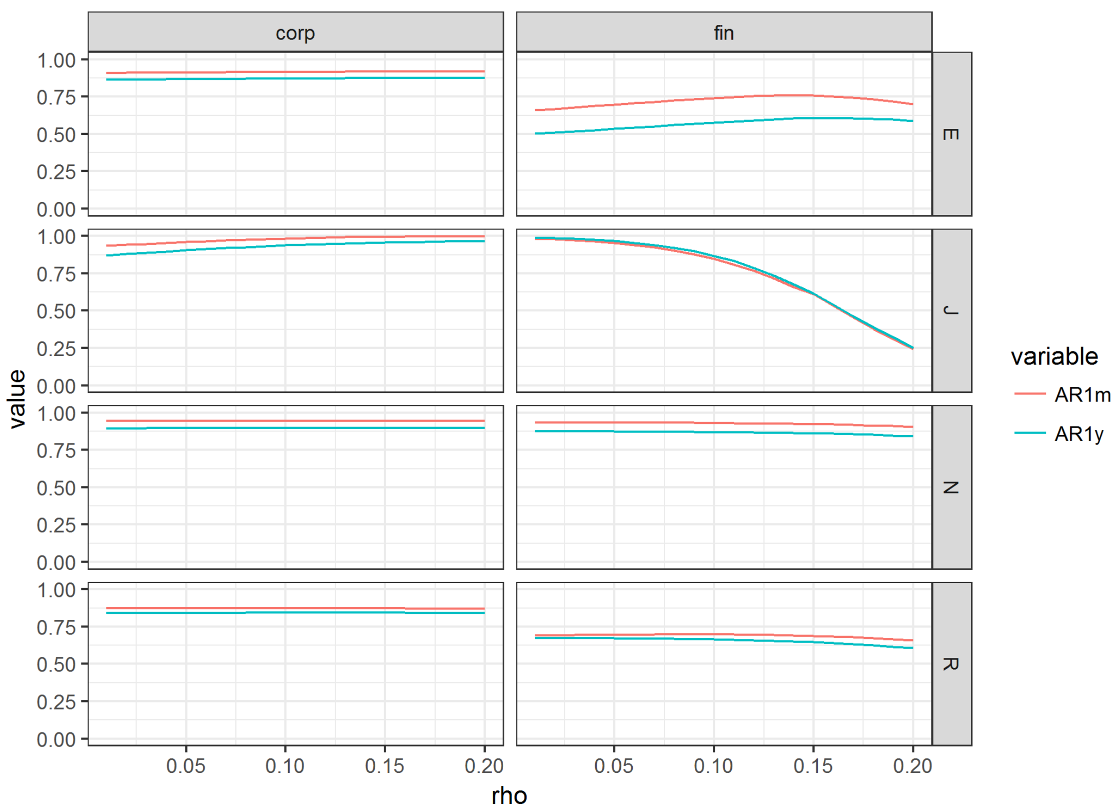


Figure 2: Accurary ratios.



4.2 Accuracy ratios

4.3 Classifiers

Classify firms in every month using the condition

$$A < DP + \rho\, A, \qquad \text{or} \qquad (1-\rho)\, A < DP.$$

class number $= \frac{DP}{A}$

rho reverse $= 1-$ class number

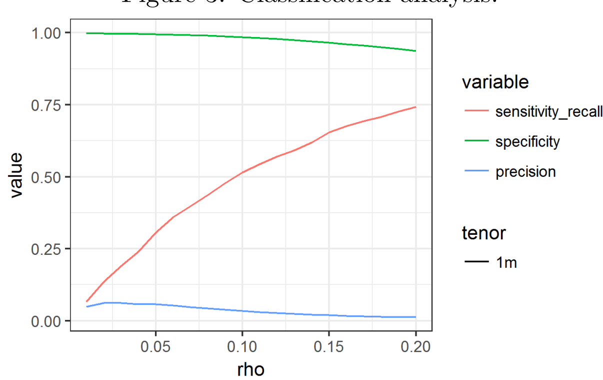
confusion matrix: TP, FP, TN, FN

Concepts:

- Recall (or sensitivity) $= \dfrac{TP}{relevant} = \dfrac{TP}{TP+FN}$. Detect positives. What fraction of defaults is detected? Cheat by saying all positive.
- Precision $= \dfrac{TP}{P} = \dfrac{TP}{TP+FP}$. What fraction of positives are defaults? Cheat by only calling securest positive.
- Specificity $= \dfrac{TN}{N} = \dfrac{TN}{TN+FP}$. Avoid false alarms. What fraction of negatives are actually non-defaulters? Cheat by calling all negative.

Most relevant here: Recall and precision. Trade-off.

Figure 3: Classification analysis.



