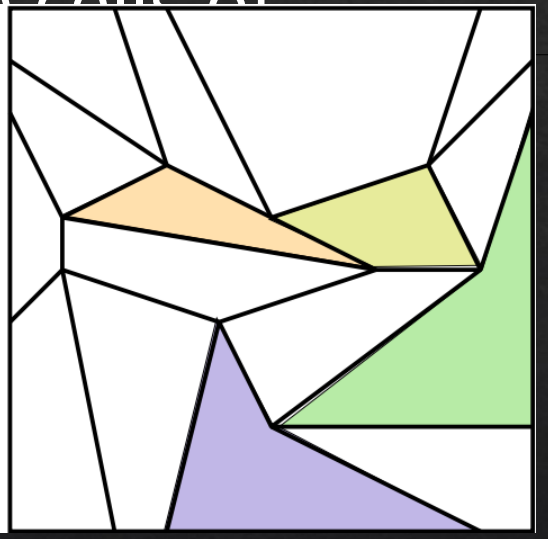
An Example of 2D cartography

Table Cartogram

Table Cartogram

A table cartogram of a two dimensional $m \times n$ table A of non-negative weights in a rectangle R, whose area equals the sum of the weights, is a partition of R into convex quadrilateral faces corresponding to the cells of

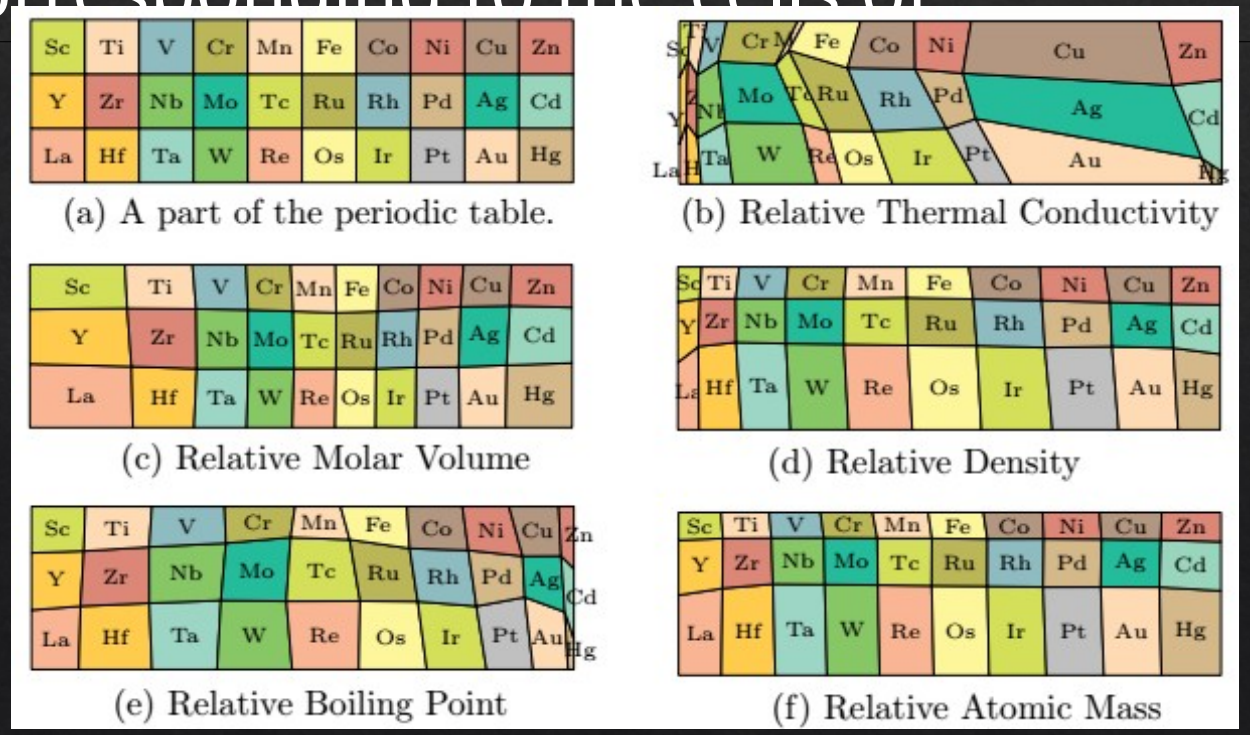
4.5	4.5	16	2.5
4	3	4.5	3
2.5	6	4.5	10.5
7	9	9	6



A such that each face has the same adjacency as its corresponding cell and has area equal to the cell's weight.

Table Cartogram

A table cartogram of a two dimensional $m \times n$ table A of non-negative weights in a rectangle R, whose area equals the sum of the weights, is a partition of R into convex quadrilateral faces corresponding to the cells of A such that each face has the same adjacency as its corresponding cell and has area equal to the cell's weight.



(a) A part of the periodic table.

