Lab 11/30

Please do not use array or global variable in this lab.

Please use recursive function to solve problem, and print out the answer in the main function.

Lab Part

1. Please write a recursive function to calculate π as the following function. Display the result to the 14th in the main function and compare the result with M_PI in math.h. Let the user continuously input the n until inputting CTRL+Z.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

n=0 n=1 n=2

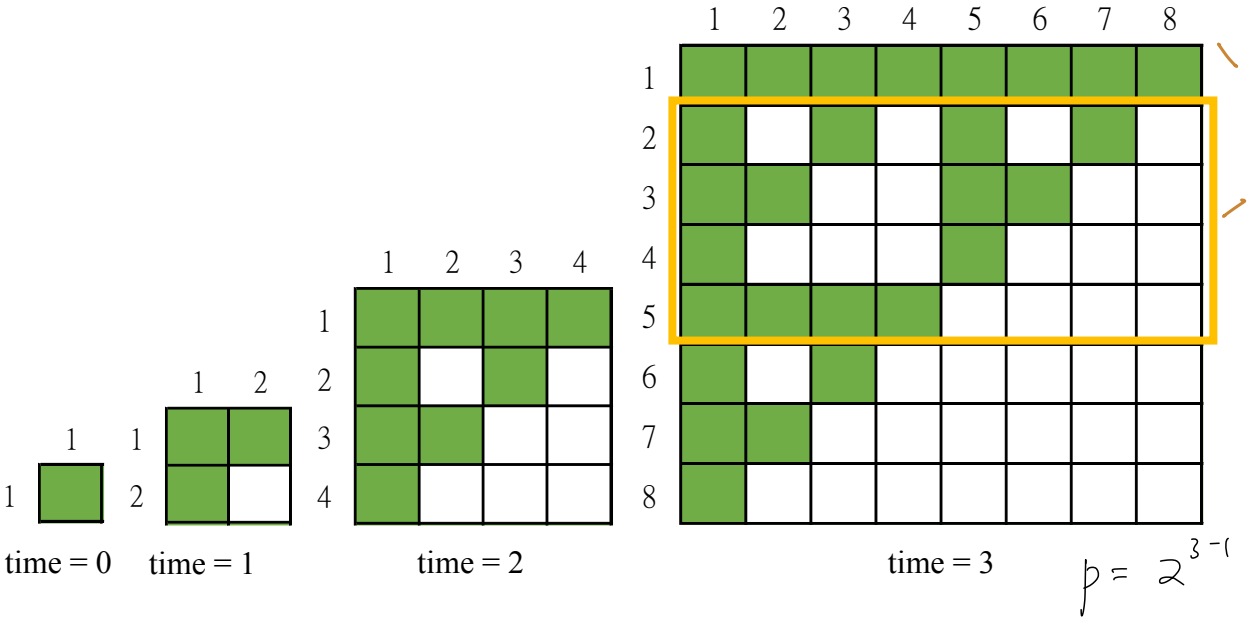
Input/Output Example

```
1
pi=3.06146745892072
M_PI=3.14159265358979
4
pi=3.14033115695475
M_PI=3.14159265358979
100
pi=3.14159265358979
M_PI=3.14159265358979
50
pi=3.14159265358979
M_PI=3.14159265358979
10
pi=3.14159234557012
M_PI=3.14159265358979
^Z

--------------------------------
```

