Skewness Managed Portfolios

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Abstract

Due to positive skewness in the distribution of monthly stock returns, a few stocks play a disproportionally outsized role in the performance of factors. Because skewed stocks can end up in either the long or short leg of the portfolio, their impact depends on how skewness is related to the characteristic used to create the anomaly. Anomalies that long skewed stocks benefit, while those short lose. In a sample of core anomaly strategies, a skewness managed strategy that seeks skewness in the long leg but avoids it in the short leg improves the average return by five to ten percent per year, while limiting downside risk as measured by strategy skewness.

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1 Introduction

The distribution of monthly stock returns exhibits a positive skewness which results in disproportionately few stocks playing a key role in the equity premium. I build portfolios that use firms' expected skewness to target these potential high performers in the long leg and avoid them in the short leg of anomaly strategies. This skewness management strategy greatly improves the performance of anomaly strategies based on book-to-market (b/m), size, momentum, investment, return on equity (ROE), and operating profitability (op).

I motivate the analysis by demonstrating the impact of positive outliers on these six fundamental anomaly strategies. I isolate the impact by reconstructing these anomalies from a monthly return distribution where the right tail has been winsorized. Limiting the best performing observations in the sample either hurts or helps the performance of an anomaly based on whether these positive outliers tend to end up on the long or short leg. Small, value stocks tend to be positive outliers, thus capping the sample *reduces* the performance of anomalies based on size and b/m. Low profitability stocks with poor past performance also tend to be positive outliers, thus capping the sample *improves* the performance of anomalies based on ROE, operating profitability (OP), and momentum.

Predicting outliers is difficult. To target these potential high performers, I create a measure for a firm's expected skewness using its characteristics. I then employ this measure to separate stocks based on their potential as outliers. The measure benefits from two features of anomaly construction. First is the long-short structure. Enhancing portfolios based on skewness means seeking skewness in the long leg and avoiding it in the short leg. This long-short framework enhances the impact of the expected skewness measure by applying its predictive power in two directions simultaneously. The second feature is the relation between return skewness and firm characteristics. This relation means skewed stocks are concentrated in certain legs of each anomaly. Thus, the firm skewness measure only needs to refine the sort on skewness already occurring when stocks are sorted on the anomaly characteristic.

My skewness management strategy takes advantage of these features by sequentially sorting anomaly portfolios by expected skewness, thus splitting portfolios into high and low expected skewness samples. The final strategy buys the long leg of an anomaly in the high expected skewness sample and the short leg in the low expected skewness sample. This simple strategy improves the average return by five to ten percent per year and Sharpe ratios between 0.10 and 0.46. The strategy also enhances portfolio skewness and Sortino ratios and produces positive and significant alphas against the original anomalies and the Carhart 4-factor model. [1] As predicted, the improvements are concentrated in the leg of the anomaly strategy most impacted by outliers (skewed stocks).

One concern is that expected skewness is simply a return predictor that contains some unmeasured risk, and indeed, a long short strategy based on expected skewness deciles does show an annual return of approximately 8.83%. I address this concern with two points. First, this concern should only arise when looking at the skewness management strategy in which the long and short legs are taken from different expected skewness buckets. I find that the effect exists as anticipated *within* the high and low expected skewness buckets. Anomalies in which skewness dominates the long leg perform better in the high expected skewness sample, while anomalies in which skewness dominates the short leg perform better in the low expected skewness sample. Second, as a robustness I run spanning regressions between the universal strategy and a long-short strategy based on the expected skewness measure. I find that variation in my skewness management strategy is not fully explained by the expected skewness portfolio. This finding generally holds to the inclusion of the original anomaly and Carhart 4-factors. Alphas range from 1.27 to 6.71 percent per year in the most strict specification.

As further evidence, I also present an alternate skewness management strategy that uses the average expected skewness of stocks in the portfolio to scale exposure to the anomaly return. This approach is similar to the method used by Moreira and Muir (2017) for managing volatility. For anomalies in which skewness has the greatest impact in the long leg, I target skewness by *increasing*

[1]The Sortino ratio is defined as the ratio of expected return to downside volatility.

exposure in periods when skewness of individual stocks in expected to be high on average. Conversely, anomalies in which skewness dominates the short leg, I set the scaling strategy such that an investor would *decrease* exposure in periods when skewness is expected to be high. As with the main skewness management strategy, each strategy improves the overall performance, though the improvements in performance for strategies based on investment and momentum are primarily in the form of downside risk. The scaling version of skewness management increases average returns by one to ten percent per year and Sharpe ratios by 0.02 to 0.5.

Finally, my choice to use realized skewness as my proxy for a stock's overall skewness is inherently an empirical one. To test how important this specific measure is for my findings I replicate the core results using firms' idiosyncratic skewness in place of realized skewness. I find that idiosyncratic skewness produces very similar results. In certain cases, idiosyncratic skewness may even act as a better proxy for the effect I aim to capture with this strategy. Managing skewness with a measure based on idiosyncratic skewness improves average returns by five to eleven percent per year and Sharpe ratios by 0.15 to 0.43.

The role of skewness in stock returns is a topic that has been discussed since Markowitz (1952) first derived the mean-variance frontier by assuming investors should not prefer the third moment. Since that time other work has expanded the CAPM framework to include stocks' contributions to skewness as an additional risk factor (Rubinstein 1973, Kraus and Litzenberger 1976). Stocks with higher coskewness (those more exposed to systematic skewness) command a return premium (Harvey and Siddique 2000). Recent work shows this effect to persist in the modern data (Harvey and Siddique 2022). Also, coskewness is a potential explanation for the alpha generated by low risk anomalies (Schneider, Wagner, Zechner 2020).

Skewness in individual stocks also plays a role in the cross-section when investors have a skewness preference (Mitton and Vorkink 2007, Brunnermeier et al., 2007, Barberis and Huang 2008). This conclusion is supported by the empirical evidence, which shows that stocks with higher idiosyncratic skewness command a negative return premium. This negative premium has been shown when skewness is measured over different time horizons using different methods (Boyer, Mitton, and Vorkink 2010, Ameya et al 2015, Bali et al 2011, Conrad et al. 2014). More recent work applies these insights to explain anomaly premiums, in which skewness preferences lead to overpricing of the short leg of strategies (Kumar, Motahari, and Taffler 2019) or idiosyncratic skewness being a proxy for growth options (Bali et al 2019). These final two papers focus on the equilibrium effect in which investors with skewness preferences bid up prices resulting in lower subsequent returns. My findings differ by focusing on the importance in realized outliers and their disproportionate role in the performance of a given portfolio.

This paper is also related to a growing literature looking at managing volatility in portfolios. Barrosa and Santa-Clara (2015) and Daniel and Moskowitz (2016) show that a strategy that uses portfolio volatility to scale exposure to momentum can help combat momentum crashes. My findings demonstrate that these momentum crashes are also related to positive outliers. Daniel and Moskowitz highlight how momentum crashes are related to the fact that stocks rebounding after a market downturn tend to end up in the short leg of momentum. I show that these rebounding stocks tend to be those outliers that drive the effect I discuss in this paper, thus limiting positive skewness helps to alleviate some of the downside risk of the momentum strategy.

Moreira and Muir (2017) use past volatility to improve the performance of several other common factor strategies, including momentum, and are able to generate alpha over the original factors. Recent work by Barrosa and Detzel (2021) shows that implementing these scaling strategies can incur extra trading costs that mitigate the benefits and propose a set of solutions that decrease the potential costs. Cederberg et al (2020) highlight that alpha may not be the best indicator of whether volatility management improves outcomes, instead the authors focus on how these strategies often fail to create a meaningful increase in Sharpe ratios.

This paper is inspired by Bessembinder (2018) who demonstrates the impact of positive outliers on the equity premium and portfolios of public equities more generally. This paper differs by showing how disproportionally few stocks can play an outsized role in anomaly strategies. I map how the effect plays out in several common anomalies and in long-short strategies more broadly. Finally, I produce a skewness management strategy that combines an outlier measure with several features of anomaly construction to improve the overall performance and downside risk profile of common anomalies.

The paper proceeds as follows. Section 2 motivates the question by showing the impact of outliers through winsorization. Section 3 outlines the model used to predicted individual stock skewness. Section 4 outlines the construction of the skewness management strategy and reports the results. Section 5 provides robustness checks. Section 6 concludes.

2 Motivation

In this section I show the role of the extreme returns in anomaly premiums by calculating the performance of common anomalies in a winsorized sample of stock returns.

2.1 Data

I draw my sample from the The Center for Research in Security Prices (CRSP) universe of common stocks, focusing on the period from July 1963 to December 2021. Anomaly strategies are constructed using data from Compustat and CRSP, with details provided in Appendix A. In the next section I look at percentiles to determine which observations are part of the "tail" of the distribution. To calculate these percentiles I use the historical distribution each month, which includes all observations from January 1926 up to each month t to minimize look-ahead bias. All the insights are robust to looking at percentiles in the full distribution.

2.2 The Role of Skewness

How skewed is the return distribution? Panel A of Figure 1 shows a histogram of monthly returns from July 1963 to December 2021. The long tail is the most striking visual feature of this graph. It accounts for a part of the right-skewed asymmetry that amounts to a skewness of 7.64 and a kurtosis of 414. These observations do not come evenly in each month, rather these extreme observations are clustered in time. Panel B of Figure 1 plots the percent of stocks above a certain return cutoff (26.6 percent, approximately the top 5% of observations). Overall, These outliers appear to be at least partially related to the rebounds that occur following downturns in the market. The most recent peaks occur in January 2001, April 2009, and April 2020 aligning with the tech bubble, the great financial crisis, and the covid crisis, respectively.

The prior literature finds that this positive skewness in the distribution of stock returns is a driver of performance, that is, disproportionately few high performing observations are key for the overall performance of a portfolio of stocks. In the case of anomalies, the long-short structure means that these important observations could either work for (long) or against (short) a potential investor. Therefore, the importance of these observations on the final performance of a characteristic sorted long-short strategy is a function of how outliers are related to the underlying characteristic.

Strategies in which the long side of the portfolio tend to have shares with a more skewed return distributions will benefit as a result. Strategies in which the short side of the portfolio tend to have shares with a more skewed return distributions will suffer as a result. It is also possible that a given characteristic underlying a strategy is unrelated to the skewness in the individual stock's distributions and thus unrelated to the ultimate realization of observations on the tail of the distribution. Therefore, I propose a simple test to explore the impact of these outliers on all considered anomalies. In this test, I recreate the anomaly portfolios from a sample of stocks in which returns are capped (ex-post). This capped sample should reflect an investment opportunity set with a limited positive tail and thus show what strategies look like absent these outliers.

The sample cap is selected from the distribution of monthly stock returns for each month based on historical data. Each month, the 90th, 95th, and 99th percentiles of the monthly return distribution are calculated using all prior return data. These percentiles are equal to approximately 15.5, 23.9, and 51.5 percent (186.5, 286.4, and 618.4 percent annualized).

I use percentiles calculated from an extending sample of common stock returns from January 1926 through December 2021. That is, each return month is assigned percentiles based on the distribution of returns from January 1926 through the prior month. These percentiles are used to winsorize the returns of samples ranked by anomaly characteristic. Therefore, the sample is winsorized after individual stocks are sorted into portfolios, but prior to the calculation of the strategy return.

Table 1 displays the outcomes from each anomaly strategy winsorized as described. The impact of winsorizing depends on which portfolio (long or short) tends to be most related to outlier return observations. What I observe is that anomalies based on size and book-to-market tend to be negatively impacted by limiting the upside on the long side of the portfolio, indicating that small and growth stocks tend to be more skewed than large, value stocks. On the other side, anomalies based on profitability and momentum tend to be positively affected by capping the best performing monthly return observations. This finding is consistent with stocks that have low profitability and poor past performance having a more skewed distribution. Finally, low investment stocks do not appear to come from a distribution that is more skewed than high investment stocks.

I draw two conclusions from this analysis to motivate my skewness enhanced strategy. First, it appears that certain anomaly strategies are harmed by skewness while others are helped depending on which leg of the anomaly tends to contain the more skewed stocks. Thus, strategies would generally benefit from being calculated in a high or low skewed sample depending on which leg is most effected by skewness. Second, I infer from these results that while the overall effect tends to come from one leg or the other, an investor will generally prefer drawing a short portfolio from the less skewed distribution and the long portfolio from the more skewed distribution. These insights motivate my skewness management strategy outlined in the following sections.

3 Predicting Skewness

I use a model of realized skewness to proxy for stocks that are more likely to exhibit extreme returns in the coming month, I call this measure expected skewness. The model uses available firm characteristics to predict realized skewness in daily returns each month. A firm i's realized skewness is calculated from the daily returns in month t as

$$rs_{i,t} = \frac{1}{N(t)} \frac{\sum_{d=1}^{N(t)} r_{i,d}^3}{rv_t^3}, \tag{1}$$

$$rv_{i,t} = \left(\frac{1}{N(t)} \sum_{d=1}^{N(t)} r_{i,d}^2 \right)^{1/2}. \tag{2}$$

In which d is from the set of trading days in month t, r is daily return of stock i on day d, N(t) is the total number of trading days in month t [2]

Unsurprisingly, this contemporaneous measure of skewness captures a large majority of tail events. 97%, of monthly return observations in the positive tail of the distribution lie above median realized skewness. [3] Conversely, approximately 97 percent of those stocks that lie in the left tail fall below median rs. This observation is visually represented in Figure 2 in which I overlay histograms from the sample of returns split out into high, medium, and low realized skewness. In this figure you can see how high realized skewness tends to dominate the right leg of the distribution, while the low expected sample dominates the left side.

Figure 3 plots the 10th, 30th, 50th, 70th, and 90th percentiles of realized skewness. I remove noise by plotting a 3-year rolling average. Realized skewness, though noisy month to month, is relatively stable over time. The most significant movement in the measure is that the highest and

[2] $N(t)$ is the number of trading days minus an adjustment for degrees of freedom of one for volatility and two for skewness.

[3] The positive tail is defined as those observations above the historic 90th percentile.

lowest percentiles disperse starting in 1970 and converge starting in the 1980's up until 2000.

The second part of the expected skewness measure is using firm characteristics to model realized skewness. Setting up an ex-ante measure of realized skewness presents a challenge. With measures of volatility, it is common to use a lagged value to proxy for future outcomes. With skewness, there is very little persistence. Thus, any ex-ante measure of future realized skewness requires a more complex method. I use the model of idiosyncratic skewness presented in Boyer, Mitton, and Vorkink (2012) as an example. To model expected skewness for stock i in month $t+1$, I first estimate separate, cross-sectional regressions for realized skewness at the end of month t.

$$rs_{i,t} = \beta_t + \lambda_t' X_{i,t-1} + \epsilon_{i,t} \tag{3}$$

in which X is a vector of firm characteristics available at the end of month $t-1$, and skewness is measured in month t. Following Boyer, Mitton, and Vorkink (2010) I run the model separately each month, which allows the relation between firm variables and skewness to vary over time. I use the coefficients from equation (3) to generate a measure of expected skewness for month $t+1$ based on characteristics available at the end of month t.

$$E[rs_{i,t+1}] = \beta_t + \lambda_t' X_{i,t}. \tag{4}$$

I choose the firm specific variables, X, based on the prior literature. The variables include skewness (RS) and volatility (RV) measured as described in equations (1) and (2), momentum measured as the return over the last year less the most recent month, prior month return, turnover, and indicators for size, Fama and French 48 industry, and membership on the Nasdaq. Momentum and prior month return are motivated by findings in Chen, Hong, and Stein (2001) and Boyer, Mitton, and Vorkink (2010) that past returns are negatively correlated with forecasted skewness. Table 2 reports the summary statistics. Panel A reports descriptive statistics and Panel B reports correlations between continuous variables.

As with the prior literature I focus on a parsimonious specification that maximizes the available observations. The final version of the model includes the above variables less turnover, which is only available for Nasdaq stocks after 1988. Prior works have also found book-to-market to be a valuable predictor (Chen, Hong, and Stein (2001)). Again, I omit this variable in favor of minimizing the number of observations lost to missing data so that the recalculated anomaly strategies have as similar a sample as possible to the original.

Table 3 reports the results from the cross-sectional regression models, with each row representing a different combination of the firms' characteristics. The reported coefficients are the average of the all the coefficients from each monthly, cross-sectional model. To represent the importance of each characteristic I report the percentage of months in which the estimated coefficents are significant at the 10% level that have the same sign as the average coefficient ($\%sig$). Based on this measure, realized volatility and prior month return appear to be the most consistent in predicting realized skewness in the upcoming month. As with prior findings, the impact of a standard deviation increase in volatility is around twice as large as that for a standard deviation increase in skewness when predicting future skewness. A standard deviation increase in prior return leads to an increase in predicted skewness that is not quite twice as large as volatility. I use model 7 through the remainder of the paper. I note that the R-squared of 4.8% is commensurate with other projections of firm skewness for shorter horizons.

4 Skewness Managed Portfolios

I use the expected skewness measure outlined in Section 3 to enhance the performance of anomaly portfolios. The primary skewness management strategy uses a second, sequential sort to split anomaly portfolios into high and low expected skewness stocks. The strategy buys the long leg of the anomaly in a high expected skewness sample and buys the short leg in the low expected skewness sample. This matches the intuition that an investor wants to seek positive outliers in a long strategy and avoid them in a short strategy. As an alternate, I put together a scaling strategy that uses next period expected skewness to determine exposure to an anomaly depending on if that anomaly performs well or poorly in a more skewed return sample.

4.1 Sequential Sorts on Expected Skewness

I construct each anomaly based on the description provided in Appendix A. I then separately estimate stock level expected skewness using the full sample of common stocks in CRSP. The full sample I consider is the intersection of stocks ranked by their anomaly characteristic and those with an available expected skewness measure. There is a small loss of observations from this process when compared to the original anomaly sample, though with only a limited difference in overall performance.

In each month, the 10 characteristic portfolios are then split into terciles (30th, 70th) based on expected skewness which results in 30 portfolios double sorted on the strategy characteristic and expected skewness. Each portfolio's return is calculated as the value-weighted return based on the lagged market cap. The average returns of each of the 30 portfolios can be found in Table 4. Part A looks at anomaly strategies in which skewness is concentrated in the long leg, while part B looks at anomalies in which skewness is concentrated in the short leg.

