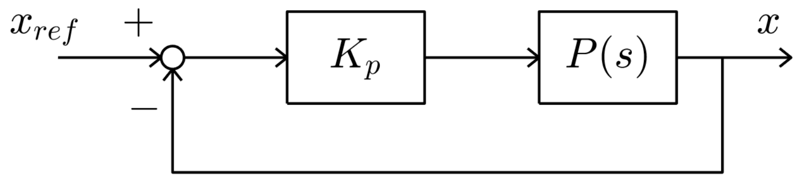
Homework 3

See materials and assigned reading from Lectures 4 and 5.

Problem 1. Block diagrams, feedback, and PD control.



$$P(s) = \frac{10}{2s^2 + 32s}$$

Figure 1: A feedback loop [for parts a and b].

Above is a block diagram of a plant, P(s), controlled by a Proportional (P) controller, K_p.

a) Show work and solve for K_p so that the resulting closed-loop poles have zeta=1, which is critically damped. (This is by definition when there are two, identical closed-loop poles.)

b) Show work and solve for K_p so that the closed-loop system has 75% overshoot.

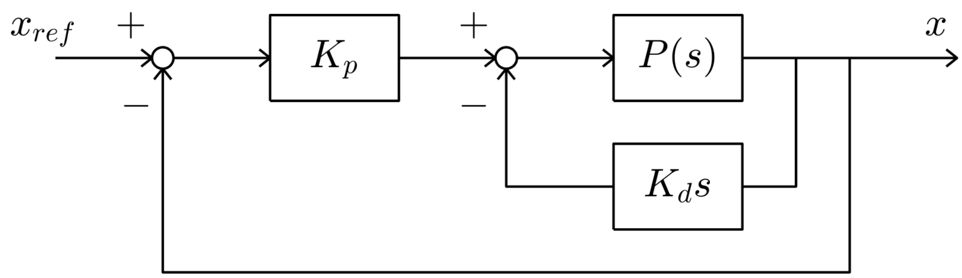


Figure 2: Two, nested feedback loops. For part c), close the inner loop first, then the outer one.

Above is a block diagram with Proportional-plus-Derivative (PD) control. P(s) is the same.

c) Show work to solve for both K_p and K_d so that the closed-loop system has poles with zeta=0.7 and a natural frequency of 100 (rad/sec). Also, what is the closed-loop transfer function, and what are the closed-loop poles for this case?

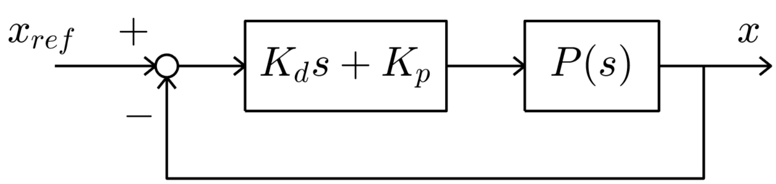
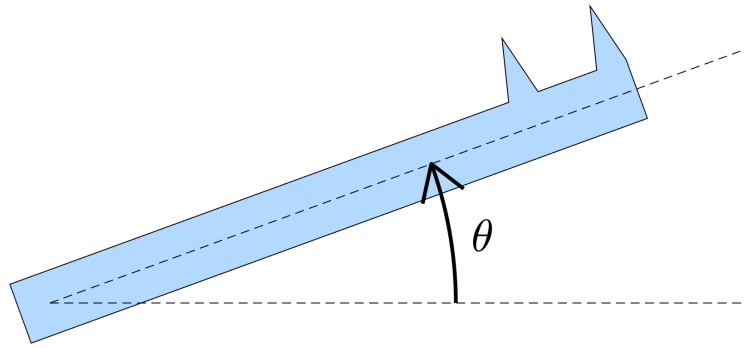


Figure 3: An alternate form for a PD controller [for part d].

d) Repeat all of part c) for the feedback loop shown in Figure 3. (Are K_p and K_d any different?)

e) On the SAME AXES, plot and label a closed-loop unit step response for the CLOSED-LOOP transfer function, $X(s)/X_{ref}(s)$, for part c) and for part d). Describe how they are the same and how they are different. (e.g., Percent overshoot? Time between peaks? Percent decay per cycle?)

Problem 2. Control law partitioning.



Above is a simple "robot arm", to be controlled by setting the torque of a motor (τ_m).

Assume the dynamics of this system are:

$$\tau_m = J_a\ddot{\theta} + B_a\dot{\theta} + \frac{1}{2}mgL^2 cos\theta + F_a sign(\dot{\theta})$$

a) Use "control law partitioning" to design a control law for τ_m (setting torque directly) so the closed-loop system has underdamped, second-order poles, with $\omega_n = 5$ (rad/sec) and $\zeta = 0.7$.