2. How many green grids

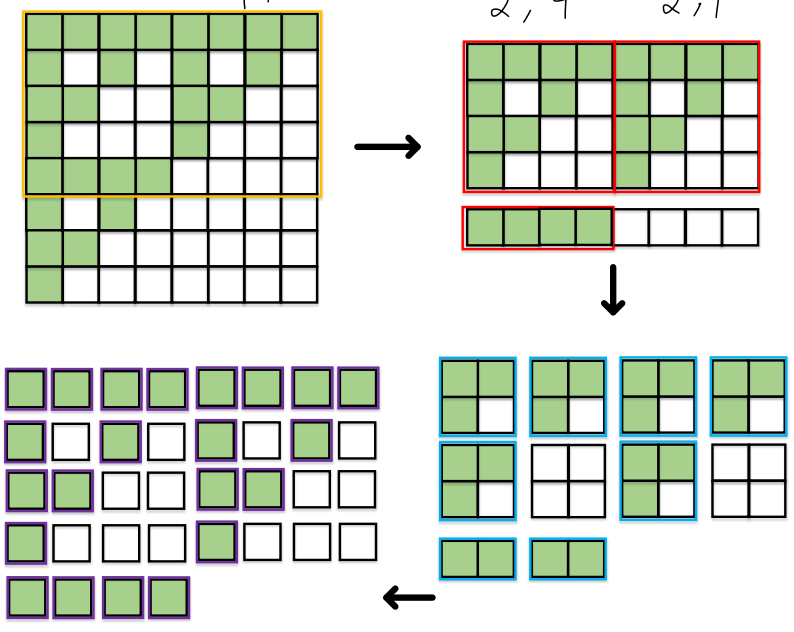
The picture below shows how the number of green grids changes over time.



Please write a recursive function that calculate the total number of green grids from row A to row B (A<=B) at time t. For example, time = 3, [A, B] = [2, 5], the total number of green grids is 14.

Please let the user input t, A and B continuously until input CTRL+Z. Please judge the illegal input A and B in recursive function, and show the error message in the main function. Please print out the answer in the main function.

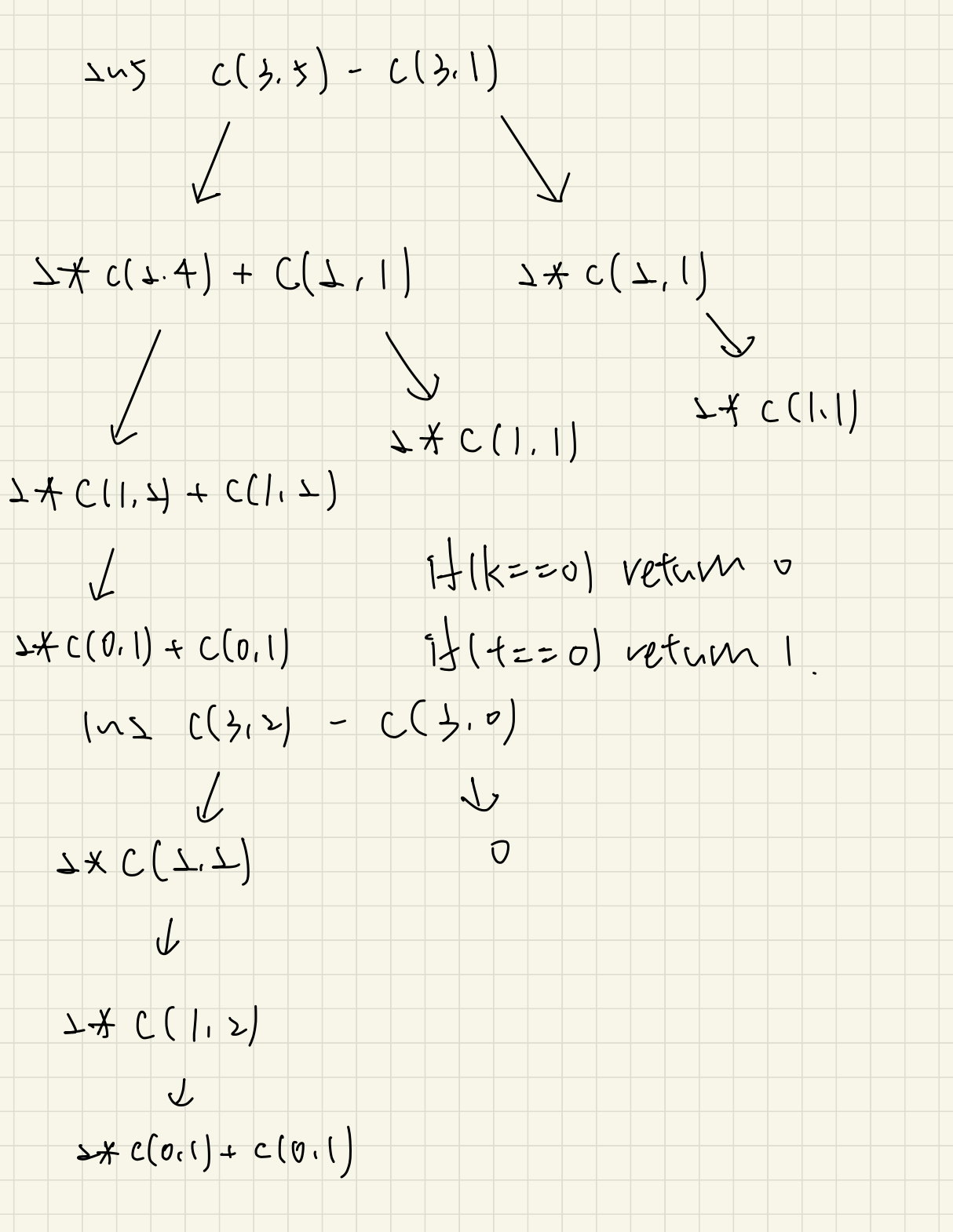
1. You can write a recursive function "count(t, k)" that calculate the number of green grids from row 1 to row k at time t.
2. The recursive relation is as follows:



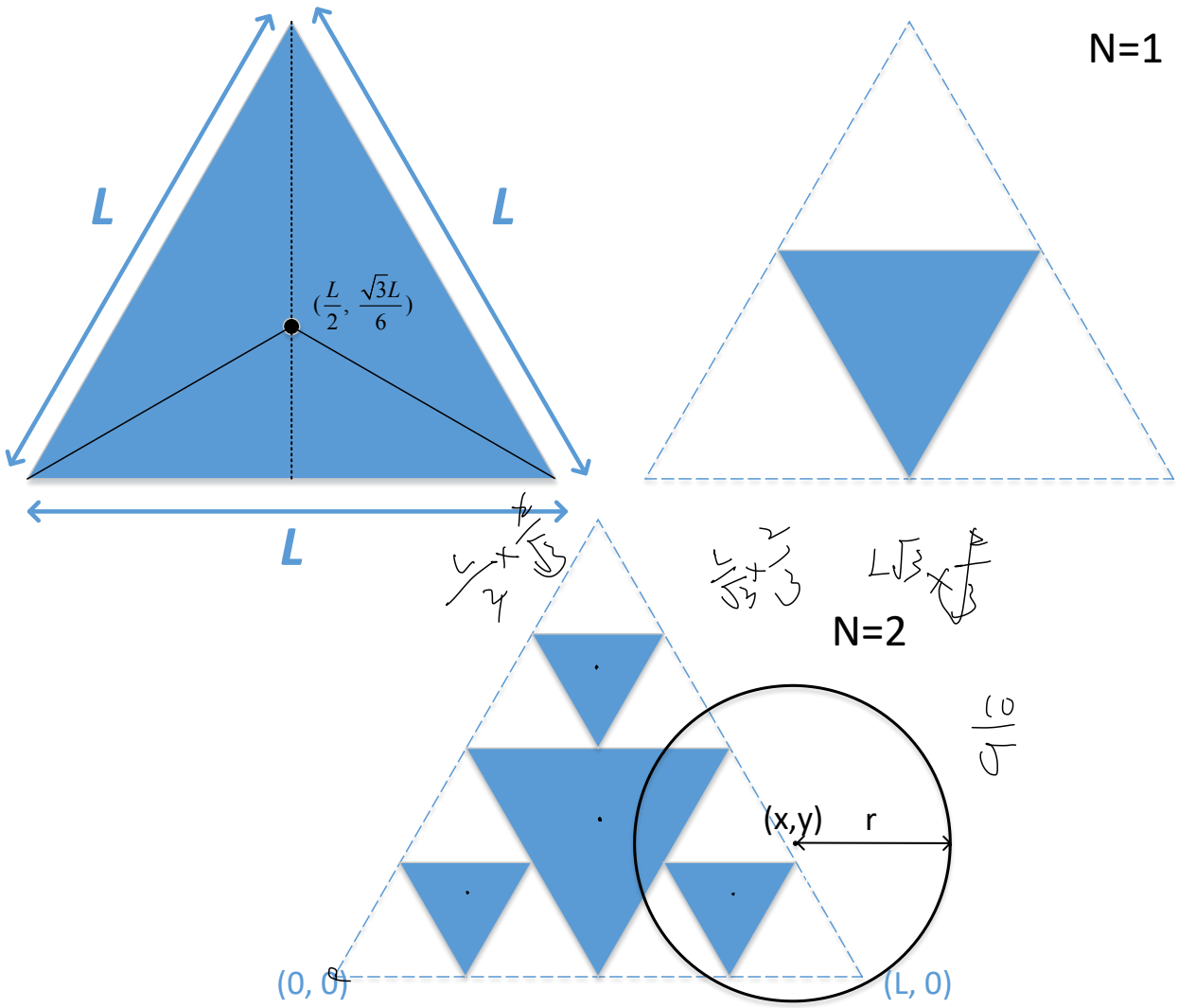
Input/Output Example:

```
Input time, [A, B]: 4 2 10
green grids #: 50
Input time, [A, B]: 0 1 1
green grids #: 1
Input time, [A, B]: 4 2 17
Out of range.
Input time, [A, B]: 4 2 16
green grids #: 65
Input time, [A, B]: ^Z

--------------------------------
```