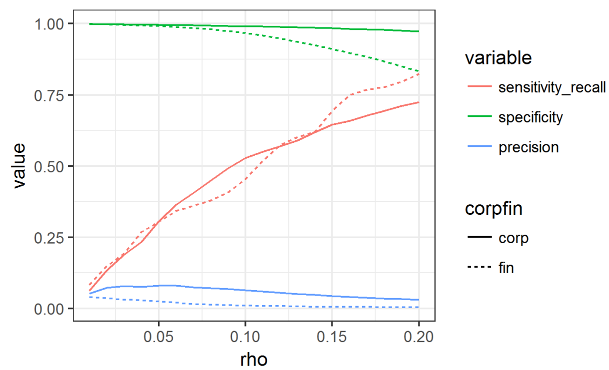
4.4 Differences

What is the difference between overall changing the default point (to say, $D^* + .6\, LBS$, or $DP/(1-\rho)$) and changing the DP only in DD (to say, $DP + \rho\, A$)?

- intuitively... the default point determines what we estimate asset value to be when equity is zero. setting a higher default point increases the asset value that we estimate.

[3]Also ran test for simple DD.

Figure 4: Classification analysis.



- just changing the default point in DD penalizes low volatility firms. Think two firms with the same DD, but one firm has higher volatility, and therefore a lower default point. Changing the default point by a fraction of assets decreases the difference between asset value and default point more (percentage-wise) for the low-volatility firm. "low volatility not as advantageous".

- increasing the default point decreases defaultable debt in the asset value equation. For given volatility, effect of DP on A can be obtained by implicit function from the asset value function function f.

$$f(A) = A - \underbrace{\left(A^* + \frac{S - S^*}{1-\gamma}\right)}_{\text{CallPlusStrike}} + \underbrace{\left(A^* - DP - \frac{S^*}{1-\gamma}\right)}_{\text{DefaultableDebt}} \frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$f_A > 0 \quad \text{(biased HR)}$$

$$f_{DP} = -\frac{F(\alpha, A/A^*)}{F(\alpha, DP/A^*)} + \text{DefaultableDebt}\frac{-F(\alpha, A/A^*)}{F(\alpha, DP/A^*)^2} F_2(\alpha, DP/A^*)$$
$$- \frac{F(\alpha, A/A^*)}{F(\alpha, DP/A^*)}\left(1 + \text{DefaultableDebt}\frac{1}{F(\alpha, DP/A^*)} F_2(\alpha, DP/A^*)\right)$$

Does not unequivocally determine sign...

- for distressed firms, asset value equal to...

- perhaps most obvious difference: if change default point generally, will estimate asset value equal to new default point when equity zero. If only change DD, will asset value will be the lower old default point when equity is zero. Thus when equity is very low, we will estimate asset value to be *lower* than the new default point.

5 The equity value in default

5.1 New boundary condition in the model

New boundary condition

$$S = E_d \Leftrightarrow A = DP. \tag{31}$$

We set up the general solution as

$$S(A) = C_1(A - A^*) + C_2 F\left(\alpha, \frac{A}{A^*}\right) + S^*.$$

The first boundary condition (31) yields

$$E_d = C_1(DP - A^*) + C_2 F\left(\alpha, \frac{DP}{A^*}\right) + S^*. \tag{32}$$

The second boundary condition as above gives $C_1 = 1 - \gamma$, so that we get

$$C_2 = \frac{E_d - (1-\gamma)(DP - A^*) - S^*}{F\big(\alpha, \frac{DP}{A^*}\big)},$$

and the solution is

$$S(A) = S^* + (1-\gamma)(A - A^*) + \Big(E_d + (1-\gamma)(A^* - DP) - S^*\Big)\frac{F\big(\alpha, \frac{A}{A^*}\big)}{F\big(\alpha, \frac{DP}{A^*}\big)} \tag{33}$$

Solving for A, we get the new asset value equation

$$S = S^* + (1-\gamma)(A - A^*) + \Big(E_d + (1-\gamma)(A^* - DP) - S^*\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$\frac{S - S^*}{1-\gamma} = (A - A^*) + \left(\frac{E_d}{1-\gamma} + (A^* - DP) - \frac{S^*}{1-\gamma}\right)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$A = \frac{S - S^*}{1-\gamma} + A^* - \left(\frac{E_d}{1-\gamma} + (A^* - DP) - \frac{S^*}{1-\gamma}\right)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

$$A = \frac{S}{1-\gamma} + DP - \frac{E_d}{1-\gamma} + \left(A^* + \frac{E_d}{1-\gamma} - DP - \frac{S^*}{1-\gamma}\right)\left(1 - \frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}\right) \tag{34}$$

The equation is the same as before except for the terms with E_d.

5.1.1 Asset value calculation

Rearrange (34) to get

$$A = \left(A^* + \frac{S - S^*}{1-\gamma}\right) - \left(A^* - DP + \frac{E_d}{1-\gamma} - \frac{S^*}{1-\gamma}\right)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})}$$

and the define the function f as

$$f(A) = A - \underbrace{\left(A^* + \frac{S - S^*}{1-\gamma}\right)}_{\text{CallPlusStrike}} + \underbrace{\left(A^* - DP + \frac{E_d}{1-\gamma} - \frac{S^*}{1-\gamma}\right)}_{\text{DefaultableDebt}}\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})} \tag{35}$$

The only difference to before is the E_d term in DefaultableDebt. Therefore the asset value calculation works in the same ways as before, with DefaultableDebt defined differently.