For part b), now assume we instead set torque indirectly, by applying a voltage, V_{in}, to the motor.

$$\tau_m = K_T i_m = K_t \frac{(V_{in} - K_T\dot{\theta})}{R}$$

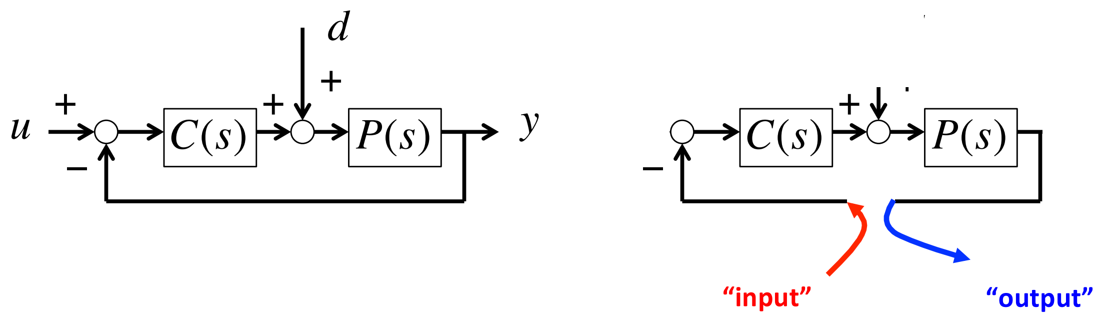
where K_T is the motor torque constant, i_m is the current through the motor, and we model the electrical impedance as a resistance, R.

b) Use "control law partitioning" to design a control law for V_{in} to give the closed-loop system dynamics an under-damped, second-order behavior, with $\omega_n = 5$ (rad/sec) and $\zeta = 0.7$.

Problem 3. Loop-shaping. See notes from Lecture 5.

Lecture 5 (and 6) covered a lot of material, but the basic take-away idea is simply that we can predict closed-loop dynamics of a system by looking at a particular "open loop" Bode plot. Also, tuning control (C(s)) to change that Bode plot will in turn change the closed-loop system.

This approach, to design a controller, C(s), for a plant, P(s), based on "reshaping" a particular open-loop Bode plot is called "loop shaping". What it turns out we care about is the frequency response of a signal that makes one full trip through the feedback "loop", which is why this is called "loop shaping".



At left above, assume u is a desired input, d is a disturbance input, and y is the output. The closed-loop poles of the system do not depend on our choice of particular input or output; they depend only on the frequency response of the feedback loop. [For example, the closed-loop transfer function Y/U = CP/(CP+1), and Y/D = P/(CP+1) – and both have the same poles.]

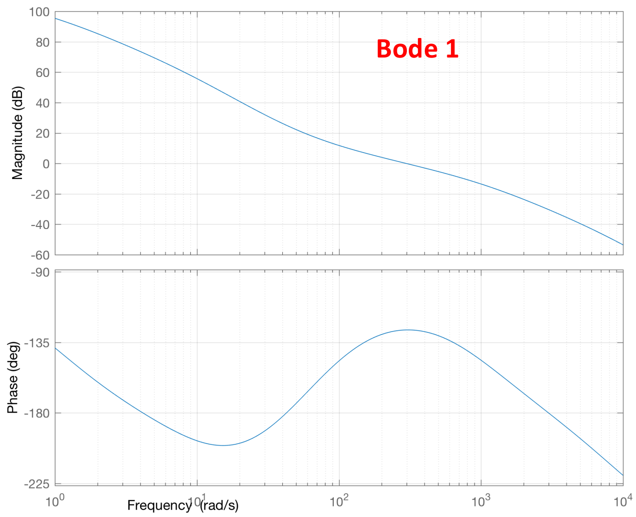
What happens to a signal in making a full trip through the feedback loop is referred to as the "loop transmission". Imagine cutting the feedback loop anywhere and injecting an **"input"** sine wave. What are the magnitude and phase of the **"output"** sine wave, after it travels through the loop? A Bode plot of **-1*C(s)*P(s)** gives the answer. Why is "-1" here? When the signal enters a summing junction, it is multiplied either by +1 (at a + sign) or -1. We have negative feedback, so there is a -1. Since we almost ALWAYS care about systems with negative feedback, control engineers tend to create Bode plots of the negative of the loop transmission, or "–LT".

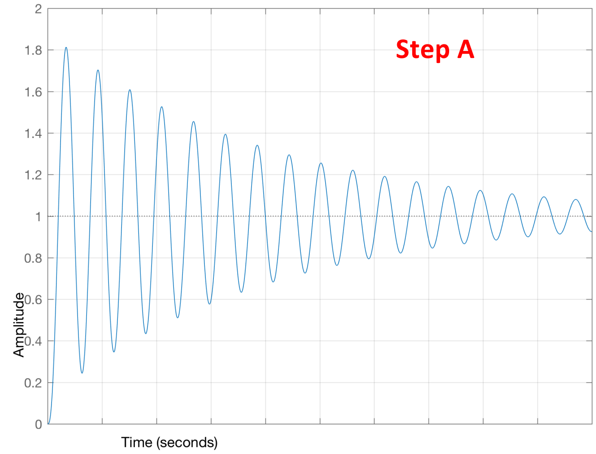
On the next page are Bode plots of the (open-loop) negative loop transmission, C(s)*P(s), for three systems. By analyzing the open-loop Bode plots for "C(s)*P(s)", you can predict damping ratio and natural frequency of the resulting closed-loop system, Y(s)/U(s).