ans C(3,5) - C(3,1)
2*C(2,4) + C(2,1)
2*C(2,1)
2*C(1,1)
2*C(1,1)
2*C(0,1) + C(0,1)
if(k==0) return 0
if(t==0) return 1.
ans C(3,2) - C(3,0)
2*C(2,2)
2*C(1,2)
2*C(0,1) + C(0,1)
0

3. As the following picture, please write a program to calculate how many triangles are in a circle with (x, y) as the center, and the radius of this circle is **r**:



(a). Input N > 0

(c) Let the user continuously input the N, L, x, y, r until inputting CTRL+Z.

Input/Output Example

```
(N,L,x,y,r): 2 5 0 0 10
4 triangle(s) in circle

(N,L,x,y,r): 2 5 1.25 0.721688 0.721689
1 triangle(s) in circle

(N,L,x,y,r): 4 5 1.25 0.721688 0.721689
3 triangle(s) in circle

(N,L,x,y,r): 6 5 1.25 0.721688 0.721689
39 triangle(s) in circle

(N,L,x,y,r): 6 5 1.25 0.721688 10
364 triangle(s) in circle

(N,L,x,y,r): ^Z

--------------------------------
```

Homework Part

4. Write a program to approximate the value of sin(x) using the formula:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^{n+1}x^{2n-1}}{(2n-1)!}$$ (x 值由 keyboard key in).

The program stops when $|\frac{(-1)^{n+1}x^{2n-1}}{(2n-1)!}| < 10^{-12}$.

Once the term is less than 10^{-12}, this term "should not" be added into the sum.

Input x in the main function and pass this data to the following functions by value:

Case I:

Write one value return function and use **one while loop** to calculate the sin value in the function body and then display the result to the **14th** decimal place in the main.

Case II:

Write one value return recursive function to calculate the sin value and display the result to the **14th** decimal place in the main**.**

Case III:

Use the same input to call the **sin** function in the **math.h** and display the result to the 14th decimal place in the main.

The input will stop when you enter Ctrl D.

