

Concordia University
Department of Computer Science and Software
Engineering
SOEN 331:
Formal Methods for Software Engineering

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March 26, 2024

Problem 2

1. CL-USER 1 > (defvar *airports* '((YUL.Montreal) (LCY.London_UK) (LHR.London_UK) (MIL.MILAN) (SFO.SAN_FRANCISCO) (SD.SAN_DIEGO))
AIRPORTS

CL-USER 2 > (print *airports*)

((YUL.MONTREAL) (LCY.LONDON_UK) (LHR.LONDON_UK) (MIL.MILAN) (SFO.SAN_FRANCISCO) (SD.SAN_DIEGO))
((YUL.MONTREAL) (LCY.LONDON_UK) (LHR.LONDON_UK) (MIL.MILAN) (SFO.SAN_FRANCISCO) (SD.SAN_DIEGO))

CL-USER 3 > *airports*

((YUL.MONTREAL) (LCY.LONDON_UK) (LHR.LONDON_UK) (MIL.MILAN) (SFO.SAN_FRANCISCO) (SD.SAN_DIEGO))

CL-USER 4 >

2. $\text{monitored} = \text{dom}(\text{airports})$

3. *airports* is a variable that holds one to many mappings of objects of type AIRPORT to objects of type CITY. $\text{dom}(\text{airports}) = \mathbb{P} \text{ AIRPORT}$. $\text{codomain}(\text{airports}) = \mathbb{P} \text{ CITY}$.
 $\text{airports} = \mathbb{P} \text{ AIRPORT} \mapsto \text{CITY}$

4. This should be an unordered structure, because order doesn't matter in this scenario, nor does it make sense. Looking up in an unordered structure would also be a lot faster than in an ordered one $O(1)$ vs $O(n)$. Assuming a Hashmap or any variation is not permitted, a set containing tuples of type (AIRPORT, CITY) should be used. There should be a restriction on the domain of this data structure, indicating no head of tuple can be a duplicate of another head. Lists of strict length 2 with types $\langle \text{AIRPORT}, \text{CITY} \rangle$ could also be used to replace the tuples.

5.

<i>AirportSystem</i>	
<i>codes</i> : $\mathbb{P} \text{ AIRPORT}$	
<i>airports</i> : $\text{AIRPORT} \rightarrow \text{CITY}$	--non-injective, surjective
<i>cities</i> : $\mathbb{P} \text{ CITY}$	
<hr/>	
<i>codes</i> = dom <i>airports</i>	
<i>deployed</i> = ran <i>airports</i>	

6.

<i>Success</i>	
$\exists \text{ AirportSystem}$	
<i>response!</i> : <i>MESSAGE</i>	
<hr/>	
<i>response!</i> = 'ok'	

<i>CodeAlreadyInUse</i>	
$\exists \text{ AirportSystem}$	
<i>code?</i> : <i>AIRPORT</i>	
<i>response!</i> : <i>Message</i>	
<hr/>	
<i>code?</i> \in dom <i>airports</i>	
<i>response!</i> = 'Code already in use'	

<i>AddAirportOK</i>	
$\Delta \text{ AirportSystem}$	
<i>code?</i> : <i>AIRPORT</i>	
<i>city?</i> : <i>CITY</i>	
<hr/>	
<i>code?</i> \notin dom <i>airports</i>	
<i>airports'</i> = <i>airports</i> $\cup \{code? \mapsto city?\}$	

$$\text{AddAirport} \hat{=} (\text{AddAirportOK} \wedge \text{Success}) \oplus \text{CodeAlreadyInUse}$$

7. CL-USER 1 > (defvar *airports*

```

'((YUL . Montreal)
  (LCY .London_UK)
  (LHR . London_UK)
  (MIL . Milan)
  (SFO . San_Francisco)
  (SDQ . Santo_Domingo)))
*AIRPORTS*

```

CL-USER 2 : 1 > *airports*

```
((YUL . MONTREAL) (LCY .LONDON_UK) (LHR . LONDON_UK) (MIL . MILAN) (SFO . SAN_FRANC
```

CL-USER 3 : 1 > (defun add-airport (code city)

```

  (if (find code *airports* :key #'car)
      (print "ok")
      (progn
        (push (cons code city) *airports*)
        (print "Code already in use."))))