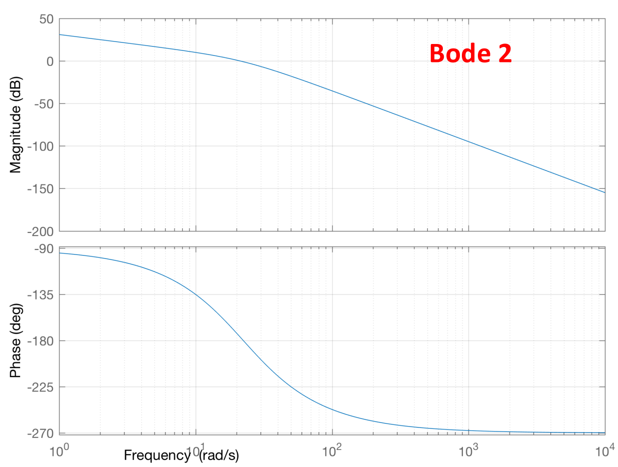
(a) For each Bode plot (next page), estimate the crossover frequency, ω_{co}, and phase margin, ϕ_m.

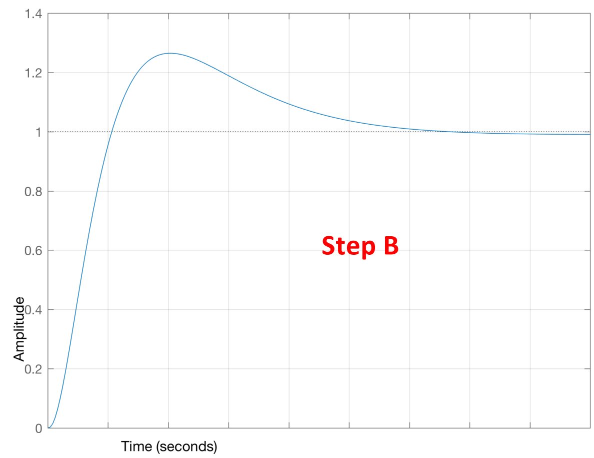
(b) More phase margin generally yields more damping (and less overshoot) in the resulting closed-loop system: CP/(CP+1), with $\zeta \approx 100 \cdot \phi_m$. Match each open-loop Bode plot (1, 2, and 3) with the corresponding closed-loop step response at right (A, B, or C). Explain your matches, briefly.

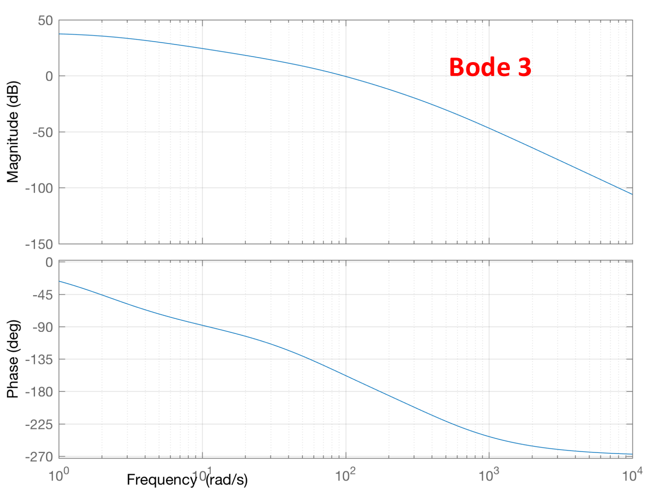
(c) Also recall that the crossover frequency is typically close to the natural frequency of the closed-loop system (which is also closed to the damped natural frequency), $\omega_{co} \approx \omega_n \approx \omega_d$. Using this, estimate time, t_{peak}, when each step response is at its maximum.

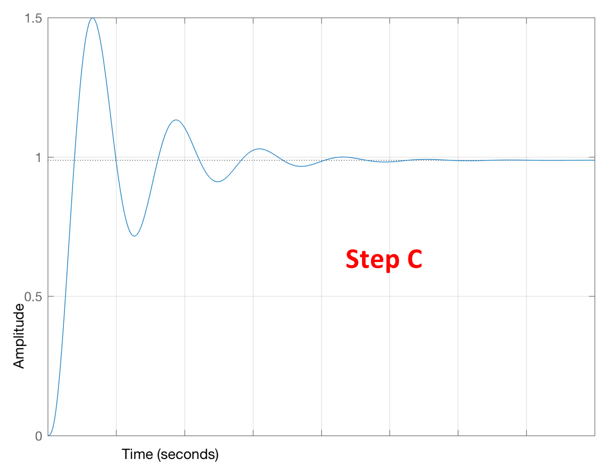

Bode 1
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step A
Amplitude
Time (seconds)


Bode 2
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step B
Amplitude
Time (seconds)


Bode 3
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step C
Amplitude
Time (seconds)