

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B6. CONDENSED-MATTER PHYSICS

TRINITY TERM 2017

Saturday, 10 June, 2.30 pm – 4.30 pm

Candidates are strongly advised to use the first 10 minutes
to read the whole paper before starting writing.

*Answer **two** questions.*

*Start the answer to each question in a **fresh book**.*

The use of approved calculators is permitted.

A list of physical constants and conversion factors accompanies this paper.

*The numbers in the margin indicate the weight that the Examiners expect to
assign to each part of the question.*

Do NOT turn over until told that you may do so.

1. A crystalline material has a face-centred cubic lattice. The conventional unit cell is a cube of side a . Without detailed calculation, describe the reciprocal lattice of this material. [3]

Calculate the spacing between the (111) lattice planes in terms of a . Describe how these relate to a particular region of the reciprocal lattice. [2]

Calculate the angle between the (111) planes and (i) the $(\bar{1}11)$ planes; (ii) the (100) planes. [4]

GaAs has the cubic zincblende structure with $a = 5.653 \times 10^{-10}$ m, which has a face-centred cubic lattice with a basis of two atoms: Ga at 0,0,0 and As at $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$. In a powder x-ray diffraction experiment the incident x-rays have an energy of 8 keV and the angle through which they are scattered is measured. Calculate the scattering angles for the lowest six reflections and estimate their relative intensities. [12]

A sample of AlAs has the same crystal structure as GaAs but $a = 5.660 \times 10^{-10}$ m. What is the difference in angle between the sixth lowest reflection for AlAs and that for GaAs? If this is too small to measure, how else might the two materials be distinguished using x-rays? [4]

[The atomic numbers of Al, Ga and As are 13, 31 and 33 respectively.]

2. The internal energy U of a free electron metal at temperature T can be written

$$U(T) = \int_0^\infty E g(E) f(E) dE.$$

Explain the meaning of the factors $g(E)$ and $f(E)$ in this expression and state which terms inside the integral depend on T . Use a rough argument to show that you would expect the heat capacity C at low temperatures to be proportional to T and to $g(E_F)$ where E_F is the Fermi energy, i.e. that

$$C = \gamma T,$$

where $\gamma = \alpha k_B^2 g(E_F)$ is a constant. You do not need to derive the numerical factor $\alpha = \pi^2/3$ in this constant. [7]

Derive an expression for the low-temperature entropy in terms of γ and T . [3]

Derive an expression for the Pauli susceptibility χ_p of a free electron metal. [5]

Sodium is a monovalent metal with a body-centred cubic lattice. The measured Hall coefficient is found to be $R_H = -25.0 \times 10^{-11} \text{ m}^3 \text{ C}^{-1}$. Use this value to deduce the lattice constant of sodium. Assuming an effective mass $m^* = 1.3m_e$, estimate values for the Fermi energy (in eV), γ (in $\text{mJ mol}^{-1} \text{ K}^{-2}$) and χ_p . [10]

3. A paramagnetic material contains n non-interacting spin- $\frac{1}{2}$ magnetic moments per unit volume. Derive an expression for the magnetization of this material in an applied magnetic field B at temperature T and hence show that the magnetic susceptibility χ is given by $\chi = C/T$ where $C = n\mu_0\mu_B^2/k_B$. [7]

If the magnetic moments interact with each other, then their effect can be modelled (in the mean-field approximation) by adding an effective magnetic field $B_{\text{eff}} = \lambda\mu_0M$, where λ is a constant. The total magnetic field is now given by $B + B_{\text{eff}}$. Show that the magnetic susceptibility then takes the form $\chi = C/(T - \theta)$ and find an expression for θ . What is the physical significance of the parameter θ ? Sketch χ as a function of T . [8]

Nickel is a face-centred cubic ferromagnetic metal with density 8908 kg m^{-3} , relative atomic mass 58.693 and critical temperature $T_C = 627 \text{ K}$. By treating nickel as a spin- $\frac{1}{2}$ system, and thus assuming the results obtained above are applicable, derive a value for λ . Hence estimate B_{eff} at very low temperature. Comment on whether or not B_{eff} could represent a real magnetic field. [7]

Specimens of nickel at room temperature often appear to have zero magnetization, even though room temperature is well below T_C . Why is this? [3]

4. Show that, in a semiconductor at temperature T , the product of the number density of electrons, n , and the number density of holes, p , is given by

$$np = A(m_e^*m_h^*)^B T^C \exp(-E_g/k_B T),$$

where m_e^* and m_h^* are the effective masses of electrons and holes respectively, E_g is the band gap, and A , B and C are constants which you should determine. [12]

[You may use the standard integral: $\int_0^\infty x^{1/2} e^{-x} dx = \sqrt{\pi}/2$]

For the intrinsic case, show that the temperature dependence of the chemical potential, $\mu(T)$, takes the form

$$\mu(T) = \frac{E_c + E_v}{2} + \alpha k_B T,$$

where E_c and E_v are the energies at the bottom of the conduction band and the top of valence band respectively, and find the constant α . [4]

Phosphorous ($Z = 15$) can be used as a dopant in silicon ($Z = 14$) to produce an n -type semiconductor. Estimate the energy required to ionize the phosphorous dopant and hence comment on whether or not dopants are likely to be ionized in silicon at 1000 K. [3]

Calculate p at 300 K for (i) pure silicon (with no dopants) and (ii) n -type silicon (with phosphorous doping $N_d = 10^{20} \text{ m}^{-3}$), and describe the position of μ within the gap in each case. [6]

[The (indirect) band gap of silicon is 1.11 eV, $m_e^* = 0.26m_e$, $m_h^* = 0.39m_e$, $\epsilon_r = 11.7$.]