Structural measures for multiplex networks

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Many real-world systems consist of simple units connected through a variety of different relationships. For instance, the same set of individuals in a social system can be connected through friendship, collaboration, kinship, communication, commercial and co-location relationships, just to name some of them, while in complex multimodal transportation systems, which are typical of large metropolitan areas, a set of locations might be reached in several different ways, e.g. using bus, underground, suburban rail, riverboat networks and the like. In these systems, each kind of interaction has associated a peculiar relevance, importance, cost, distance or meaning, so that such systems can be represented as multi-layer networks, or *multiplexes*, where each node appears in all layers and each layer includes all the edges of the same kind. In this work we propose several basic measures to characterise the structure of multiplexes, including their degree distributions, edge overlap, node clustering, reachability and centrality. All the proposed measures are tested and validated on a genuinely multiplex real-world social data set, the Top Noordin Terrorist Network, which includes detailed information about mutual trust, common operations, exchanged communications and business involvement of 78 Indonesian terrorists. We quantify the importance of each node in the network by taking into account either his total number of connections and his participation in the different activities, distinguishing individuals focused on given activities from multi-tasking individuals. We show that one of the four layers, namely the trust layer, acts as a driver for the others, with the existence of trust between two individuals fostering the formation of links in other layers. This result can be explained in terms of social reinforcement, and reveals important details about the overall dynamics of edge formation and strengthening in the multiplex. Since in multiplex networks transitivity may arise by the interaction of different layers, we also introduce and study multi-layer clustering showing the existence of a highly non-trivial triadic organisation across the different levels of the system.