Digital Image Processing - Homework Assignment#1

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Due: 10/13/2025

1 Exercise 1 - Scaling

1.1

1.2

1.3

1.4 Bicubic Interpolation

Bicubic interpolation estimates a value at a 2D grid point using a 4×4 neighborhood (16 points) for smoother results. It extends 1D cubic splines separably:

1. For each of 4 rows, compute 1D cubic interpolation across 4 columns using offset t:

$$p_k = \sum_{m=0}^{3} c_m(t) \cdot f(i + m - 1, j + k - 1), \quad k = 0 \ldots 3$$

(e.g., Catmull-Rom basis: $c_0(t) = -0.5t^3 + t^2 - 0.5t$, etc.)

2. Interpolate the 4 p_k vertically using offset s:

$$p(x, y) = \sum_{k=0}^{3} c_k(s) \cdot p_k.$$

This approximates a degree-3 surface, reducing blur/artifacts in tasks like image scaling.

In bilinear interpolation, we uses 2×2 neighborhood (4 points) for linear weighting:

$$p(x, y) = (1 - t)(1 - s)f(i, j) + t(1 - s)f(i + 1, j) + (1 - t)sf(i, j + 1) + tsf(i + 1, j + 1).$$

Separable: horizontal linears, then vertical.

The comparison between the complexity of two methods can be summarized as follows, we can see bicubic trades $\sim 4\times$ computations for better detail preservation; bilinear prioritizes speed.

Aspect	**Bilinear**	**Bicubic**
Pixels Used	4 (2×2)	16 (4×4)
Operations per Pixel	∼4 mult + ∼2 add	∼20 mult + ∼15 add (4× cubic horiz + 1 vert)
Speed	Very fast ($O(1)$, real-time ok)	4 5× slower ($O(1)$, but higher cost)
Quality	Basic, can blur	Sharper, smoother gradients

2 Exercise 2 - Distortion

2.1

1. Brown–Conrady Model of Radial Distortion

The Brown–Conrady model expresses the relation between the ideal (undistorted) image point (x, y) and the distorted image point (x_d, y_d) as:

$$x_d = x \cdot \left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \dots\right)$$
$$y_d = y \cdot \left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \dots\right)$$

where:

- (x, y) are normalized image coordinates (centered at the principal point),
- $r^2 = x^2 + y^2$ is the squared radial distance from the optical axis,
- $k_1, k_2, k_3, \dots$ are the radial distortion coefficients.

The sign and magnitude of the coefficients determine whether the lens exhibits *barrel distortion* or *pincushion distortion*.

2. Barrel Distortion

- **Definition:** Straight lines appear to bulge outwards, like the sides of a barrel.
- **Mathematical explanation:** Occurs when $k_1 < 0$ (dominant case). The scaling factor

$$1 + k_1 r^2 + k_2 r^4 + \dots$$

  becomes *smaller* as r increases. Thus, points farther from the image center are mapped closer inward, compressing the edges and making straight lines look convex.

Typical example: wide-angle or fisheye lenses.

3. Pincushion Distortion

- **Definition:** Straight lines appear to bend inward, like the edges of a pincushion.
- **Mathematical explanation:** Occurs when $k_1 > 0$ (dominant case). The scaling factor

$$1 + k_1 r^2 + k_2 r^4 + \dots$$

  grows with r. Thus, points farther from the image center are pushed outward, stretching the edges and making straight lines bow inward.

Typical example: telephoto lenses.

4. Visual Summary

- If the radial factor decreases with r: **Barrel distortion** ($k_1 < 0$).
- If the radial factor increases with r: **Pincushion distortion** ($k_1 > 0$).

2.2

2.3

2.4