Modification:

(a). ~~sin(x)~~ => $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{(n)!}$

(b). ~~14^{th} decimal place~~ => 11^{th} decimal place

Input/Output Example

```
1
The exp(1.000000) is 2.71828182846
The exp(1.000000) is 2.71828182846
The exp(1.000000) is 2.71828182846
2
The exp(2.000000) is 7.38905609893
The exp(2.000000) is 7.38905609893
The exp(2.000000) is 7.38905609893
3
The exp(3.000000) is 20.08553692319
The exp(3.000000) is 20.08553692319
The exp(3.000000) is 20.08553692319
^D

--------------------------------
```

5. Please use recursive function to compute the following:

(a). $S = \frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + \cdots + \frac{1}{n*(n-1)}$

(b). $\pi = 4 * \frac{2}{3} * \frac{4}{3} * \frac{4}{5} * \frac{6}{5} * \ldots * \frac{2n}{2n+1} * \frac{2n+2}{2n+1}$

Let user input the number n and Stop the program when inputting CTRL+Z in (a)(b)

(c). Please write a recursive function double approxiPI() to find the approximated value of π until the error between M_PI in math.h and your value is smaller than 10^{-4}. (judge the error and calculate n in approxiPI())

(You can see more detail in homework9)

Please show the answer to the 8th decimal place.

Modification:

(a). $S = \frac{1}{1*4} + \frac{1}{2*5} + \frac{1}{3*6} + \cdots + \frac{1}{n*(n+3)}$

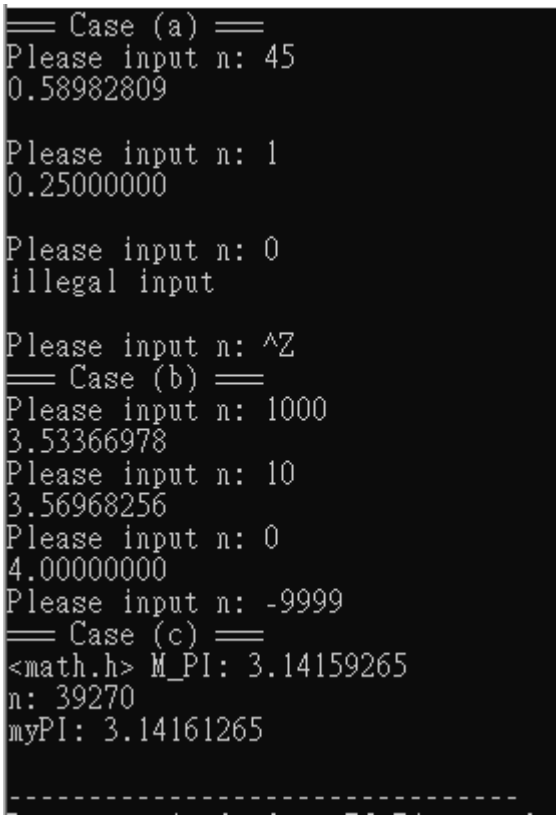
You have to judge all illegal n in the recursive function.

(b). $Q = 4 * \frac{3}{4} * \frac{5}{4} * \frac{6}{7} * \frac{8}{7} * \ldots * \frac{3n}{3n+1} * \frac{3n+2}{3n+1}$

Stop the program when inputting CTRL+Z or -9999 in (a)(b)

(c). smaller than 10^{-4} => smaller than $2*10^{-5}$

Input/Output Example:



6. Suppose that we have a 2 × n rectangular board divided into 2n squares. Please write a recursive function that computes the number of ways to cover this board exactly by 1 × 2 dominoes. Let the user input the number n. Please print out the answer in the main function. Stop the program when inputting CTRL+D.

Modification:

(a). ~~2 × n rectangular board~~ => 3 × n rectangular board

(b). ~~exactly by 1 × 2 dominoes~~ => by 1 × 3 dominoes

Input/Output Example: