INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by

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STATEMENT OF DISSERTATION APPROVAL

The dissertation of **Jane Doe**

has been approved by the following supervisory committee members:

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and by **Alice B. Toklas** , Dean of The Graduate School.

ABSTRACT

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For my parents, Alice and Bob.

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LIST OF FIGURES

LIST OF TABLES

NOTATION AND SYMBOLS

α	fine-structure (dimensionless) constant, approximately $1/137$
α	radiation of doubly-ionized helium ions, He++
β	radiation of electrons
γ	radiation of very high frequency, beyond that of X rays
γ	Euler's constant, approximately $0.577\,215\,\ldots$
δ	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
ϵ	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
$\ldots$	$\ldots$
$\psi(x)$	logarithmic derivative of the gamma function
ω	frequency

CHAPTER 1

INTRODUCTION

In 1952 L.A. Hiller and L.M Issacson ushered forth a new era of the study of both music and computer science when they introduced the Illiac Suite – the first composition that was created solely by a computer [**?**]. The computational study of music presents unique challenges to both fields given the highly subjective nature of music paired with the highly quantitative and mathematical nature of computer science. There have been impressive advances in the field, particularly in digital technology, which have led to the rise of entirely new musical genres and musical production. This technology has fundamentally changed the interface between music and the way we listen to it - but it is yet to be discovered how computation can change the actual way we study, compose, and perform music on a more fundamental level. It is possible that new developments could reshape the way we think about music itself.

To reach such a point, it is necessary to view the field from the lens of Artificially Intelligent musical systems that are able to reason themselves about music. In general, Artificially Intelligent systems have seen immense progress in the last decade due to the rise of Machine Learning (particularly with Deep Learning) and it's applications in several different domains. Music has been one of these domains and has seen impressive advances in several musical tasks such as musical composition[**?**], instrument sound synthesis[**?**], and musical analysis[**?**].

One of the more intriguing problems in computer music is the creation of an expressive performance generation system. There are several commercially available notation and playback software systems [1] that are able to automatically generate musical performances from a purely symbolic musical representation in the form of a score (more commonly known as sheet music). The systems are built based on a predefined set of rules that create

[1] musescore.com and www.finalemusic.com

deterministic performances given a score. Although the performances are technically an ”accurate” rendering of the score, they don’t contain the *human* element.

CHAPTER 2

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CHAPTER 3

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CHAPTER 4

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4.1 More on the topic

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4.2 Even more on the topic

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4.3 Summary and conclusions

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APPENDIX A

THE FIRST

This is an appendix. Notice that the LaTeX markup for an appendix is, surprisingly, `\chapter`. The `\appendix` command does not produce a heading; instead, it just changes the numbering style from numeric to alphabetic, and it changes the heading prefix from **CHAPTER** to **APPENDIX**.

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APPENDIX B

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APPENDIX C

THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in LaTeX `\cite{...}` commands with their BibTeX citation labels in reverse alphabetical order, thanks to the `citesort` package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their BibTeX entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,
  author =        "Albert Einstein",
  title =         "{Eine Neue Bestimmung der Molek{\"u}ldimensionen}.
                  ({German}) [{A} new determination of molecular
                  dimensions]",
  type =          "Inaugural dissertation",
  school =        "Bern Wyss.",
  address =       "Bern, Switzerland",
  year =          "1905",
  bibdate =       "Fri Dec 17 10:46:57 2004",
```

```
  bibsource =     "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
  note =          "Published in \cite{Einstein:1906:NBM}.",
  acknowledgement = ack-nhfb,
  language =      "German",
  advisor =       "Alfred Kleiner (24 April 1849--3 July 1916)",
  URL =           "http://en.wikipedia.org/wiki/Alfred_Kleiner",
  remark =        "Received August 19, 1905 and published February 8,
                  1906.",
  Schilpp-number = "6",
}
```

The `note` field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in BIBTEX files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long `note` field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that ..., multiple passes of LATEX and BIBTEX are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

BIBTEX entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes BIBTEX entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In BIBTEX `title` fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842 by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16, 18]. She was the assistant to Charles Babbage (1791–1871), and they are the world's first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index (except that this sample thesis doesn't have one!)

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