Integrating Classic Combinatorial Optimization Problems for Drone Wildfire Monitoring
(Paper Proposal)

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Abstract

Wildfires pose an increasingly significant threat to both human populations and natural ecosystems. Advancements in drone technology provide a promising platform for proactive wildfire monitoring, yet efficient deployment strategies are vital. This paper explores how three well-known combinatorial optimization problems—the Nurse Scheduling Problem (NSP), the Knapsack Problem (KP), and the Traveling Salesman Problem (TSP)—can be adapted to address different aspects of drone-based wildfire surveillance. We demonstrate how (1) the NSP framework ensures adequate and balanced coverage of a wildfire-prone area, (2) the KP formulation optimizes drone resource allocation (e.g., flight time or energy), and (3) the TSP-based approach reduces the total travel distance or time required for monitoring. By combining these three methods, we present a comprehensive solution that maximizes coverage of high-risk regions while respecting operational constraints.

1 Introduction

Wildfires are becoming more frequent and severe, fueled by climate change and land-use practices. Early detection and continuous monitoring are crucial to mitigate potential damage. Unmanned Aerial Vehicles (UAVs), commonly known as drones, offer a flexible and cost-effective alternative to traditional aerial surveillance methods. However, deploying multiple drones across vast, wildfire-prone regions requires careful consideration of:

- Ensuring that all high-risk grid cells are appropriately monitored.
- Respecting each drone's operational constraints, such as limited battery life or flight time.
- Optimizing the travel routes to minimize unnecessary flight distance or energy consumption.

Addressing these challenges effectively necessitates robust mathematical models and optimization techniques. In this paper, we explore three classic combinatorial optimization problems and adapt them to drone wildfire monitoring:

1. The **Nurse Scheduling Problem (NSP)**: Assigns resources to tasks (or shifts), ensuring coverage requirements are met and constraints are satisfied.
2. The **Knapsack Problem (KP)**: Maximizes value (e.g., coverage of high-risk cells) subject to capacity constraints (e.g., drone battery or flight time).
3. The **Traveling Salesman Problem (TSP)**: Minimizes total travel distance or time for visiting a set of locations.

We demonstrate how these three methods can complement each other to form a holistic solution for wildfire surveillance.

2 Nurse Scheduling Problem (NSP) for Drone Assignment

2.1 NSP Overview

The Nurse Scheduling Problem (NSP) is a well-known combinatorial optimization challenge where nurses (resources) are assigned to shifts (tasks) in a way that meets patient or operational requirements while respecting constraints such as rest periods and staff fairness. The NSP often employs a weighted objective function:

$$H(X) = H_1(X) + \lambda H_2(X) + \gamma H_3(X),$$

where:

- $H_1(X)$ penalizes consecutive shifts assigned to the same nurse, ensuring sufficient rest.
- $H_2(X)$ ensures staffing levels meet the required demand for each shift.
- $H_3(X)$ encourages equitable distribution of shifts among nurses.
- λ, γ are weighting factors controlling the importance of these terms.

2.2 Adapting NSP to Drone Monitoring

Analogous to nurses being scheduled for shifts, drones must be *assigned* to monitor specific grid cells over a planning horizon. We divide the wildfire-prone region into m grid cells, each with a fire risk level p_i. In a simplified version:

- Drones are "resources," similar to nurses.
- Grid cells (or time slots for monitoring each cell) are "shifts."
- Constraints include ensuring each grid cell is covered (akin to staffing requirements) and preventing overuse of any single drone (akin to rest requirements).

2.3 NSP-Based Formulation for Drone Coverage

We formulate the drone assignment problem to maximize weighted coverage of high-risk cells:

$$\text{Maximize: } H(X) = \sum_{d=1}^{n} \sum_{i=1}^{m} p_i \, x_{d,i},$$

subject to:

$$\sum_{d=1}^{n} x_{d,i} \leq 1, \qquad \forall i \in \{1, \ldots, m\}, \tag{1}$$

$$\sum_{i=1}^{m} x_{d,i} \leq t_d, \qquad \forall d \in \{1, \ldots, n\}, \tag{2}$$

$$x_{d,i} \in \{0, 1\}, \qquad \forall d,\, i. \tag{3}$$

Here:

- p_i is the fire risk level of cell i.
- $x_{d,i}$ is a binary decision variable set to 1 if drone d monitors cell i, and 0 otherwise.
- Constraint (1) ensures each grid cell is monitored by at most one drone.
- Constraint (2) prevents exceeding each drone's maximum operational capacity t_d.

2.4 Discussion

Using the NSP structure provides a powerful way to ensure global coverage with balanced workload distribution among drones. In real-world scenarios, more nuanced constraints (e.g., mandatory rest times, flight corridor restrictions) can be added similarly to how additional constraints are introduced in classical NSP formulations.

3 Knapsack Problem (KP) for Resource Allocation

3.1 KP Overview

The Knapsack Problem (KP) involves selecting items, each with a value and cost, to maximize total value without exceeding a capacity. It is often stated as:

$$\text{Maximize: } \sum_{i=1}^{m} v_i\, x_i \quad \text{subject to: } \quad \sum_{i=1}^{m} w_i\, x_i \leq W,$$

where v_i is the value of item i, w_i is its cost (weight), W is the knapsack capacity, and x_i is a binary decision variable.

