Homework 9

Please do not use array in this lab.

Please print out the answers in the main function

1. Write a program to approximate the value of sin(x) using the formula:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!}$$

(x 值由 keyboard key in).

The program stops when $\left|\frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!}\right| < 10^{-12}$.

Once the term is less than 10^{-12}, this term "should not" be added into the sum.

Input x in the main function and pass this data to the following functions by value:

Case I:

Write one value return function and use **one while loop** to calculate the sin value in the function body and then display the result to the **14th** decimal place in the main.

Case II:

Write one value return recursive function to calculate the sin value and display the result to the **14th** decimal place in the main**.**

Case III:

Use the same input to call the **sin** function in the **math.h** and display the result to the 14th decimal place in the main.

The input will stop when you enter Ctrl D.

Input/Output Example:

```
-0.1
The sin(-0.100000) is -0.09983341664683
The sin(-0.100000) is -0.09983341664683
The sin(-0.100000) is -0.09983341664683
2
The sin(2.000000) is 0.90929742682564
The sin(2.000000) is 0.90929742682564
The sin(2.000000) is 0.90929742682568
-2
The sin(-2.000000) is -0.90929742682564
The sin(-2.000000) is -0.90929742682564
The sin(-2.000000) is -0.90929742682568
3.1415926
The sin(3.1415926) is 0.00000005358962
The sin(3.1415926) is 0.00000005358962
The sin(3.1415926) is 0.00000005358979
^D
```

2. Please use **recursive** function to compute the following:

(a). $S = \frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + \ldots + \frac{1}{n*(n-1)}$

Let the user input the number n and show the answer to the 8th decimal place.
Stop the program when inputting **CTRL+Z**.
Hint: Think about when **n** becomes to 2 what the value of this term

Input/Output Example:
Please input n: 30
0.96666667

Please input n: 1
illegal input

Please input n: 15
0.93333333

Please input n: ^Z

(b). $\pi = 4 * \frac{2}{3} * \frac{4}{3} * \frac{4}{5} * \frac{6}{5} * \ldots * \frac{2n}{2n+1} * \frac{2n+2}{2n+1}$

Please write a recursive function to calculate the equation. Let the user input the number n and show the answer to the 8th decimal place.
Stop the program when inputting CTRL+Z.

Hint: You may consider $(\frac{2}{3} * \frac{4}{3})$, $(\frac{4}{5} * \frac{6}{5})$, …as one pair and let n=0 as 4

Input/Output Example:
Please input n: 1000
3.14237737
Please input n: 10000
3.14167119
Please input n: ^Z

(c). Please write a recursive function double approxiPI() to find the approximated value of π until the error between M_PI in math.h and your value is smaller than 10^{-4}. (judge the error and calculate n in approxiPI()).

Please print out the number of n in 2(b) equation for your approximated π.
Please show your approximated π to the 8th decimal place. In this problem, you can print 'n' in either the main function or approxiPI().

Hint:To use the constant M_PI in the math.h you need to add the following into your

program:

```
#define _USE_MATH_DEFINES
#include<math.h>
```

Input/Output Example:

<math.h> M_PI: 3.14159265
n: 7854
myPI: 3.14169264

3. Write a program that inputs two numbers: x and y. (data are all integers) in the main program and passes these two numbers (pass by value) to the recursive function: ***power*** that returns the $\mathbf{x}^{\mathbf{y}}$.

If y ≧0,

$$power(x, y) = \begin{cases} 1 & \text{if } y=0 \\ x & \text{if } y=1 \\ x*power(x,y-1) & \text{if } y>1 \end{cases}$$

If y<0

$$power(x, y) = \frac{\mathbf{1}}{\boldsymbol{power(x,y)}}$$

Please print the result in the main program and show the answer to the 8th decimal place. The program should be able to execute repeatedly until user entering Ctrl-D.

Input/Output Example:

x,y = 2 10
power(x, y)= 1024.00000000
x,y = 2 -10
power(x, y)= 0.00097656
x,y = ^D

4. A robot can take steps of 1 meter, 2 meters and 3 meters. Write a recursive function to evaluate the number of ways the robot can walk n meters. Let the user input the number **n.** Please print out the answers in the main function. Stop the program when inputting **CTRL+Z.**

Input/Output Example:

n = 3
4 ways
n = 5
13 ways
n = 10
274 ways
n = ^Z

5. Suppose that we have a 2 × n rectangular board divided into 2n squares. Please write a recursive function that computes the number of ways to cover this board exactly by 1 × 2 dominoes. Let the user input the number n. Please print out the answer in the main function. Stop the program when inputting CTRL+D.

   Input/Output Example:

   n= 10
   89 ways
   n= 43
   701408733 ways
   n= ^D