(b) Relative Thermal Conductivity

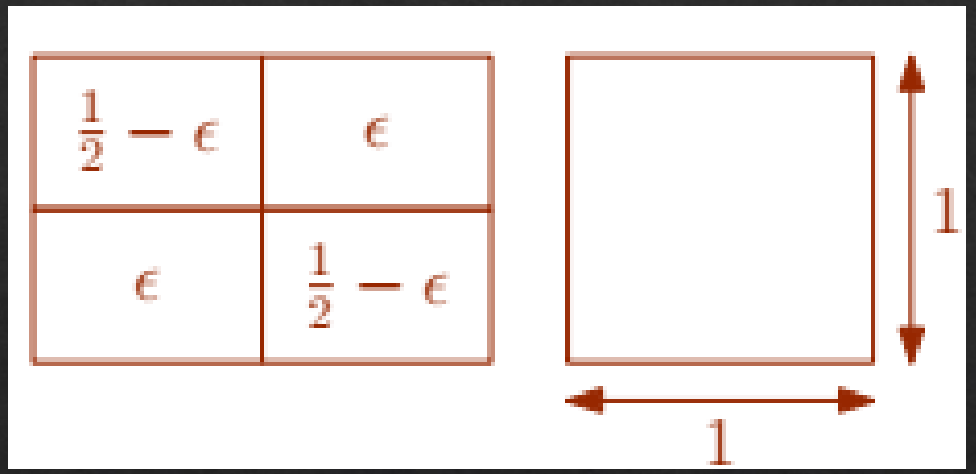
(c) Relative Molar Volume

(d) Relative Density

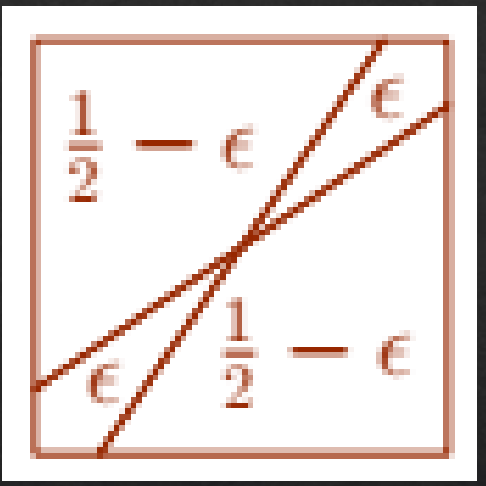
(e) Relative Boiling Point

(f) Relative Atomic Mass

Table Cartogram



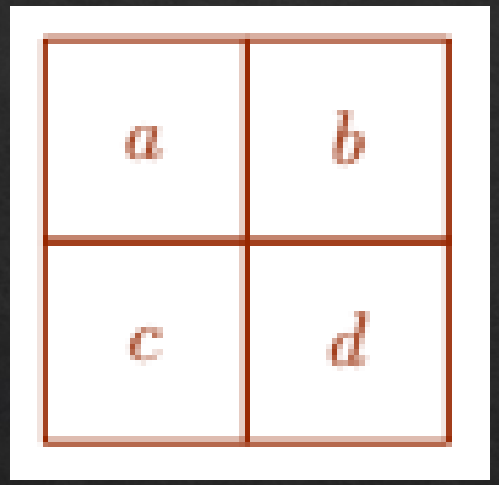
Input

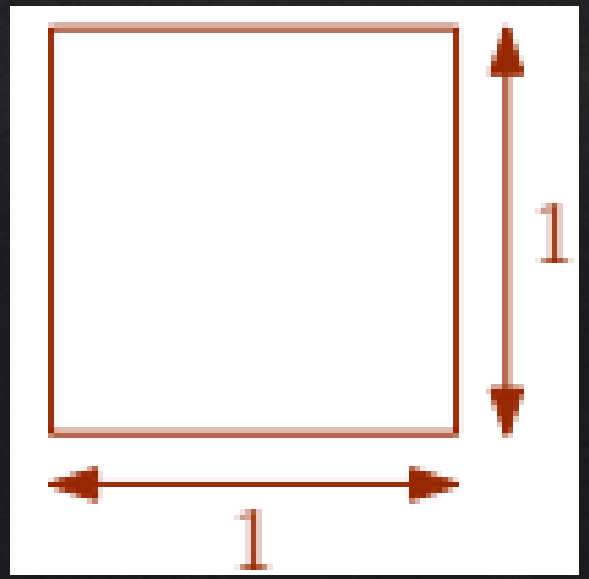


Output

How to Compute this?

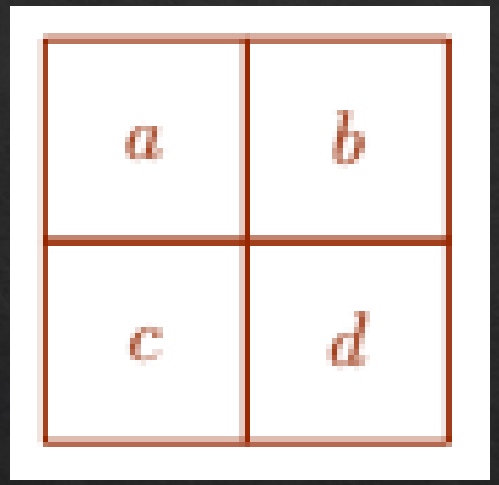
Table Cartogram: Case 2 x 2 table





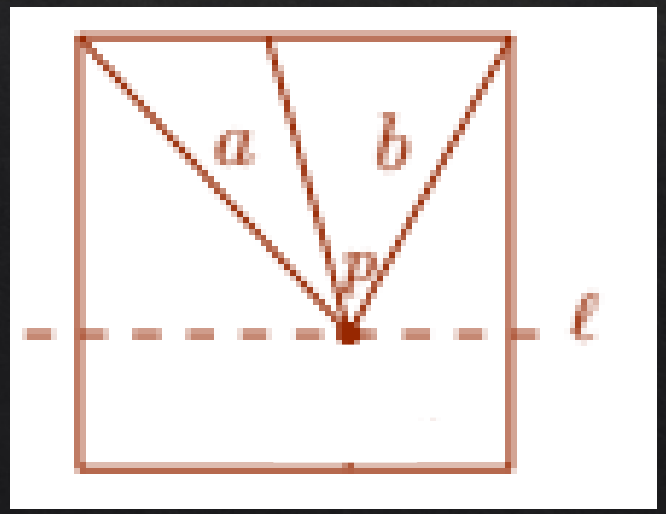
Without loss of generality assume that $a + b + c + d = 1$ and $a + b \leq (1/2)$

Table Cartogram: Case 2 x 2 table



Without loss of generality assume that $a + b + c + d = 1$ and $a + b \leq (1/2)$

Now consider the horizontal line with the property that every triangle *T (p)* with top side equal to the top side of *R* and one corner *p* on has area $a + b$.



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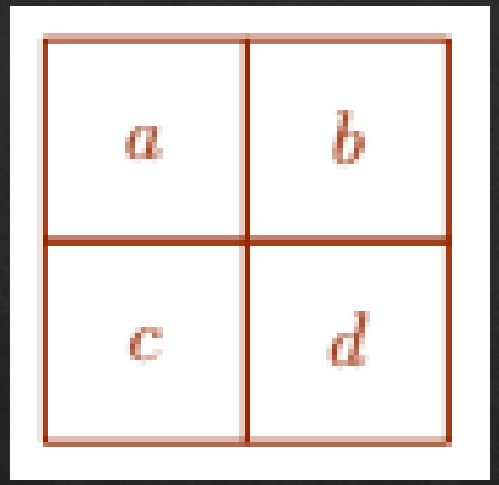
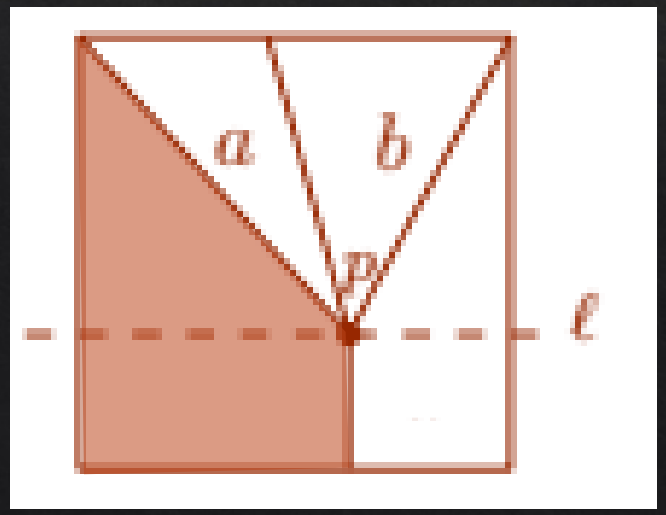


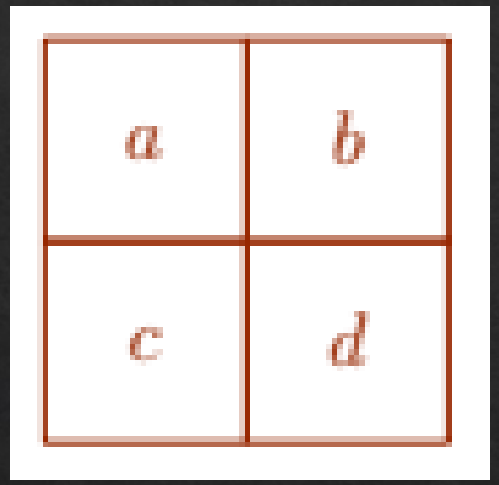


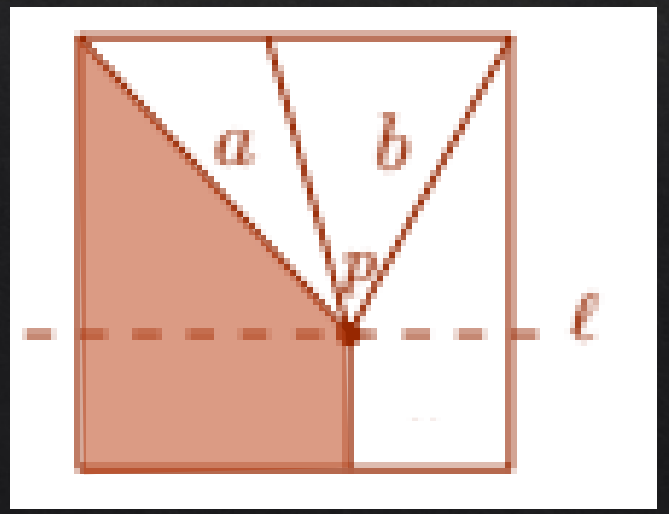
Without loss of generality assume that $a + b + c + d = 1$ and $a + b \leq (1/2)$

