# HW1

Tuesday, October 8, 2019 8:37 AM

2. a) $\xi = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$\delta = \frac{1}{4} \ln\left(\frac{X_0}{X_4}\right) = \frac{1}{4} \ln\left(\frac{3-0}{3-0.5}\right) = \frac{1}{4} \ln\left(\frac{3}{2.5}\right) = 0.04558$

$\xi = \dfrac{0.04558}{\sqrt{4\pi^2 + 0.04558^2}} = \boxed{0.007254} = \zeta$

$w_n = \dfrac{w_d}{\sqrt{1-\xi^2}}$

$w_d = \dfrac{2\pi}{T_d} = \dfrac{2\pi}{0.25} = 8\pi \text{ rad/s}$

$w_n = \dfrac{8\pi}{\sqrt{1-(0.007254)^2}} = \boxed{25.1334 \text{ rad/s} = W_n}$

$H(s) = K \dfrac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$ $\quad K = 3$, $w_n = 25.1334$, $\xi = 0.007254$

$\boxed{H(s) = 3 \dfrac{(25.1334)^2}{s^2 + 2(0.007254)(25.1334)s + (25.1334)^2}}$

b) $\zeta = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$\delta = \frac{1}{n} \ln\left(\frac{X_0}{X_n}\right) = \frac{1}{2} \ln\left(\frac{10}{1}\right) = 1.1513$

$\xi = \dfrac{1.1513}{\sqrt{4\pi^2 + (1.1513)^2}} = \boxed{0.1802}$

$w_n = \dfrac{W_d}{\sqrt{1-\xi^2}}$ $\quad W_d = \dfrac{2\pi}{T_d} = \dfrac{2\pi}{0.5} = 4\pi$

$$w_n = \frac{4\pi}{\sqrt{1-(0.1802)^2}} = \boxed{12.7756 \text{ rad/s}}$$

$$H(s) = K \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

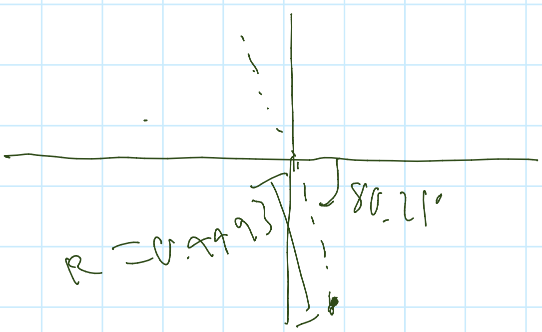
$K = 10$
$w_n = 12.7756$
$\zeta = 0.1802$

$$\boxed{H(s) = 10\left(\frac{(12.7756)^2}{s^2 + 2(0.1802)(12.7756)s + (12.7756)^2}\right)}$$

c) $e^{-0.8-1.4j}$

$R = e^{-0.8} = 0.4493$

angle $= -1.4$ radians $\approx -80.21°$




d) $s_1 = a + wj$

$w = \frac{2.5 \cdot 2\pi}{10} = \frac{\pi}{2}$

$R = 1 \to 0.2$ $\quad R = 0.5$ where $t = 4$ $\quad e^{4a} = 0.5$ $\quad a = -0.173$