The formula for the hedge ratio is obtained by taking the derivative of (33) (using equation (13))

$$\frac{\partial S}{\partial A} = 1 - \gamma - \frac{\big(E_d + (1-\gamma)(A^* - DP) - S^*\big)\Gamma\big(\alpha+1, \frac{\alpha A^*}{A}\big)}{A^* F\big(\alpha, \frac{DP}{A^*}\big)} \tag{36}$$

5.2 What should the value for equity in default be?

5.2.1 Li, Zhong (2013)

602 chapter 11 filings

equity value measured on first trading day after Chapter 11 filing

calculate Black-Scholes option value

$$C(V, X, r, T, \sigma^2) = VN(d_1) - Xe^{-rT}N(d_2), \quad d_1 \frac{log(\frac{V}{X} + r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T},$$

where V total assets (last filing), X sum of short-term and long-term debt (last filing), r risk-free rate, σ asset volatility, T expected time until resolution (maturity of the option).

regress equity value on

- option value
  - significant, $R^2 = 27.5\%$
- inputs to Black-Scholes: total assets, volatility, short-term debt, long-term debt, risk-free rate, expected duration.
  - assets, volatility, long-term debt, risk-free rate, duration significant, $R^2 = 36.8\%$

or

-

5.2.2 My regressions

public default database

15 jan 1999 to 31 dec 2017, no covenant defaults

call in calibration data if in that month have EDF there

for one value for every default

oe: before = up to same day, after = strictly after. use 6 days before to day of default.

om: 6 days before to day of default

os: statement date 199 days before to day before default

We can think of equity in default as an out-of-the-money call option on the assets of the firm, with strike price the total face value of the firm's debt. The spot price is the default point. The volatility of the underlying is *csg*. The maturity of the option is uncertain, since the period until the firm emerges from bankruptcy protection is uncertain.

Reduced form model for the price of the option

$$log(S) = log(DP) + log(TL) + log(csg) + X \tag{37}$$

Another possibility is to predict the equity value as a fraction of some other quantity, e.g. the default point or total liabilties. Call this fraction τ.

$$log(\tau) = log(TL) + log(csg) + X \tag{38}$$

We can then predict

$$\hat{E_d} = \hat{\tau}\, DP$$

or

$$\hat{E_d} = \hat{\tau}\, TL$$

If we observe a higher equity value in default E_d, this means that the out-of-the-money option has a larger value for this firm, since the likelihood of firm recovery is larger. If we can predict the value of the out-of-the-money option for an arbitrary firm at an arbitrary point in time, then we can use the predicted value as boundary condition for that firm at that point in time when solving for asset value and volatility.

In order to run a regression of equity value in default on explanatory variables, the former values need to be all on the same scale.

The value of the out-of-the-money option can also be modeled explicitly. It is a call option on the assets of the firm, with strike price the total face value of the firm's debt. The volatility of the underlying is csg, but the maturity of the option is uncertain, since the period until the firm emerges from bankruptcy protection is uncertain.

(39)

(40)

Table 2:

	Dependent variable:				
	log(EVL)	log(EVL/DP)		log(EVL/TL)	
	(1)	(2)	(3)	(4)	(5)
log(DP)	−0.048			−0.232	0.136***
	(0.601)			(0.581)	(0.037)
log(TL)	1.227**	0.143***		0.368	
	(0.599)	(0.037)		(0.579)	
log(DP/TL)			−1.413**		−0.368
			(0.585)		(0.579)
TL/BA		−1.059***	−1.163***	−1.074***	−1.074***
		(0.150)	(0.149)	(0.150)	(0.150)
log(csg)	1.271***	1.136***	0.854***	1.103***	1.103***
	(0.149)	(0.145)	(0.130)	(0.145)	(0.145)
Financial	−0.258	−0.317	−0.217	−0.326	−0.326
	(0.234)	(0.227)	(0.226)	(0.226)	(0.226)
Constant	−2.914***	−1.542***	−1.182***	−1.880***	−1.880***
	(0.324)	(0.306)	(0.290)	(0.344)	(0.344)
Observations	695	695	695	695	695
R^2	0.613	0.200	0.189	0.198	0.198
Adjusted R^2	0.611	0.195	0.184	0.193	0.193
MSE	2.59	2.43		2.41	
Note:					*p<0.1; **p<0.05; ***p<0.01

