

Assignment 1

Due Feb 9th, 2026 at 9:00 am. Late submissions will be graded with a 1. The assignment will be graded based on effort!

1. (4 points) Show that Assumptions 1.2 and 1.4 (see Hayashi or lecture notes) imply

$$Var(\varepsilon_i) = \sigma^2, \quad (i = 1, \dots, n) \quad (1)$$

$$Cov(\varepsilon_i, \varepsilon_j) = 0, \quad (i, j = 1, \dots, n, i \neq j) \quad (2)$$

2. (2 points) A study relating college GPA (grades average) to time spent in various activities has a data set which includes, for each student observed, the average number of hours spent each week in four activities: studying (st), sleeping (sl), working (w), leisure (l). Any activity is put into one of four categories, so that for each student the sum of hours in the four activities must be 168. It is proposed that the following model should be used:

$$GPA_i = \beta_0 + \beta_1 st_i + \beta_2 sl_i + \beta_3 w_i + \beta_4 l_i + \varepsilon_i$$

- (a) Explain clearly why this model cannot be used to obtain parameter estimates? Consider this in the context of the four assumptions.
- (b) How could the model be reformulated so that it can be used to estimate the parameters?
3. (4 points) Consider the following two regressions

$$y_i = \beta_1 + \beta_2 \ln(x_{i2}) + \beta_3 x_{i3} + \varepsilon_i$$

$$y_i = \alpha_1 + \alpha_2 \ln(x_{i2}^*) + \alpha_3 x_{i3}^* + \varepsilon_i$$

where $x_{i2}^* = x_{i2}/1000$ and $x_{i3}^* = x_{i3}/1000$. Stacking the n observations gives:

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \text{ and}$$

$$\mathbf{y} = \mathbf{Z}\alpha + \varepsilon$$

where $\mathbf{y} = (y_1, \dots, y_n)'$; $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)'$, $\beta = (\beta_1, \beta_2, \beta_3)'$, $\alpha = (\alpha_1, \alpha_2, \alpha_3)'$ and

$$\mathbf{X} = \begin{pmatrix} 1 & \ln(x_{12}) & x_{13} \\ 1 & \ln(x_{22}) & x_{23} \\ \dots & \dots & \dots \\ 1 & \ln(x_{n2}) & x_{n3} \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} 1 & \ln(x_{12}^*) & x_{13}/1000 \\ 1 & \ln(x_{22}^*) & x_{23}/1000 \\ \dots & \dots & \dots \\ 1 & \ln(x_{n2}^*) & x_{n3}/1000 \end{pmatrix}$$

- (a) Show that the columns of \mathbf{Z} can be written as linear combinations of the columns of \mathbf{X} , i.e. $\mathbf{Z} = \mathbf{X}\mathbf{A}$ for some (3×3) matrix \mathbf{A} . Find the elements of \mathbf{A} .
- (b) Let $\mathbf{b} = (b_1, b_2, b_3)' = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ the OLS estimator for β and $\mathbf{a} = (a_1, a_2, a_3)' = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}$ the OLS estimator for α . Show that
- (i) $\mathbf{a} = \mathbf{A}^{-1}\mathbf{b}$
 - (ii) $a_2 = b_2$
 - (iii) $a_1 = b_1 + b_2 \ln(1000)$