Table 4 shows how calculating anomalies in different expected skewness samples leads to very different average returns that follow the predictions from the motivation in Section 2. In Panel A, b/m and size based anomalies perform better in the high expected skewness sample as shown in the final row in each table. The column on the far right of each table represents the difference between each anomaly portfolios between the high and low sample. The difference column shows that the improvement comes primarily from the long leg, that is, the leg most impacted by skewness. Investment presents an interesting case, in that the effect is weaker that others in the winorizing exercise from Section 2, but returns change with the sort on expected skewness. Panel B shows that anomalies based on momentum and profitability perform best in the low expected skewness sample. Again, the difference columns demonstrates that the largest impact on returns comes from the short leg of the strategy, as predicted. Momentum tends to be the weakest of the strategies that are more skewed in the short leg.

To address anomaly downside risk, Table 5 reports skewness in the return of the 30 portfolios. Again, Panel A focuses on anomalies in which skewed stocks tend to occupy the long leg, and Panel B focus on anomalies in which skewed stocks tend to occupy the short leg. The portfolio that is more impacted by skewness is placed at the top of the table. The results show that the more affected leg of the strategy has large changes in its return skewness between the high and low expected skewness sample. Commensurate with predictions, moving from high to the low expected skewness sample decreases the skewness in the more effected leg (long or short). Table 6 shows how this translates to improvement in skewness for the overall strategy.

The findings confirm that sorting on expected skewness has the greatest impact on the most skewed anomaly portfolio. It also shows that strategies constructed in the ideal skewness sample generally outperform their originals in terms of returns and measures of portfolio skewness. The primary skewness management strategy is calculated by taking the long leg of an anomaly from the high expected skewness sample, and the short leg from the low expected skewness sample. This arrangement takes advantage of the insight that you would prefer to draw a long portfolio from a more skewed sample and the short portfolio from a less skewed sample. Table 6 reports the performance of the skewness management strategy. Panel A presents the full sample version of the anomaly, calculated using observations with non-missing expected skewness. Panel B presents the results from the skewness management strategy, and Panel C is the difference between the two.

Managing skewness improves the performance of all considered anomaly strategies. This includes improving anomaly average returns, Sharpe ratios, and the downside risk profile in the forms of skewness and Sortino ratios. The Sortino ratio is the expected return divided by the downside volatility, that is, volatility calculated using only months in which the return is negative. Increases in average return are large, ranging from 5% to 10% annually. Sharpe ratios improve between 18% (momentum) and 332% (operating profitability). Improvements in skewness are almost completely positive, apart from operating profitability, which also has the largest return improvement. The combination of return improvements and increases in return skewness means that Sortino ratios increase more than their Sharpe counterparts, ranging from 28% to 338% indicating a favorable improvement in the trade-off between return and downside risk.

Table 7 reports alphas when the skewness managed version of an anomaly is compared to the original anomaly and a Carhart 4-factor model. All strategies produce significant, positive alphas in each of the three models. Panel A shows how the skewness managed strategy compares to the original anomaly. All strategies produce large alphas of five to eleven percent when compared to the original anomaly in the full sample. Panel B reports alphas when the skewness management strategy is compared to the Carhart 4-factor model. Again, all strategies produce alphas ranging from seven to sixteen percent. Panel C compares the skewness management strategy to the original anomaly and a Carhart 4-factor model. Again, the skewness management strategy produces positive and significant alphas for all six anomalies. Overall, I find evidence that skewness management results in broad performance improvements.

4.2 Skewness Scaling

The recent literature on volatility management strategies scale exposure to the returns of the original strategy by conditional volatility (Mueria and Muir 2017). In doing so, these strategies change risk exposure each month based on the information carried in current volatility. In this section I generate an analogous strategy for skewness using stock level expected skewness to control an investor's exposure to the original anomaly. Because each anomaly differs in whether you should seek or avoid skewness, I create two separate scalars. The first increases exposure with expected skewness and the second decreases exposure. This allows the investor to select a strategy that is appropriate for each anomaly.

I measure expected skewness as the average of individual stock expected skewness in the long and short portfolio of a given anomaly, with equal weights given to the long and short portfolio.

Using this value, I calculate two scalars.

$$\theta_{long} = percentile_t(E[Skew]_t)/0.5 \tag{5}$$

$$\theta_{short} = (percentile_t(E[Skew]_t) - 1)/(-0.5) \tag{6}$$

in which $percentile_t(E[Skew])$ represents the percentile of the average expected skewness for a given anomaly in month t. The percentile is calculated using only historical data. This percentile is then mapped onto a range of [0-2], with 0 representing no exposure to the anomaly return and 2 representing double exposure through a leveraged position. When expected skewness is at the historical median the scalar should be equal to 1, which represents a full investment in the original anomaly. Using one of these scalars, the skewness managed scalar strategy is calculated as

$$f_{t+1}^{skew} = \theta f_{t+1} \tag{7}$$

in which θ is selected depending on whether the impact of skewness is more important in the long or short leg of the strategy.

Table 8 reports the results for the scaling version of the skewness management strategy. Overall, the improvements in average returns and Sharpe ratios are smaller in magnitude, and improvements in overall skewness remain positive and of a similar magnitude. Panels A and B report the original and scaled strategies, respectively. Panel C reports the difference between the original and skewness managed strategy. Improvements in returns range from one to ten percent, and improvements in Sharpe ratios range from 0.02 to 0.45. Overall, momentum and investment have the smallest improvements overall.

5 Robustness

5.1 Replacing Realized Skewness with Idiosyncratic Skewness

For the main results I choose realized skewness in daily returns as my proxy for return skewness, here I recreate the skewness management strategy using idiosyncratic skewness as an alternate. Overall, using idiosyncratic skewness proves equally as effective, and in certain cases more effective at improving performance.

I define idiosyncratic skewness as

$$is_{i,t} = \frac{1}{N(t)} \frac{\sum_{d=1}^{N(t)} \epsilon_{i,d}^{3}}{iv_t^3}, \tag{8}$$

$$iv_{i,t} = \left(\frac{1}{N(t)} \sum_{d=1}^{N(t)} \epsilon_{i,d}^{2} \right)^{1/2}. \tag{9}$$

In which d comes from the set of trading days in month t, ϵ is the residual taken from regressing the daily return of stock i on the market factor, $N(t)$ is the total number of trading days in month t. [4]

I use an identical method for modeling expected skewness. I first estimate the cross-sectional regressions for idiosyncratic skewness at the end of month t.

$$is_{i,t} = \beta_t + \lambda_t' X_{i,t-1} + \epsilon_{i,t} \tag{10}$$

in which X is a vector of firm characteristics available at the end of month $t-1$, and skewness is measured in month t. Following Boyer, Mitton, and Vorkink (2010) I run the model separately each month. I use the coefficients to generate a measure of expected skewness for month $t+1$ based on characteristics available at the end of month t as in the following equation:

$$E[is_{i,t+1}] = \beta_t + \lambda_t' X_{i,t} \tag{11}$$

[4] $N(t)$ is the number of trading days minus an adjustment for degrees of freedom of one for volatility and two for skewness.

Table 9 summarizes the regression results. As with realized skewness, I find that volatility and prior month return are both consistent predictors of expected skewness. I find that idiosyncratic skewness is harder to explain, as shown by the lower adjusted R-squared.

Idiosyncratic skewness provides a very similar performance outcome to using realized skewness. The results from Table 10 indicate that a skewness management strategy based on idiosyncratic skewness increases performance of all considered anomalies. Panels A and B report the outcomes from the original and skewness managed strategies, respectively. Panel C reports the difference between the two strategies. Skewness Management based on idiosyncratic skewness produces overall improvements of a similar magnitude to the results using realized skewness, with some anomalies having a slightly better improvement in their performance (value, investment, and momentum), and others with smaller improvements (size and profitability). This also applies to downside risk. Managed portfolios using size, investment, and ROE have more muted gains or mild declines in skewness while value and momentum have larger improvements.

5.2 Validating Expected Skewness

In this section I aim to verify that my measure of expected skewness captures the intended effect. I address this in two layers. First, sorting on expected skewness should should have the greatest impact on the portfolio most affected by skewness. I look at the impact of sorting on the differences in portfolios' average returns and the differences for skewness in portfolio returns. Second, sorting on expected skewness should lead to differences in firms' realized skewness. That is, portfolios with high expected skewness should contain stocks that have higher realized skewness on average.

Table 4 shows that an additional sort on expected skewness creates a return spread between the high and low expected skewness samples. As predicted, this return spread is largest in the portfolio that is most impacted by skewness. The difference between the high and low sample is reported in the H-L column in each table. Panel A shows that portfolios based on B/M, size, and investment have the largest return spread in the long portfolio. Panel B shows that portfolios based on momentum and profitability have the largest return spread in the short portfolio.

The same intuition applies to skewness in portfolio returns. Table 5 reports the return skewness of each of the 30 portfolio for each anomaly. The difference in skewness between the high and low sample is reported in the H-L column. Again, Panel A shows that portfolios based on B/M, size, and investment have the largest difference in skewness in the long portfolio. Panel B shows that portfolios based on momentum and profitability have the largest difference in skewness in the short portfolio. Thus, both portfolio skewness and portfolio returns respond predictably based on the finding that skewness tends to dominate the long or short leg of anomaly strategies.

As an additional test, I look at how sorting on expected skewness impacts the average realized skewness in each of the 30 portfolios generated in my strategy. Table 11 reports the time series average of stocks' realized skewness in each of the 30 portfolios. As with portfolio returns and return skewness, the average realized skewness is consistently higher in the high expected skewness sample, and the difference seems to be largest in the leg of the strategy most affected by skewness. Panel A shows that portfolios based on B/M, size, and investment have the largest difference in realized skewness in the long portfolio. Panel B shows that portfolios based on momentum and profitability have the largest difference in realized skewness in the short portfolio.

5.3 Expected Skewness Predicts Returns

The task of anticipating positive skewness predicts returns. Firms with higher expected skewness tend to outperform those that have lower expected skewness. The average premium for a long-short strategy based on decile portfolios of expected skewness is 8.32%, with a positive and significant Carhart 4-factor alpha. This finding is consistent with the empirical fact that realized skewness is positively associated with monthly returns when measured in the same month, that is, stocks with higher monthly returns have more positively skewed daily returns. Nevertheless, there exists the possibility that the model of expected skewness I use introduces an unmeasured risk that accounts for the return improvement. I provide two points to address this potential explanation.

First, unmeasured risks should only be an issue in the main skewness management strategy in which the long and short legs are taken from different expected skewness bins. The performance of an anomaly within an expected skewness bin should avoid any potential unmeasured risks. Table 12 shows results in which anomalies are purchased within the high or low expected skewness sample depending on whether the long or short portfolio is more effected by skewness. Anomalies in which skewness has a greater effect on the the long leg (B/M and size) perform better when calculated in the high skewness sample. Anomalies in which the short leg is more affected (momentum, profitability) generally perform better in the low expected skewness sample. Table 12 shows that isolating anomalies to their optimum skewness sample improves returns, Sharpe ratios, and the downside risk profile as measured by skewness and Sortino ratios. Momentum is the exception, in which the positive impact is limited to the improvement in skewness.

Second, I calculate alphas for each skewness management strategy while controlling for the return predictability of expected skewness. As a control, I employ a long-short portfolio based on decile sorts of expected skewness. The portfolio is long high expected skewness stocks and short low expected skewness stocks. This portfolio is included alongside the original anomaly and the Carhart 4-factor model in a spanning test against the skewness managed portfolios. Table 13 reports the results. Panel A shows that all skewness managed anomalies generate alpha above what is predicted by a simple sort on expected skewness. Panels B and C show that including the original factor leaves all anomalies with positive alphas, though without statistical significance for investment and momentum.

5.4 Does realized skewness capture what we want?

In this section I look two questions. First, does realized skewness match the findings that skewness has different impacts of each anomaly? Second, how well would the skewness strategy perform if an investor could perfectly model skewness? To answer these two questions I recreate the skewness management strategy outlined in Section 3 using realized skewness as the second sort variable in place of expected skewness. Table 14 reports the results of each anomaly strategy in samples with high realized skewness, low realized skewness, and the skewness management strategy that combines the best of both.

Panels A, B, and C emphasize the different role skewness plays across anomalies. When Skewness in concentrated in the long portfolio (B/M, Size, Inv) stocks perform best in the high expected skewness sample. When skewness is concentrated in the short portfolio (MOM, ROE, OP), the strategy performs best in the low expected skewness sample. The different impact between the two samples validates the findings outlined in Section 2 in which positive outliers tend to be concentrated in either the long or short leg of the portfolio.

Panel E reports the performance of the skewness management strategy with the benefit of perfect information. The ability to perfectly predict the target skewness measure results in large performance gains both overall and in terms of downside risk. The incredible performance of these portfolios are of course the results of perfect hindsight, but show that the strategy has much room to grow as the model for individual stock skewness is improved.

6 Conclusions

This paper explores the role of skewness in monthly stock returns on anomaly strategies. I find these strategies are disproportionately affected by positive outliers, and that the direction of this effect is determined by which leg of the strategy, long or short, tends to be more impacted by positive outliers. This impact can be exploited to improve portfolio performance by targeting stocks with higher predicted skewness on the long leg and avoiding the same stocks on the short leg. This skewness management strategy improves the overall performance as measured by the Sharpe ratio, as well as measures of downside risk such as the Sortino ratio and strategy return skewness. Future work can further improve upon these strategies by improving the prediction of individual stock skewness though additional methods.

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Table 1: **Winsorized Performance**

This table shows the performance of anomalies calculated in samples in which the right tail of monthly returns has been winsorized at the 90th, 95th and 99th percentiles. Winsorization takes place after portofolio construction, but before performance calculations are made. The three panels show the average return premium, the Sharpe ratio, and the return skewness of each strategy. In each panel, the statistics from every sample is provided along with the difference between the original and the 90th percentile samples.

Sample	**B/M**	**Size**	**Asset Growth**	**Momentum**	**ROE**	**OP**
Return						
Original	5.18	3.19	4.32	13.65	7.95	2.08
win99	4.21	-1.12	3.89	15.70	9.80	3.66
win95	1.89	-11.30	3.39	19.99	14.90	8.82
win90	-0.44	-19.43	3.42	22.50	19.43	13.87
Difference	-5.62	-22.61	-0.90	8.84	11.48	11.79
Sharpe						
Original	0.32	0.19	0.38	0.55	0.44	0.14
win99	0.26	-0.07	0.35	0.67	0.57	0.25
win95	0.12	-0.83	0.32	0.98	0.94	0.64
win90	-0.03	-1.57	0.34	1.22	1.31	1.08
Difference	-0.35	-1.76	-0.05	0.66	0.87	0.94
Skewness						
Original	0.10	0.75	0.30	-1.41	0.14	0.15
win99	-0.05	0.24	0.29	-0.96	0.47	0.38
win95	-0.22	-0.16	0.38	-0.35	0.91	0.70
win90	-0.39	-0.29	0.48	-0.05	1.17	0.85
Difference	-0.49	-1.04	0.18	1.35	1.03	0.70

Table 2: **Summary Statistics**

The following table summarizes the main continuous variables used in creating expected skewness. In which rv is realized volatility and rs is realized skewness. Momentum is defined as the prior year return omitting the most recent month, and prior is the last month return. The sample includes all common stocks trading on the AMEX, Nasdaq, and NYSE from July 1963 - December 2021.

Panel A: Summary Stats							
	mean	**std**	**min**	**25%**	**50%**	**75%**	**max**
rv	0.03	0.03	0.00	0.01	0.02	0.04	4.16
rs	0.26	1.29	-4.91	-0.48	0.27	1.03	4.91
momentum	1.14	0.75	0.00	0.82	1.06	1.31	437.68
prior	0.01	0.17	-0.99	-0.06	0.00	0.07	24.00
Panel B: Correlation table							
	rv	**rs**	**momentum**				
rs	0.14						
momentum	-0.09	-0.01					
prior	0.16	0.55	0.00				

Table 3: **Predictive Regressions**

This table reports the time series average from the first regression in the predictive model. %sig is the percent of coefficients that are statistically significant at the 5% level (and of the same sign as the average coefficient). Sample is the collection of common stocks trading on the AMEX, NYSE, and Nasdaq. Momentum (mom) is the prior year return omitting the most recent month, prior is the return in the last month, dummy variables are included for NASDAQ stocks, stocks in the small (sm) and medium (med) terciles, and ff48 industries (ind). The adjusted R-squared and Nobs are the cross-sectional average.

Model		**rv_t-1**	**rs_t-1**	**mom**	**turnover**	**nasdaq**	**sm**	**med**	**prior**	**ind**	**adjrsq**	**nobs**
1	**Avg**	1.87	-0.01							No	0.009	6008
	%Sig	(0.51)	(0.39)									
2	**Avg**	1.94	-0.02	0.02	1.58	-0.05	0.06	0.05		No	0.021	4822
	%Sig	(0.53)	(0.44)	(0.35)	(0.17)	(0.28)	(0.43)	(0.41)				
3	**Avg**									yes	0.025	5492
	%Sig											
4	**Avg**	1.83	-0.03	0.01	1.08	-0.06	0.06	0.04		yes	0.044	4773
	%Sig	(0.52)	(0.48)	(0.30)	(0.15)	(0.26)	(0.41)	(0.40)				
5	**Avg**								-0.37	no	0.004	5652
	%Sig								(0.59)			
6	**Avg**	2.09	0.01	0.01	2.90	-0.06	0.04	0.04	-0.58	yes	0.47	4771
	%Sig	(0.55)	(0.26)	(0.28)	(0.18)	(0.26)	(0.39)	(0.38)	(0.80)			
7	**Avg**	2.02	0.02	0.01		-0.02	0.03	0.04	-0.61	yes	0.044	5032
	%Sig	(0.57)	(0.33)	(0.30)		(0.30)	(0.39)	(0.38)	(0.82)			

Table 4: **Average Returns by Portfolio**

This panel shows the average return of all 30 portfolios for each of the 6 anomaly strategies we consider. Portfolios are first sorted on the anomaly characteristic and then merged with stock level expected skewness. The second sort is done sequentially within the characteristic portfolio. The final result is 30 portfolios, based on Sequential sorts. Returns are calculated using value weighting by prior month market cap. Differences are calculated in the last column and row, while the bottom right corner provides the diagonal, long/high minus short/low.