6 Implementation

6.1 production

6.1.1 Emp-Vol2_delta

- read in from output_mktfs
  - mktfs.hist has columns: pid, xdate, curliab, curdebt, eret, yieldm, bondreturn,... and aret, ratio from output_asset_return. Is for relevant period before and including initym.
  - mktfs.curr has columns: pid, xdate, curliab, curdebt, eret, yieldm, bondreturn,... Is for individual ym, starting in month after initym.
- call PreProcess
  - The only thing PreProcess is doing regarding aret and ratio is create columns of NA if no such columns exists.
  - adds columns aret, ratio, Dstar, DefaultPoint, DefaultableDebt
- rbind mktfs.hist and mktfs.curr to aret.in, also define aret.all (weekly36 + monthly60) and aret.actv (*)
- define initASG in aret.actv
- call ARet on aret.actv to get aret.out
- merge aret.out into aret.all to get vol.in, adding columns AssetReturn, Converged, ratio2
  - Have asset return time series now
- in vol.in, redefine **aret** from AssetReturn, and redefine **ratio** from ratio2
- call EmpVol on vol.in to get vol.out
  - adds columns ASG, ASG.n,...
  - vol.out has fewer rows than vol.in
  - save results in output, one row per pid (by construction of function EmpVol)
- in vol.in, rename aret aret2 and ratio ratio3
- rbind mktfs.hist and mktfs.curr to mktfs.hist, same data as in (*)
- merge aret2 and ratio3 from vol.in, then make aret aret2 and ratio ratio3, and delete aret2 and ratio3
- select from mktfs.hist if weekly returns and 36 months, or monthly returns and 60 months

6.1.2 ARet

- Idea: get asset returns using (30). For this need estimate of asset value and hedge ratio, so we need to go through the full iteration.
- call EventRatio, which adjusts aret.actv
- Loop
  - initial value of CSG
  - call AssetValue
    - get asset value and hedge ration based on CSG
  - calculate asset returns with ARE
  - call ARetVol
    - new CSG
    - check convergence
- for non-converged do smth...

6.2 My implementation

- Define number of PIDs, query that many distinct PIDs from output_mktfs (and input_entity).
- For PIDs, query asg from output_emp_vol by yearmon as initial values (vol.old)
- Have defined init.ym and final.ym
- Get mktfs.hist. I cannot simply query historical asset returns, I have to calculate them.
  - Query from output_mktfs (and input_entity) for yearmon $\leq$ init.ym to get mktfs.hist
  - Define EquityInDefault
  - Run PreProcess (my version, only difference DefaultableDebt and DefaultPoint is defined with EquityInDefault)
  - Define aret.in and aret.all the same as mktfs.hist, aret.actv with some conditions...
  - Call ARet
    - Initial asg merged from vol.old on yearmon
    - Use my version of EventRatio, which is the same except for how DefaultableDebt is defined
  - merge aret.out into aret.all to get vol.in, adding columns AssetReturn, Converged, ratio2
  - in vol.in, do aret = AssetReturn and ratio = ratio2
    - for all rows
    - (columns aret and ratio already existed from PreProcess)
    - where yearmon = init.ym and either Converged = NA or Converged = 0, set aret = NA
  - in vol.in, rename aret aret2 and ratio ratio3

- merge vol.in into mktfs.hist (on pid, xdate, retfreq), adding columns aret2 and ratio3
- set aret = aret2 and ratio = ratio3, and delete aret2 and ratio3

- Loop ym over yearmon after init.ym to final.ym
  - Query from output_mktfs (and input_entity) for yearmon = ym to get mktfs.curr
  - Define EquityInDefault
  - Run PreProcess (my version, only difference DefaultableDebt is defined with EquityInDefault)
  - Define aret.in as rbind(mktfs.hist, mktfs.curr) (*), aret.all as aret.in with (weekly36 + monthly60) condition, aret.actv as aret.all with some conditions...
  - Call ARet
    - Initial asg merged from vol.old on yearmon
    - Use my version of EventRatio, which is the same except for how DefaultableDebt is defined
  - merge aret.out into aret.all to get vol.in, adding columns AssetReturn, Converged, ratio2
  - in vol.in, do and
    - set aret = AssetReturn for all rows where yearmon> a year before ym and Converged is not NA and Converged is not 0, but where yearmon = ym and either Converged = NA or Converged = 0, set aret = NA
    - set ratio = ratio2 where yearmon> a year before ym and ratio2 is not NA
    - (columns aret and ratio already existed from PreProcess)
  - define xdate as maximal xdate in vol.in
  - set mktfreq = 0 in vol.in
  - set ratio to 1 where is NA in vol.in
  - call EmpVol on vol.in to get vol.out
    - with xdate = xdate
    - produces columns ASG, ASG.n,...
    - vol.out has fewer rows than vol.in, one row per pid (by construction of function EmpVol)
    - save results in output, with yearmon = ym and last_mktcap = xdate
  - in vol.in, rename aret aret2 and ratio ratio3
  - Define like (*) mktfs.hist as rbind(mktfs.hist, mktfs.curr)
  - merge vol.in into mktfs.hist (on pid, xdate, retfreq), adding columns aret2 and ratio3
  - set aret = aret2 and ratio = ratio3, and delete aret2 and ratio3
  - reduce mktfs.hist with (weekly36 + monthly60) condition
- Now move to calculating DD
- need mkt_datetime
- function DD my version: DefaultPoint is defined with EquityInDefault

- xdate = mkt_datetime
- xdate_last_year a year earlier
- ym derived from xdate

- big query
  - the base table is input_mktcap
    - input_mktcap is not properly populated on DEV
    - query (everything) from PROD mirror instead
  - don't query asg, asgn, last_mktcap, last_aret
    - in original query are from output_emp_vol, but only the observations where last_mktcap = max(last_mktcap) such that (asof_datetime $\leq$ mkt_datetime or (last_mktcap $\leq$ 2015-01-01 and last_mktcap $\leq$ mkt_datetime)) and yearmon $\geq$ month before ym
  - timing
    - datadate (in input_mktcap) = mkt_datetime (AT TIME ZONE 'US/Pacific')

-

old...

- Define list of pids, in output_mktfs and input_entity
- read in from output_mktfs to get mktfs.hist
  - weekly36 + monthly60 relative to initym
  - columns: pid, xdate, yearmon, curliab, curdebt, eret, yieldm, bondreturn,...
- add EquityInDefault
- call PreProcess
  - The only thing PreProcess is doing regarding aret and ratio is create columns of NA if no such columns exists.
  - adds columns aret, ratio, Dstar, DefaultPoint, DefaultableDebt
- call mktfs.hist (89 columns) (*) aret.in, also define aret.all (here same, normally weekly36 + monthly60) and aret.actv
- define initASG in aret.actv
  - download pid, yearmon, asg from output_emp_vol for pids, merge to aret.actv on pid, yearmon
  - populate almost all
- call ARet on aret.actv to get aret.out
  - Idea: get asset returns using (30). For this need estimate of asset value and hedge ratio, so we need to go through the full iteration.

- call EventRatio, which adjusts aret.actv
- Loop
  - initial value of CSG
  - call AssetValue
    - get asset value and hedge ration based on CSG
  - calculate asset returns with ARE
  - call ARetVol
    - new CSG
    - check convergence
- for non-converged do smth...
- aret has columns pid, xdate, yearmon, AssetReturn, ratio2, retfreq, mktfreq,...
-
- merge aret.out into aret.all to get vol.in, (on pid, uid, xdate, retfreq, errcode, mktfreq, yearmon) adding columns AssetReturn, Converged, ratio2
  - Have asset return time series now
- in vol.in, redefine **aret** from AssetReturn, and redefine **ratio** from ratio2
  - don't understand conditions...
- xdate = maximal xdate in vol.in

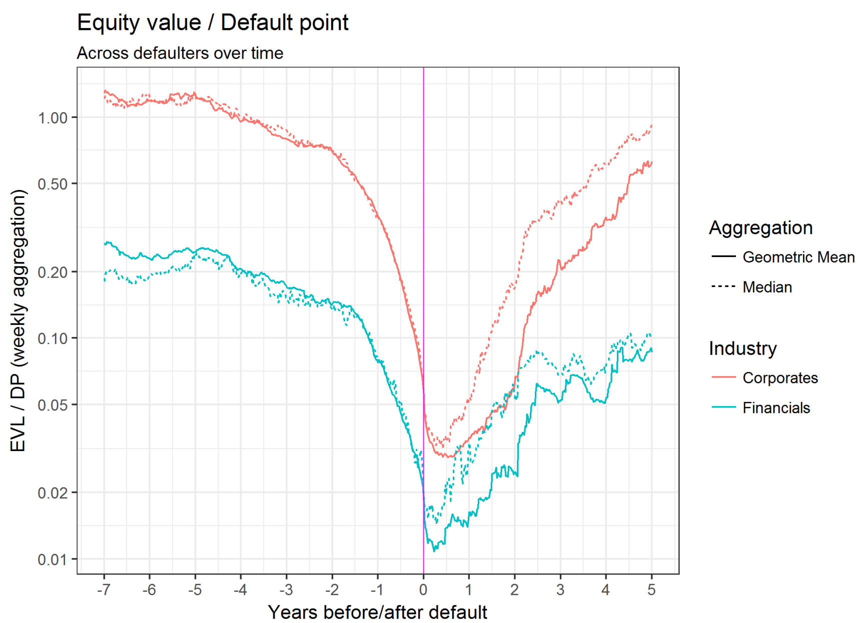
* in vol.in, rename aret to aret2 and ratio to ratio3
* merge aret2 and ratio3 from vol.in into mktfs, which has same data as (*)
* set aret = aret2 and ratio = ratio3, then delete aret2 and ratio3
* mktfs.hist 89 columns
*

6.2.1 big query

asg, asgn, last_mktcap, last_aret from ev
ev is a left join to the main on pid of
pid, uid, last_mktcap, last_aret, asg, asgn, yearmon from output_emp_vol
with an inner join of
pid, uid, max(last_mktcap) AS empvol_date from output_emp_vol where
asof_datetime is not null and asof_datetime ¡= ’”, mkt_datetime , ”’ or (last_mktcap ¡= ’2015-01-01’ and last_mktcap ¡= date(’”, mkt_datetime , ”’))) ”, ”and yearmon ¿= ”, ymM2, ” ”, ifelse(nchar(input$szpids) > 6, cc('andpidIN', input$szpids), cc(” and MOD(ev.uid, ”, input$num.of.par, ") = ", input$partition)), ”group by pid, uid) e_max ”, ”ON e_max.pid = ev.pid and e_max.empvol_date = ev.last_mktcap) ev ”), ”ON ev.pid = m.pid ”),

7 We observe positive equity values in default

Equity can have a positive value (which is equivalent to the asset value being above the default point in the current framework) for multiple reasons. Equity holders can expect there to be a government bailout or other form of cash injection that leaves them with (partial) ownership of the firm. Equity holders may put a positive probability to the possibility that the firm will continue operating, possibly under bankruptcy protection, and that at some date in the future the asset value will be larger than the face value of the debt, so that the equity option is in the money again. In this view equity holders are holding an out-of-the-money option.



The possibility of a cash injection can be modeled by positing that in default the equity value is equal to the expected value (the occurence of the bailout has a certain probability) of the equity value after the bailout that still belongs to the original shareholders. This is then a boundary condition to the ODE relating equity value and asset value.

7.1 Literature

7.1.1 Betker 1995

investigate equity's absolute priority deviation in 75 bankruptcies

deviation = (what equity holders got - what they would have got)/total value after reorganization

value securities issued in reorganization, use market value where can

determinants investigated: shareholder power, managers' incentives

Find

the closer firm is to solvency them more equityholders get. Because need to be paid more not to delay reorganization, delay is costly to creditors equity gets more if more claims are held by banks. Banks are more effective at enforcing priority

equity gets more if CEO holds more shares

equity gets more if CEO pay and shareholder wealth are positively related (dummy). Means creditors where not able to influence managers.

equity gets more if firm (meaning management) retains right to propose bankruptcy plan

7.2 Bailout modeling

A bailout is a cash injection to a firm, at the time when it otherwise would have defaulted, that allows it to keep operating as a going concern. Typically, the money comes from the government, which also takes on ownership of a part of the equity.

We assume a bailout happens with probability P_{BO} every time a firm's assets are equal to the default point. When it happens, the firm receives cash in the amount of $(\pi - 1)DP$, where $\pi > 1$. At the same time, the government assumes ownership of a fraction $1 - \mu$ of the firm's equity.

The possibility of a bailout leads shareholders to assign a greater value to the firm's stock compared to otherwise.

In order to model the possibility of a bailout, we change the left boundary condition (7) to

$$S = P_{BO}\mu\, S_{BO} \Leftrightarrow A = DP, \tag{41}$$

where S_{BO} is the equity value of the firm right after a bailout. (This value is not constant over time.) Condition (41) means that when the asset value hits the default point, equity is worth its expected value calculated as the probability of the cash injection times the stock value that the original equity holders keep. (Equity can in this case be interpreted as a call option with a rebate of $P_{BO}\mu\, S_{BO}$.) The value of equity right after the bailout (including the part not belonging to the original shareholders anymore), S_{BO}, is determined by the differential equation (9) with the new boundary condition, since we assume the bailout probability to be unconditional. For now assume the value of S_{BO} to be known at all times.

To solve (9) with boundary conditions (41) and (8), similar to above, we first guess the solution

$$S(A) = (1-\gamma)(A - A^*) + S^* - \Big((1-\gamma)(DP - A^*) + S^* - P_{BO}\mu\, S_{BO}\Big), \tag{42}$$

which again does not work, so we use a setup like equation (**??**)

$$S(A) = (1-\gamma)(A - A^*) + S^* - \psi\Big((1-\gamma)(DP - A^*) + S^* - P_{BO}\mu\, S_{BO}\Big)F\bigg(\alpha, \frac{A}{A^*}\bigg). \tag{43}$$

Plugging in DP for A and solving for ψ with $S(DP) = P_{BO}\mu\, S_{BO}$ yields

$$P_{BO}\mu\, S_{BO} = (1-\gamma)(DP - A^*) + S^* - \psi\Big((1-\gamma)(DP - A^*) + S^* - P_{BO}\mu\, S_{BO}\Big)F\bigg(\alpha, \frac{DP}{A^*}\bigg).$$

After solving for ψ (same as above), the solution is

$$S(A) = S^* + (1-\gamma)(A - A^*) + \Big((1-\gamma)(A^* - DP) - S^* + P_{BO}\mu\, S_{BO}\Big)\frac{F(\alpha, \frac{A}{A^*})}{F(\alpha, \frac{DP}{A^*})} \tag{44}$$

For known α (and π, P_{BO} and μ), equation (44) can be used to solve for S_{BO}, with $A = \pi DP$.

7.2.1 Jan's implementation

We have a number of bailout observations. For these, we empirically observe S_{BO}.

Assume $\mu = 1$.

Take P_{BO} from the SF team's scorecard, or use a constant value.

