Money and Banking
Introduction: Financial System

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Outline

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   - It influences long-term financing decisions of business, (e.g., corporate bonds) and that of government (e.g., Treasury bonds).
   - It influences households' financing decisions as well, e.g., mortgage rate.

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3. However, it is not the only way. We can employ **discount yield** and **current yield** and so on. Significant differences among those concepts
4. In addition, yield to maturity is different from rate of return.

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2. Real interest rate, by definition, excludes the influence of **inflation** (more precisely, expected inflation rate) from nominal interest rate, known as Fisher's equation.
3. In depth, real interest rate is determined by real economy, e.g., production and investment.
4. Fluctuations in real rates causes cycles of booms and busts.

Measuring Interest Rates

Basic Concepts

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3. Why do you need the way of thinking as present value?
4. Various investment has different **expected cash flows** , **maturity date**, and imbedded **risks**. Only today's value is certain.

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4. You will have $100 \times (1 + \frac{5\%}{2}) = \$102.5$, where \$2.5 is interest payment for six months. How about you deposit the proceeds in Bank of China for another six months?

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4. In general, we have

$$i_{\text{effective}} = (1+i)^n - 1\,, \tag{1}$$

where n is compounding periods.

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2. On short-term instruments there is usually only the one interest payment on maturity, hence simple interest is received when the instrument expires.
3. The terminal value of an investment with simple interest is given by

$$FV = PV(1+i)\,, \tag{2}$$

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$$FV = PV(1+i)^n \,, \tag{3}$$
3. If compounding takes place m times per year, then at the end of n years mn interest payments will have been made and the future value of the principle is given by
$$FV = PV(1+\frac{i}{m})^{mn} \,, \tag{4}$$

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   - monthly: $(1+\frac{i}{12})^{12}$,
   - daily: $(1+\frac{i}{365})^{365}$.
2. If compound interest rate continuously, we have

$$FV = PV\left\{\left(1+\frac{i}{m}\right)^{\frac{m}{i}}\right\}^{in} = PV\left\{\left(1+\frac{1}{m/i}\right)^{\frac{m}{i}}\right\}^{in}, \\ = PV\left\{\left(1+\frac{1}{n}\right)^{n}\right\}^{in}, \tag{5}$$

where $n = m/i$.

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2. Therefore, the continuous compounding can be approximately by

$$FV = PVe^{in}, \tag{7}$$

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4. Such convention makes the comparison between deposits and loans of different maturities and different instruments possible.

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4. If interest is just earned by the proportion of days during the investment period, we have

$$PV = \frac{FV}{(1+i\times\frac{\text{days}}{\text{year}})}, \tag{9}$$

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3. In discounting formula, $PV = FV(1+i)^{-n}$, the term $(1+i)^{-n}$ is known as the n-year **discount factor**.

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3. It is much common to come across a regular stream of future payments, i.e, **annuity**. In such case, expected cash flows C are identical in each period.

$$FV = C \sum_{n=1}^{N} (1+i)^{N-n} = C\left(\frac{(1+i)^N - 1}{i}\right), \tag{11}$$

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2. $$\begin{aligned} FV-(1+i)FV &= C\left(\sum_{n=1}^{N}(1+i)^{N-n}-\sum_{n=1}^{N}(1+i)^{N-n+1}\right), \\ &= -C\left((1+i)^{N}-1\right). \end{aligned} \tag{12}$$

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3. With some simple algebraic rearrangement, we will have the formula in last slide.

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2. $$\begin{aligned} PV &= \frac{FV}{(1+i)^N}, \\ &= C\left(\frac{(1+i)^N - 1}{i}\right)\left(\frac{1}{(1+i)^N}\right), \\ &= C\left(\frac{1-(1+i)^{-N}}{i}\right), \end{aligned} \tag{13}$$

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Bond Cash Flows

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$$\begin{aligned} P &= \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{M}{(1+i)^n}, \\ &= \sum_{n=1}^{N} \frac{C}{(1+i)^N} + \frac{M}{(1+i)^N}, \end{aligned} \tag{14}$$

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3. The first part of the formula, we can use annuity

$$C\left\{\frac{1-\frac{1}{(1+i)^n}}{i}\right\}, \tag{15}$$

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1. Example 1: Consider a 20-year 10% coupon bond with a par value of $1,000. Let's suppose that the required yield on this bond is 11%. Suppose the bond issuer pays investors coupon semiannually. What is the fair price of this bond?

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4. Then the present value of the bond is $919.77.

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4. The present value is $252.12.

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4. $\frac{70}{769.42} = 9.10\%$.

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4. The time value of money is also ignored.

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3. Yield to maturity computed on the basis of this market convention is called the **bond-equivalent yield**.

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2. In addition, the yield to maturity considers the timing of the cash flows.

Coupon Rate, Required Yield, and Price

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2. Economic interpretation (the third scenario): Investors have the opportunity to purchase the bond at par would be getting a coupon rate in excess of what the market requires.
3. As a result, investors would bid up the price of the bond because its yield is so attractive.

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2. The last component of the potential dollar return is referred to as **reinvestment income**. It is also known as **interest-on-interest component**.

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4. Then, interests-on-interests is $C\left[\frac{(1+i)^N-1}{i}\right]-nC$.

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   - coupon: the higher the coupon rate, the more dependent on bond's total dollar return will be on the reinvestment of the coupon payments in order to produce the yield to maturity anticipated at the time of purchase.

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2. **total return** is a measure of yield that incorporates an explicit assumption about the reinvestment rate.
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Bond	Coupon(%)	Maturity (years)	Yield to Maturity(%)
A	5	3	9.0
B	6	20	8.6
C	11	15	9.2
D	8	5	8.0

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   - Sum the values computed in step 1 and step 2.
   - obtain the total return, use the formula

$$\left[\frac{\text{total future dollars}}{\text{purchase price of bonds}}\right]^{1/n} - 1\,, \tag{19}$$

   where n is periodic number of coupon payments.

Computing the Total Return

A Numerical Example

1. Suppose that an investor with a three-year investment horizon is considering purchasing 20-year 8% coupon (paid semiannually) bond for \$828.40. The yield to maturity for this bond is 10%. The investor expects be to able to reinvest the coupon interest payments at an annual interest rate of 6% and that at the end of the planned investment horizon the then 17-year bond will be selling to offer a yield to maturity of 7%.

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2. Question: What is the total return on this bond investment?

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1. Compute the total coupon payments plus the interest on interest, assuming an annual reinvestment rate of 6%.

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5. Annualized rate of return is $2 \times 8.58\% = 17.16\%$.

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4. Continue and we have
$$1 + r_t + \pi^e_{t+1} + r_t\pi^e_{t+1} = 1 + i_t\,, \tag{22}$$

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4. Thus, asset bubble and general price level move up together.