# Lab 10/18

**Please do not use math functions, if/else, self-defined function, and array in this Lab.**

## Lab Part

1. Calculate sin, cos using following formulas:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Calculate above functions simultaneously

Eg.

n=0: $\sin(x) = x, \cos(x) = 1$

n=1: $\sin(x) = x - \frac{x^3}{3!}, \cos(x) = 1 - \frac{x^2}{2!}$

n=2: …

Note that $\sin^2(x) + cos^2(x) = 1$

stop when the error $|\sin^2(x) + \cos^2(x) - 1|$ is less than $10^{-10}$

The last term is added.

1) Let user continuously input x until Ctrl+D
2) Show following outputs to the $12^{th}$ decimal place :

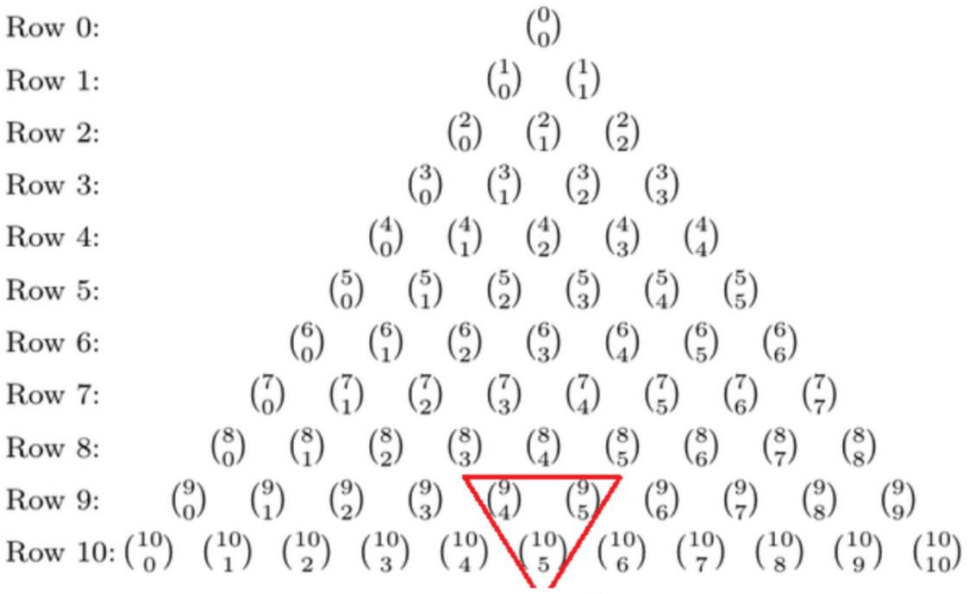
Input/Output Example:

```
Input x: 1
sin = 0.841470984809
cos = 0.540302305880
sin^2 + cos^2 = 1.000000000014
err = 0.000000000014

Input x: 0.5
sin = 0.479425538604
cos = 0.877582561890
sin^2 + cos^2 = 0.999999999999
err = -0.000000000001

Input x: ^D
```

2. Please write a program which lets user continuously input an integer n, stop when inputting CTRL+Z and prints out the first n rows of reverse Pascal's triangle.




$$\binom{n}{k} = \mathrm{C}_k^n = \frac{n!}{k!\,(n-k)!}$$

Input/Output Example:

```
n = 3

    1   2   1
      1   1
        1


n = 5

    1   4   6   4   1
      1   3   3   1
        1   2   1
          1   1
            1


n = 10

    1   9  36  84 126 126  84  36   9   1
      1   8  28  56  70  56  28   8   1
        1   7  21  35  35  21   7   1
          1   6  15  20  15   6   1
            1   5  10  10   5   1
              1   4   6   4   1
                1   3   3   1
                  1   2   1
                    1   1
                      1


n = ^Z
```

3. Estimate the following fun1ction:

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3n)!}{(2n+3)(n!)^4 4^{2n}} x^{2n}$$

Stop when the added term is less than $10^{-13}$ (don't add this term)

(a) Output answer to the 15th decimal place
(b) Let user input x continuously until Ctrl+D

Input/Output Example:

```
Input x: 1
ans = 0.277469354165504
Input x: 2
ans = 0.207433403655143
Input x: 0.1
ans = 0.332585836918589
Input x: ^D
```

## Homework Part

Please **use one while loop in problem 4(a), 4(b), 5** (**nested loop is not allowed**), but you can use additional one while loop for user continuous input purpose.

4. Write a program to approximate the value of e using the formula

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

(a). Stop when the added term is less than $10^{-10}$ (do not add this term)
(b). Stop when the difference between the two successive terms is less than 0.00001. (do not add the smaller term among the last two successive terms)

1. Please show the answer to the 11th decimal place.
2. In (b), you can use while or do/while.

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Modification:

1) Compute $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.
2) (a). Stop when the added term is not larger than $10^{-8}$.
3) (b). Stop when the difference between the two successive terms is less than $10^{-3}$.

Input/Output Example:

```
x = 1
(a). e^1.000 = 2.71828182620
(b). e^1.000 = 2.71825396825
```

```
x = 3.14159
(a). e^3.142 = 23.14063122272
(b). e^3.142 = 23.14049941084
```

5. Write a program to approximate the value of sin(x) using the formula:

$$sin\,(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n x^{2n-1}}{(2n-1)!}$$

The program stop when absolute value of last term is less than $10^{-9}$ (don't add/subtract this term)

a. Please let user continuously input x and stop when inputting CTRL+Z.

b. Please output to the 11th decimal place.

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Modification:

1) Compute $\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$
2) Stop the program when the absolute value of last term is not larger than $10^{-10}$.
3) Please output to the 14th decimal place.

Input/Output Example:

```
x = 1
sinh(1.000000) = 1.17520119364303
x = 5
sinh(5.000000) = 74.20321057776712
x = ^Z
```

6. Write a program that reads an integer and converts it to multiples of 50, 20, 10, and 1. For example, if the user enters the number 285, the program should display 5*50, 1*20, 1*10, 5*1.

   a. Please let user continuously input x and stop when inputting CTRL+Z.

   b. Please use two while loops to complete this program, one for user continuous input, the other for main problem.

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Modification:

1) Let user input four number a, b, c, d instead of 50, 20, 10, 1 at the beginning. (d is always 1)
2) Stop when inputting CTRL+D

Input/Output Example:

```
a, b, c, d = 100 50 10 1
x = 3030
3030 = 30*100 + 0*50 + 3*10 + 0*1

x = 3081
3081 = 30*100 + 1*50 + 3*10 + 1*1

x = ^D
```

```
a, b, c, d = 200 100 50 1
x = 8763
8763 = 43*200 + 1*100 + 1*50 + 13*1

x = 1234
1234 = 6*200 + 0*100 + 0*50 + 34*1

x = ^D
```

7. In cryptarithmetic puzzles, mathematical equations are written using letters. Each letter can be a digit from 0 to 9, but no two letters can be the same. Write a program that finds solutions to the cryptarithmetic puzzle of:

TOO + TOO + TOO + TOO = GOOD

The simplest technique is to use a nested loop for each unique letter (in this case T, O, G, D). The loops would systematically assign the digits from 0-9 to each letter.

In the loop body test that each variable is unique and that the equation is satisfied.

Output the values for the letters that satisfy the equation.
You can use ternary operator or if/else if needed in this problem.

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Modification:

Please find the solution of TOD + TOD + TOD + TOD = GOOG

Input/Output Example:

```
T=1, O=6, G=0, D=5
```