With this information, and using csg from production, we can solve

$$S_{BO} = S^* + (1-\gamma)(\pi DP - A^*) + \Big((1-\gamma)(A^* - DP) - S^* + P_{BO}\mu\, S_{BO}\Big)\frac{F(\alpha, \frac{\pi DP}{A^*})}{F(\alpha, \frac{DP}{A^*})} \tag{45}$$

for π for each bailout case, giving us a distribution of π values. We find that mean $\pi = 1.1$.

With this value of π we can estimate S_{BO} for any firm at each point in time using (45). We find that the higher the probability of bailout P_{BO}, the higher the bailout equity value S_{BO}, which makes sense since S_{BO} is again an option value.

With estimated S_{BO}, and the other parameter values, Jan reestimated asset value and asg, and a simple version of DD using asg.

7.2.2 General cash injection

The above framework for bailouts can be more generally used to model any cash injection into a firm that allows it to keep operating as a going concern, at the time when it otherwise would have defaulted. Such an injection can be in the form of a government bail-out, but it can also be a distressed debt exchange where part of the debt is (effectively) forgiven, or converted into equity.

The possibility of a cash injection induces shareholders to assign a greater value to the firm's stock compared to otherwise. This may explain that at the time of default many firms have positive market cap, which is what we observe in the data.

To estimate parameters we could the "Distressed Exchange Offer" default cases in the public default database.

7.3 Boundary equity value as fraction of default point

$$S = \tau DP \Leftrightarrow A = DP, \tag{46}$$

We set up the general solution as

$$S(A) = C_1(A - A^*) + C_2 F\left(\alpha, \frac{A}{A^*}\right) + S^*. \tag{47}$$

The first boudary condition (46) yields

$$\tau DP = C_1(DP - A^*) + C_2 F\left(\alpha, \frac{DP}{A^*}\right) + S^*. \tag{48}$$

The second boundary condition as above gives $C_1 = 1 - \gamma$, so that we get

$$C_1 = \frac{\tau DP - (1-\gamma)(DP - A^*) - S^*}{F\big(\alpha, \frac{DP}{A^*}\big)}, \tag{49}$$

and the solution is

$$S(A) = S^* + (1-\gamma)(A - A^*) + \Big(\tau DP + (1-\gamma)(A^* - DP) - S^*\Big)\frac{F\big(\alpha, \frac{A}{A^*}\big)}{F\big(\alpha, \frac{DP}{A^*}\big)} \tag{50}$$

7.4 Liquidity defaults / Cash in default

defconsid = Liquidity issues

populated 4 times

looked at 3: 1 bank cancelled line of credit, and that meant a sale that was supposed to cover debt was not possible anymore. 1 mentions cash flow problems. 1 has a lot of assets, but are mostly tech intellectual property which is hard to sell.

asdfasd

8 bailout, bailin, distressed exchange, rebate

rebate barrier option.

option holder receives part of the premium back if the value of the underlying reaches the barrier (which is a value unfavorable to the option holder)

discontinuity in value

I think Jan and James use the probability of bailout to smoothen out the discontinuity

in a bailout, equity holders receive an equity injection when assets hit the default point. Cash injection?

At the same time, the goverment assumes ownership of a fraction of the equity, but neglect that for now.

Solving the model with a different left boundary condition of the form

$$S = S_0 > 0 \Leftrightarrow A = D^* \tag{51}$$

does not seem feasible, since S is of a different scale for each firm. Perhaps

$$\frac{S}{D^*} = \xi \Leftrightarrow A = D^* \tag{52}$$

James

$$S = P_{BO}\mu S_{BO} \Leftrightarrow A = D^* \tag{53}$$

.

if the we raise the boundary condition to a positive number for equity, that means we predict default to happen quicker/earlier/more often.

intuitively, if there is the possibility of a bail-out/-in, default should happen less frequently

$$S(A) = (1-\gamma)(A - A^*) + S^* - X((1-\gamma)(D^* - A^*) + S^*)F\left(\alpha, \frac{A}{A^*}\right) \tag{54}$$

Since $F(\alpha, \infty) = 0$, the equation satifsies the right boundary condition.

For the left boundary condition, plug in D^* for A and solve for X with $S(D^*) = 0$.

$$X = \frac{1}{F(\alpha, \frac{D^*}{A^*})}.$$

.

In the case of a bailout

when asset value hits the default point, cash is injected into the firm so that $A = \pi DP$

equity is a call option on the assets of the firm (underlying) with strike price the *book*[4] value of the liabilities

[4]EDF9 methodology

8.1 Jan

S_{BO} equity value immediately after the bailout

$(\pi-1)DP$ is the cash injection

$$S_{BO} = S^* + (1-\gamma)(\pi DP - A^*) + \\ (P_{BO}\mu\, S_{BO} + (1-\gamma)(A^* - D^*) - S^*)\frac{F(\alpha, \frac{\pi DP}{A^*})}{F(\alpha, \frac{D^*}{A^*})} \quad (55)$$