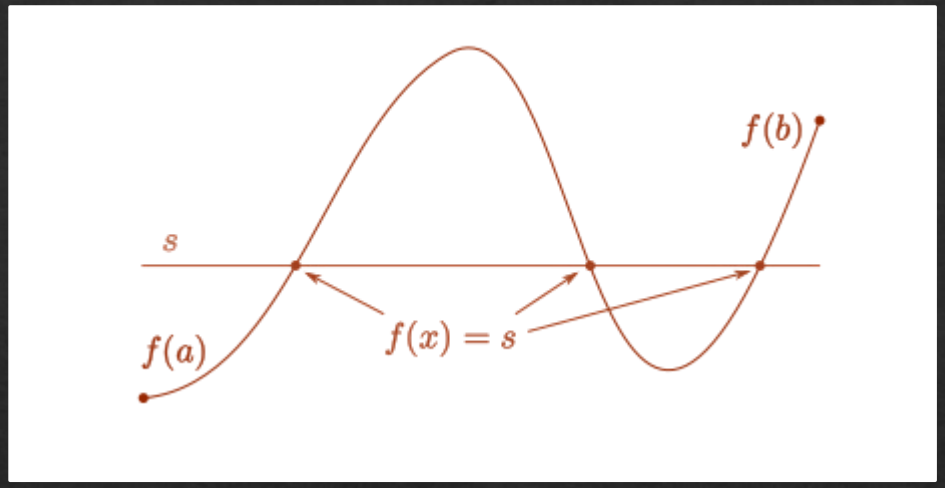
Now consider the horizontal line with the property that every triangle *T (p)* with top side equal to the top side of *R* and one corner *p* on has area $a + b$.

Consider a line through p and think what would happen to the left region if you move *p* along the line!

Table Cartogram: Case 2 x 2 table

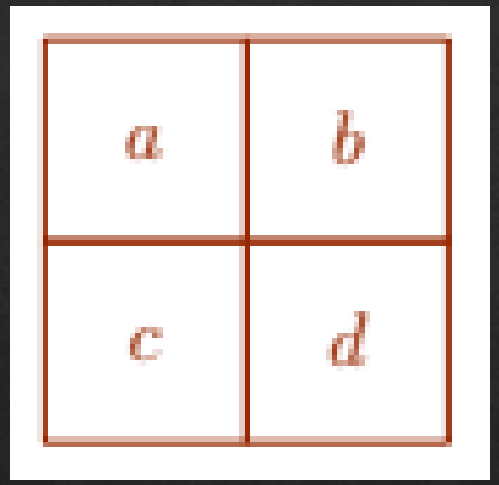




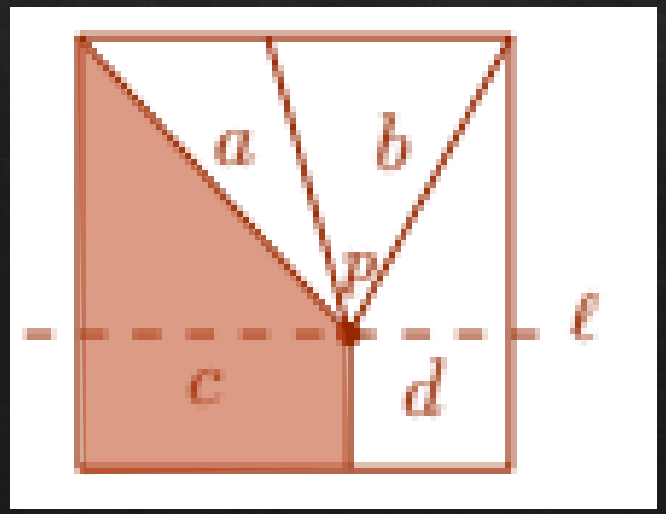


Intermediate value theorem: Let *f* be a defined continuous function on [*a*, *b*] and let *s* be a number with $f(a) < s < f(b)$. Then there exists at least one *x* with $f(x) = s$

Table Cartogram: Case 2 x 2 table



By intermediate value theorem there is a place for *p* on *l* such that the left region is has exactly *c* area, and the right side with *d* area

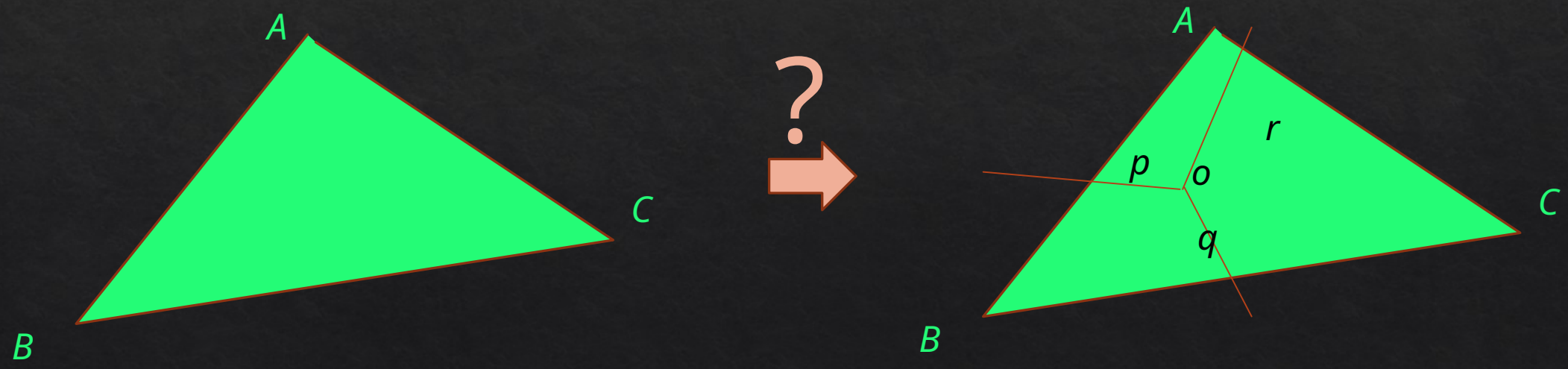


Think – how to compute that place for *p*?

Table Cartogram: General Case

Sidetrack: Barycentric Coordinates

Given ΔABC such with area 1, and three numbers p, q, r that sum to 1.
Can you split the triangle into three triangles ABo, BCo, ACo with area equal to p, q, r, respectively?



The idea relates to barycentric coordinates – explore on your own interest
https://en.wikipedia.org/wiki/Barycentric_coordinate_system

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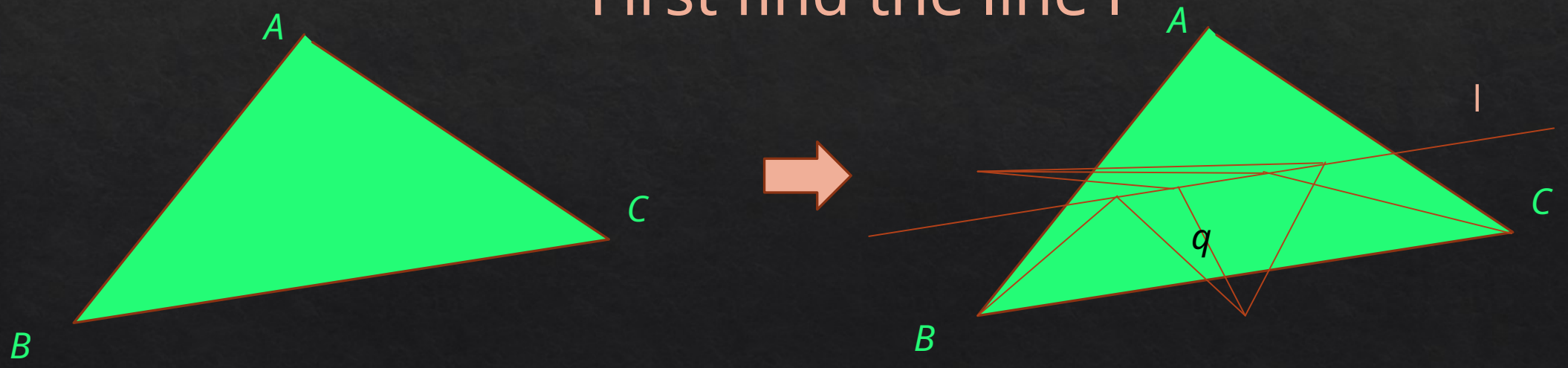
YES!

