$$\frac{\partial \mathcal{L}}{\partial q_k} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_k}\right) = \frac{dp_k}{dt}$$

$$H = \sum_k p_k \dot{q}_k - \mathcal{L}(q, \dot{q})$$

Define $p_k = \frac{\partial \mathcal{L}}{\partial \dot{q}_k}$

$$\delta H = \sum_k \left(\delta p_k \dot{q}_k + \cancel{p_k \delta \dot{q}_k} - \delta q_k \frac{\partial \mathcal{L}}{\partial q_k} - \cancel{\delta \dot{q}_k \frac{\partial \mathcal{L}}{\partial \dot{q}_k}} \right)$$

$$\therefore \quad \frac{\partial H}{\partial p_k} = \dot{q}_k \qquad \frac{\partial H}{\partial q_k} = -\frac{\partial \mathcal{L}}{\partial q_k} = -\dot{p}_k$$

Define A Poisson bracket,

$$[A, B] = \sum_k \left(\frac{\partial A}{\partial q_k} \frac{\partial B}{\partial p_k} - \frac{\partial A}{\partial p_k} \frac{\partial B}{\partial q_k} \right)$$

e.g. $[q_k, p_{k'}] = \delta_{kk'}$

Then consider some

$$\frac{dA}{dt} = \sum_k \left(\frac{\partial A}{\partial q_k} \frac{dq_k}{dt} + \frac{\partial A}{\partial p_k} \frac{dp_k}{dt} \right)$$

$$= \sum_k \left(\frac{\partial A}{\partial q_k} \frac{\partial H}{\partial p_k} - \frac{\partial A}{\partial p_k} \frac{\partial H}{\partial q_k} \right)$$

$$= [A, H]$$

(1) There exist some Hilbert space $\mathcal{H}$ with states $|\psi\rangle \in \mathcal{H}$

(2) Physical quantities, A and B, are promoted to operators $\hat{A}$, $\hat{B}$

(3) This is the trippy rule: Canonical Quantization. Poisson brackets become commutators (up to a factor of $i\hbar$). e.g. $[x_k, p_{k'}] = i\hbar\,\delta_{kk'}$
This is the Heisenberg eq. of motion,

$$i\hbar \frac{d\hat{A}}{dt} = [A, H]$$

(4) Expectation values are given by

$$\langle A\rangle(t) = \frac{\langle\psi|\,A(t)\,|\psi\rangle}{\langle\psi|\psi\rangle}$$

(5) Measurement collapse.

Comment:

This is the Heisenberg picture. Here,

$$\frac{\partial}{\partial t} |\psi_H\rangle = 0$$

$$A_H(t) = e^{iHt/\hbar} A_H(0) e^{-iHt/\hbar}$$

So when we consider

$$A_H(t) |\psi\rangle$$

we might want to force $|\psi(t)\rangle$ and A as some operator. Then,

$$|\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}} |\psi(0)\rangle$$

which is the solution to the TDSE

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle$$