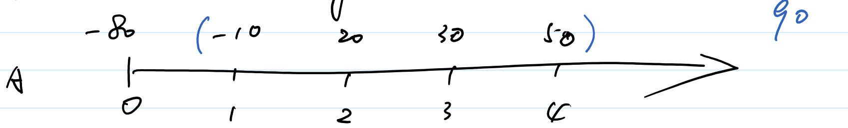
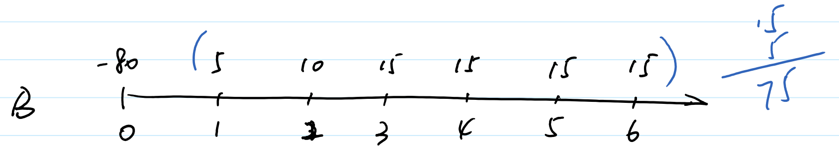
Lecture 2 Understanding Interest Rates





\$100 ¥100

present value

$100(1+3\%) = \underline{100} + 100 \times 3\% = 103$

$$PV(1+i)^n = FV \quad \text{or} \quad PV\left(1+\frac{i}{m}\right)^{m \times n} = FV$$

interests reward to you for the time you don't use the money.

$100 + 100 \times 3\%/2 = 100\left(1+\frac{3\%}{2}\right) = 101.5$

$101.5 \times \left(1+\frac{3\%}{2}\right) = 100\left(1+\frac{3\%}{2}\right) + 1.5 \times \left(1+\frac{3\%}{2}\right)$

$100 \times \left(1+\frac{3\%}{4}\right)^4 =$

bonds

maturity e.g. 1 year, 3 year

coupon rate fixed in maturity

face value / par value principle

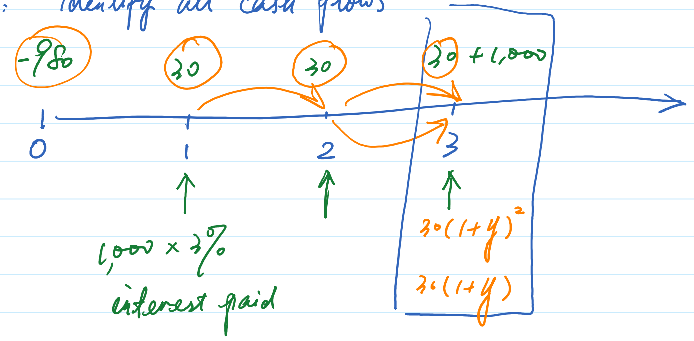
yield to maturity

suppose Treasury department issues a 3-year bond with face value of \$1,000

3% (p.a.) coupon paid ~~annually~~ semi-annually, today trading price is \$980, Q:

what is the yield to maturity?

step 1: identify all cash flows



step 2: discount all cash flows

30 reinvestment assume y

$S = 30(1+y)^2 + 30(1+y) + 30.$ interest on interest

trick $(1+y)S = 30(1+y)^3 + 30(1+y)^2 + 30(1+y)$

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$(1+y)S - S = 30(1+y)^3 - 30$

$yS = 30\left[(1+y)^3 - 1\right]$

$S = 30\left[\frac{(1+y)^3-1}{y}\right]$

$$S = C\left[\frac{(1+y)^n - 1}{y}\right]$$

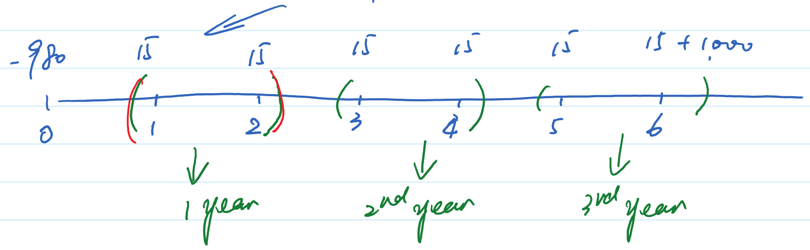
$$980 = \frac{S + 1{,}000}{(1+y)^3} = \frac{30\left[\frac{(1+y)^3-1}{y}\right]}{(1+y)^3} + \frac{1{,}000}{(1+y)^3}$$

$$\frac{\frac{(1+y)^3-1}{y}}{(1+y)^3} = \frac{(1+y)^3-1}{y(1+y)^3} = \frac{\cancel{(1+y)^3}}{y\cancel{(1+y)^3}} - \frac{1}{y(1+y)^3} = \frac{1}{y} - \frac{\frac{1}{(1+y)^3}}{y} = \frac{1}{y} - \frac{(1+y)^{-3}}{y} = \frac{1-(1+y)^{-3}}{y}$$

$$980 = 30\left[\frac{1-(1+y)^{-3}}{y}\right] + \frac{1000}{(1+y)^3}$$

$$\frac{30}{(1+y)} + \frac{30}{(1+y)^2} + \frac{1030}{(1+y)^3} = 980$$

step 1: cash flows $1000 \times 3\%/2$



step 2:

$$15\left[\frac{(1+y)^6 - 1}{y}\right] \quad \frac{y}{2}$$

y annualized rate

$15(1+y)$

$15(1+\frac{y}{2})$

$$\frac{15\left[\frac{(1+\frac{y}{2})^6 - 1}{\frac{y}{2}}\right] + 1{,}000}{(1+\frac{y}{2})^6}$$

$$980 = 15\left[\frac{1-(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right] + \frac{1{,}000}{(1+\frac{y}{2})^6}$$

Suppose a 3-year gov. bond traded @ 950, coupon rate is 5%, coupon paid semi-annually, if the yield to maturity is 6%, do you think the trade price is reasonable?

$$P = \overset{1000 \times 5\%/2}{25}\left[\frac{1-(1+3\%)^{-6}}{3\%}\right] + \frac{1000}{(1+3\%)^6}$$

if $P > 950$. long

$P < 950$ short