First find the line l



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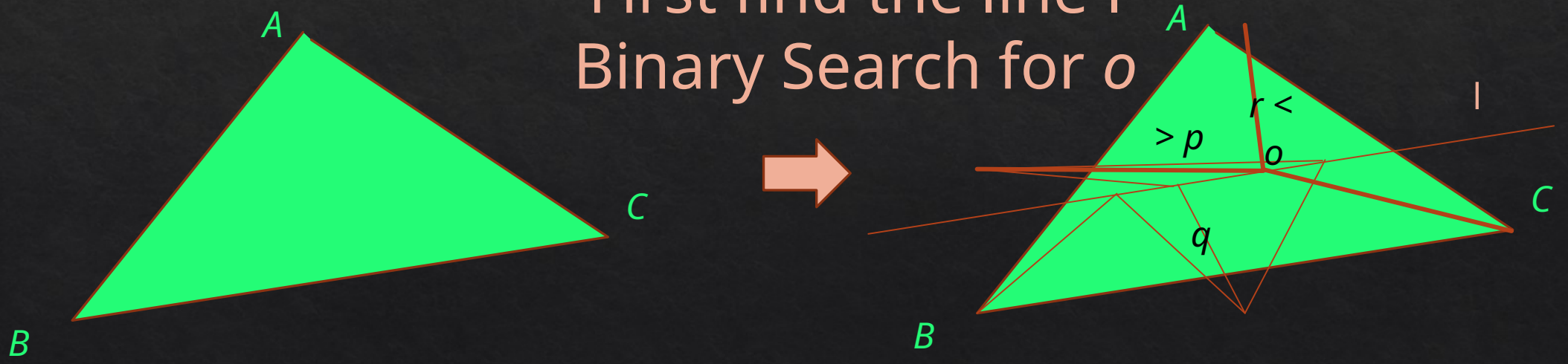
Binary Search for *o*



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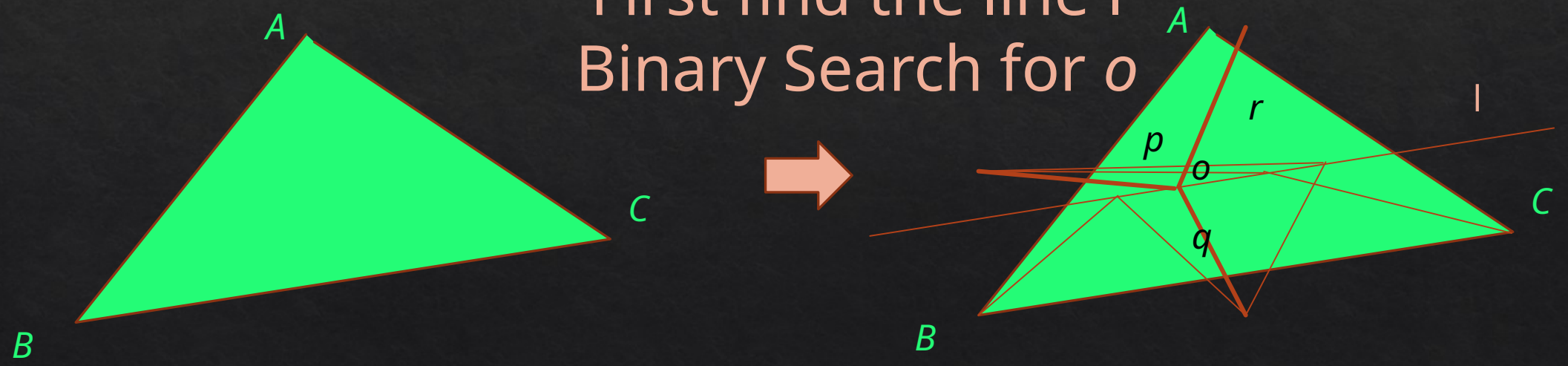

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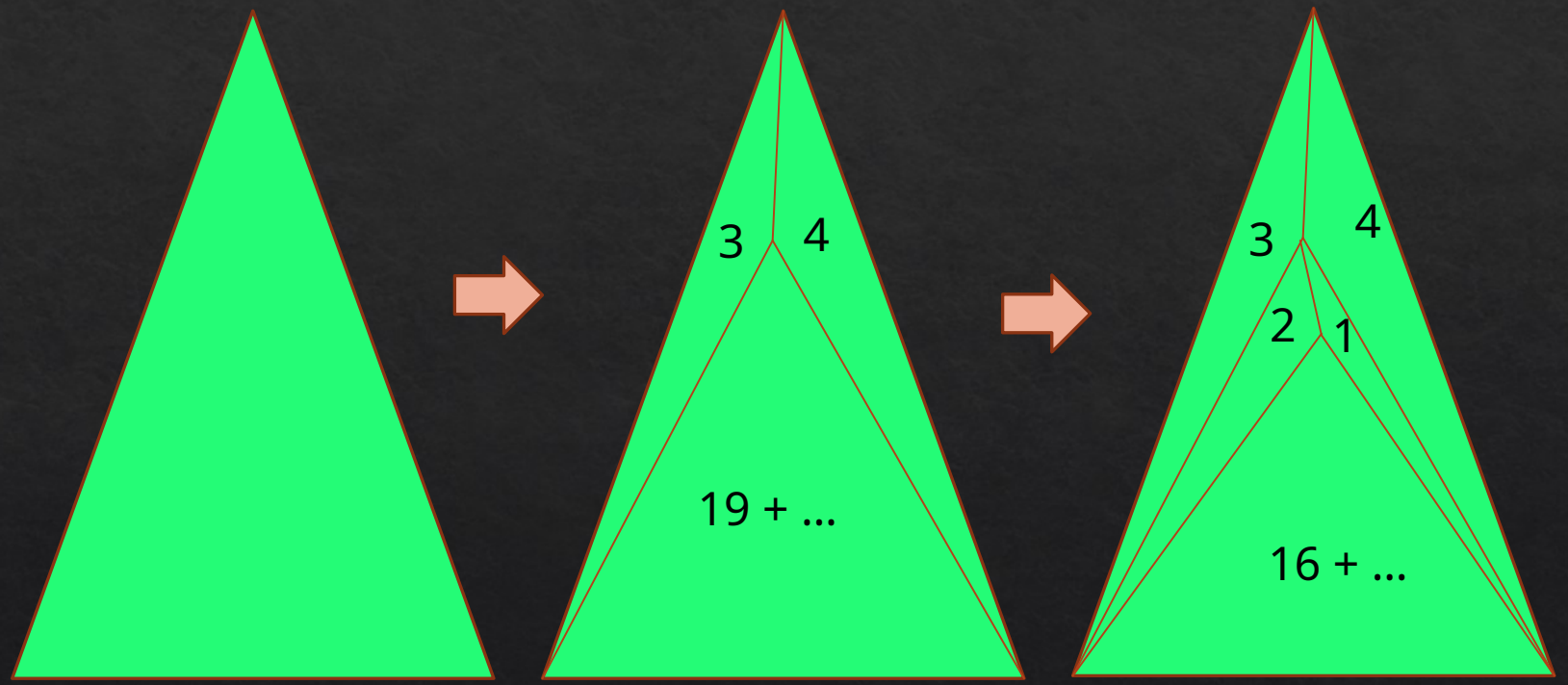
Sidetrack: Barycentric Coordinates