Panel A: Skewness in Long Leg

B/M

port	high	med	low	H-L
Long	24.56	14.87	12.80	11.76
9	19.42	16.32	10.28	9.14
8	17.76	14.89	9.79	7.96
7	15.68	13.91	9.75	5.92
6	14.96	13.50	10.84	4.12
5	15.45	12.44	10.67	4.77
4	13.70	12.82	8.77	4.93
3	14.51	11.33	9.59	4.93
2	14.21	12.52	9.88	4.33
Short	12.32	9.59	9.60	2.72
L-S	12.25	5.29	3.20	14.96

Size

port	high	med	low	H-L
Long	22.85	14.65	5.86	16.98
2	20.83	14.99	6.42	14.42
3	20.70	14.66	8.07	12.63
4	19.70	13.46	8.00	11.71
5	19.77	13.74	9.56	10.20
6	17.98	12.11	8.74	9.23
7	17.70	12.70	8.87	8.83
8	16.86	12.74	9.23	7.64
9	15.50	12.04	8.80	6.70
Short	12.51	10.96	9.00	3.51
L-S	10.34	3.69	-3.13	13.85

Inv

port	high	med	low	H-L
Long	20.61	14.33	8.33	12.28
2	18.58	13.20	11.93	6.65
3	17.32	13.43	10.09	7.23
4	15.99	12.24	8.83	7.16
5	13.65	11.94	9.94	3.71
6	14.59	12.49	8.40	6.19
7	14.00	12.43	10.07	3.93
8	14.62	11.90	8.95	5.67
9	16.05	12.16	8.74	7.31
Short	11.57	9.07	7.04	4.54
L-S	9.04	5.26	1.29	13.57

Panel B: Skewness in Short Leg

MOM

port	high	med	low	H-L
Long	20.25	18.56	15.03	5.22
9	15.42	14.48	12.54	2.88
8	16.16	14.17	9.04	7.12
7	15.07	11.39	9.82	5.26
6	13.58	11.36	8.43	5.16
5	14.05	11.71	7.50	6.55
4	13.99	11.27	8.69	5.30
3	15.30	11.33	6.86	8.43
2	12.64	9.32	7.29	5.35
Short	10.72	2.76	1.32	9.40
L-S	9.54	15.80	13.71	18.94

ROE

port	high	med	low	H-L
Long	17.97	15.57	13.31	4.66
9	14.50	13.02	12.28	2.22
8	13.95	13.13	11.52	2.43
7	10.99	13.67	10.64	0.35
6	13.21	12.25	10.44	2.76
5	12.39	14.72	9.64	2.75
4	12.20	10.02	8.67	3.53
3	15.12	11.09	6.39	8.74
2	15.46	12.56	5.68	9.77
Short	16.32	8.16	0.08	16.24
L-S	1.65	7.41	13.23	17.89

OP

port	high	med	low	H-L
Long	15.35	10.64	11.17	4.18
9	15.89	13.19	10.24	5.65
8	15.89	11.87	10.90	4.99
7	13.78	12.14	9.09	4.69
6	13.77	12.03	8.45	5.32
5	14.17	13.46	9.70	4.47
4	14.66	12.77	9.74	4.92
3	15.67	10.60	8.35	7.31
2	15.47	10.14	5.68	9.79
Short	16.42	10.85	4.16	12.26
L-S	-1.07	-0.21	7.00	11.19

Table 5: **Return Skewness by Portfolio**

Portfolios are first sorted on the anomaly characteristic and then merged with stock level expected skewness. The second sort is done sequentially within characteristic portfolio. The final result is 30 portfolios, based on sequential sorts. Returns are calculated using value weighting by prior month market cap and skewness in those returns is reported here. Differences are calculated in the last column and row, while the bottom right corner provides the diagonal, long/high minus short/low.

Panel A: Skewness in Long Leg

B/M

port	high	med	low	H-L
Long	0.80	-0.32	-0.48	1.28
9	0.40	-0.54	-0.45	0.85
8	0.15	-0.39	-0.26	0.41
7	-0.27	-0.26	-0.50	0.22
6	-0.10	-0.48	-0.33	0.23
5	0.09	-0.54	-0.20	0.29
4	-0.42	-0.45	-0.22	-0.20
3	0.58	-0.43	-0.20	0.77
2	-0.01	-0.44	-0.39	0.37
Short	-0.39	-0.21	-0.21	-0.18
L-S	1.19	-0.10	-0.27	1.01

Size

port	high	med	low	H-L
Long	0.59	-0.06	-0.09	0.69
2	0.37	-0.22	0.09	0.28
3	-0.02	-0.38	-0.45	0.43
4	-0.17	-0.52	-0.41	0.24
5	-0.17	-0.52	-0.40	0.23
6	-0.18	-0.55	-0.41	0.23
7	-0.03	-0.55	-0.35	0.32
8	-0.18	-0.49	-0.28	0.11
9	-0.16	-0.37	-0.45	0.29
Short	-0.32	-0.31	-0.28	-0.04
L-S	0.91	0.25	0.18	0.87

Inv

port	high	med	low	H-L
Long	1.63	-0.46	-0.20	1.82
2	0.26	-0.09	-0.20	0.47
3	-0.02	-0.45	-0.21	0.19
4	0.12	-0.28	-0.33	0.44
5	-0.14	-0.30	-0.26	0.13
6	-0.09	-0.64	-0.48	0.39
7	-0.41	-0.49	-0.31	-0.10
8	0.03	-0.07	-0.35	0.38
9	-0.10	-0.30	-0.44	0.34
Short	0.10	-0.51	-0.37	0.47
L-S	1.53	0.05	0.17	2.00

Panel B: Skewness in Short Leg

MOM

port	high	med	low	H-L
Short	1.75	0.63	0.66	1.09
2	0.80	0.27	0.04	0.77
3	0.03	0.12	0.09	-0.05
4	-0.05	-0.07	-0.38	0.32
5	0.04	-0.19	-0.30	0.34
6	-0.23	-0.45	-0.32	0.10
7	-0.21	-0.44	-0.24	0.02
8	-0.46	-0.30	-0.19	-0.26
9	-0.33	-0.52	-0.33	0.00
Long	-0.25	-0.36	-0.30	0.05
S-L	1.99	0.99	0.96	2.05

ROE

port	high	med	low	H-L
Short	1.36	0.16	-0.30	1.66
2	1.99	-0.10	-0.23	2.22
3	0.82	-0.14	0.04	0.78
4	-0.33	-0.35	-0.14	-0.18
5	-0.42	-0.23	-0.25	-0.17
6	0.02	-0.53	0.41	-0.39
7	-0.25	-0.27	-0.14	-0.11
8	-0.28	-0.31	-0.26	-0.02
9	-0.11	-0.30	-0.17	0.06
Long	0.13	-0.24	-0.33	0.47
S-L	1.22	0.40	0.03	1.69

OP

port	high	med	low	H-L
Short	0.83	-0.49	-0.18	1.00
2	0.30	-0.51	-0.30	0.60
3	-0.11	-0.59	-0.46	0.35
4	0.45	-0.59	-0.14	0.59
5	-0.18	-0.46	0.07	-0.25
6	-0.18	-0.40	-0.53	0.35
7	0.06	-0.33	-0.41	0.46
8	0.23	-0.28	-0.30	0.52
9	-0.13	-0.38	-0.16	0.03
Long	-0.44	-0.37	-0.41	-0.03
S-L	1.26	-0.12	0.23	1.23

Table 6: **Skewness Management Strategy Performance**

This table compares the original to the skewness managed versions of each anomaly. Panel A lists performance in the full sample in which both the characteristic and expected skewness are non-missing. Panel B lists the performance of the skewness managed version of each anomaly in which the long leg comes from the high expected skewness sample and the short leg comes from the low expected skewness sample. Panel C reports the difference. The Sharpe ratio t-statistics from Jobson and Korkie (1981) are listed in parenthesis.

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Panel A: Original Strategy						
mean	3.63	5.03	4.44	13.69	7.48	2.15
Sharpe	0.21	0.30	0.40	0.55	0.42	0.14
skew	0.81	0.10	0.29	-1.41	0.09	0.17
Sortino	0.24	0.33	0.42	0.46	0.43	0.14
Panel B: Skewness Management Strategy						
mean	13.85	14.96	13.57	18.94	17.80	11.19
Sharpe	0.60	0.65	0.58	0.66	0.81	0.60
skew	1.85	1.04	3.10	-0.61	0.36	-0.01
Sortino	0.73	0.73	0.70	0.59	0.94	0.63
Panel C: Difference						
mean	10.22	9.93	9.13	5.24	10.32	9.04
Sharpe	0.39	0.34	0.18	0.10	0.39	0.46
	(5.46)	(3.40)	(1.24)	(1.38)	(3.26)	(4.24)
skew	1.03	0.93	2.81	0.79	0.27	-0.18
Sortino	0.49	0.41	0.28	0.13	0.51	0.49

Table 7: **Skewness Management Alphas**

This table reports alphas from spanning tests of skewness managed anomalies vs several benchmarks. Panel A compares the original anomaly portfolios on their skewness managed counterparts. Also included are spanning tests against a 4-factor model (Mkt, SMB, HML, UMD) and a 4-factor model + the original anomaly. heteroskedasticity robust t-stats are in parenthesis.

