A13485W1

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B6. CONDENSED-MATTER PHYSICS

TRINITY TERM 2016

Saturday, 11 June, 2.30 pm – 4.30 pm

10 minutes reading time

Answer **two** *questions.*

Start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. Write down the *structure factor*, $F(hkl)$, for the hkl x-ray reflection from a crystal containing atoms at positions (x_j, y_j, z_j) in the unit cell with atomic form factor f_j. Draw a graph showing how f_j varies with scattering angle for typical elements. How does the diffracted intensity from the (hkl) family of planes depend on $F(hkl)$? [6]

CsCl adopts a simple cubic lattice with a basis consisting of a Cs^+ ion at 000 and a Cl^- ion at $\frac{1}{2}\frac{1}{2}\frac{1}{2}$. Derive a formula for the intensity of the reflection hkl from this structure. How would your expression be modified if both Cs and Cl are replaced by Fe atoms? Compare the intensity of the 111 reflections for the two cases. How would you describe the Fe lattice in this case? [8]

Fe has an isothermal compressibility given by $\kappa = 0.0059\ \mathrm{GPa}^{-1}$. To measure this, an x-ray experiment is performed on a sample of Fe inside a pressure cell held at constant temperature. The incident beam is deflected through an angle 2θ. Show that the pressure dependence of θ is given by

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}p}\right)_T = \alpha\kappa\tan\theta,$$

where α is a numerical constant that you should determine. At ambient pressure, Fe has a cubic lattice constant given by $a = 0.287$ nm. Find the value of 2θ for the 110 reflection in an experiment using $\lambda = 0.154$ nm x-rays

(i) at ambient pressure and

(ii) with an applied pressure of 100 GPa.

[11]

2. What is the *free electron model* for a metal? What is meant by the terms *Fermi energy* and *Fermi temperature*? Why are the Fermi temperatures of conventional metals typically much higher than room temperature, even though such metals held at room temperature feel cold to the touch? [6]

A d-dimensional sample ($d = 1, 2$ or 3) with volume L^d contains N electrons and can be described by the free electron model. Show that the Fermi energy, E_{F}, is given by

$$E_{\mathrm{F}} = \frac{\hbar^2}{2m}(n a_d)^{2/d},$$

where $n = N/L^d$ and find the values of the numerical constants a_1, a_2 and a_3. Show that $g(E_{\mathrm{F}}) = b_d(n/E_{\mathrm{F}})$, where $g(E_{\mathrm{F}})$ is the density of states per unit energy and per unit volume at the Fermi energy, and find the value of the numerical constants b_1, b_2 and b_3. [10]

Assuming the applicability of the free electron model, estimate the Fermi energy (in eV), Fermi temperature, scattering time and mean-free path for the following materials:

(i) Aluminium, a trivalent metal, with a face-centred cubic structure for which the side of the conventional unit cell is 0.405 nm, and for a sample with resistivity $2.8 \times 10^{-8}\,\Omega\,\mathrm{m}$.

(ii) A two-dimensional electron gas with an electron density of $3 \times 10^{15}\,\mathrm{m}^{-2}$ and a (two-dimensional) resistivity of $10^3\,\Omega$. [9]

3. Show that the magnetization M of a spin-$\frac{1}{2}$ paramagnet comprising N non-interacting spin-$\frac{1}{2}$ particles in volume V, which is placed in a magnetic field with flux density B at temperature T, is given by

$$M = M_{\mathrm{s}} \tanh\left(\frac{\eta B}{T}\right),$$

and give expressions for the constants M_{s} and η. Derive an expression for the magnetic susceptibility χ. [10]

The magnetic susceptibility of copper sulphate pentahydrate (chemical formula $CuSO_4 \cdot 5H_2O$) is dominated by Cu^{2+} ($3d^9$) ions contained within it. Use Hund's rules to determine the magnetic moment of a single Cu^{2+} ion. When such an ion is placed in a crystal, the magnetic moment of Cu^{2+} behaves as if it only has the spin part of the angular momentum. Why do the other atoms in the formula help copper sulphate pentahydrate to behave as a paramagnet? How would pure elemental copper (Cu) behave differently? Calculate the magnetic susceptibility of copper sulphate pentahydrate at 300 K and 2 K. At each of these temperatures, evaluate the fraction of moments aligned with the applied magnetic field when (i) $B = 0.05\,\mathrm{T}$ and (ii) $B = 5\,\mathrm{T}$. [15]

[The molar volume of $CuSO_4 \cdot 5H_2O$ is $1.092 \times 10^{-4}\,\mathrm{m}^3$.]

4. Give a brief account of experimental methods that can be used to investigate the lattice vibrations of crystals. [5]

A model one-dimensional monatomic crystal, with lattice constant a, contains atoms of mass m. Each atom is connected to its neighbours by a spring of force constant C. Explain why the equation of motion for the n^{th} atom is given by

$$m\frac{\mathrm{d}^2 u_n}{\mathrm{d}t^2} = C(u_{n+1} - 2u_n + u_{n-1})$$

where u_n is the displacement of the n^{th} atom from equilibrium. Hence derive an expression for the phonon dispersion relation. [8]

(i) Draw a labelled sketch of the dispersion relation for a monatomic chain for which $a = 0.2\,\mathrm{nm}$ and the speed of sound is $4\,\mathrm{km\,s^{-1}}$.

(ii) On a separate diagram, draw a labelled sketch of $v_{\mathrm{g}}(k)$, the group velocity, as a function of phonon wavevector k.

(iii) Draw a labelled sketch of $g(\omega)$, the density of vibrational states per unit frequency interval and per unit length, as a function of ω.

(iv) Draw a labelled sketch of the heat capacity of the chain as a function of temperature. [12]