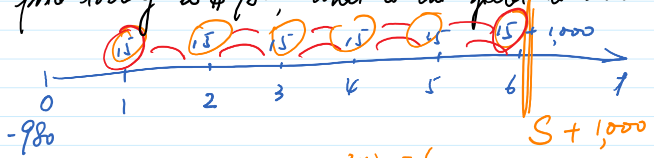
Lecture Review

government bond with 3-year maturity, coupon rate 3% (semi-annually), trading price today is \$980, what is the yield to maturity?



$1000 \times 3\% / 2 = 15$

$3 \times 2 = 6$

$$PV = C\left[\frac{1-(1+y)^{-n}}{y}\right] + \frac{\text{face value}}{(1+y)^n}$$

$$980 = 15\left[\frac{1-(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right] + \frac{1,000}{(1+\frac{y}{2})^6}$$

$$PV(1+i)^n = FV$$

$$980\left(1+\frac{y}{2}\right)^6 = S + 1,000$$

$$980 = \frac{S+1,000}{(1+\frac{y}{2})^6} = \frac{S}{(1+\frac{y}{2})^6} + \frac{1000}{(1+\frac{y}{2})^6}$$

$$= \frac{15\left[\frac{(1+\frac{y}{2})^6-1}{\frac{y}{2}}\right]}{(1+\frac{y}{2})^6} + \frac{1,000}{(1+\frac{y}{2})^6}$$

When investor receive \$15 in the first six-month, do reinvestment @ $\frac{y}{2}$ by the end of 3 years, how much is the interest on interest?

$$= 15\left[\frac{\cancel{(1+\frac{y}{2})^6}-1}{\frac{y}{2}\cancel{(1+\frac{y}{2})^6}}\right]$$

$$= 15\left[\frac{1-(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right] + \frac{1,000}{(1+\frac{y}{2})^6}$$

$15(1+\frac{y}{2})^5$ first coupon payment

$15(1+\frac{y}{2})^4$ second

⋮

$$\frac{1}{x} = x^{-1}$$

$$\left[\frac{(1+\frac{y}{2})^6}{\frac{y}{2}(1+\frac{y}{2})^6} - \frac{1}{\frac{y}{2}(1+\frac{y}{2})^6}\right]$$

$$\left[\frac{1}{\frac{y}{2}} - \frac{(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right]$$

$$S = 15\cancel{(1+\frac{y}{2})^5} + 15\cancel{(1+\frac{y}{2})^4} + \cdots + 15 \quad \cdots ①$$

$$\left(1+\frac{y}{2}\right)S = 15\left(1+\frac{y}{2}\right)^6 + 15\cancel{(1+\frac{y}{2})^5} + \cdots + \cancel{15(1+\frac{y}{2})} \quad \cdots ②$$

② − ① Left hand side: $(1+\frac{y}{2})S - S = \frac{y}{2}S$

Right hand side: $15(1+\frac{y}{2})^6 - 15$

$$\Rightarrow \frac{y}{2}S = 15\left[\left(1+\frac{y}{2}\right)^6 - 1\right]$$

$$\Rightarrow S = 15\left[\frac{(1+\frac{y}{2})^6-1}{\frac{y}{2}}\right] \quad \text{equivalent to ①}$$

$$= \left[\frac{1-(1+\frac{y}{2})^{-6}}{\frac{y}{2}}\right]$$

multiply −1 on both sides of ①: $-S = \cancel{-15(1+\frac{y}{2})^5} - 15\cancel{(1+\frac{y}{2})^4} - 15\cancel{(1+\frac{y}{2})^3} - 15(1+\frac{y}{2})^2 - 15(1+\frac{y}{2}) - 15$

$$\left(1+\frac{y}{2}\right)S = 15\left(1+\frac{y}{2}\right)^6 + 15\cancel{(1+\frac{y}{2})^5} + 15\cancel{(1+\frac{y}{2})^4} + 15\cancel{(1+\frac{y}{2})^3} + 15\cancel{(1+\frac{y}{2})^2} + 15\cancel{(1+\frac{y}{2})}$$

$$\frac{y}{2}S = 15\left[\left(1+\frac{y}{2}\right)^6 - 1\right] = 15\left[\frac{(1+\frac{y}{2})^6-1}{\frac{y}{2}}\right]$$

	A	B	C
1	-980		
2	15		
3	15		
⋮	⋮		
6	15		
7	1015		
8			

= 2 × IRR(A1:A7)

Internal Rate of Return = yield to maturity

Expectation Theory

indifference holding one-year bond for 2 years and holding a 2-year bond

$$1(1+i_{1,t})(1+i^{e}_{1,t+1})$$

1 year

Expectation (forecast)

return from one-year bond investment

$$1(1+i_{2,t})(1+i_{2,t})$$

2 year

How about the $i_{2,t} > i^{e}_{1,t+1}$

risk-free arbitrage

say $: ¥ 1:8 | $: ¥ 1:10

US i_u 3% | borrow $ @ 3% — convert $ -> ¥

CHINA i_c 6% | ¥ @ 6% for 1 year

$$(1+i_{1,t})(1+i^{e}_{1,t+1}) = (1+i_{2,t})^2$$

3% × 6%

$$\cancel{1} + i^{e}_{1,t+1} + i_{1,t} + i_{1,t}\, i^{e}_{1,t+1} = \cancel{1} + 2i_{2,t} + i^{2}_{2,t}$$

$$i^{e}_{1,t+1} + i_{1,t} \approx 2i_{2,t}$$

$$\Rightarrow i_{2,t} \approx \frac{i_{1,t} + i^{e}_{1,t+1}}{2}$$

0% 100%