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SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B6: CONDENSED-MATTER PHYSICS

TRINITY TERM 2015

Monday, 15 June, 2.30 pm – 4.30 pm

10 minutes reading time

Answer **two** *questions.*

Start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. Derive expressions for the Fermi Temperature, $T_{\rm F}$, and Debye Temperature, $\theta_{\rm D}$, of a monovalent metal containing n atoms per unit volume, and within which the speed of sound averaged over polarisations is c. [6]

Outline a simple argument showing that the heat capacity per unit volume owing to the electrons within this metal is given by

$$C_{\rm e} = A\, n\, k_{\rm B} \left(\frac{T}{T_{\rm F}} \right) \ ,$$

where A is a constant that need not be evaluated. [4]

State the assumptions that underpin the Debye theory of specific heats, and obtain an expression for the heat capacity per unit volume due to the phonons, $C_{\rm ph}$, for a crystal with Debye Temperature $\theta_{\rm D}$ at a temperature $T \ll \theta_{\rm D}$. [8]

The effective speed of sound in copper, which is a face-centred cubic monovalent metal with a lattice parameter of 0.36 nm, is 2700 m s^{-1}. Given that $A = \pi^2/2$, evaluate the temperature range over which you would expect $C_{\rm e}$ to dominate over $C_{\rm ph}$. Estimate $C_{\rm e}/C_{\rm ph}$ at room temperature. [7]

[You may use $\int_0^\infty \frac{x^3}{{\rm e}^x - 1}\, {\rm d}x = \frac{\pi^4}{15}$.]

2. Explain what is meant by the *structure factor*, and how it is used in the determination of crystal structures by diffraction of X-rays and neutrons. State how the intensity of a Bragg reflection in a diffraction pattern is related to the structure factor, and write down a formula for the structure factor, $S_{(h,k,l)}$, associated with diffraction from planes with Miller indices (h, k, l) of a crystal with N atoms in the unit cell, where the $j^{\rm th}$ atom has an atomic form factor f_j, and is situated at a position x_j, y_j, z_j within the unit cell, where x_j, y_j and z_j are expressed as fractions of the unit cell edge lengths. [8]

Al, KCl, and Si all crystallize with a face-centred-cubic lattice. Al (atomic number $Z = 13$) is metallically bonded, and its basis is such that a single atom is associated with each lattice point. K ($Z = 19$) bonds ionically with Cl ($Z = 17$), and KCl has a basis of a K atom at $(0, 0, 0)$ and a Cl atom at $(\frac{1}{2}, 0, 0)$. Si ($Z = 14$) has covalent bonding, and a basis of one Si atom at $(0, 0, 0)$, and one at $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. Powder samples of all three materials are studied with short wavelength X-ray diffraction in the Debye-Scherrer geometry. For the Al sample, 6 strong diffraction rings are found with Miller indices such that $h^2 + k^2 + l^2 \leq 16$, whereas, in the case of both the KCl and Si samples, only 4 strong diffraction rings are found that satisfy this condition. Identify, giving your reasoning, the Miller indices corresponding to the strong diffraction rings for each of the three materials. [12]

In the above experiment, would the number of observed rings remain the same in each case if neutrons of the same wavelength were used instead of X-rays? [5]

3. Explain the meaning of the term *phonon*. Describe briefly a method by which the dispersion curves of phonons in a crystal may be measured. [6]

A one-dimensional monotomic lattice with period a contains atoms of mass M. The atoms interact via long-range interatomic forces obeying Hooke's law, such that the force on atom p caused by atom $p+n$ is proportional to the difference of their longitudinal displacements. The force constant C_n between them depends on $|n|$ but not p. Derive the equation of motion for atom p and show that the dispersion relation for lattice vibrations of angular frequency ω and wavevector magnitude k is

$$\omega(k) = \left(\frac{4}{M}\right)^{\frac{1}{2}} \left[\sum_{n>0} C_n \sin^2\left(\frac{nka}{2}\right)\right]^{\frac{1}{2}} \quad . \qquad [7]$$

Consider two different chains of such atoms. For the first chain only nearest-nearest neighbour interactions are present (i.e. $C_n = 0$ for $|n| > 1$). The second chain has the same value of force constant C_1 as the first chain, but now includes interactions with the next nearest neighbour such that $C_2 = C_1/2$, and $C_n = 0$ for $|n| > 2$. Sketch the dispersion relations for both chains for all k. What is the ratio of the speed of sound in the second chain to that in the first chain? Determine the ratio of the maximum frequency of phonons in the second chain to that in the first chain. [12]

4. State what you understand by the terms *intrinsic* and *extrinsic* when describing semiconductors. [4]

Show that the number of electrons per unit volume in the conduction band of an intrinsic gap semiconductor at a temperature T is given by

$$n = A\,T^{3/2} \exp\left(\frac{-E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right) \quad ,$$

where E_{g} is the energy gap and A is a constant that need not be evaluated. [10]

Define the Hall coefficient, R_{H}, and derive a simple expression for R_{H} in a semiconducting material having only one type of carrier. The absolute magnitude of the Hall coefficient for a sample of undoped silicon at 300 K is 625 $\mathrm{m^3\ C^{-1}}$. The coefficient is found to decrease by a factor of 4×10^4 as the temperature is increased from 300 K to 500 K. Assuming only one carrier type contributes to R_{H}, deduce a value for E_{g}. Why is it a reasonable approximation to assume that the majority carriers in intrinsic silicon are electrons? [6]

Estimate the minimum concentration and type of dopant atoms required, so that the silicon sample exhibits an approximately constant Hall coefficient, opposite in sign to that of the intrinsic sample, over the whole of the above temperature range. For this doped sample, estimate the electron concentration at 300 K. [5]