3.2 KP in Drone Monitoring

In drone wildfire monitoring:

- **Items:** The grid cells to be monitored.
- **Value (v_i):** Fire risk level (p_i), emphasizing high-risk cells.
- **Cost (w_i):** Time or energy required to monitor cell i (c_i).
- **Capacity (W):** Each drone's operational limit (C_d), typically battery or flight time.

3.3 Formulating KP for Drone Monitoring

Each drone solves an individual knapsack problem:

$$\text{Maximize: } H_d(X) = \sum_{i=1}^{m} p_i \, x_{d,i},$$

subject to:

$$\sum_{i=1}^{m} c_i \, x_{d,i} \leq C_d, \quad x_{d,i} \in \{0, 1\}.$$

Thus, drones prioritize high-risk cells while staying within resource limits.

3.4 Advantages of KP in This Context

- **Prioritized Coverage:** Focus on cells that yield the greatest *value* (fire risk).
- **Efficient Resource Use:** Ensures each drone's capacity is allocated to the highest-priority cells.
- **Modular Computation:** Each drone can solve its own knapsack problem independently, facilitating scalability.

4 Traveling Salesman Problem (TSP) for Route Optimization

4.1 TSP Overview

The Traveling Salesman Problem (TSP) seeks the shortest route to visit a set of cities (locations) and return to the origin, often formulated as:

$$\text{Minimize: } \sum_{i=1}^{k} \text{dist}(v_i, v_{i+1}) + \text{dist}(v_k, v_1),$$

where $\text{dist}(v_i, v_j)$ is the distance between locations v_i and v_j.

4.2 TSP in Drone Monitoring

After determining which cells a drone should monitor (via NSP or KP), one must decide the *order* in which the drone visits these cells:

- **Locations:** Grid cells assigned to a given drone.
- **Distance:** Travel cost between any two cells (could be flight distance, flight time, or energy consumption).
- **Objective:** Minimize total travel to conserve battery and reduce monitoring time.

4.3 Formulating TSP for Drone Routing

For drone d, if it has k_d assigned cells $\{v_1, \ldots, v_{k_d}\}$, then:

$$\text{Minimize: } D(X) = \sum_{i=1}^{k_d - 1} \text{dist}(v_i, v_{i+1}) + \text{dist}(v_{k_d}, v_1).$$

Solving this TSP (or approximate variations) yields an optimal (or near-optimal) route for covering those k_d cells.

4.4 Benefits of TSP Integration

- **Reduced Operational Cost:** Less distance flown translates into fewer battery replacements and lower maintenance.
- **Time Efficiency:** Faster route completion allows more frequent monitoring updates.
- **Scalability:** Each drone solves its own TSP on its subset of cells, avoiding the combinatorial explosion of a single large TSP instance.

5 Combining NSP, KP, and TSP for Drone Wildfire Monitoring

5.1 High-Level Workflow

1. **NSP for Coverage Assignment:** Assigns each grid cell to a drone (or determines which drone is *primarily responsible*), ensuring coverage constraints are met.
2. **KP for Resource Allocation:** Each drone, given its assigned cells, selects which cells to monitor within its capacity (if not all can be covered), prioritizing high-risk cells.
3. **TSP for Route Optimization:** Once a drone finalizes *which* cells it will monitor, it determines the optimal route to visit these cells.

5.2 Illustrative Example

Imagine a region with $m = 10$ cells, each with a risk level p_i and a monitoring cost c_i. Suppose $n = 3$ drones, each with capacity $C_d = 100$ units of energy. A combined approach might proceed as follows:

1. **NSP-like Assignment:** Ensure that each of the 10 cells is assigned to at most one drone, balancing the total number of cells among the 3 drones.

2. **KP Resource Check:** Each drone examines its assigned cells. If the total cost exceeds its capacity, it chooses a subset of cells that maximizes the sum of fire risk levels.

3. **TSP Routing:** For the chosen subset of cells, each drone solves a TSP to find the sequence of visits that minimizes flight distance.

This integrated methodology achieves broad coverage, prioritizes high-risk areas, and ensures minimal travel time.

6 Practical Considerations and Extensions

- **Real-Time Updates:** Wildfire conditions can change rapidly; dynamic or rolling-horizon versions of NSP, KP, and TSP allow re-optimization.

- **Heterogeneous Drones:** Different drones may have varying capacities, speeds, or sensor types, necessitating slight modifications to the formulations.

- **Environmental Factors:** Terrain, wind, and weather can alter travel cost functions in the TSP or reduce battery capacity in the KP.

- **Communication Constraints:** Drone-to-drone or drone-to-base communication can impose additional constraints on scheduling and routing.

7 Conclusion

Drone-based wildfire monitoring is a complex but critical task that can significantly benefit from combinatorial optimization frameworks. We have shown:

- How the **Nurse Scheduling Problem (NSP)** conceptually maps to assigning drones to cells in a balanced manner.

- How the **Knapsack Problem (KP)** ensures that each drone's limited resources are used optimally by prioritizing high-risk cells.

- How the **Traveling Salesman Problem (TSP)** framework optimizes the route for each drone, minimizing travel distance and time.

By integrating these three methods, decision-makers can design a robust, multi-drone strategy for wildfire surveillance, with potential for real-time updates in dynamic fire situations. Future work could explore machine learning-based risk modeling, multi-objective optimization (e.g., balancing coverage with drone safety), and decentralized solution strategies.

References

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