Panel A: Skewness Managed vs Original

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Size	1.16					
	(16.92)					
B/M		1.00				
		(17.14)				
Inv			0.79			
			(10.77)			
MOM				0.98		
				(28.56)		
ROE					0.82	
					(12.96)	
OP						0.79
						(15.91)
Alpha	9.64	9.94	10.05	5.48	11.59	9.49
tstat	(6.51)	(4.70)	(3.50)	(2.60)	(4.72)	(5.13)
adjrsq	0.74	0.51	0.14	0.71	0.43	0.43

Panel B: 4-factor alpha

Alpha	8.40	7.63	9.88	7.29	16.06	11.38
tstat	(4.19)	(3.06)	(3.23)	(2.98)	(5.39)	(4.52)

Panel C: 4-factor alpha + original

Alpha	9.14	9.14	7.80	4.16	9.17	6.90
tstat	(5.38)	(3.86)	(2.81)	(2.00)	(3.61)	(3.29)

Table 8: **Scaling Strategy Performance**

This table details the outcome when managing skewness via the scaling strategy. Exposure to the anomaly is scaled by the average expected skewness in the long and short portfolio. Strategies in which skewness dominates the long leg increase exposure to the strategy as expected skewness increases, while those in which skewness dominates the short leg decrease exposure. The first row in Panel B tells which scalar is used. Panel A contains the performance of the original factor in which the sample includes all observations in which both the characteristic and expected skewness measure is present. Panel B details the scalar strategy outcome for the correct version of the strategy. Long strategies seek skewness, short focused anomalies avoid skewness. Panel C reports the difference between the original and best scaling strategies. Sharpe ratio t-statistics reported in parenthesis are calculated using Jobson and Korkie (1981).

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Panel A: Original Factor						
mean	3.63	5.03	4.44	13.69	7.48	2.15
sharpe	0.21	0.30	0.40	0.55	0.42	0.14
skew	0.81	0.10	0.29	-1.41	0.09	0.17
sortino	0.24	0.33	0.42	0.46	0.43	0.14
Panel B: Best Scaling Strategy						
Strategy	Long	Long	Long	Short	Short	Short
mean	13.76	8.62	5.40	16.38	11.31	4.87
sharpe	0.66	0.48	0.42	0.58	0.58	0.27
skew	1.69	1.11	0.88	-1.07	0.88	0.38
sortino	1.00	0.60	0.47	0.51	0.66	0.29
Panel C: Difference						
Mean	10.14	3.59	0.95	2.69	3.83	2.72
Sharpe	0.45	0.17	0.03	0.02	0.15	0.13
	(7.21)	(2.37)	(0.41)	(0.35)	(2.18)	(2.21)
Skew	0.88	1.01	0.59	0.34	0.79	0.21
Sortino	0.76	0.27	0.05	0.05	0.23	0.15

Table 9: **Predictive Regressions on Idiosyncratic Skewness**

This table reports the time series average from the first regression in the predictive model: (insert regression equation). %sig is the percent of coefficients that are statistically significant at the 5% level (and of the same sign as the average coefficient). Sample is the collection of common stocks trading on the AMEX, NYSE, and Nasdaq. Momentum is the prior year return omitting the most recent month, prior is the return in the last month, dummy variables are included for NASDAQ stocks, stocks in the small and medium terciles, and ff48 industries. The adjusted R-squared and nobs are the cross-sectional average.

Model		**iv_t-1**	**is_t-1**	**mom**	**turnover**	**nasdaq**	**sm**	**med**	**prior**	**ind**	**adjrsq**	**nobs**
1	**Avg**	2.72	0.02							No	0.009	5976
	%Sig	(0.69)	(0.28)									
2	**Avg**	2.30	0.01	-0.01	2.48	-0.03	0.10	0.08		No	0.016	4805
	%Sig	(0.71)	(0.25)	(0.24)	(0.20)	(0.27)	(0.61)	(0.61)				
3	**Avg**									yes	0.012	5480
	%Sig											
4	**Avg**	1.98	0.01	-0.02	1.76	-0.05	0.09	0.07		yes	0.024	4756
	%Sig	(0.68)	(0.20)	(0.26)	(0.14)	(0.28)	(0.56)	(0.54)				
5	**Avg**								-0.20	no	0.002	5637
	%Sig								(0.39)			
6	**Avg**	2.28	0.02	-0.02	2.69	-0.05	0.08	0.06	-0.32	yes	0.026	4754
	%Sig	(0.71)	(0.42)	(0.24)	(0.18)	(0.30)	(0.52)	(0.51)	(0.75)			
7	**Avg**	2.21	0.03	-0.02		-0.01	0.07	0.07	-0.35	yes	0.026	5010
	%Sig	(0.71)	(0.46)	(0.25)		(0.32)	(0.53)	(0.50)	(0.79)			

Table 10: **Skewness Management Using Idiosyncratic Skewness**

This table compares the original to the skewness managed versions of each anomaly (using idiosyncratic skewness). Panel A lists the performance in the full sample in which both the characteristic and expected skewness are non-missing. Panel B lists the performance of the skewness managed version of each anomaly in which the long leg comes from the high expected skewness sample and the short leg comes from the low expected skewness sample. Panel C reports the difference. The Sharpe ratio t-statistics from Jobson and Korkie (1981) are listed in parenthesis.

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Panel A: Original Strategy						
mean	3.56	5.03	4.44	13.68	7.48	2.15
Sharpe	0.21	0.30	0.40	0.55	0.42	0.14
skew	0.81	0.10	0.29	-1.41	0.09	0.17
Sortino	0.24	0.33	0.42	0.46	0.43	0.14
Panel B: Skewness Management Strategy						
mean	9.27	16.30	15.84	19.30	14.17	10.73
Sharpe	0.37	0.63	0.64	0.70	0.69	0.57
skew	1.56	1.41	1.67	-0.32	-0.02	0.16
Sortino	0.44	0.77	0.77	0.65	0.76	0.59
Panel C: Difference						
Mean	5.71	11.27	11.40	5.62	6.69	8.58
Sharpe	0.16	0.32	0.25	0.15	0.27	0.43
	(2.49)	(3.15)	(1.65)	(2.04)	(2.32)	(3.83)
skew	0.75	1.31	1.38	1.09	-0.11	-0.01
Sortino	0.20	0.44	0.36	0.19	0.33	0.45

Table 11: **Average Realized Skewness by Portfolio**

Portfolios are first sorted on the anomaly characteristic and then merged with stock level expected skewness. The second sort is done sequentially within characteristic portfolio. The final result is 30 portfolios, based on sequential sorts. Returns are calculated using value weighting by prior month market cap. Differences are calculated in the last column and row, while the bottom right corner provides the diagonal, long/high minus short/low.

Panel A: Skewness in Long Leg

B/M

port	high	med	low	H-L
Long	0.43	0.34	0.25	0.18
9	0.38	0.31	0.23	0.16
8	0.36	0.29	0.23	0.13
7	0.35	0.28	0.21	0.14
6	0.33	0.28	0.21	0.12
5	0.33	0.27	0.20	0.13
4	0.32	0.26	0.19	0.13
3	0.32	0.25	0.20	0.12
2	0.32	0.25	0.19	0.13
Short	0.32	0.25	0.19	0.14
L-S	0.10	0.09	0.06	0.24

Size

port	high	med	low	H-L
Long	0.42	0.32	0.22	0.21
2	0.36	0.30	0.23	0.13
3	0.34	0.28	0.23	0.11
4	0.32	0.27	0.22	0.10
5	0.30	0.26	0.22	0.09
6	0.29	0.25	0.22	0.07
7	0.28	0.24	0.21	0.08
8	0.27	0.23	0.19	0.08
9	0.24	0.21	0.19	0.05
Short	0.22	0.21	0.20	0.03
L-S	0.20	0.12	0.02	0.23

Inv

port	high	med	low	H-L
Long	0.43	0.34	0.23	0.20
2	0.37	0.30	0.22	0.15
3	0.36	0.29	0.21	0.15
4	0.34	0.27	0.21	0.13
5	0.33	0.27	0.21	0.12
6	0.31	0.26	0.21	0.11
7	0.33	0.27	0.20	0.12
8	0.33	0.26	0.20	0.13
9	0.33	0.26	0.20	0.12
Short	0.33	0.27	0.20	0.14
L-S	0.10	0.08	0.03	0.23

Panel B: Skewness in Short Leg

MOM

port	high	med	low	H-L
Short	0.42	0.31	0.20	0.22
2	0.34	0.28	0.20	0.14
3	0.33	0.26	0.19	0.15
4	0.33	0.27	0.19	0.14
5	0.34	0.27	0.19	0.14
6	0.33	0.27	0.20	0.13
7	0.34	0.27	0.21	0.13
8	0.33	0.28	0.21	0.12
9	0.35	0.28	0.23	0.12
Long	0.36	0.30	0.25	0.11
S-L	0.06	0.00	-0.04	0.17

ROE

port	high	med	low	H-L
Short	0.41	0.31	0.23	0.17
2	0.34	0.28	0.21	0.13
3	0.33	0.27	0.22	0.10
4	0.31	0.27	0.22	0.10
5	0.30	0.26	0.22	0.08
6	0.30	0.25	0.22	0.07
7	0.29	0.26	0.23	0.06
8	0.28	0.25	0.21	0.07
9	0.27	0.24	0.22	0.05
Long	0.32	0.27	0.23	0.09
S-L	0.08	0.04	0.00	0.17

OP

port	high	med	low	H-L
Short	0.41	0.32	0.22	0.20
2	0.35	0.29	0.22	0.14
3	0.35	0.28	0.21	0.14
4	0.33	0.27	0.21	0.12
5	0.32	0.26	0.20	0.11
6	0.32	0.26	0.21	0.12
7	0.31	0.26	0.20	0.11
8	0.31	0.25	0.20	0.11
9	0.30	0.25	0.20	0.10
Long	0.32	0.26	0.22	0.10
S-L	0.09	0.06	0.00	0.19

Table 12: **Performance in High or Low Expected Skewness Sample**

This table compares the original to a version calculated in either the high or low expected skewness sample. Panel A lists performance in the full sample in which both the characteristic and expected skewness are non-missing. Panel B lists the performance of the high or low version of each anomaly based in which sample they are predicted to perform best. Panel C reports the difference. The Sharpe ratio t-statistics from Jobson and Korkie (1981) are listed in parenthesis.

