Homework for Lecture 2

1 Introduction

How to measure interest rates are very challenging. We confront with different interest rates in our life. In the lecture 2, we have discussed the way of calculate *yield to maturity*, an important way to measure interest rate.

Interest rate can be considered as the price of fund borrowing from fund surplus side (e.g., households) to fund deficit side (e.g., governments and corporations). In the lecture, we introduced one way that a government agency i.e., Treasury department, borrowing funds. Treasury is able to issue Treasury bonds to raise funds from the public (including overseas investors).

The Treasury bonds have *face value*, i.e., the amount of money will pay back at maturity date, *maturity*, e.g., 1-year, or 5-year, or 10-year, *coupon*, i.e., the fixed amount of money that Treasury promises to pay until the bonds are redeemed.

An numerical example: three-year Treasury bonds with face value of $1,000, coupon rate is 3% p.a. (paid annually). It indicates that the Treasury will pay prospective investors within those three years with $30 per year ($1,000 \times 3\% = 30$), and by the maturity date, investors will receive $1,000.

When those bonds issued, investors will bid the price for holding the bonds, say $980. For Treasury, in the following three years, it is supposed to pay $90 coupons plus $1,000, while it receives $980 on the issuance date. Here comes the question: *What is the cost of borrowing money from investors?*

Yield to maturity is the simple way to do this measurement. It has two assumptions:

1. Investors are assumed to hold the bond until it matures. In this case, for three years.
2. Investors are able to reinvest the coupon with the rate equal to yield to maturity.

As we did in class, the yield to maturity (denoted as y) can be measured in the following way.

$$980 = 30\left[\frac{1-(1+y)^{-3}}{y}\right] + \frac{1,000}{(1+y)^3}, \tag{1.1}$$

$y = 3.72\%$.

2 Questions

Just now, we reviewed the way to calculate the yield to maturity with investors wiling to get this bond for \$980. Yet, when the economy is booming, investors will have many investment opportunities and they may not interested in buying government bonds. In that case, the bond price falls down to \$950. If so, what is the yield to maturity? (Hint: you can use Excel function IRR to get the yield to maturity.)

$$950 = 30\left[\frac{1-(1+y)^{-3}}{y}\right] + \frac{1,000}{(1+y)^3},$$

$y = 4.83\%$.

What is the yield to maturity when the bidding price of holding the bonds rises to \$1020?

What is the yield to maturity when the price of bonds is just \$1,000?

$$1,000 = 30\left[\frac{1-(1+y)^{-3}}{y}\right] + \frac{1,000}{(1+y)^3},$$

$y = 3\%$.

From those calculations, we learn that *yield to maturity* varies as the price of bonds change. In other words, the cost of borrowing for government depends on the supply and demand in financial market. *Coupon rate* is one way to compensate the time value for investors. For investors, they do not only care about the coupons they receive, but also how quickly the face value they could receive. That is why *yield to maturity* matters.

Based on those three numerical questions, *do you find any interesting relationship between yield to maturity and bond price*?

Suppose Treasury department now pays coupon semiannually, i.e., \$15, for every 6 months. Is that going to affect the yield to maturity?

$$980 = 15\left[\frac{1-(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right] + \frac{1,000}{(1+\frac{y}{2})^6},$$

$y = 1.86\%$.

Is there any difference in yield to maturity between the bonds with semiannually paid coupon and ones with annually paid bonds? How do you interpret such difference?