```

ADD-AIRPORT

CL-USER 4 : 1 >

8.

$ \begin{array}{l} \textit{UpdateAirportOK} \\ \Delta \textit{AirportSystem} \\ \textit{code?} : \textit{AIRPORT} \\ \textit{city?} : \textit{CITY} \end{array} $	
$ \begin{array}{l} \textit{code?} \in \text{dom } \textit{airports} \\ \textit{airports}' = \textit{airports} \oplus \{ \textit{code?} \mapsto \textit{city?} \} \end{array} $	

CodeNotInUse $\exists \text{AirportSystem}$ $\text{code?} : \text{AIRPORT}$ $\text{response!} : \text{Message}$
$\text{code?} \notin \text{dom airports}$ $\text{response!} = \text{'Code not in use'}$

$$\text{UpdateAirport} \hat{=} (\text{UpdateAirportOK} \wedge \text{Success}) \oplus \text{CodeNotInUse}$$

9. $\text{airports} = \{ \text{YUL} \mapsto \text{Dorval},$
 $\text{LCY} \mapsto \text{LondonUK},$
 $\text{LHR} \mapsto \text{LondonUK},$
 $\text{MIL} \mapsto \text{Milan},$
 $\text{SFO} \mapsto \text{SanFrancisco},$
 $\text{SDQ} \mapsto \text{SantoDomingo}$
 $\}$

Remove Everything After this line _____

Part 1: Temperature monitoring system with the Z specification

Consider a system called 'TempMonitor' that keeps a number of sensors, where each sensor is deployed in a separate location in order to read the location's temperature. Before the system is deployed, all locations are marked on a map, and each location will be addressed by a sensor. The formal specification of the system introduces the following three types:

SENSOR_TYPE, LOCATION_TYPE, TEMPERATURE_TYPE

We also introduce an enumerated type *MESSAGE* which will assume values that correspond to success and error messages.

Provide a formal specification in Z, with the following operations:

- **DeploySensorOK**: Places a new sensor to a unique location. You may assume that some (default) temperature is also passed as an argument.
- **ReadTemperatureOK**: Obtain the temperature reading from a sensor, given the sensor's location.

Provide appropriate success and error schemata to be combined with the definitions above to produce robust specifications for the following interface:

- **DeploySensor**,
- **ReadTemperature**.

Solution:

TempMonitor

$deployed' : \mathbb{P} \text{ SENSOR_TYPE}$ $map : \text{SENSOR_TYPE} \rightarrow \text{LOCATION_TYPE} \quad \text{--partial bijective}$ $read : \text{SENSOR_TYPE} \rightarrow \text{TEMPERATURE_TYPE}$
$deployed = \text{dom } map$ $deployed = \text{dom } read$

DeploySensorOK

$\Delta \text{TempMonitor}$ $sensor? : \text{SENSOR_TYPE}$ $location? : \text{LOCATION_TYPE}$ $temperature? : \text{TEMPERATURE_TYPE}$
$sensor? \notin deployed$ $location? \notin \text{ran } map$ $deployed' = deployed \cup \{sensor?\}$ $map' = map \cup \{sensor? \mapsto location?\}$ $read' = read \cup \{sensor? \mapsto temperature?\}$

ReadTemperatureOK

$\exists \text{TempMonitor}$ $location? : \text{LOCATION_TYPE}$ $temperature! : \text{TEMPERATURE_TYPE}$
$location? \in \text{ran } map$ $temperature! = read(map^{-1}(location?))$

Success

$\exists \text{TempMonitor}$ $response! : \text{MESSAGE}$
$response! = 'ok'$

$\text{SensorAlreadyDeployed}$ $\exists \text{TempMonitor}$ $\text{sensor?} : \text{SENSOR_TYPE}$ $\text{response!} : \text{Message}$
$\text{sensor?} \in \text{deployed}$ $\text{response!} = \text{'Sensor deployed'}$

$\text{LocationAlreadyCovered}$ $\exists \text{TempMonitor}$ $\text{location?} : \text{LOCATION_TYPE}$ $\text{response!} : \text{Message}$
$\text{location?} \in \text{ran map}$ $\text{response!} = \text{'Location already covered'}$