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Original						
mean	3.63	5.03	4.44	13.69	7.48	2.15
Sharpe	0.21	0.30	0.40	0.55	0.42	0.14
skew	0.81	0.10	0.29	-1.41	0.09	0.17
Sortino	0.24	0.33	0.42	0.46	0.43	0.14
Ideal Sample Performance						
Sample	High	High	High	Low	Low	Low
mean	10.34	12.25	9.04	13.71	13.17	7.00
Sharpe	0.48	0.55	0.51	0.53	0.68	0.41
skew	1.38	1.01	1.18	-1.09	0.04	0.01
Sortino	0.57	0.65	0.62	0.46	0.73	0.42
Difference						
Mean	6.71	7.21	4.60	0.02	5.69	4.86
Sharpe	0.27	0.24	0.12	-0.02	0.26	0.27
	(4.91)	(2.37)	(0.88)	-(0.38)	(2.73)	(3.04)
Skewness	0.57	0.91	0.89	0.32	-0.05	-0.15
Sortino	0.32	0.32	0.20	0.00	0.30	0.28

Table 13: **Spanning Tests with $E[Skew]$ Anomaly**

This table reports alphas from spanning tests of skewness managed anomalies vs several benchmarks. Panel A compares the original anomaly portfolios to a portfolio representing the return predictability of expected skewness. Also included are spanning tests against the E[skew] anomaly + the original factor and E[skew] + original + 4-factor model. heteroskedasticity robust t-stats are in parenthesis.

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Panel A: E[Skew]						
Beta	0.76	0.61	0.76	0.34	0.24	0.23
	7.11	7.66	7.11	1.86	2.36	4.25
Alpha	8.00	9.54	6.83	15.92	16.06	9.15
tstat	3.12	3.49	2.85	4.52	5.31	3.74
adjrsq	0.24	0.21	0.31	0.04	0.04	0.04
Panel B: original + E[Skew]						
Alpha	5.99	5.67	3.25	1.16	7.90	5.54
tstat	5.30	3.14	1.55	0.60	3.80	3.37
Panel C: 4-factor + original + E[Skew]						
Alpha	6.22	5.75	3.24	1.27	6.71	3.97
tstat	4.65	2.99	1.50	0.67	2.88	2.09

Table 14: **Realized Skewness Performance**

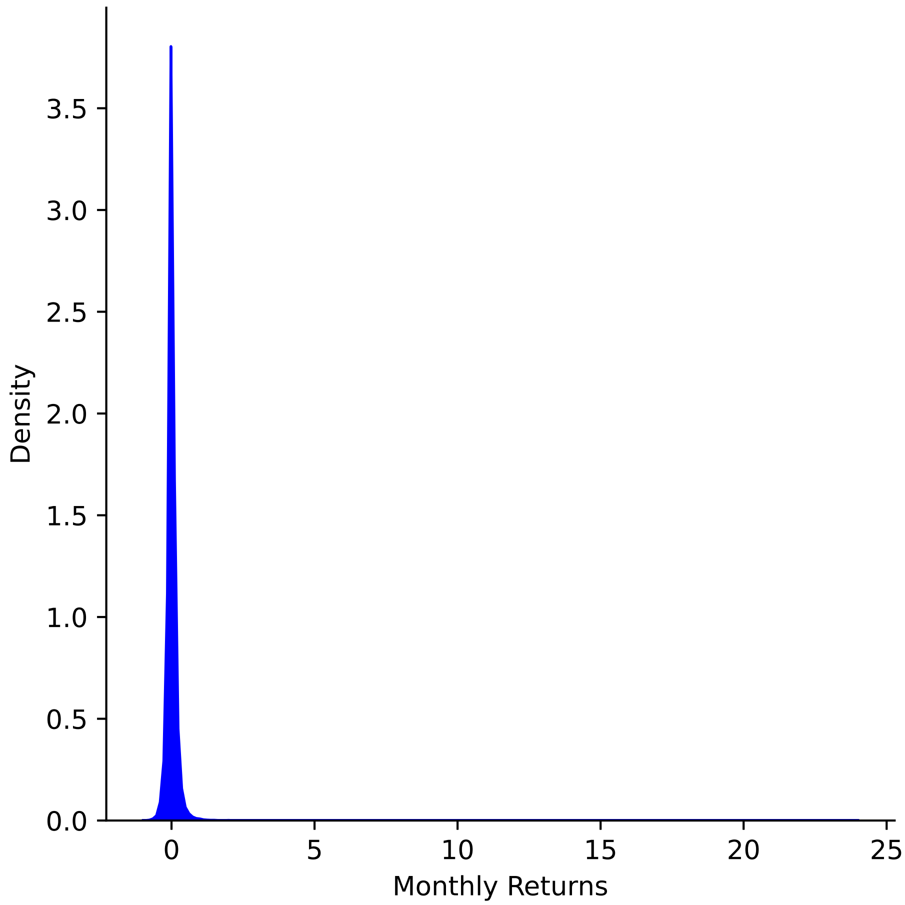
This table outlines the performance of strategies that are sequentially sorted on the anomaly characteristics and realized skewness. Note that realized skewness contains future information, and these results provide an upper-bound for how well the expected skewness strategy could perform if an investor could perfectly predict realized skewness. Panel A displays the original anomaly. Panel B is the anomaly calculated in the high realized skewness sample. Panel C is the anomaly calculated in the low expected skewness sample. Panel D compares the Sharpe performance in the high and low samples to the original. panel E looks at the combined skewness managed strategy that is long in the high expected skewness sample and short in the low expected skewness sample, along with comparisons to the original.

	Size	**B/M**	**Inv**	**MOM**	**ROE**	**OP**
Panel A: Original Factor						
mean	3.22	5.10	4.25	13.85	7.74	2.22
Sharpe	0.19	0.31	0.38	0.56	0.43	0.15
skew	0.78	0.07	0.28	-1.41	0.14	0.12
Sortino	0.22	0.33	0.40	0.47	0.43	0.15
Panel B: Factor, High Skewness Sample						
mean	90.87	28.18	9.98	-22.00	-44.97	-51.78
Sharpe	4.00	1.07	0.62	-0.63	-1.88	-2.51
skew	1.50	5.24	0.25	-3.21	-1.12	-1.25
Sortino	5.75	1.84	0.70	-0.50	-1.47	-1.91
Panel C: Factor, Low Skewness Sample						
mean	-70.84	-6.96	8.13	40.45	51.95	45.35
Sharpe	-5.08	-0.44	0.63	1.87	3.03	3.23
skew	-0.04	-0.59	0.36	0.08	0.48	0.67
Sortino	-2.82	-0.41	0.72	2.03	3.66	4.61
Panel D: Performance Comparison						
Sharpe Diff: High	3.81	0.76	0.24	-1.19	-2.31	-2.65
Sharpe Diff: Low	-5.27	-0.75	0.25	1.31	2.60	3.09
Panel E: Combined (Long High Skewness, Short Low Skewness)						
mean	207.11	185.93	204.99	235.53	209.03	190.70
Sharpe	7.77	6.33	9.40	9.03	9.25	10.01
skew	2.17	4.40	1.54	0.64	1.94	1.01
Sortino	20.88	15.81	NA	8.84	NA	19.21
Difference						
mean	203.89	180.83	200.74	221.69	201.29	188.48
Sharpe	7.58	6.02	9.02	8.47	8.82	9.87
skew	1.39	4.33	1.26	2.05	1.80	0.89
Sortino	20.66	15.48	-0.40	8.38	-0.43	19.06

Figure 1

Panel A: Histogram of Monthly Returns

This figure plots a kernel density estimate of monthly return observations for the sample from July 1963- December 2021. Monthly returns are plotted as decimals. The thin tail of the distribution extends out 24 (2400% monthly return).



Panel B: Outlier Percentage by Month

This Figure plots the percentage of stocks in a given month with returns greater than 26.6%, representing those above the 95th percentile during the period from July 1963 - December 2021. Percent is represented as a decimal. The orange line is a reference point representing 5% of stocks each month. The last 3 spikes in the blue line coincide with the periods coming out of the Tech Bubble, the Financial Crisis, and the Covid19 crash, respectively.