$$\boxed{s_1 = -0.173 + \frac{\pi}{2}j}$$

$s_2 = a + wj$

$w = \frac{-2.5 \cdot 2\pi}{10} = \frac{-\pi}{2}$

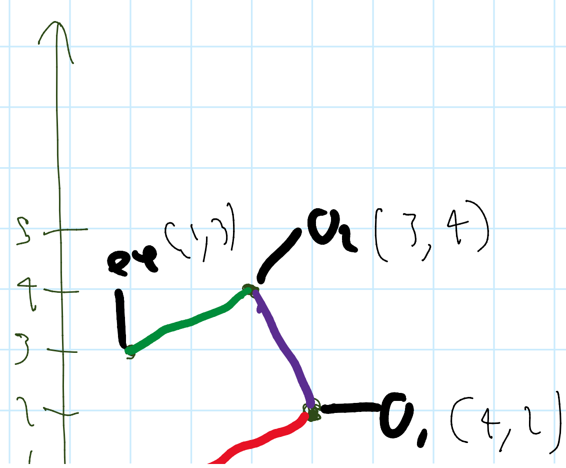
K is the same, $a = -0.173$

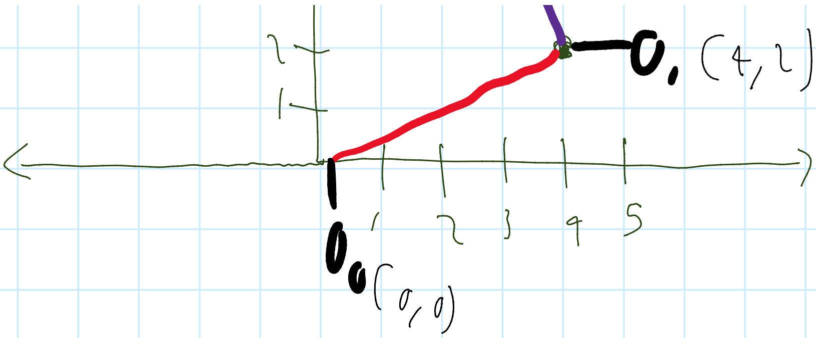
$\boxed{s_2 = -0.173 - \frac{\pi}{2} j}$

2a) $\xi = J\dot{q} = \begin{bmatrix} -3 & -1 & 1 \\ 1 & -3 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -3\dot{\theta}_1 - \dot{\theta}_2 + \dot{\theta}_3 \\ \dot{\theta}_1 - 3\dot{\theta}_2 + 2\dot{\theta}_3 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$

$y_{diff} = [3, 1, -1]$

$x_{diff} = [1, -3, -2]$



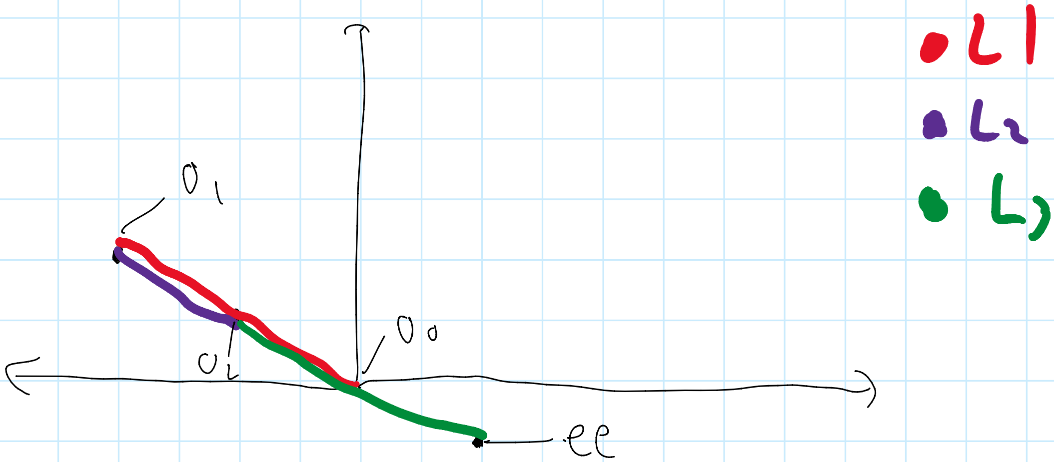



b) $\boxed{J_1 \text{ is not Singular}}$

$$\left[\begin{array}{ccc|c} 1 & 5 & 0 & 3 \\ 0 & -1 & 2 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow \boxed{\dot{q} = \begin{bmatrix} -2.7143 \\ 1.1429 \\ 1.5714 \end{bmatrix}}$$