LocationUnknown $\exists \text{TempMonitor}$ $\text{location?} : \text{LOCATION_TYPE}$ $\text{response!} : \text{Message}$
$\text{location?} \notin \text{ran map}$ $\text{response!} = \text{'Location not covered'}$

$$\text{DeploySensor} \hat{=} (\text{DeploySensorOK} \wedge \text{Success}) \oplus (\text{SensorAlreadyDeployed} \vee \text{LocationAlreadyCovered})$$

$$\text{ReadTemperature} \hat{=} (\text{ReadTemperatureOK} \wedge \text{Success}) \oplus \text{LocationUnknown}$$

Part 2: A booking system with the Object Z specification

We introduce the basic types $[Person, SeatType]$. We also introduce an enumerated type *Message* which will assume values (feel free to define your own) that correspond to success and error messages. Consider a system to book seats for a theater play. A customer can book a single seat, and a seat can only accommodate a single customer. The booking system keeps a log of the customers that have booked a seat. The system publishes a plan of the theater and it allows customers to access it online and make a booking or cancel a booking.

Class Booking

Define a formal specification in Object-Z for class *Bookingt* to support the following operations:

- **BookOK**: Reserves a seat for a given customer.
- **CancelOK**: Frees a seat for a given customer.

You will also need to provide appropriate success and error schemata to be combined with the definitions above to produce *robust specifications* for the following interface:

- **Book**, and
- **Cancel**.

Class Booking2

Subclassify *Booking* to introduce class **Booking2** that behaves exactly like *Booking*, while introducing the following operations:

- **GetNumberOfCustomers** returns the total number of customers who have made a booking.

- `ModifyBookingOK` assigns an existing customer to a different seat. Provide any additional schema(ta) in order to extend the interface to include a robust operation `ModifyBooking`.

The extended interface will now include operations

- `GetNumberOfCustomers`, and
- `ModifyBooking`.

Solution:

The main functionality for class **Booking** together with the success and error schemata are given below:

<i>Booking</i>
$\uparrow (Book, Cancel)$
$booked : \mathbb{P} Person$ $booking : Person \leftrightarrow SeatType$ $capacity : \mathbb{N}$ $count : \mathbb{N}$
$booked = dom\ booking$ $capacity > 0$ $count \geq 0$
<i>INIT</i>
$booked = \emptyset$ $capacity = 100$ $count = 0$
<i>BookOK</i>
$\Delta(booking, count)$ $customer? : Person$ $seat? : SeatType$
$customer? \notin booked$ $seat? \notin ran\ SeatType$ $count < capacity$ $booking' = booking \cup \{customer? \mapsto seat?\}$ $count' = count + 1$
<i>CancelOK</i>
$\Delta(booking, count)$ $customer? : Person$
$customer? \in booked$ $count > 0$ $booking' = \{customer?\} \triangleleft booking$ $count' = count - 1$
...

Booking/cont.

...

Success

$response! : Message$

$response! = 'ok'$

CustomerExists

$customer? : Person$

$response! : Message$

$customer? \in booked$

$response! = 'Customer\ already\ exists'$

CustomerUnknown

$customer? : Person$

$response! : Message$

$customer? \notin booked$

$response! = 'Customer\ unknown'$

SeatTaken

$seat? : SeatType$

$response! : Message$

$seat? \in ran\ booking$

$response! = 'Seat\ taken'$

TheaterFull

$response! : Message$

$count = capacity$

$response! = 'Theater\ full'$

TheaterEmpty

$response! : Message$

$count = 0$

$response! = 'Theater\ empty'$

$Book \triangleq (BookOK \wedge Success) \oplus CustomerExists \oplus (SeatTaken \vee TheaterFull)$

$Cancel \triangleq (CancelOK \wedge Success) \oplus (CustomerUnknown \vee TheaterEmpty)$

<i>Booking2</i>	
$\uparrow (Book, Cancel, GetNumberOfCustomers)$	
<i>Booking</i>	
<i>GetNumberOfCustomers</i>	
$result! : \mathbb{N}$	
$result! = count$	
<i>ModifyBookingOK</i>	
$\Delta(booking)$	
$customer? : Person$	
$seat? : SeatType$	
$customer? \in booked$	
$seat? \notin ran\ booking$	
$booking' = booking \oplus \{customer? \mapsto seat?\}$	
$ModifyBooking \hat{=} (ModifyBookingOK \wedge Success) \oplus (CustomerUnknown \vee SeatTaken)$	