

CS186 Discussion #1

(Introduction, external sorting & hashing)

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UC BERKELEY COLLEGE OF ENGINEERING
Master
of Engineering

B*i*D

Now it's your turn!

- Meet the people around you: name, major, year, interests, etc.
- Partners are **recommended** in this course. If you don't have one, take this opportunity to find one!

Enrollment

- We will not be expanding the course.
- We cannot control the processing of the waitlist.
- CS186 will be offered again Spring 2016
 - (Joe Hellerstein is awesome!)

Logistics questions?

Out-of-core? External?

```
graph LR; Disk1[Disk] --> Memory[Limited memory]; Memory --> Disk2[Disk]
```

Disk

Limited
memory

Disk

Time-space Rendezvous

- When items are in memory at the same time
 - Ex: Aggregation or eliminating duplicates
- Implemented through:
 - External sorting
 - External hashing

External Sorting

External Sorting

- Want to sort data that does not fit in memory
- Minimize number of I/O's (especially random I/O's)

Terminology: Sorted Runs

- A sorted subset of a table
- Size is denoted by how many pages it spans

(name = Bob; sid = 1)
(name = Joe; sid = 2)
(name = Ann; sid = 3)

(name = Jill; sid = 6)
(name = Mia; sid = 9)
(name = Ted; sid = 10)

(name = Bill; sid = 12)
(name = Van; sid = 13)
(name = Jon; sid = 15)

(name = Sam; sid = 2)
(name = Jen; sid = 4)
(name = Dan; sid = 5)

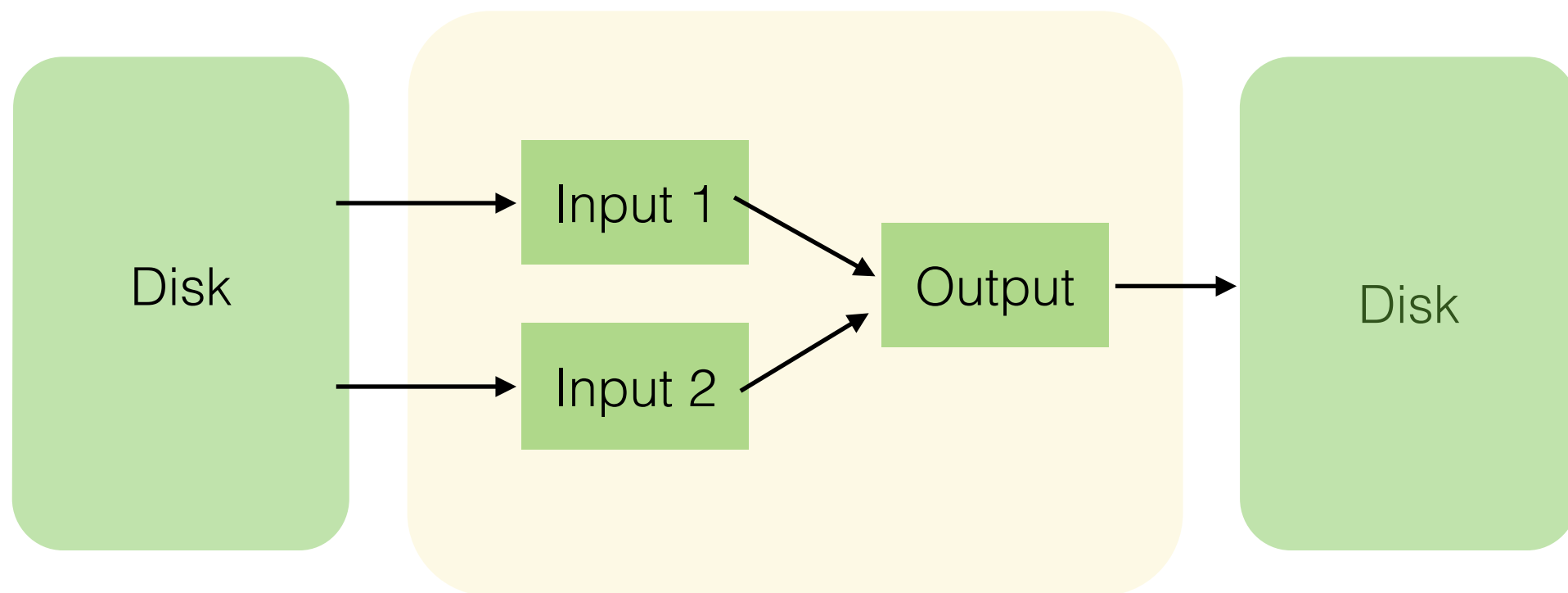
(name = Ned; sid = 6)
(name = Ed; sid = 10)
(name = Lou; sid = 11)

(name = Al; sid = 14)
(name = Kev; sid = 15)
(name = Sue; sid = 20)

Pages with tuple size = 3

There are two sorted runs,
both with a length of 3
pages.

2-Way Merge Sort



Use 3 pages of buffer

Input

3,4

6,2

9,4

8,7

5,6

6,5

1,4

4,2

Input

Pass 0

Output

3,4

6,2

9,4

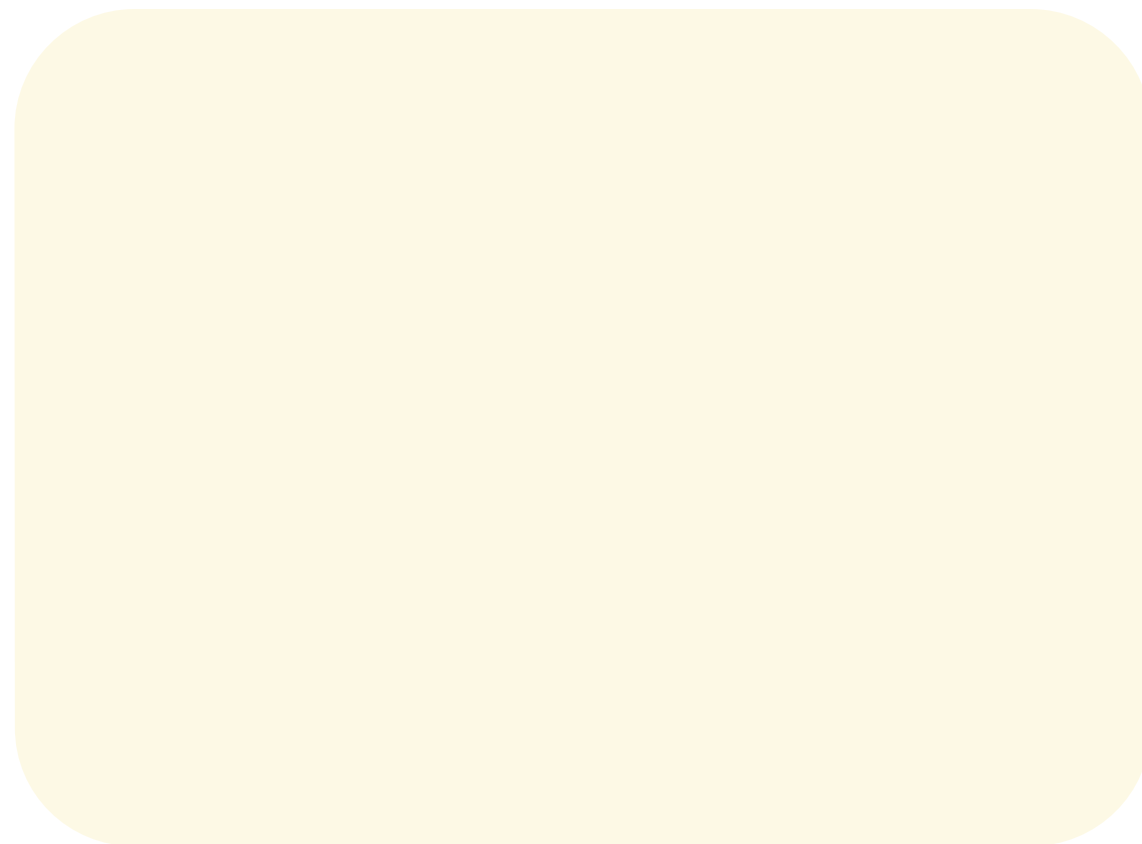
8,7

5,6

6,5

1,4

4,2



Input

Pass 0

Output

6,2

9,4

8,7

5,6

6,5

1,4

4,2

3,4

Input

Pass 0

Output

6,2

9,4

8,7

5,6

6,5

1,4

4,2

3,4

Input

Pass 0

Output

9,4

8,7

5,6

6,5

1,4

4,2

6,2

3,4

Input

Pass 0

Output

9,4

8,7

5,6

6,5

1,4

4,2

2,6

3,4

Input

Pass 0

Output

9,4

8,7

5,6

6,5

1,4

4,2

3,4

2,6

Input

Pass 0

Output

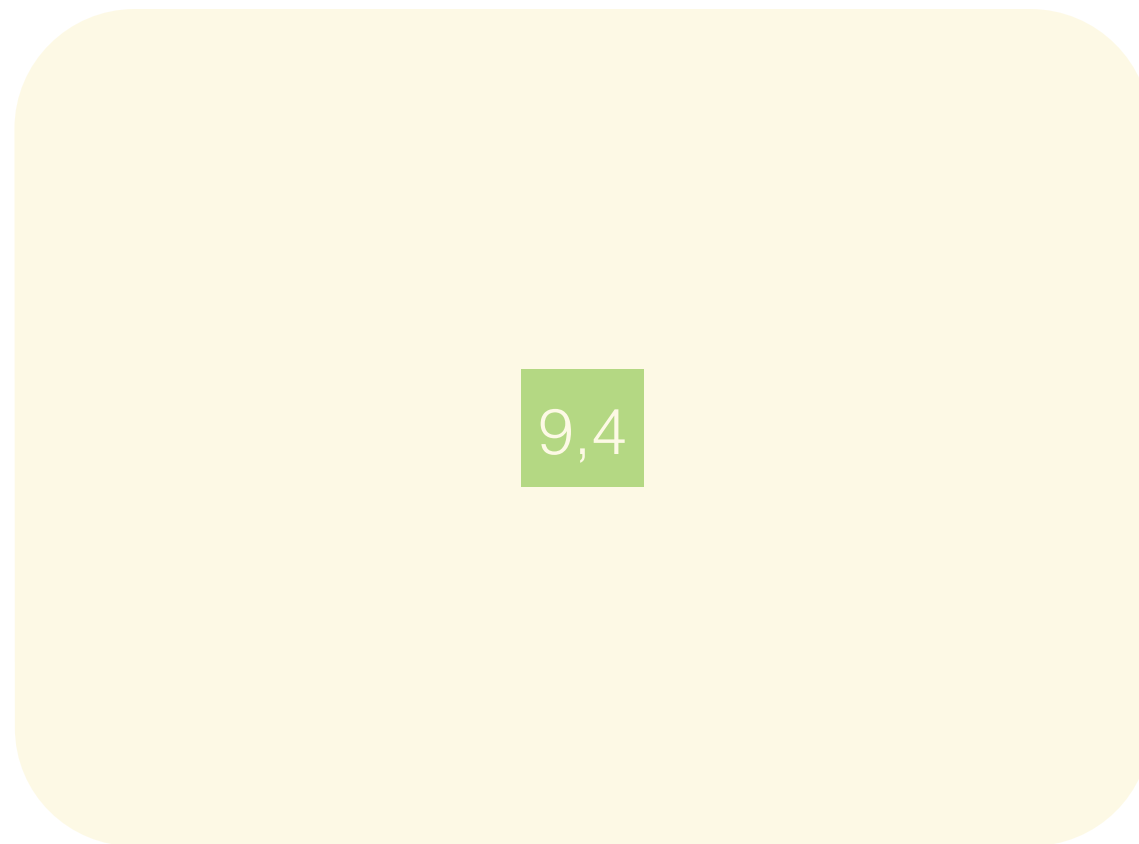
8,7

5,6

6,5

1,4

4,2



3,4

2,6

Input

Pass 0

Output

8,7

5,6

6,5

1,4

4,2

4,9

3,4

2,6

Input

Pass 0

Output

8,7

5,6

6,5

1,4

4,2

3,4

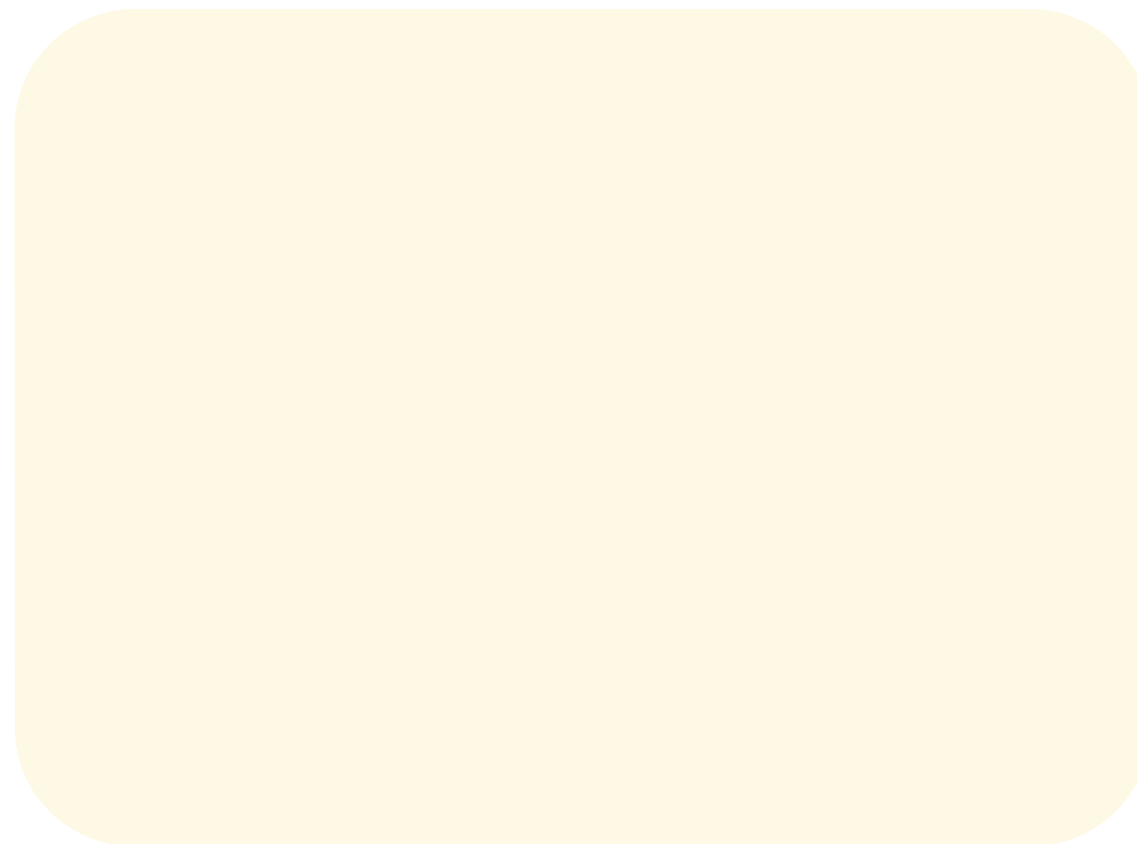
2,6

4,9

Input

Pass 0

Output



3,4

2,6

4,9

7,8

5,6

5,6

1,4

2,4

Input

Pass 0

3,4

3,4

6,2

2,6

9,4

4,9

8,7

7,8

5,6

5,6

6,5

5,6

1,4

1,4

4,2

2,4

1 page runs

Input

Pass 1

Output

3,4

2,6

4,9

7,8

5,6

5,6

1,4

2,4

Input 1



Output



Input 2



Input

Pass 1

Output

4,9

7,8

5,6

5,6

1,4

2,4

Input 1

3,4

Input 2

2,6

Output



Input

Pass 1

Output

4,9

7,8

5,6

5,6

1,4

2,4

Input 1

3,4

Input 2

6

Output

2

Input

Pass 1

Output

4,9

7,8

5,6

5,6

1,4

2,4

Input 1

4

Input 2

6

Output

2,3

Input

Pass 1

Output

2,3

4,9

7,8

5,6

5,6

1,4

2,4

Input 1

4

Input 2

6

Output

Input

Pass 1

Output

2,3

4,9

7,8

5,6

5,6

1,4

2,4

Input 1



Input 2



Output

4,6

Input

Pass 1

Output

4,9

7,8

5,6

5,6

1,4

2,4

Input 1



Input 2



Output



2,3

4,6

Input

Pass 1

Output

2,3

4,6

Input 1

4,9

Output



Input 2

7,8

5,6

5,6

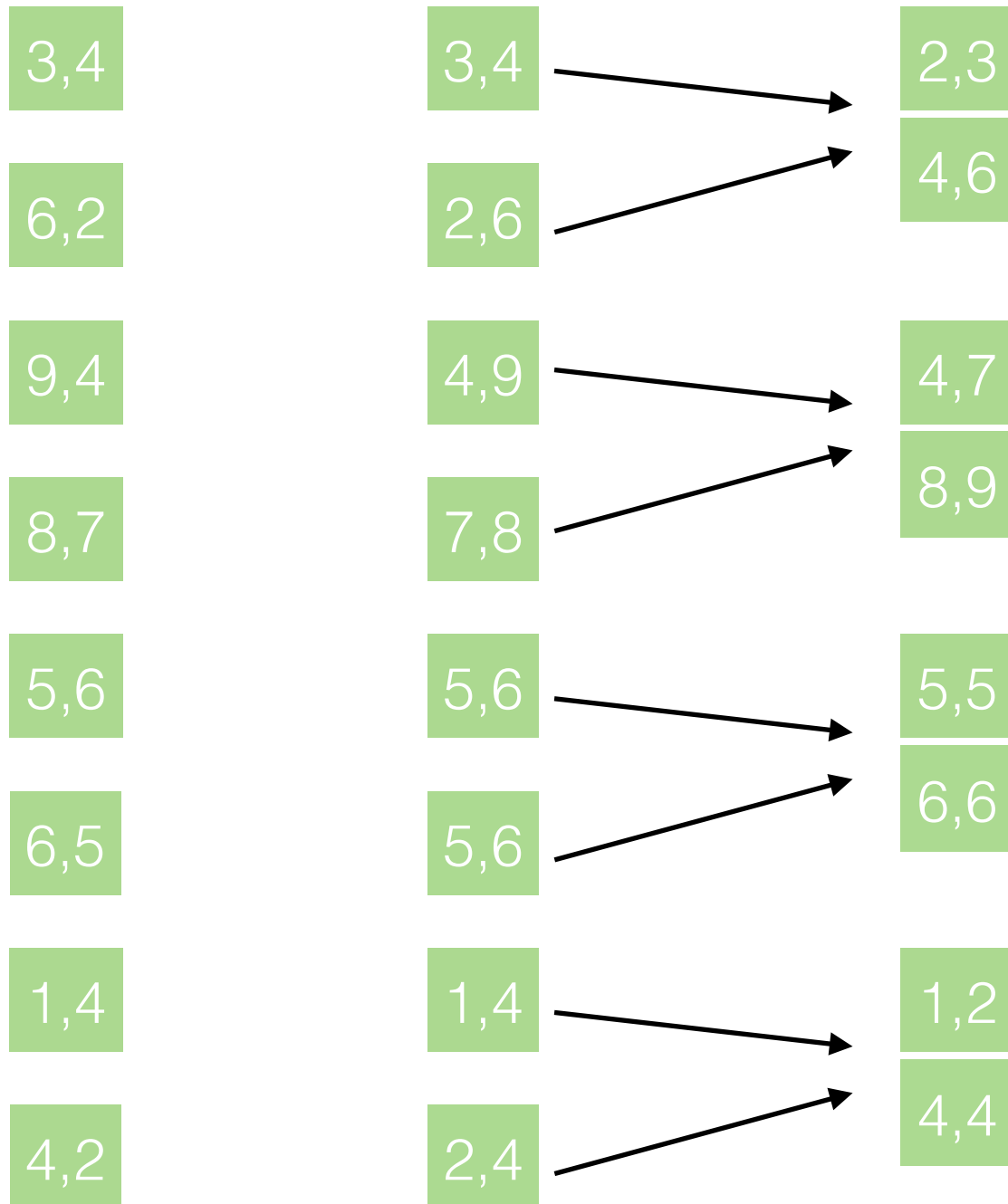
1,4

2,4

Input

Pass 0

Pass 1



1 page runs

2 page runs

Pass 2

Input

2,3

4,6

4,7

8,9

5,5

6,6

1,2

4,4

Output

Input 1



Output



Input 2



Input

Pass 2

Output

4,6

8,9

5,5

6,6

1,2

4,4

Input 1

2,3

Input 2

4,7

Output



Input

Pass 2

Output

4,6

8,9

5,5

6,6

1,2

4,4

Input 1



Input 2



Output

2,3

Input

Pass 2

Output

2,3

8,9

5,5

6,6

1,2

4,4

Input 1

4,6

Input 2

4,7

Output



Input

Pass 2

Output

2,3

8,9

5,5

6,6

1,2

4,4

Input 1

6

Output

4,4

Input 2

7

Input

Pass 2

Output

8,9

5,5

6,6

1,2

4,4

Input 1

6

Input 2

7

Output

2,3

4,4

Input

Pass 2

Output

8,9

5,5

6,6

1,2

4,4

Input 1



Input 2



Output

6,7

2,3

4,4

Input

Pass 2

Output

8,9

5,5

6,6

1,2

4,4

Input 1



Input 2



Output



2,3

4,4

6,7

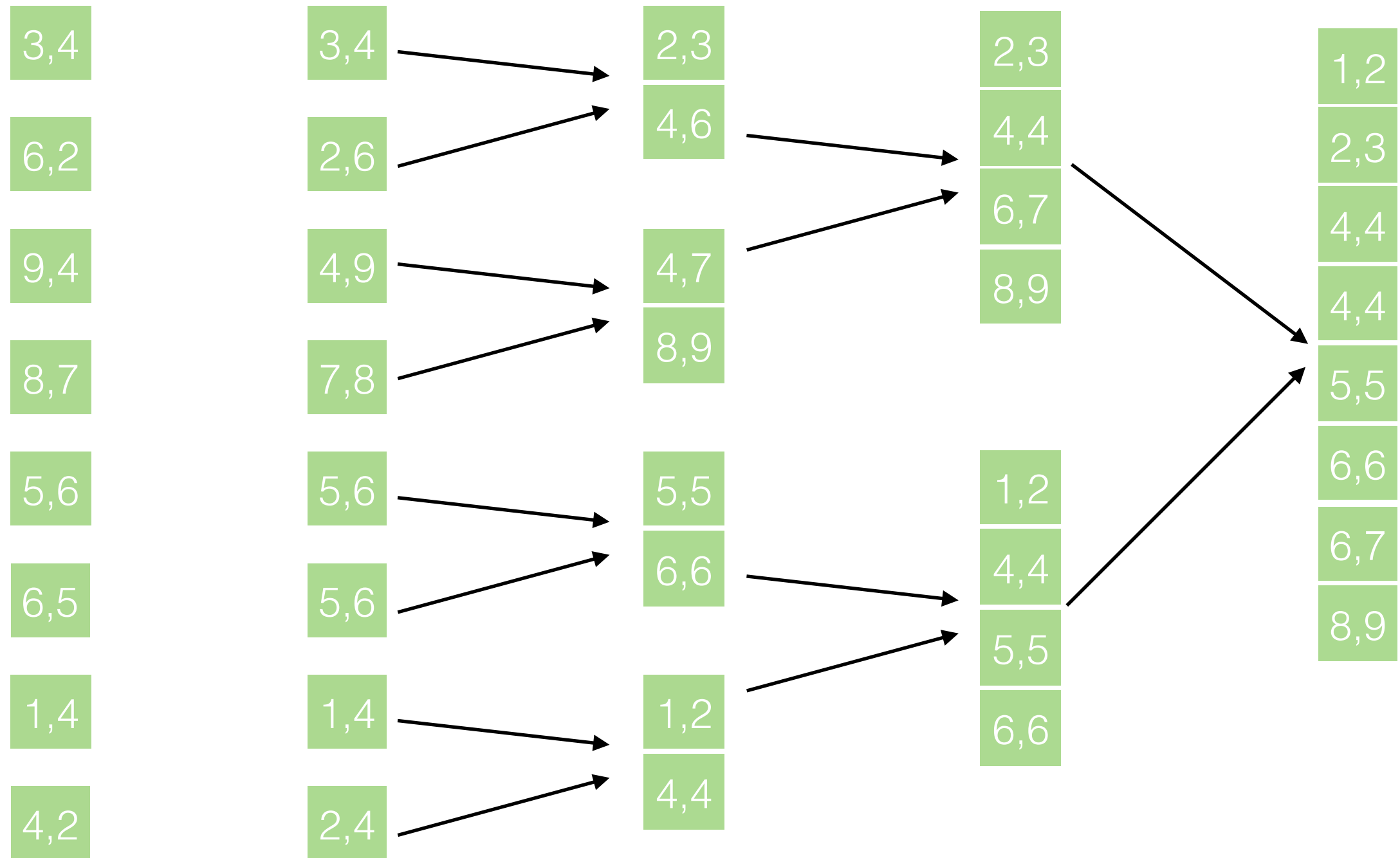
Input

Pass 0

Pass 1

Pass 2

Pass 3



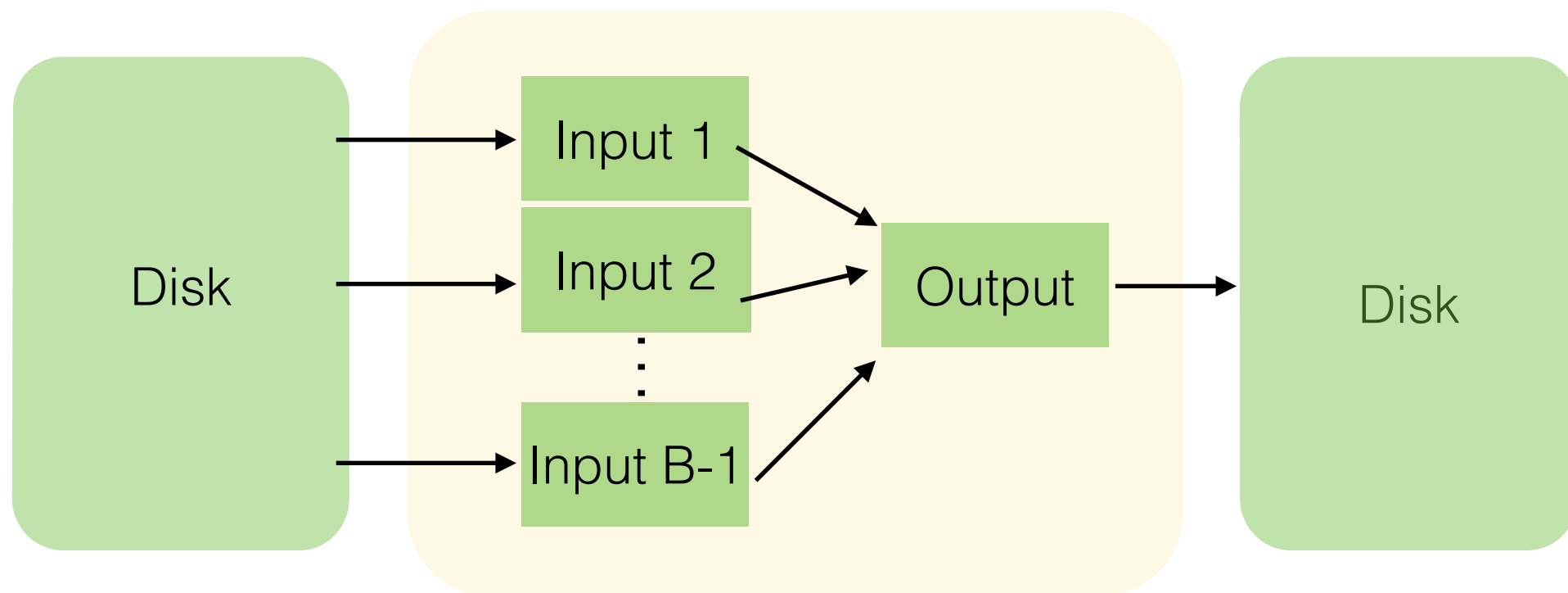
1 page runs

2 page runs

4 page runs

8 page runs

Generalized Merge Sort



Buffer size of B pages

Generalized Merge Sort

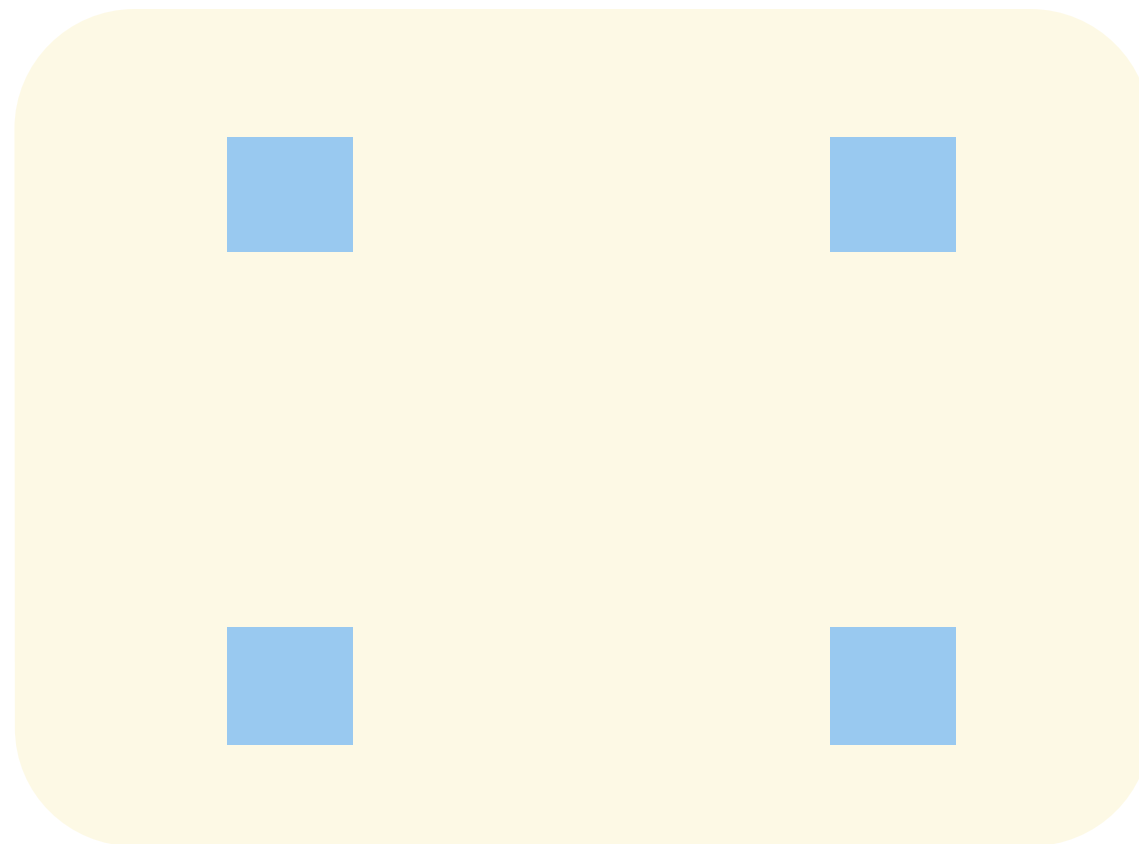
- Pass 0: Use all B buffers to sort, giving N/B sorted runs
- Pass 1, 2, ..., etc: Merge $B-1$ runs
- # Passes: $\text{ceil}(\log_{B-1}(\text{ceil}(N/B))) + 1$
- # I/O's: $2N * (\text{ceil}(\log_{B-1}(\text{ceil}(N/B))) + 1)$

Input

Pass 0

Output

	3,4
	6,2
5,3	9,4
1,2	8,7
3,3	5,6
9,2	6,5
	1,4
	4,2



Input

Pass 0

Output

5,3

1,2

3,3

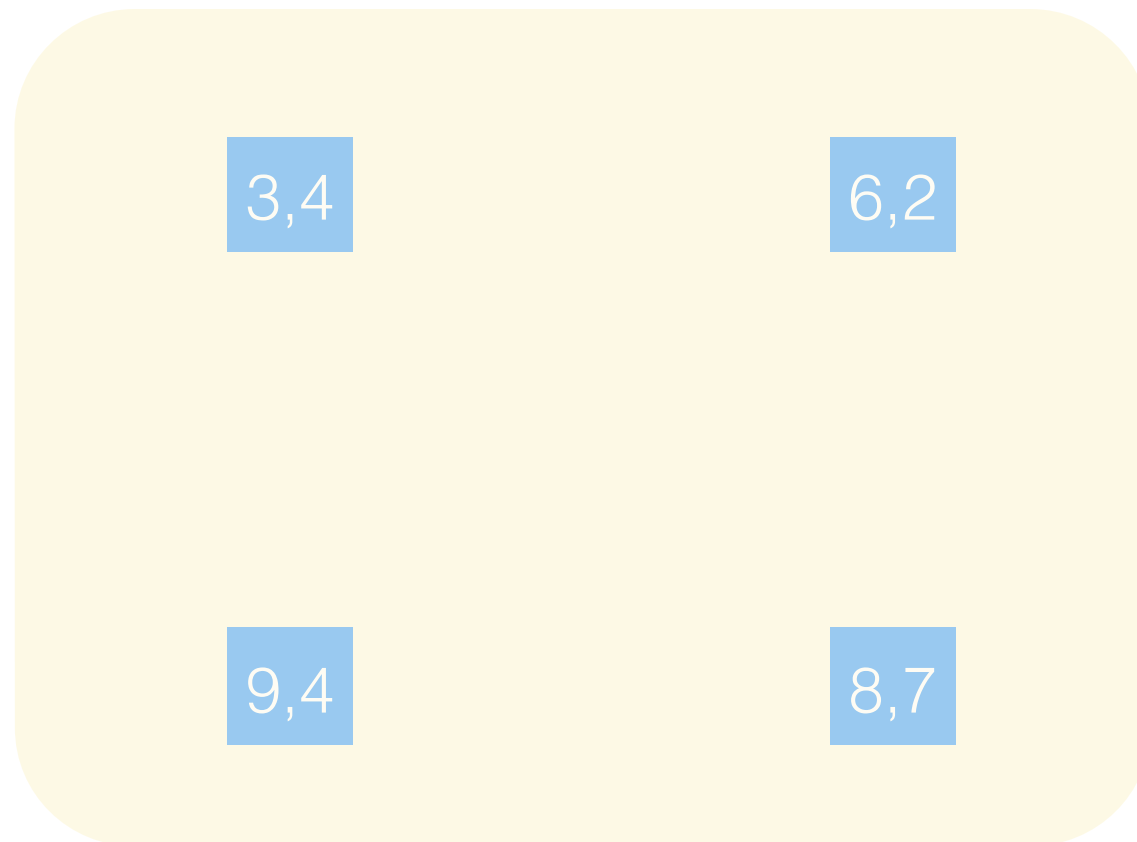
9,2

5,6

6,5

1,4

4,2



B = 4

Input

Pass 0

Output

(Use a sort algorithm from 61B!)

5,3

1,2

3,3

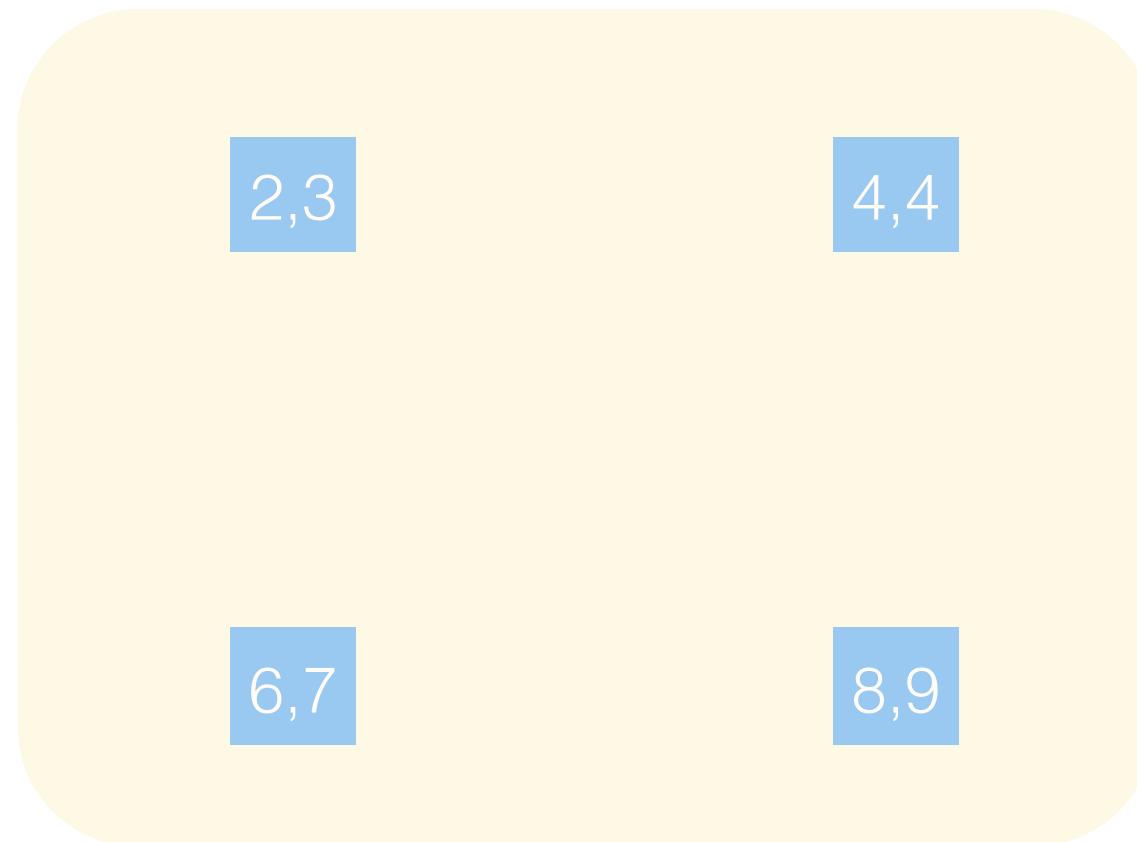
9,2

5,6

6,5

1,4

4,2



$B = 4$

Input

Pass 0

Output

(Use a sort algorithm from 61B!)

5,3

1,2

3,3

9,2

5,6

6,5

1,4

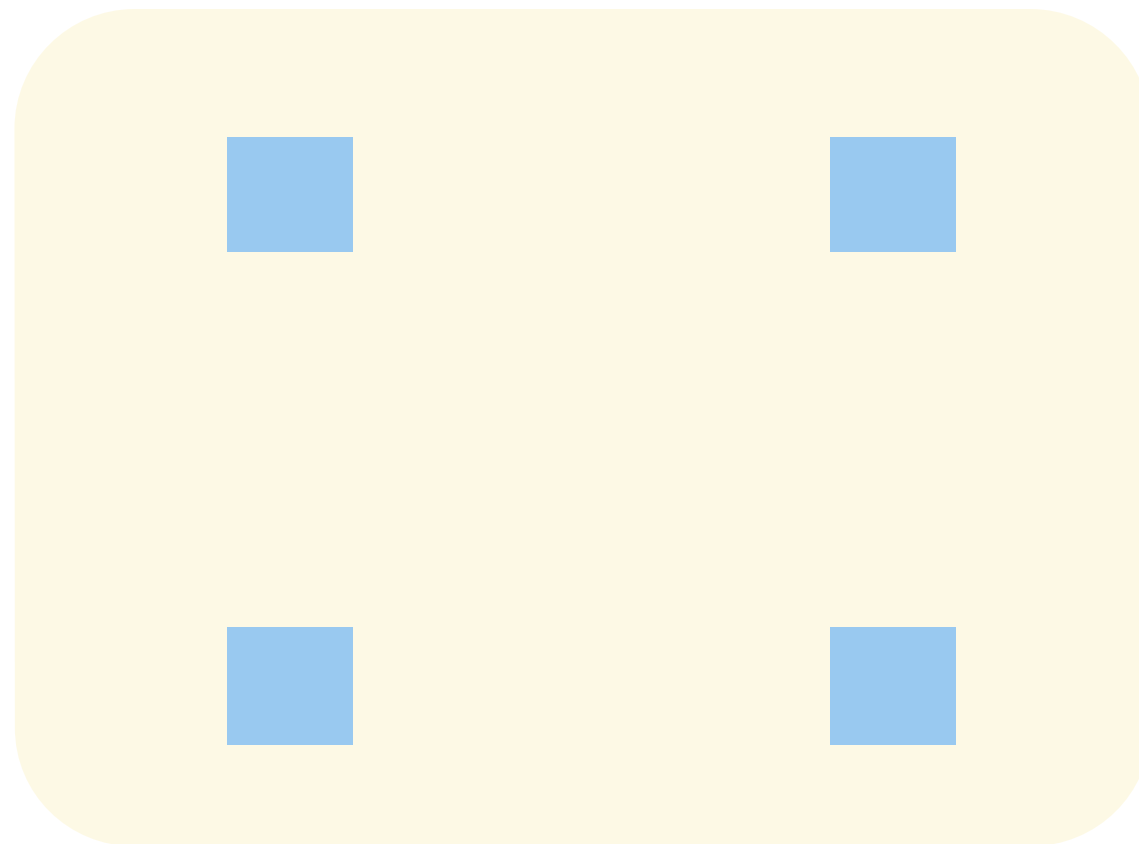
4,2

2,3

4,4

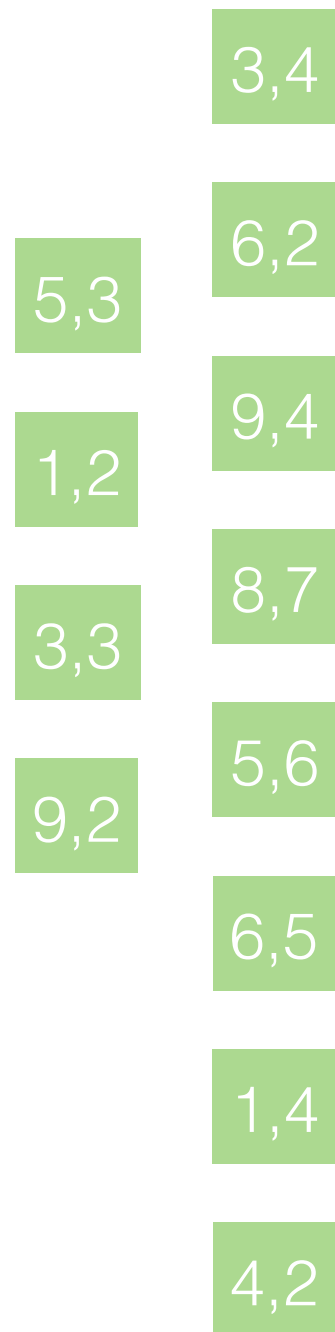
6,7

8,9

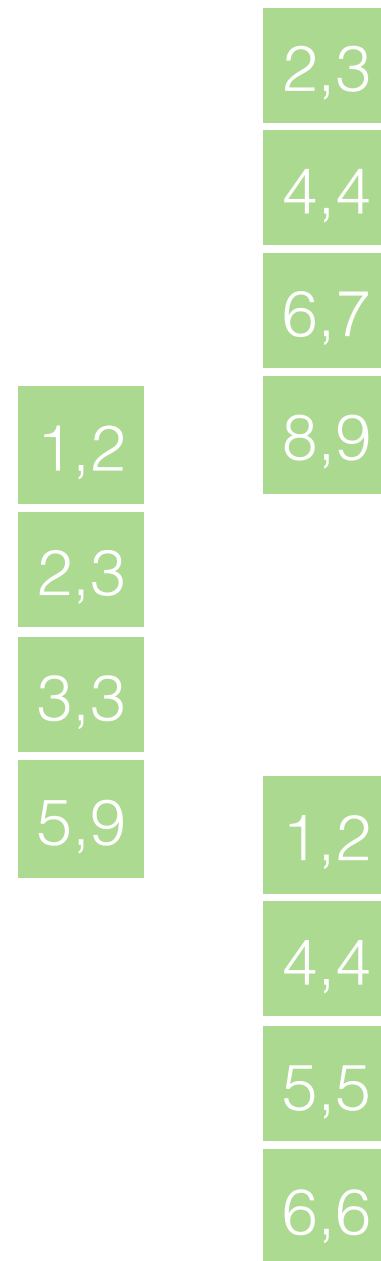


$B = 4$

Input



Pass 0

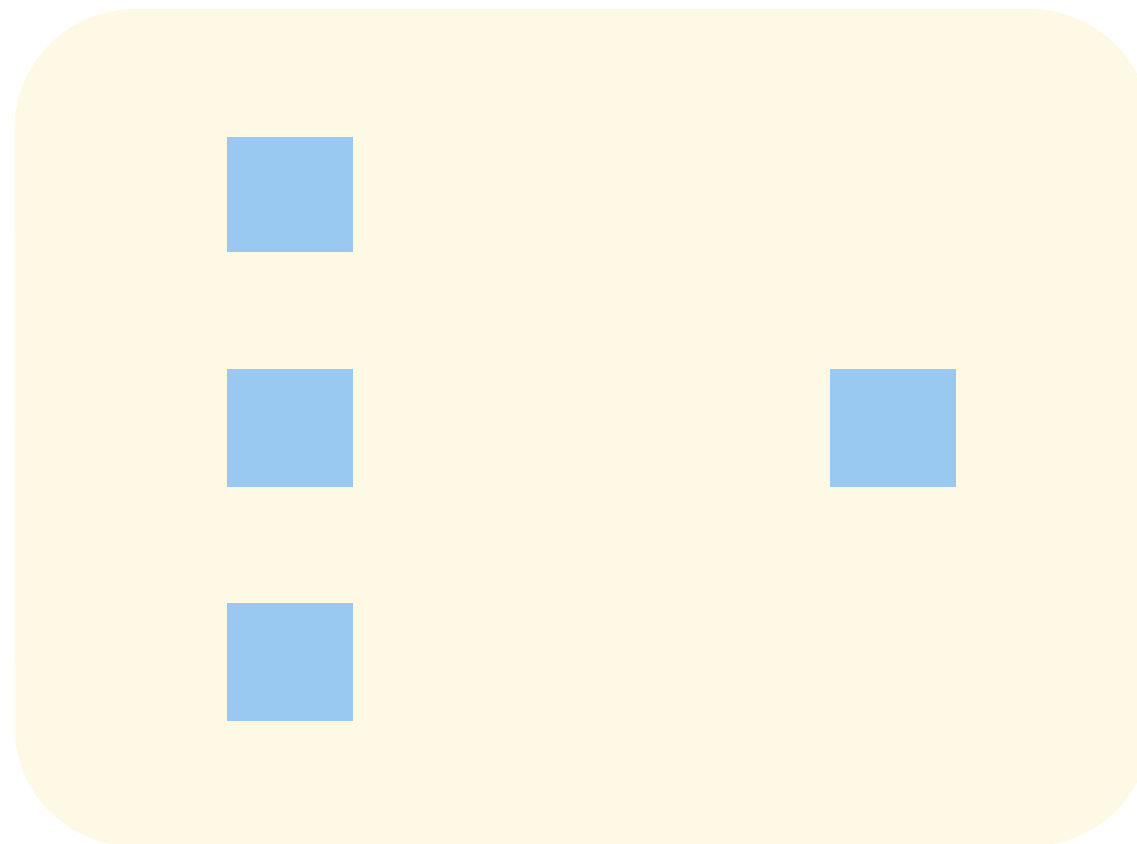
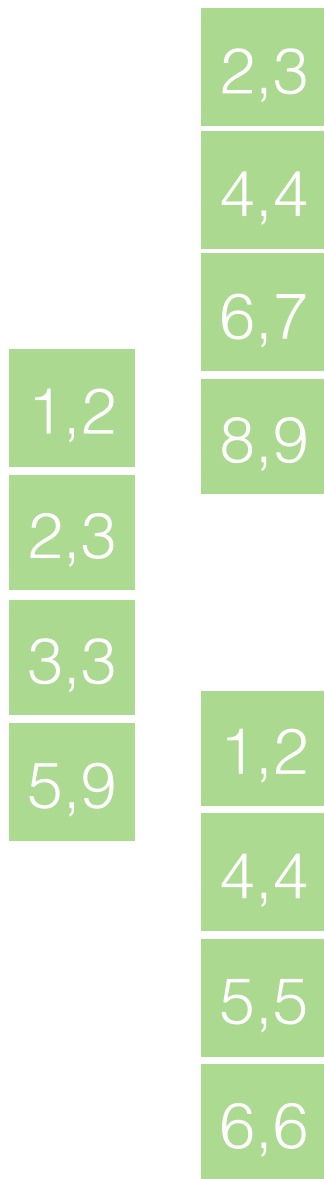


4 page runs

Input

Pass 1

Output

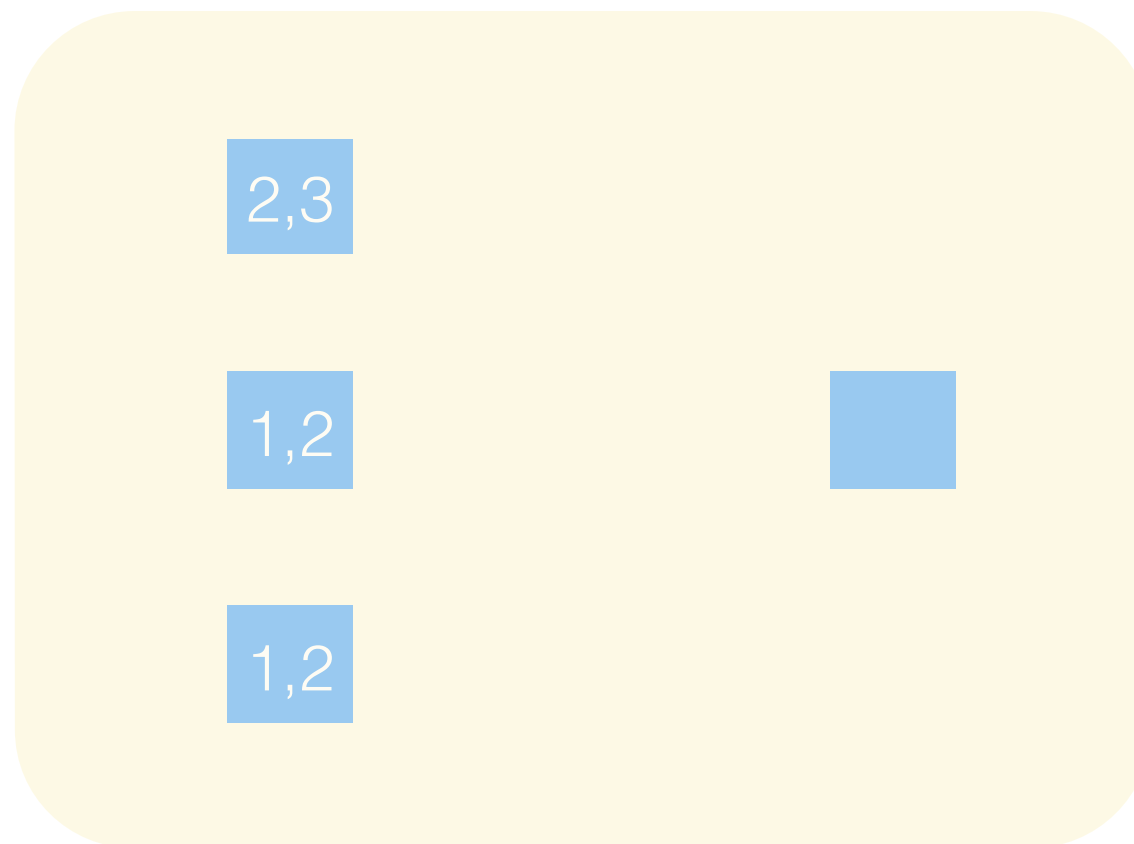
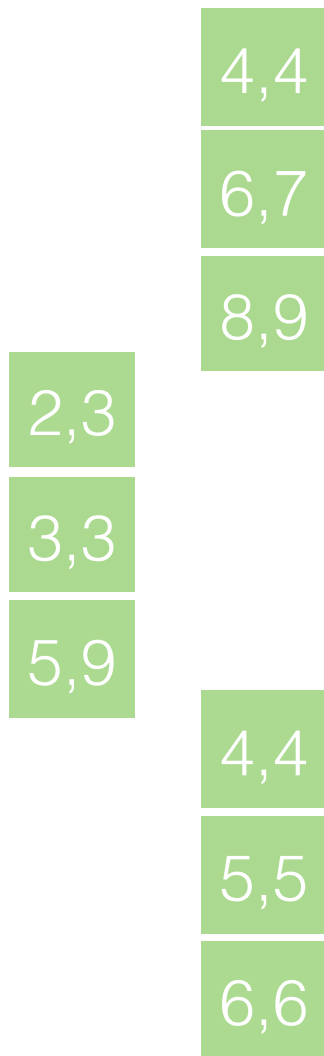


$B = 4$

Input

Pass 1

Output

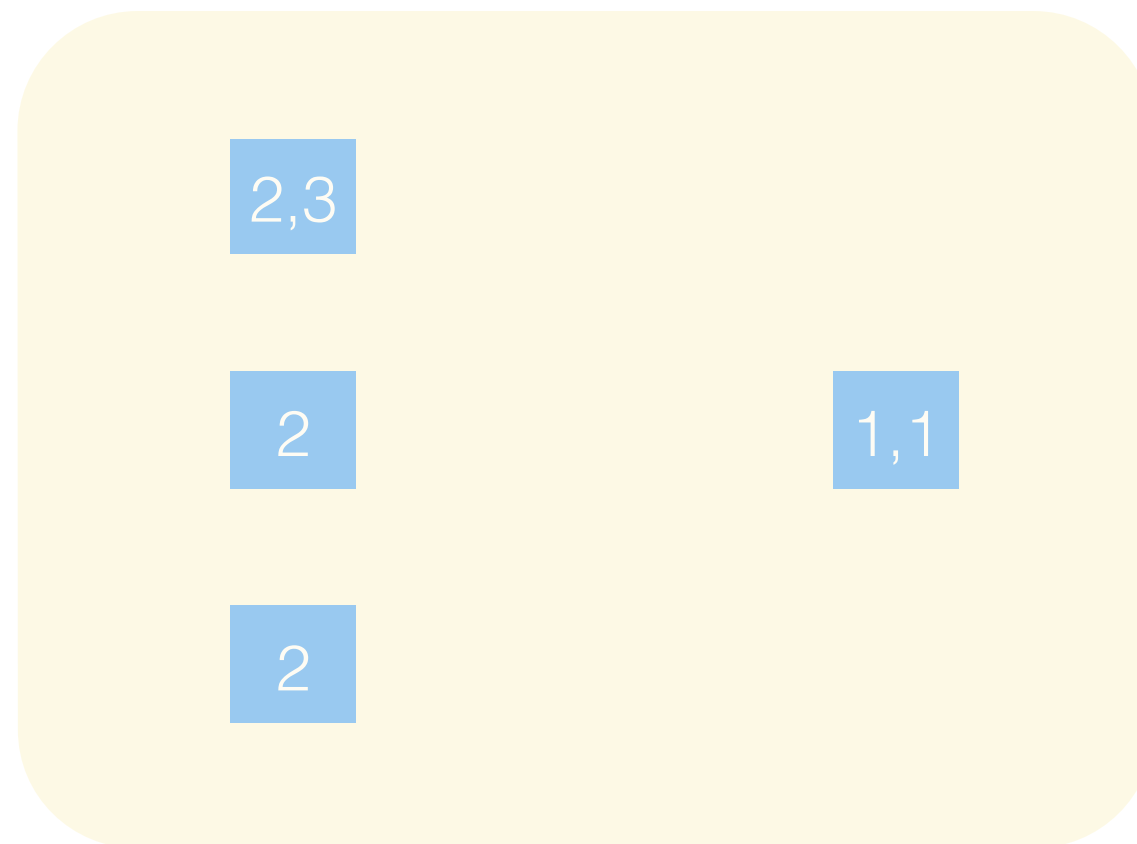
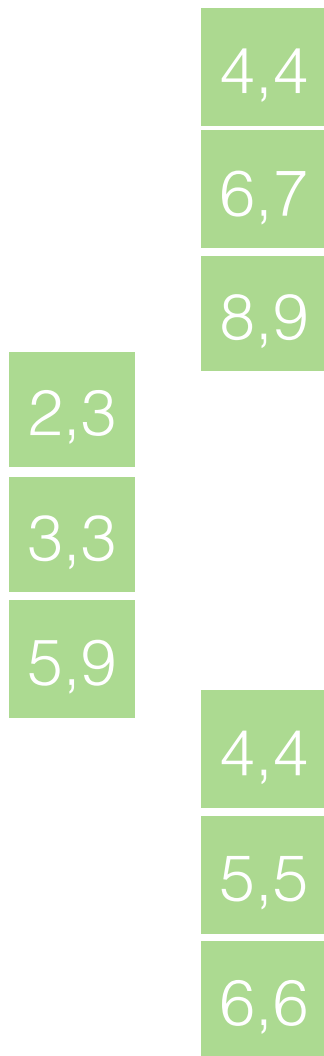


$B = 4$

Input

Pass 1

Output



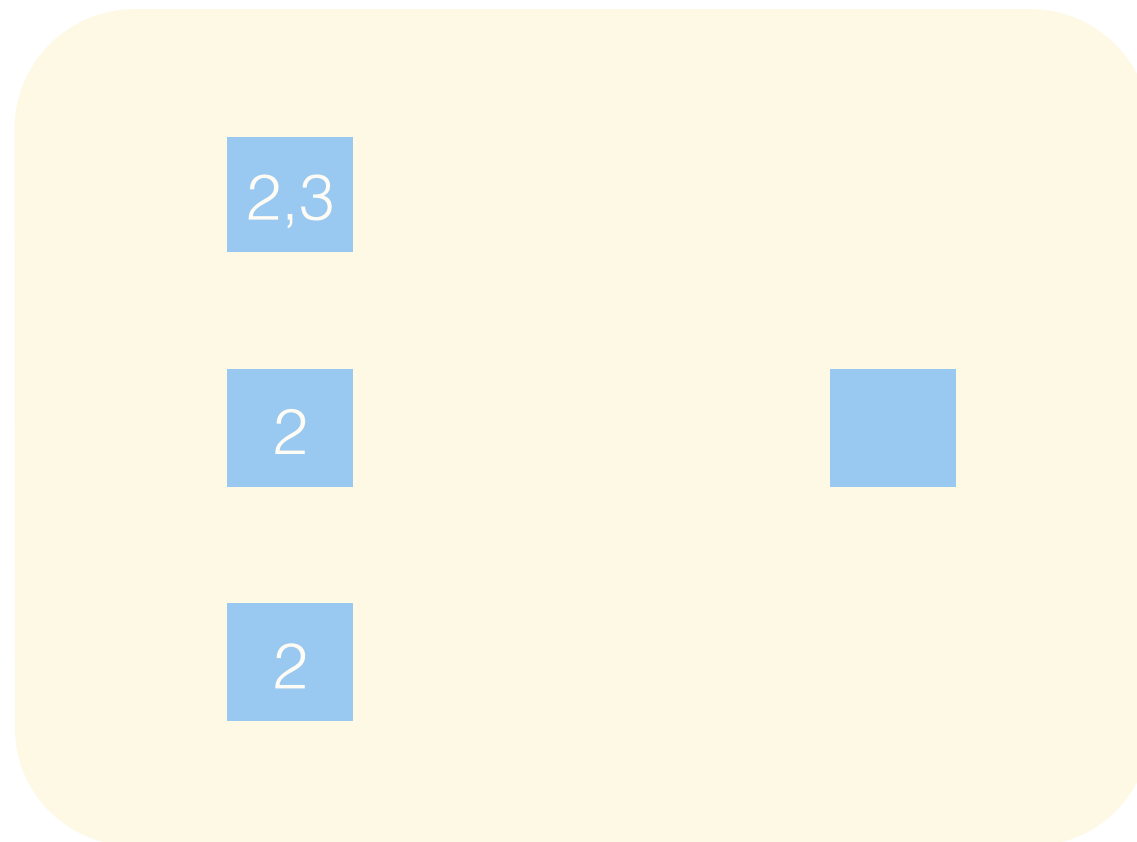
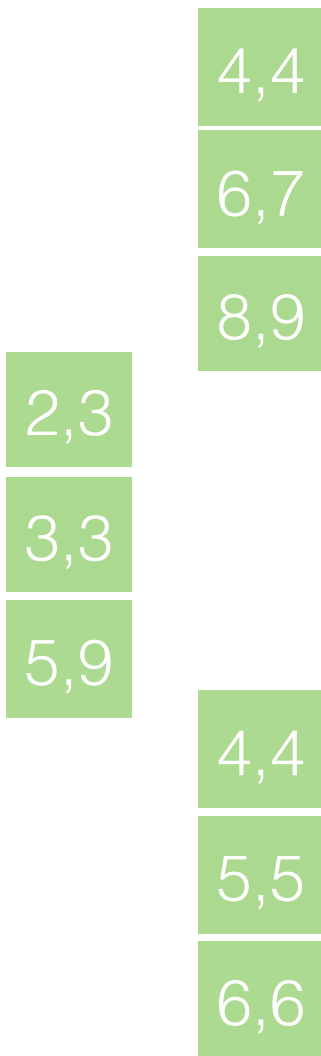
$B = 4$

Input

Pass 1

Output

1,1



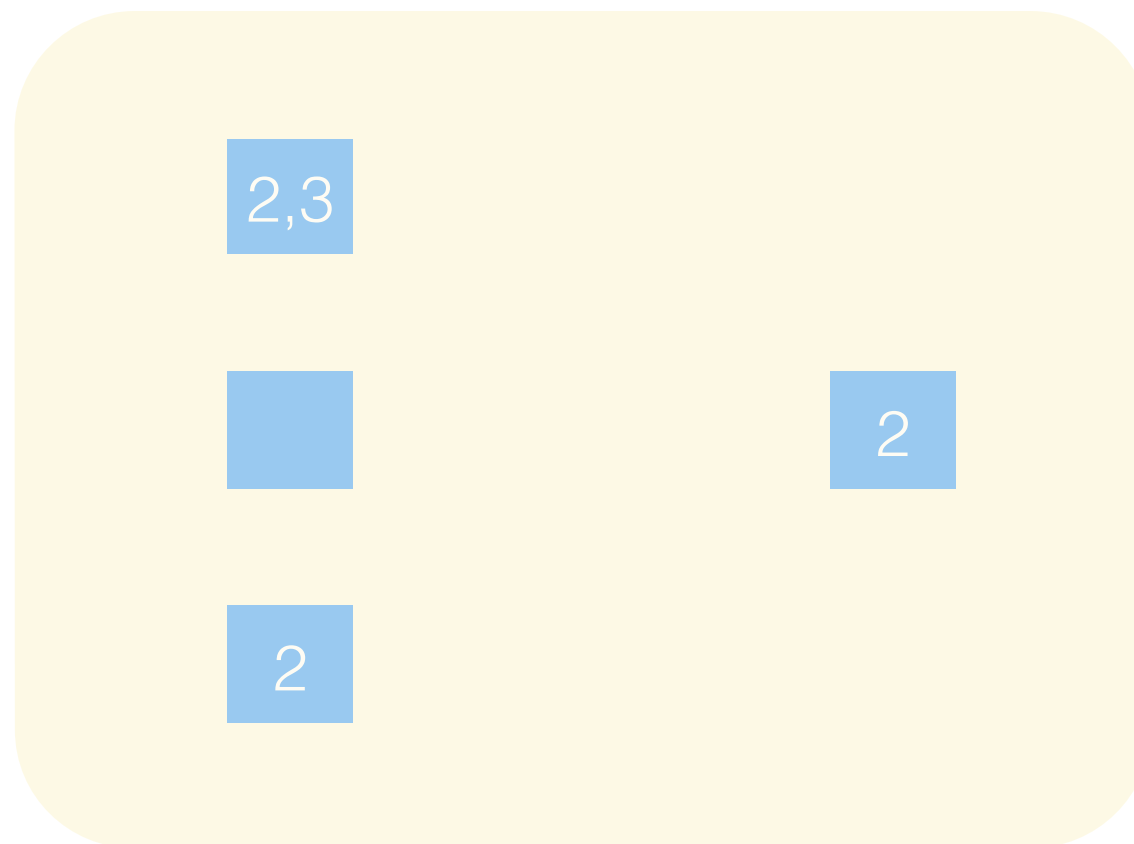
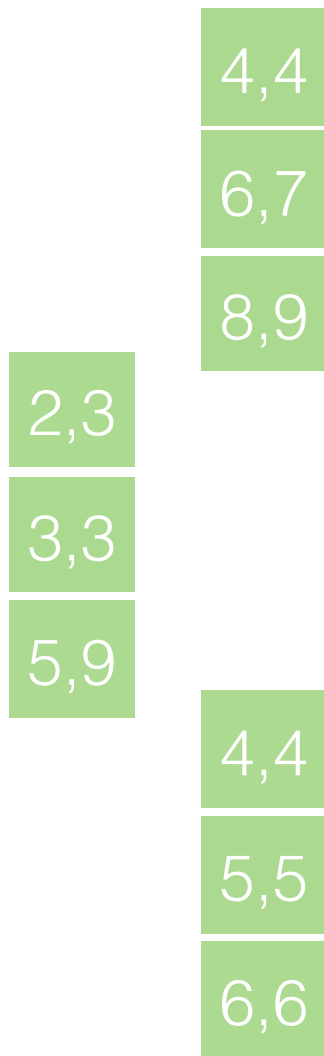
B = 4

Input

Pass 1

Output

1,1



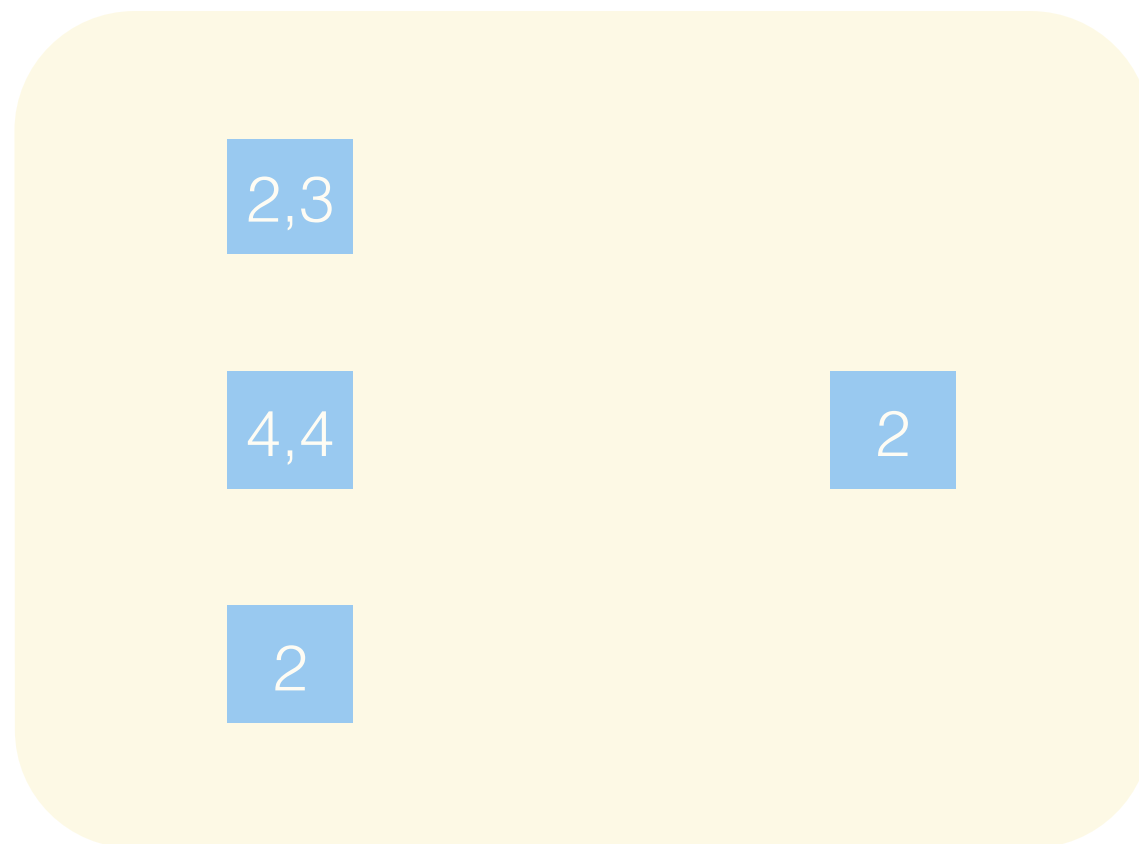
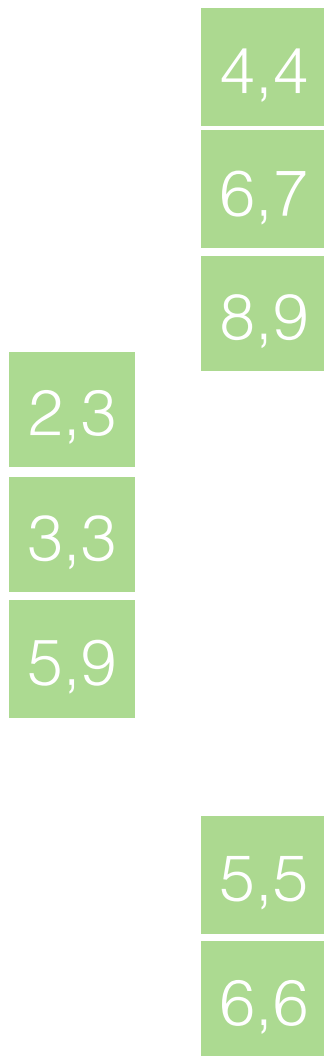
$B = 4$

Input

Pass 1

Output

1,1



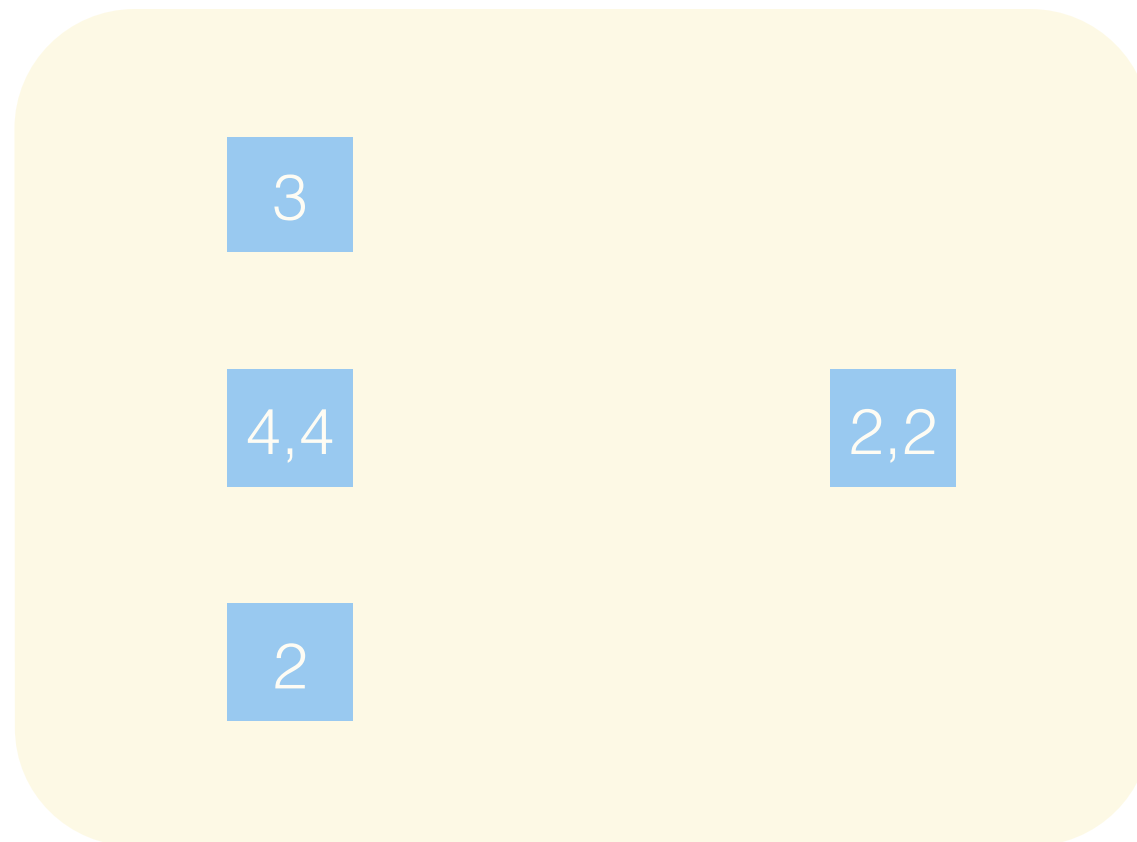
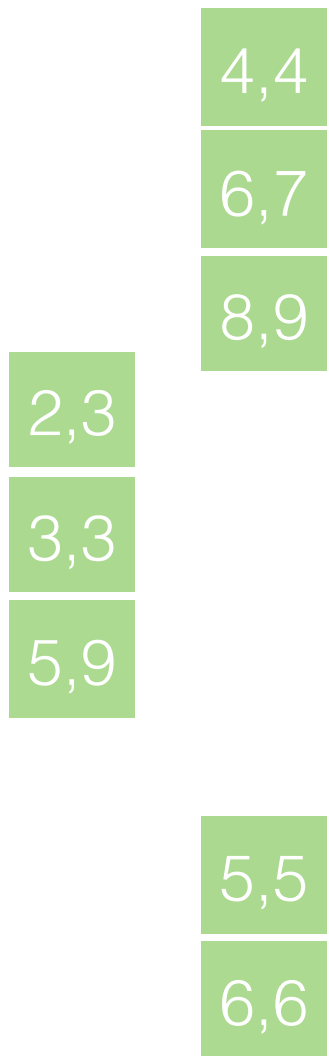
B = 4

Input

Pass 1

Output

1,1

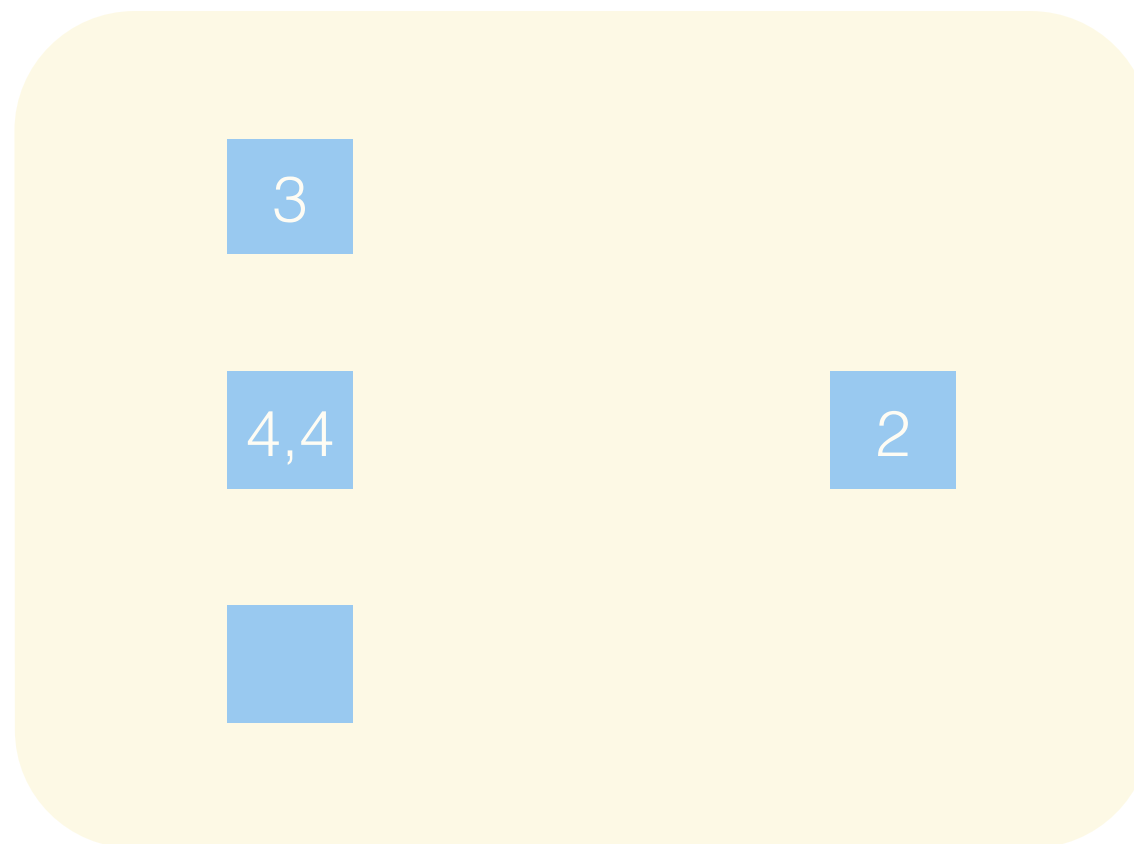
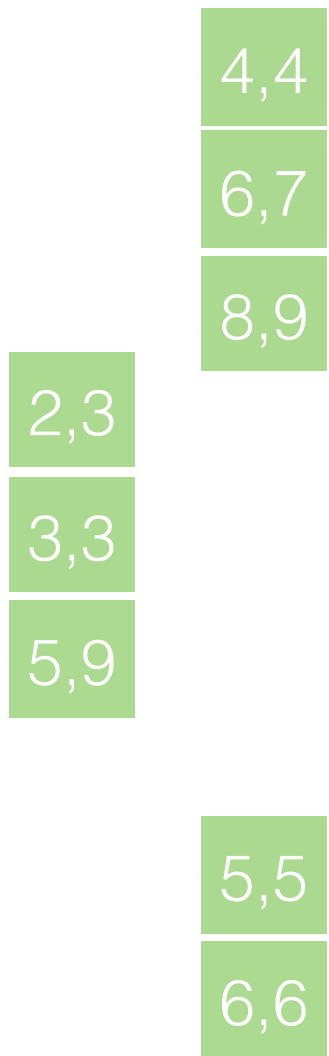


B = 4

Input

Pass 1

Output

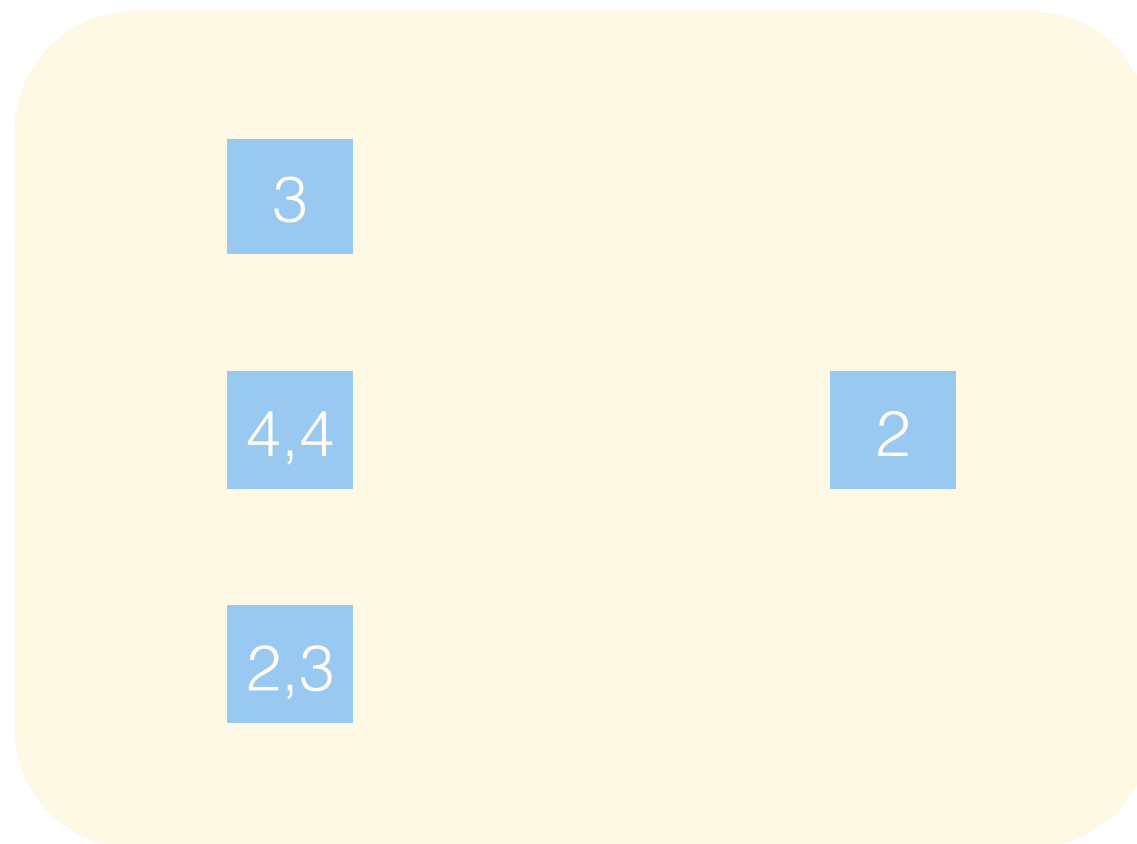


B = 4

Input

Pass 1

Output



B = 4

Generalized Merge Sort

- Pass 0: Use all B buffers to sort, giving N/B sorted runs
- Pass 1, 2, ..., etc: Merge $B-1$ runs
- # Passes: $\text{ceil}(\log_{B-1}(\text{ceil}(N/B))) + 1$
- # I/O's: $2N * (\text{ceil}(\log_{B-1}(\text{ceil}(N/B))) + 1)$

Worksheet #3, 4

List the differences between 2-way external merge sort and general external merge sort.

List the differences between 2-way external merge sort and general external merge sort.

- 2-way only utilizes 2 input buffers, general utilizes $B-1$
- During pass 0, 2-way only uses 1 page to sort files. General external merge sort uses all B pages in its buffer to sort the initial runs.

Buffer: 640 KB

Page: 64 KB

How many pages can your buffer hold?

Buffer: 640 KB

Page: 64 KB

How many pages can your buffer hold?

- $640 \text{ KB} * (1 \text{ page} / 64 \text{ KB}) = \mathbf{10 \text{ pages}}$

Buffer: 640 KB

Page: 64 KB

How many pages are in a 4 MB file?

Buffer: 640 KB

Page: 64 KB

How many pages are in a 4 MB file?

- $4 \text{ MB} * (1024 \text{ KB} / 1\text{MB}) * (1 \text{ page} / 64 \text{ KB}) = \mathbf{64 \text{ pages}}$

Buffer: 10 Pages

File: 64 Pages

How many passes would it take to externally
merge sort a 4 MB file?

Buffer: 10 Pages

File: 64 Pages

How many passes would it take to externally merge sort a 4 MB file?

- $\text{ceil}(\log_{10-1} \text{ceil}(64 / 10)) + 1 = \text{ceil}(\log_9(7)) + 1 = 1 + 1 = 2$ passes

Buffer: 10 Pages

File: 64 Pages

How many I/O's are needed to to externally
merge sort a 4 MB file?

Buffer: 10 Pages

File: 64 Pages

How many I/O's are needed to to externally merge sort a 4 MB file?

- $(\# \text{ of passes}) * 2 * (\# \text{ of pages in file}) = 2 * 2 * 64 = 256 \text{ I/O's}$

Buffer: 10 Pages

File: 64 Pages

What is the maximum file size that can be sorted with just 2 passes in this system?

Buffer: 10 Pages

File: 64 Pages

What is the maximum file size that can be sorted with just 2 passes in this system?

- $(\# \text{ of buffer pages}) (\# \text{ of buffer pages} - 1) = 10 * 9$
 $= 90 \text{ pages} = 5760 \text{ KB} \approx 5.6 \text{ MB}$

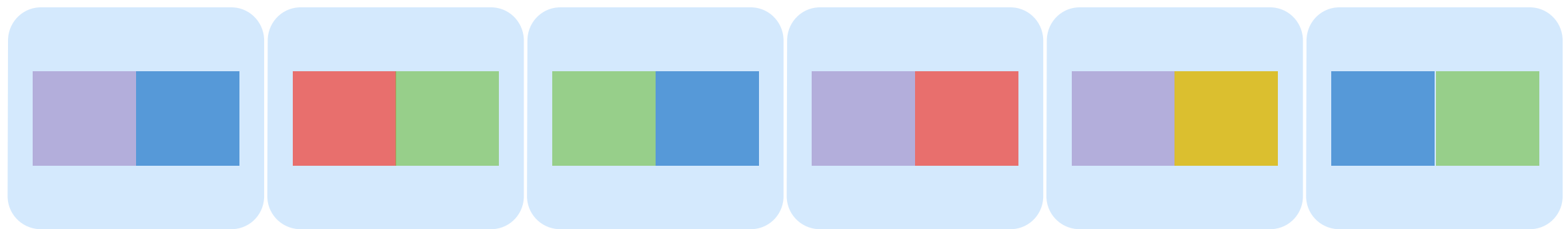
External Hashing

External Hashing

- Want to aggregate data that does not fit in memory
- Minimize number of I/O's (especially random I/O's)

Aggregating Colors

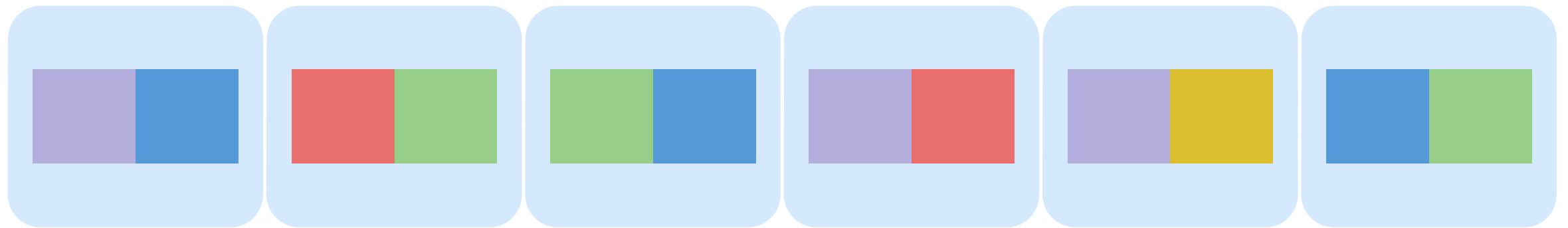
- Goal: Group squares by color
- Setup: 12 squares, 2 can fit per page. We can hold 8 squares in memory.
- $N=6$, $B=4$