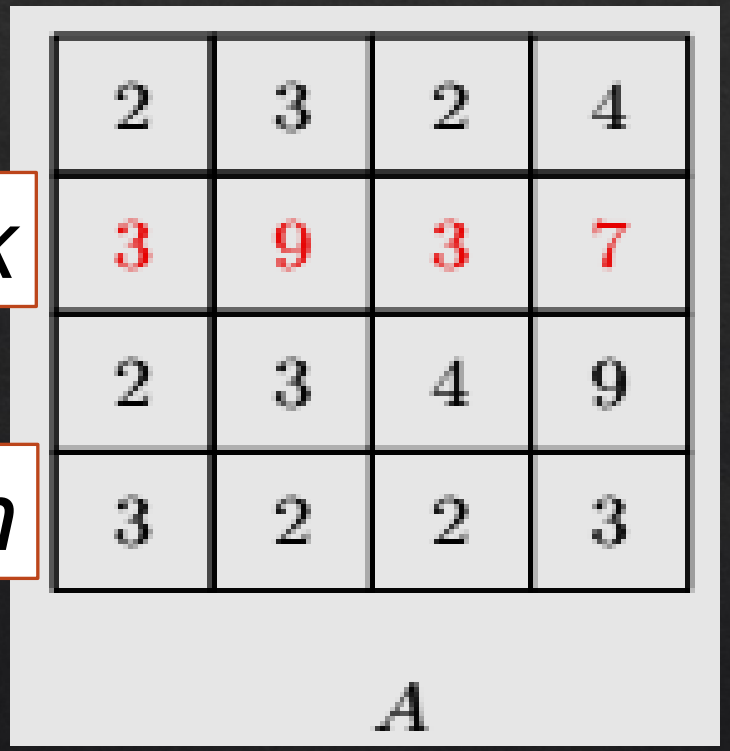
Given 2 x n Table, we can find a split-triangle cartogram!

3	4
2	1
4	5
3	4
…	…



Total area 26+…

Table Cartogram: General Case



S = total sum

Let S_i be the sum of the ith row $S_i = \sum_{1 \le j \le n} A_{i,j}$

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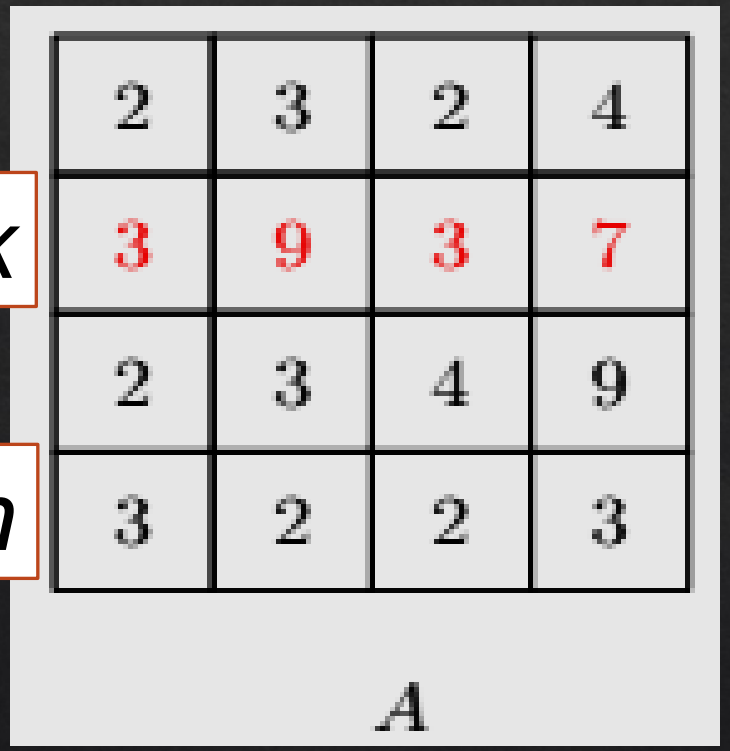
Split a row to get two smaller tables.



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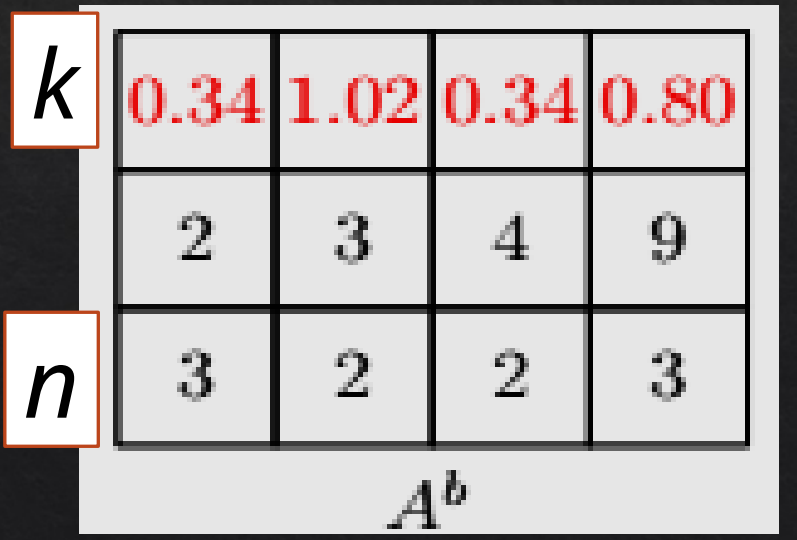

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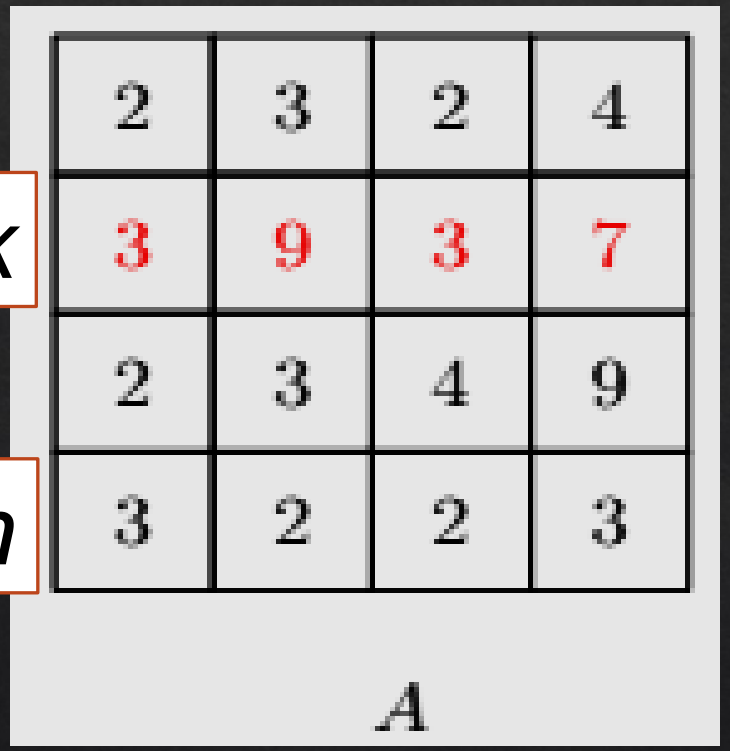

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$\sum_{k+1 \le i \le n} S_i + (1-\lambda) S_k = S/2.$

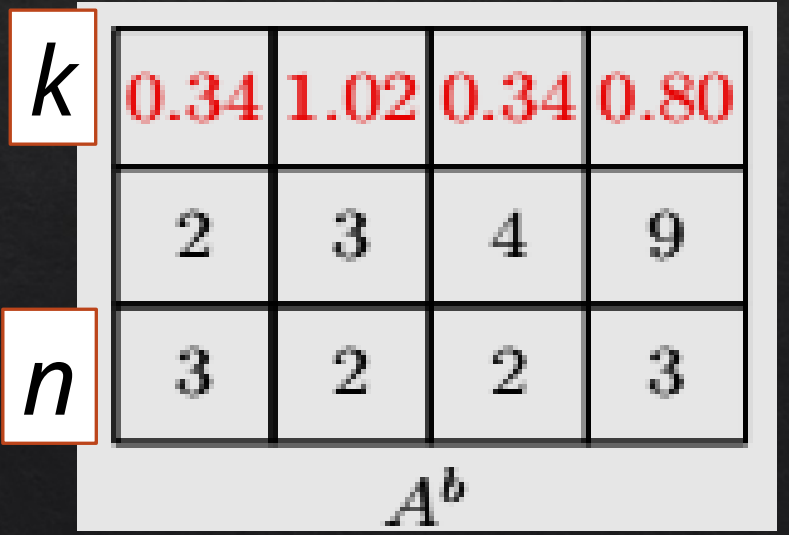

Table Cartogram: General Case

2	3	2	4
3	9	3	7
2	3	4	9
3	2	2	3

k n

A

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2	3	2	4
2.66	7.98	2.66	6.20

k

A^t

0.34	1.02	0.34	0.80
2	3	4	9
3	2	2	3

k n

A^b

Table Cartogram: General Case

2	3	2	4
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In this example $\lambda \approx 0.886$

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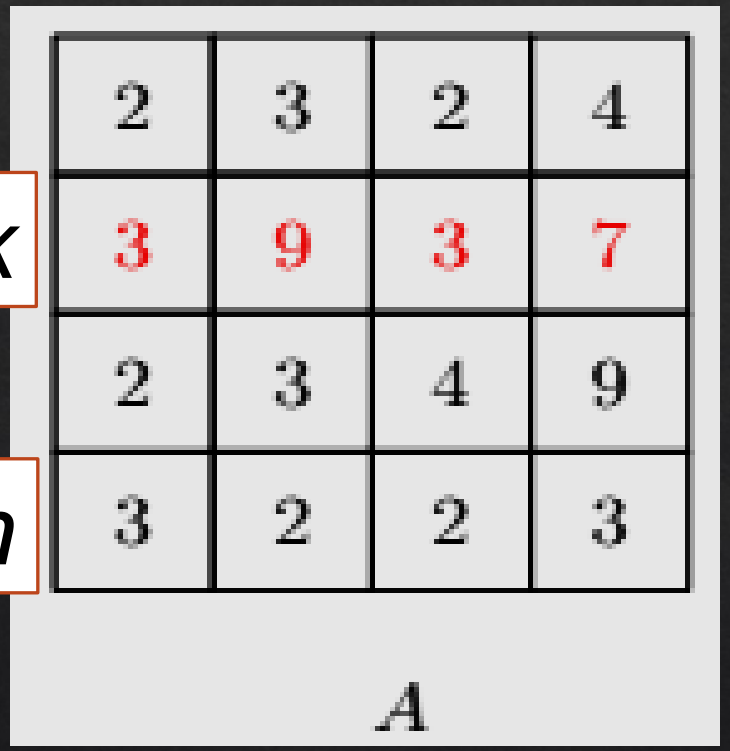
Table Cartogram: General Case



S = total sum

How to Compute λ ?

Split a row to get two smaller tables.

$$\sum_{1 \le i \le k-1} S_i + \lambda S_k = S/2.$$

$$\sum_{k+1 \le i \le n} S_i + (1-\lambda) S_k = S/2.$$

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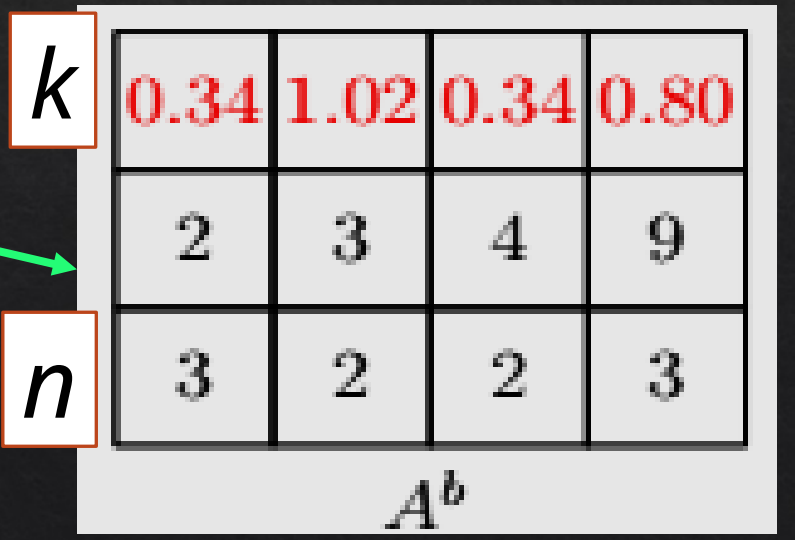

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k n

A

S = total sum

How to Compute λ ?

Split a row to get two smaller tables.

$$\sum_{1 \le i \le k-1} S_i + \lambda S_k = S/2.$$

Guess λ between 0 and 1, say λ = 0.5

If $\sum_{1 \le i \le k-1} S_i + \lambda S_k > S/2$

Guess between 0 and 0.5

else Guess between 0.5 and 1

2	3	2	4
2.66	7.98	2.66	6.20

k

A^t

0.34	1.02	0.34	0.80
2	3	4	9
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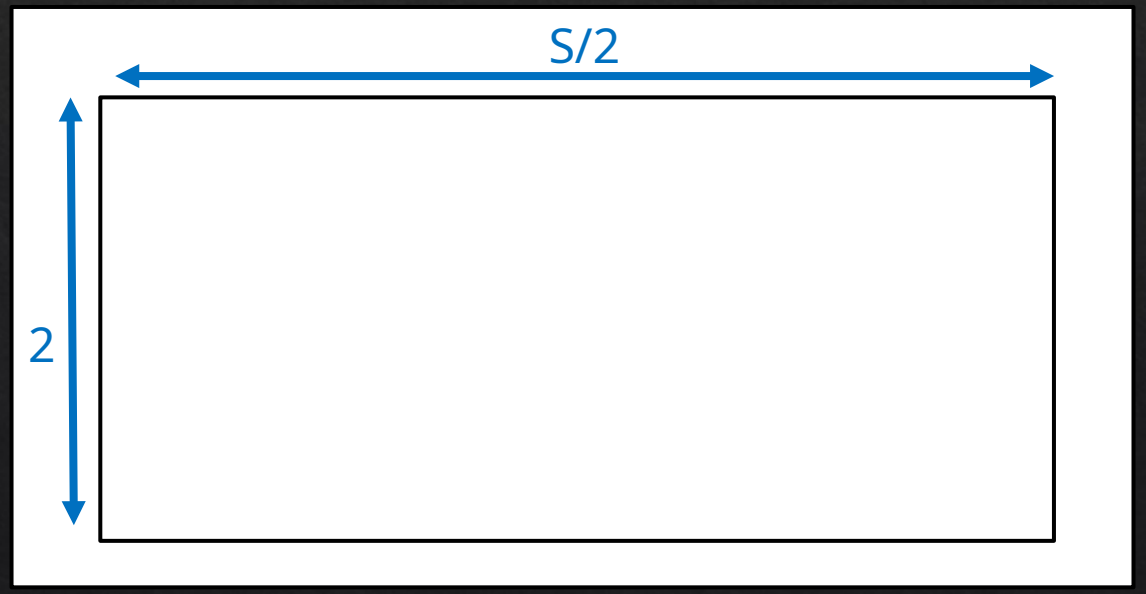
k n