$\boxed{J_2 \text{ is singular}}$

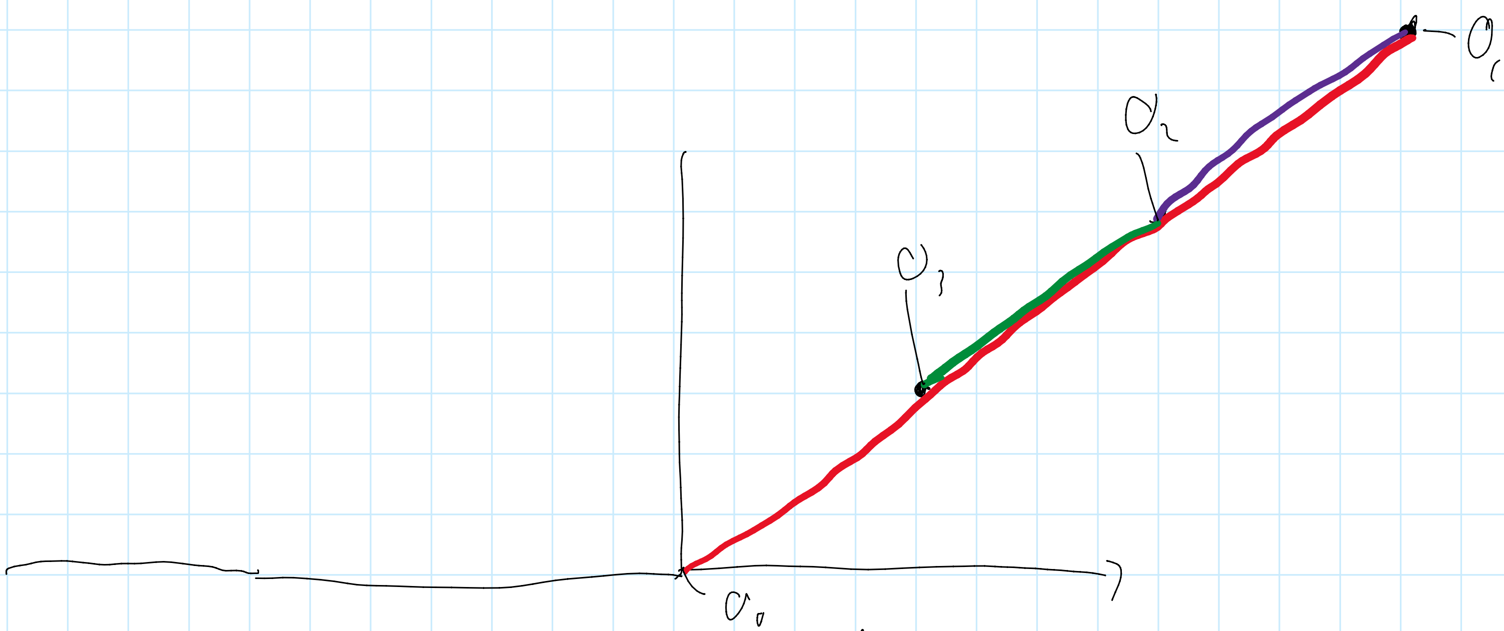
$\dot{y}$ is always going to be $2\dot{x}$




The end effector is not at the edge of the reachable work space, but because all links are lined up in the same direction, any angular change will only move the e.e. along the $y = 2x - 5$ line.

$$J_3 = \begin{bmatrix} -3 & 6 & 3 \\ 4 & -8 & -9 \\ 1 & 1 & 1 \end{bmatrix}$$

$\boxed{J_3 \text{ is singular}}$




The end effector is not at the edge of the reachable work space, but because all links are lined up in the same direction, any angular change will only move the e.e. along a slope of $-\frac{4}{3}$ from the e.e.

$$J_4 = \begin{bmatrix} 4 & 2 & 5 \\ 5 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$\boxed{\text{Not singular}}$

$$\left[\begin{array}{ccc|c} 4 & 2 & 5 & 3 \\ 5 & 2 & 4 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow \boxed{a = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}$$

2c) Because angles are absolute, we can treat each piece of the Jacobian as if it's affecting the next point, not the e.e.

$$J = \begin{bmatrix} 1 & -3 & -2 \\ 2 & 4 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

2d) RWS: $r_{min} = 0$ $\quad r_{max} = 11\,m$

DWS: $r_{min} = 0$ $\quad r_{max} = 9\,m$

2e) R.W.S: $L_3 = 0.7$ $\quad r_{min} = 0.3$ $\quad r_{max} = 4 + 0.7 = 4.7$

$$L_1 + L_2 = 4$$
$$L_1 - L_2 = 1$$
$$\text{or } L_2 - L_1 = 1$$

$\boxed{L_1 = 2.5\,m \quad L_2 = 1.5\,m}$

This solution is not unique. $L_2 = 2.5m$, $L_1 = 1.5m$ would also work.