James: this is the solution to the new problem

$$S(A) = S^* + (1-\gamma)(A_{adj} - A^*) + \\ (P_{BO}\mu\, S_{BO} + (1-\gamma)(A^* - D^*) - S^*)\frac{F(\alpha_{adj}, \frac{A_{adj}}{A^*})}{F(\alpha_{adj}, \frac{D^*}{A^*})} \quad (56)$$

$$\quad (57)$$

A Gamma and F functions

$$\Gamma(b) = \int_0^\infty t^{b-1}e^{-t}dt \quad (58)$$

$$\Gamma(b,y) = \frac{1}{\Gamma(b)}\int_0^y t^{b-1}e^{-t}dt \quad (59)$$

$$\Gamma'(b,y) = \Gamma_y(b,y) = \frac{1}{\Gamma(b)}y^{b-1}e^{-y} \quad (60)$$

$$\begin{aligned}\Gamma''(b,y) = \Gamma_{yy}(b,y) =& \frac{1}{\Gamma(b)}\Big((b-1)y^{b-2}e^{-y} + -y^{b-1}e^{-y}\Big) \\ =& \frac{1}{\Gamma(b)}e^{-y}\Big((b-1)y^{b-2} - y^{b-1}\Big) \\ =& -\Gamma'(b,y) + (b-1)\frac{\Gamma'(b,y)}{y} \\ =& \Big(\frac{b-1}{y} - 1\Big)\Gamma'(b,y)\end{aligned} \quad (61)$$

$$F(\alpha,x) = (1-x)\Gamma\Big(\alpha+1, \frac{\alpha}{x}\Big) + \Gamma'\Big(\alpha+1, \frac{\alpha}{x}\Big) \quad (62)$$

Properties

$$F(\alpha,0) = 1, \quad F(\alpha,\infty) = 0, \quad F_2(\alpha,\cdot) < 0$$

B Linear ODE

Consider the second order linear ODE

$$y_{tt} = p(t)y_t + q(t)y = g(t). \tag{63}$$

The homogenous equation is

$$y_{tt} = p(t)y_t + q(t)y = 0. \tag{64}$$

Principle of superposition: a linear combination of two solutions to the homogenous equation is again a solution.

Table 3:

	Dependent variable:							
	log(Market cap)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(Book assets)	0.587***	0.454***	0.611***	0.708***	0.788***	0.808***	0.570***	0.789***
	(0.020)	(0.022)	(0.027)	(0.006)	(0.007)	(0.026)	(0.028)	(0.009)
CSG	4.229***					4.914***		
	(0.099)					(0.150)		
log(Total liabilities)	0.393***	0.278***						
	(0.022)	(0.024)						
log(Current liabilities)			0.194***					
			(0.023)					
log(Long-term liabilities)			−0.001					
			(0.014)					
log(Current debt)						0.167***	0.188***	
						(0.017)	(0.019)	
log(Long-term debt)						0.062***	0.049***	
						(0.016)	(0.018)	
Risk-free rate	−9.209***	−6.734***	−3.143**			1.810	8.808***	
	(1.385)	(1.534)	(1.556)			(2.521)	(2.869)	
Constant	−3.519***	−0.423***	−0.985***	−0.557***	−1.148***	−4.015***	−0.970***	−1.086***
	(0.096)	(0.069)	(0.077)	(0.039)	(0.044)	(0.140)	(0.120)	(0.067)
Observations	7,925	7,925	6,990	7,925	6,990	3,534	3,534	3,534
R^2	0.706	0.639	0.676	0.632	0.673	0.752	0.677	0.667
Adjusted R^2	0.706	0.639	0.676	0.632	0.673	0.752	0.676	0.667

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4:

	Dependent variable:					
	log(Market cap/Total liabilities)					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Book assets)	0.581***	0.452***	0.449***	−0.183***	0.667***	0.716***
	(0.020)	(0.022)	(0.022)	(0.006)	(0.022)	(0.020)
CSG	4.075***					3.500***
	(0.100)					(0.100)
log(Total liabilities)	−0.603***	−0.708***	−0.702***		−0.850***	−0.719***
	(0.021)	(0.023)	(0.023)		(0.023)	(0.021)
Risk-free rate	−9.320***	−7.052***			−2.961**	−6.073***
	(1.376)	(1.513)			(1.439)	(1.342)
Financial firm	−0.665***	−1.050***	−1.045***	−1.128***	−1.013***	−0.693***
	(0.064)	(0.069)	(0.069)	(0.073)	(0.066)	(0.062)
|log(Total liabs/Book assets)|					0.890***	0.637***
					(0.029)	(0.028)
Constant	−3.364***	−0.356***	−0.616***	−0.892***	−1.447***	−3.720***
	(0.096)	(0.068)	(0.040)	(0.041)	(0.074)	(0.095)
Observations	7,925	7,925	7,925	7,925	7,925	7,925
R^2	0.358	0.223	0.221	0.132	0.304	0.397
Adjusted R^2	0.358	0.223	0.221	0.132	0.303	0.396

Note: *p<0.1; **p<0.05; ***p<0.01