A^b

Table Cartogram: General Case

Take a rectangle with height 2 and width S/2
Area is equal to the area of the table *A*



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k	2.66	7.98	2.66	6.20

A^t

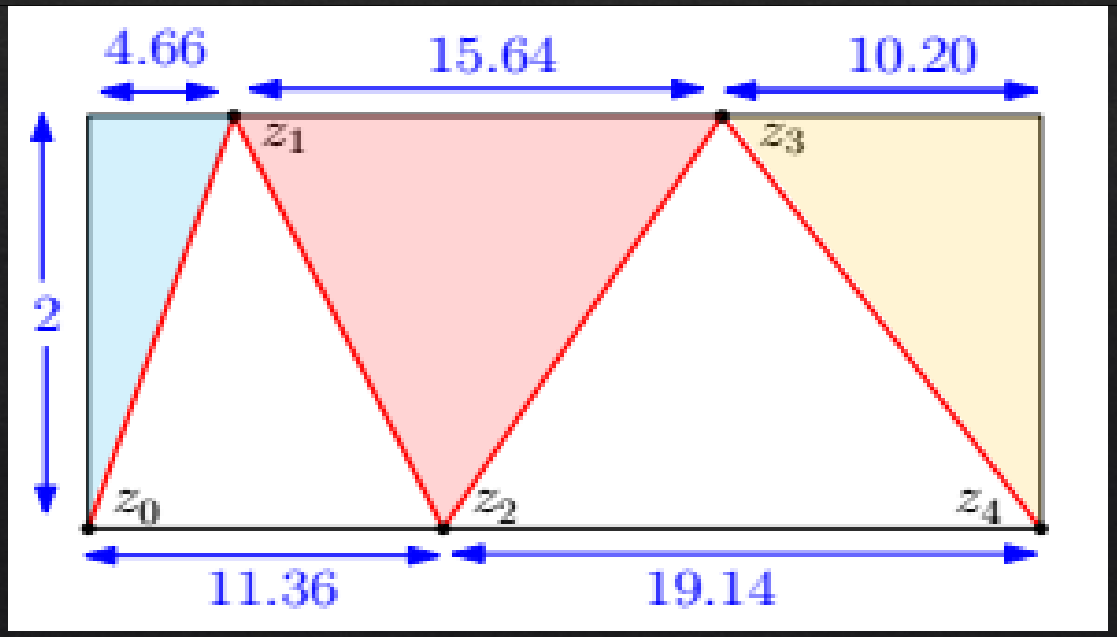
k	0.34	1.02	0.34	0.80
	2	3	4	9
n	3	2	2	3

A^b

Table Cartogram: General Case

Take a rectangle with height 2 and width S/2
Area is equal to the area of the table *A*

Draw a zigzag: The first triangle has area equal to the sum of the first column of A^t, and each subsequent triangle has area equal to the next 2 columns, and the last triangle may correspond to 1 or 2 columns.



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A^t

k	0.34	1.02	0.34	0.80
	2	3	4	9
n	3	2	2	3

A^b

Table Cartogram: General Case

The zigzag will end at one of the two rightmost corners.

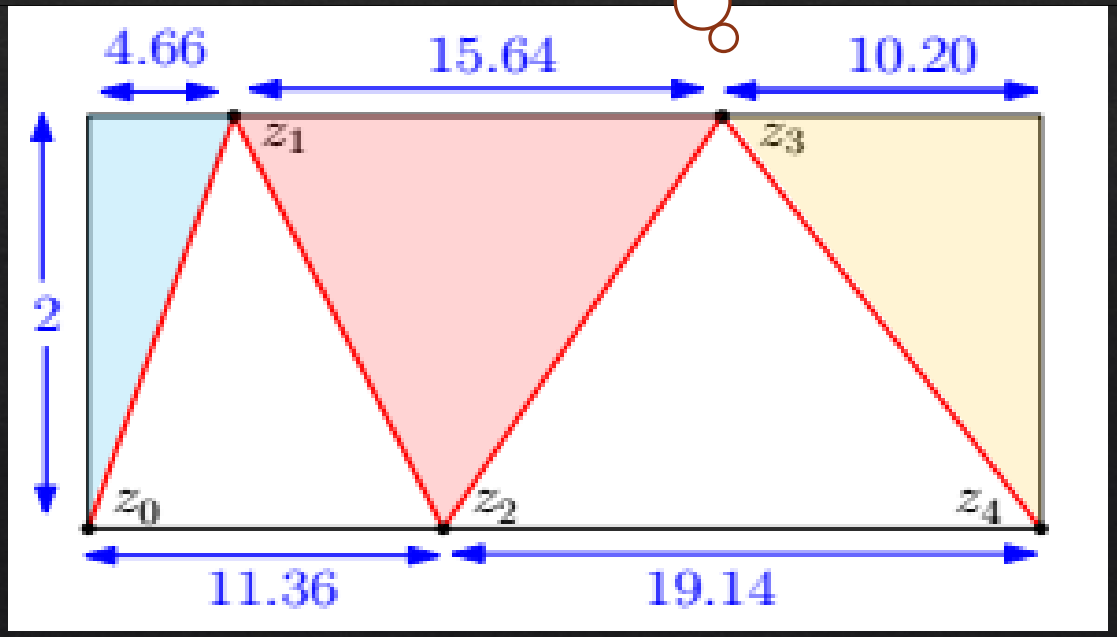
Why?

2	3	2	4
2.66	7.98	2.66	6.20

k

A^t

sum of the first … ent triangle has area equal to the next 2 columns, and the last triangle may correspond to 1 or 2 columns.



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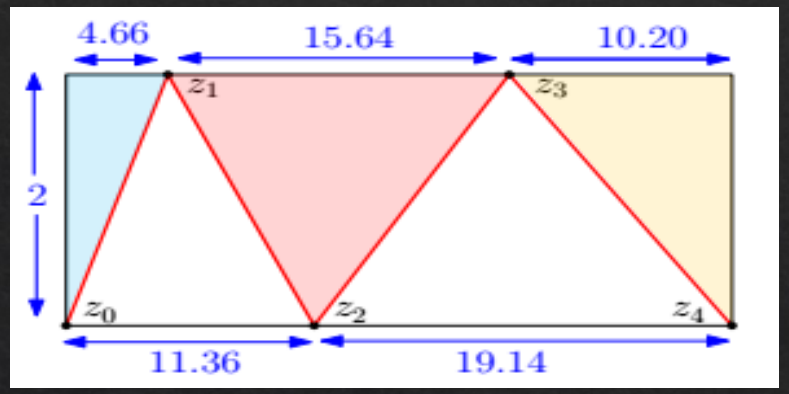
k

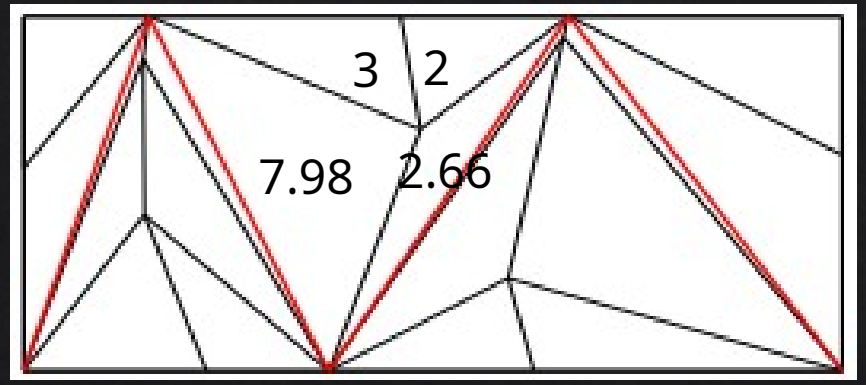
n

A^b

Table Cartogram: General Case

Split the triangles into good areas using the idea of barycentric coordinates.