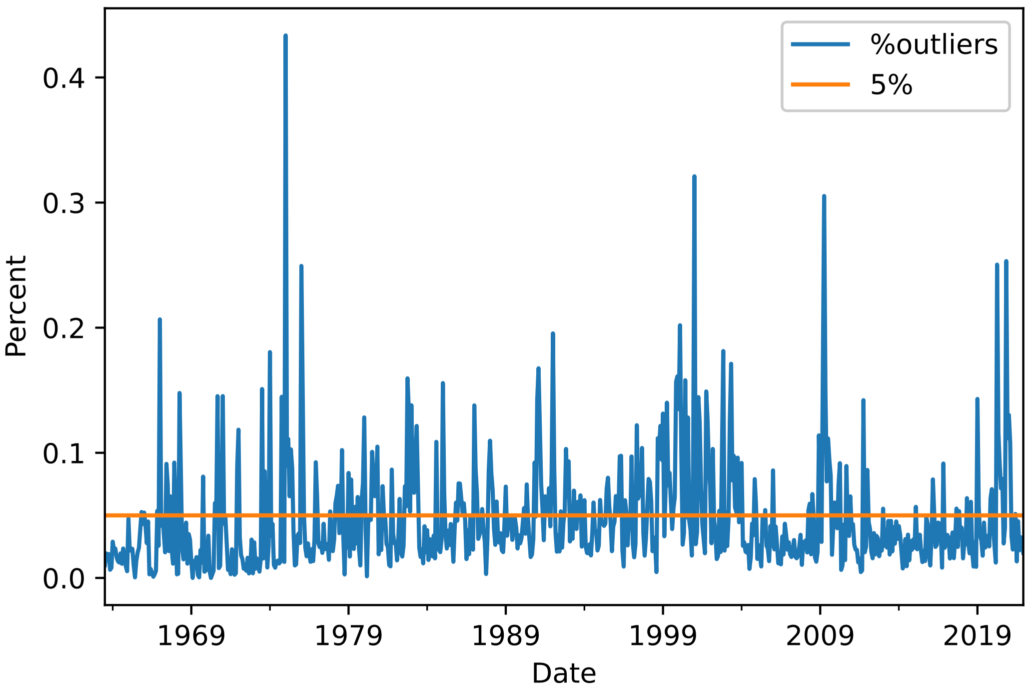


Figure 2: Monthly Return Histogram Split by Realized Skewness

Realized skewness is used as the target of the prediction model to calculate *expected* skewness. This Figure illustrates how realized skewness is related to monthly return outliers. The plots are stacked. The x-axis plots the monthly return as a decimal, and is limited at 3 (300%). In the true distribution the tail continues past 20 as with figure 1a. The orange density plot shows how high realized skewness in daily returns coincides with the right tail of the monthly return distribution.

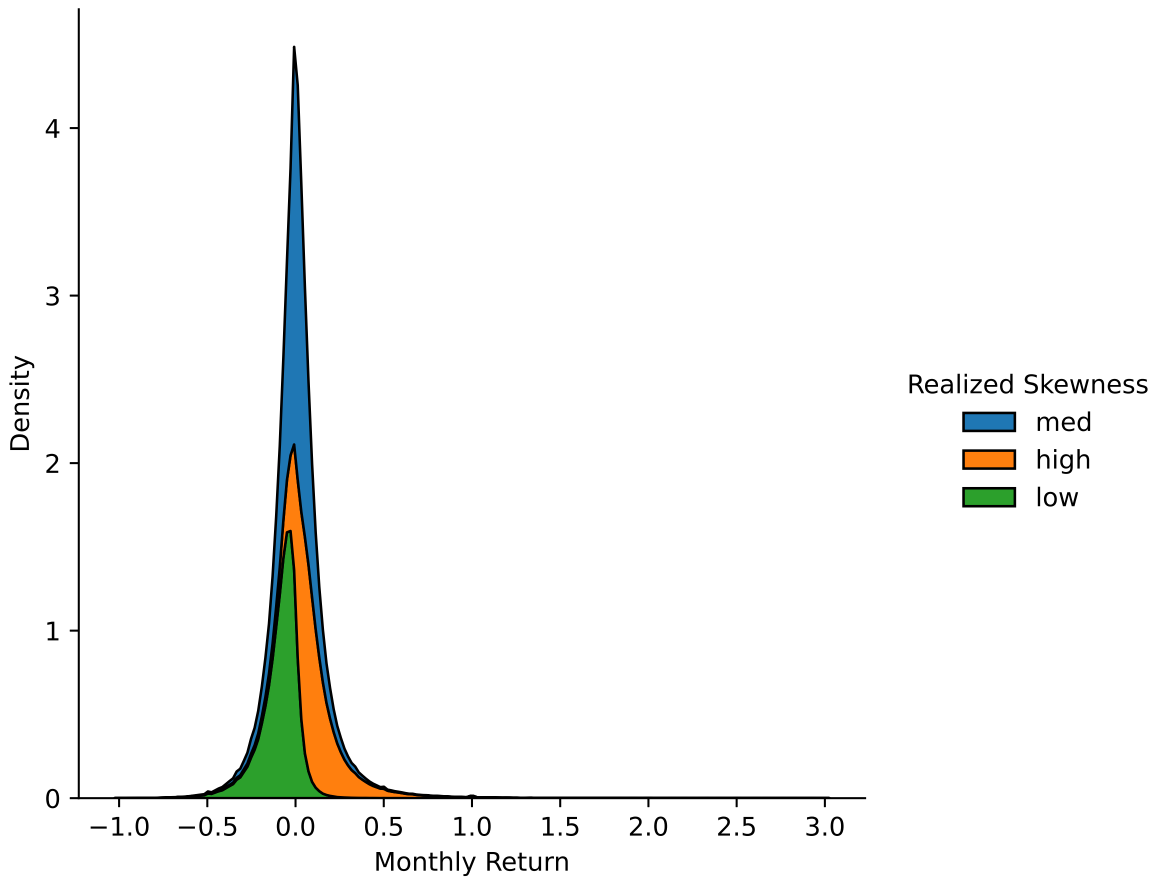
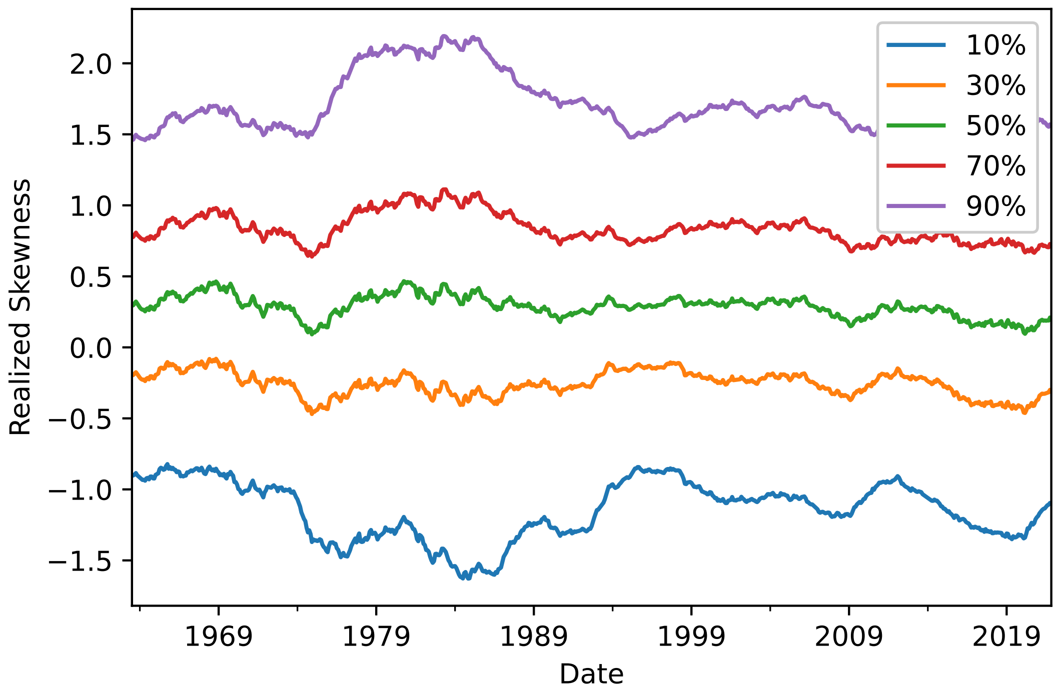


Figure 3: Realized Skewness distribution

This figure graphs the percentiles of realized skewness over time. The plot is of a 36 month moving average of each percentile to smooth out noise. Apart from monthly noise, realized skewness is stable over time.



Internet Appendix

Appendix A: Anomaly Construction

This appendix discusses the details of how the anomalies are constructed. Each signal is constructed as a long-short, self financing strategy based on deciles of firm characteristics. Breakpoints are constructed using NYSE stocks. For a majority of signals I focus on the period from July 1963 to December 2021. Return on Equity uses quarterly files and starts in January 1972. For strategies using annual Compustat files, accounting data for fiscal year-end of year t is matched with stock returns data from July of year t+1 until June of year t+2 to avoid look-ahead bias. ROE uses earnings data from Compustat quarterly files. The data are used in the months immediately after the most recent public quarterly earnings announcement dates (Compustat quarterly item RDQ) where the end of the fiscal quarter that corresponds to its announced earnings is within six months prior of portfolio formation.

The sample includes all common stocks, share code 10 and 11, with available characteristic data and returns. Book equity of firms is calculated by adding the deferred taxes and investment tax credits where available, and preferred stock values were incorporated in the following order of availability: redemption value, liquidation value, or par value of preferred stock. Book-to-market equity is calculated using the December of year t - 1 value for market equity. Tax credits are only added prior to 1993 due to changes in the treatment of deferred taxes. Returns are adjusted for delisting when data are available.

Additional Details:

- Size: Follows Fama and French (1993). size is price times shares outstanding using June Crsp Data.
- Book-to-market (b/m): Follows Fama and French (1993). Book equity from previous fiscal year and market equity from December of prior year. Rebalanced annually.
- Investment (asset growth): Growth in assets from the prior fiscal year, $AssetGrowth = AT/AT_{t-12}$. Rebalanced annually.
- Momentum: Follows Jegadeesh and Titman (1993). Momentum is the cumulative return from the prior year omitting the most recent month. Rebalanced monthly.
- Return on Equity (ROE): Roe is income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity where quarterly data book equity data are described above. Rebalanced monthly. Financial Firms omitted.
- Operating Profitability (op): Following Fama and French (2015). Equal to annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in t-1. Rebalanced annually.