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k	0.34	1.02	0.34	0.80
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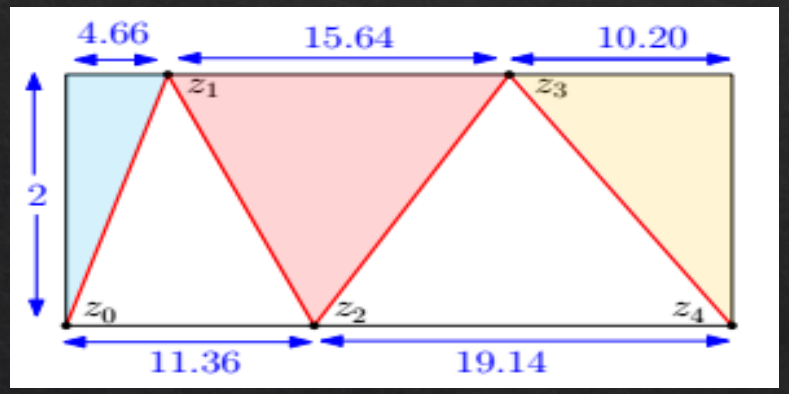

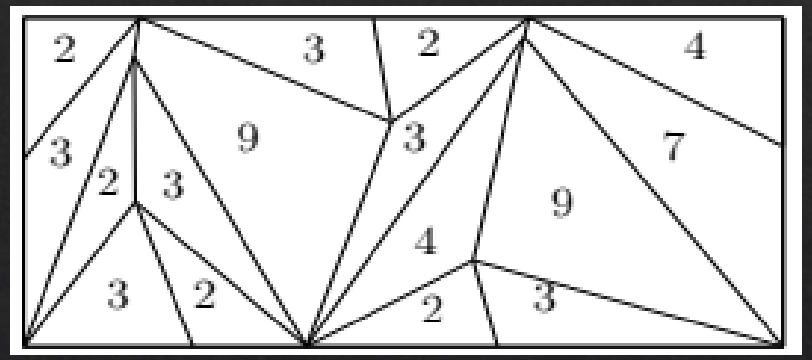

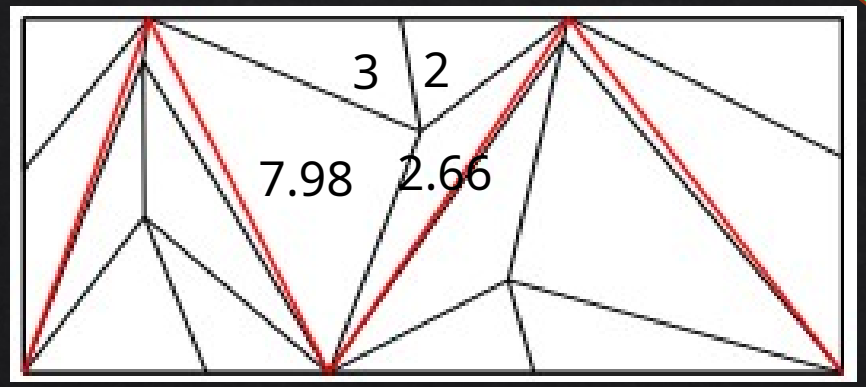

Remove the zigzag to get your cartogram!

k	2	3	2	4
	2.66	7.98	2.66	6.20

A^t

k	0.34	1.02	0.34	0.80
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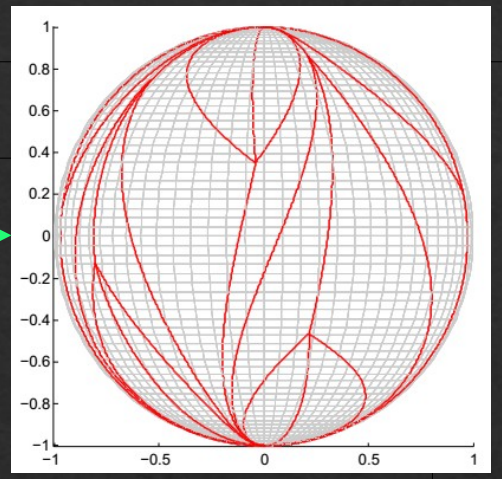
n	3	2	2	3

A^b

Table Cartogram: General Case



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- Inside arbitrary convex quadrilaterals
- The corresponding problem in 3D does not have a feasible solution – not yet published (from authors).
- Can be used for Maps

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WA 6.725	MT 0.989	ND 0.673	MN 5.304	WI 5.687	NY 19.378	VT 0.626	ME 1.328
OR 3.831	ID 1.568	SD 0.814	IA 3.046	NI 9.884	PA 12.702	NH 1.316	MA 6.548
NV 2.701	WY 0.564	NE 1.826	IL 12.831	IN 6.484	OH 11.537	CT 3.574	RI 1.053
UT 2.764	CO 5.029	KS 2.853	MO 5.989	KY 4.339	WV 1.853	MD 5.774	NJ 8.792
CA 37.254	NM 2.059	OK 3.751	AR 2.916	TN 6.346	SC 4.625	VA 8.001	DE 0.898
AZ 6.392	TX 25.146	LA 4.533	MS 2.967	AL 4.780	GA 9.688	FL 18.801	NC 9.535

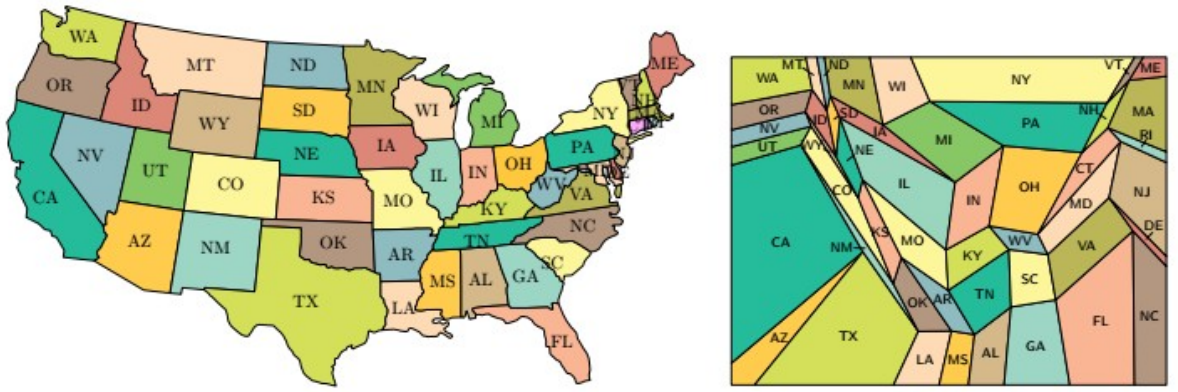

Fig. 3. A table cartogram of USA according to the population of the states in 2010, using the grid map of [9].

Table Cartogram: General Case