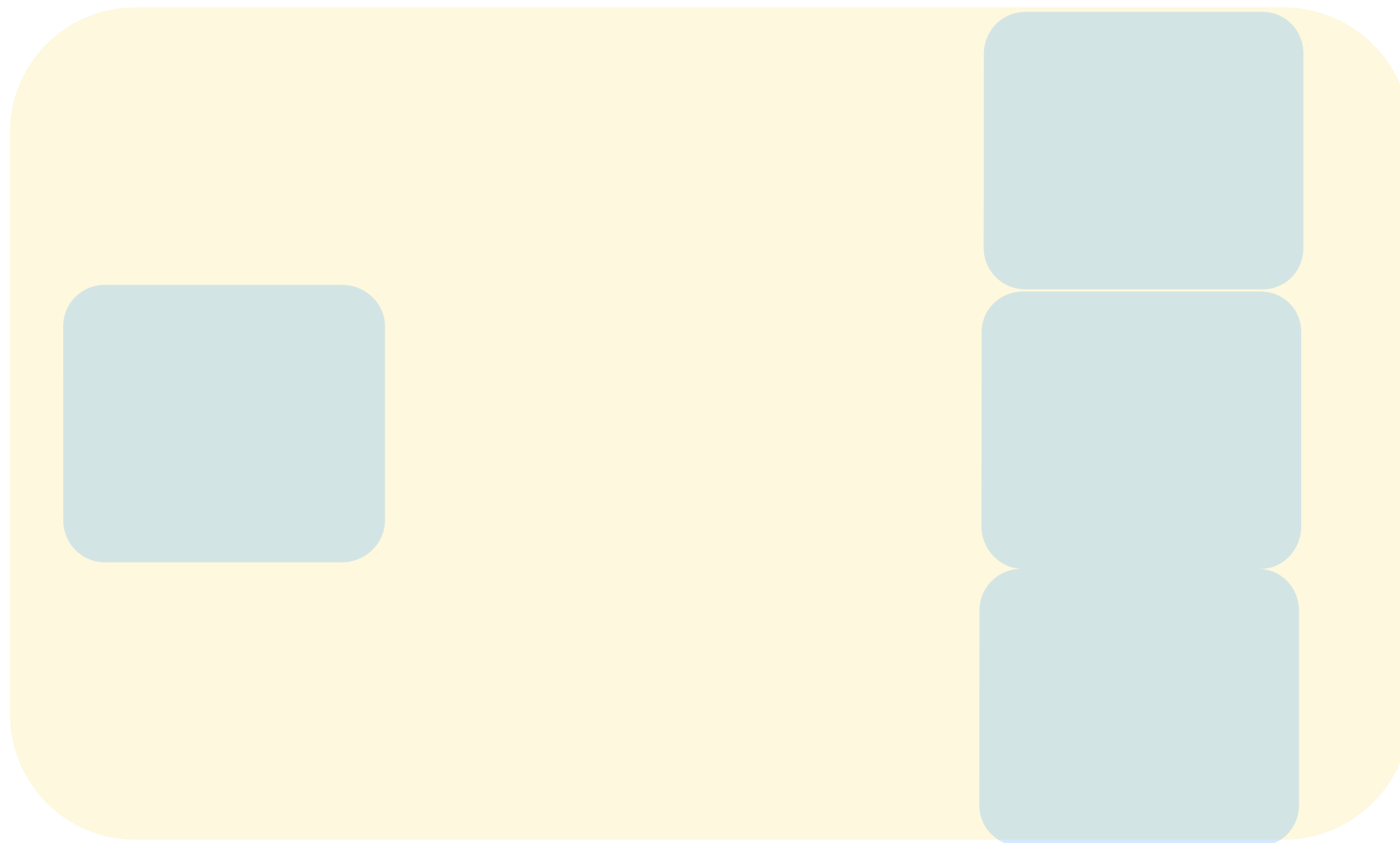
Pass 1: Divide

- Read all pages in, hash to B-1 partitions/buckets so that each group guaranteed to be in same partition.
- May not be a whole partition for each group.
- # I/O's = $2N$

Pass 1: Divide



$N=6$, $B=4$



Assign colors to 3 partitions
using hash function.

Our hash function:

$\{G, P\} \rightarrow 1$

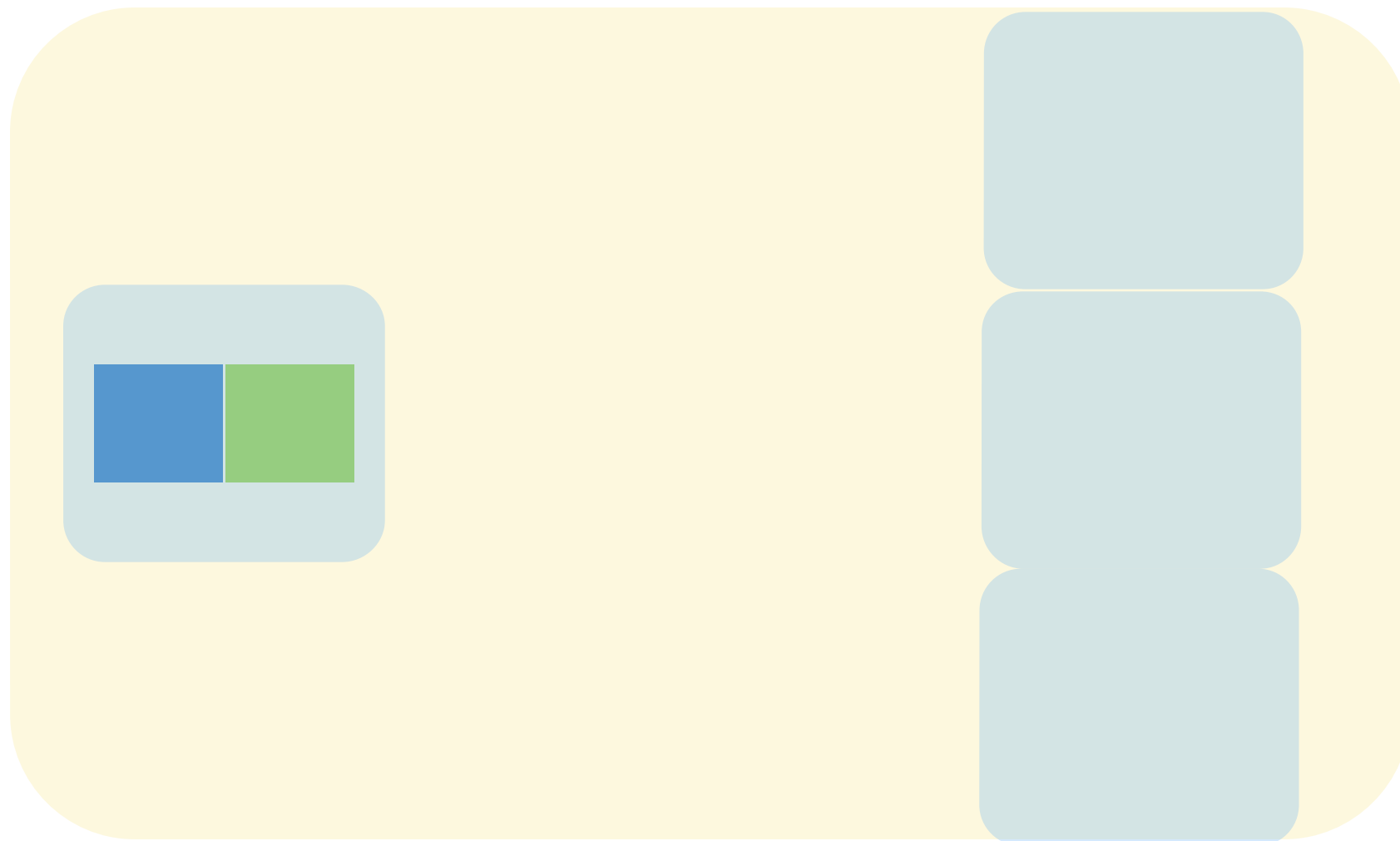
$\{B\} \rightarrow 2$

$\{R, Y\} \rightarrow 3$

Pass 1: Divide



$N=6$, $B=4$



Assign colors to 3 partitions
using hash function.

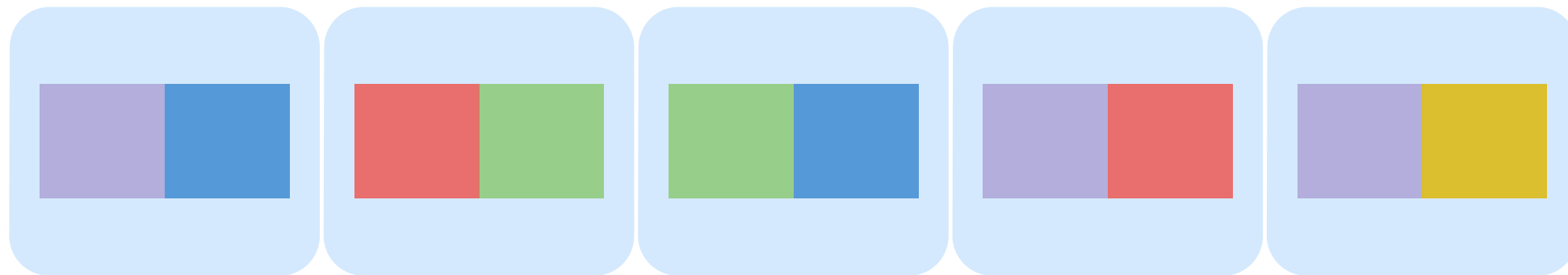
Our hash function:

$\{G, P\} \rightarrow 1$

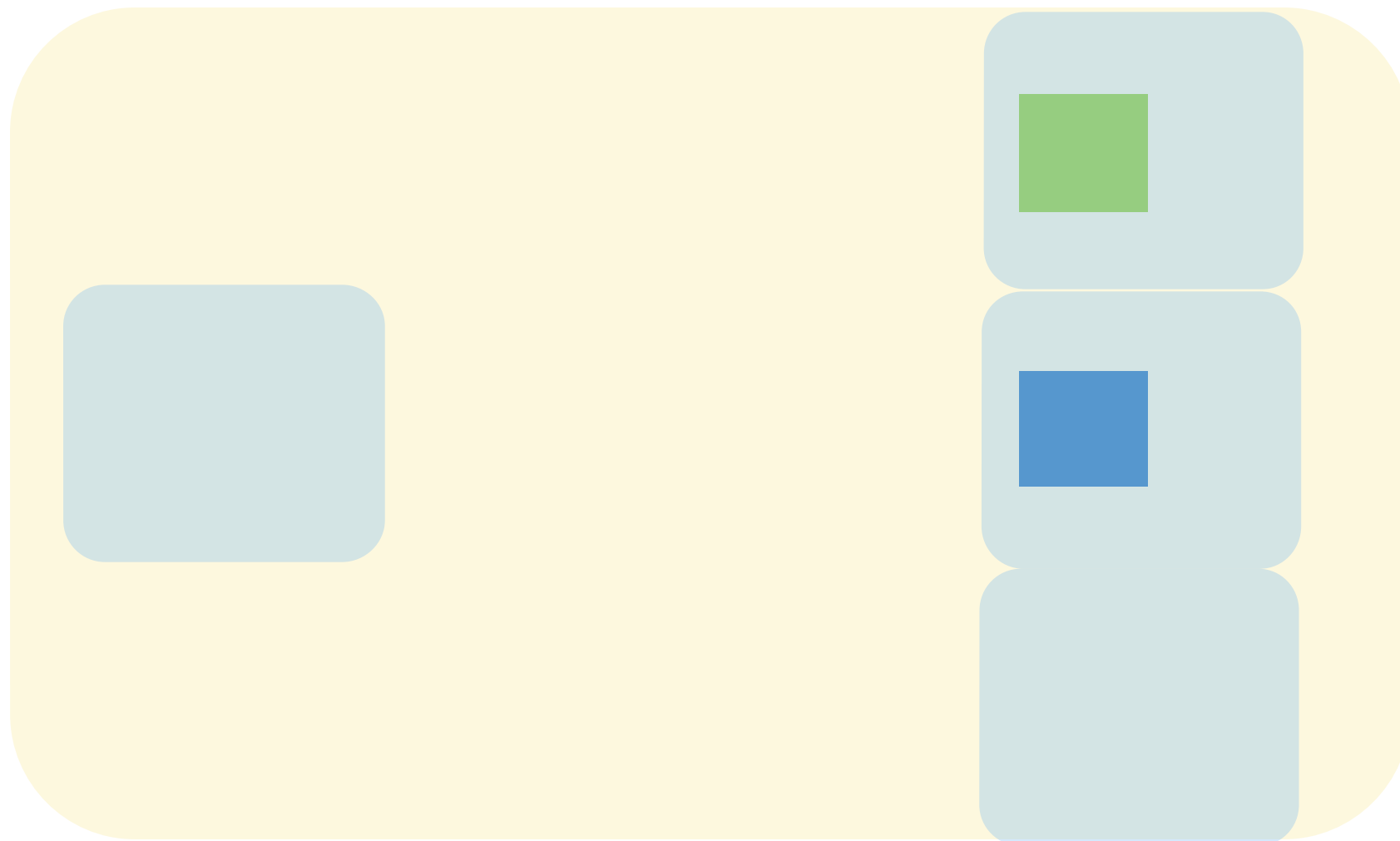
$\{B\} \rightarrow 2$

$\{R, Y\} \rightarrow 3$

Pass 1: Divide



$N=6, B=4$



Assign colors to 3 partitions
using hash function.

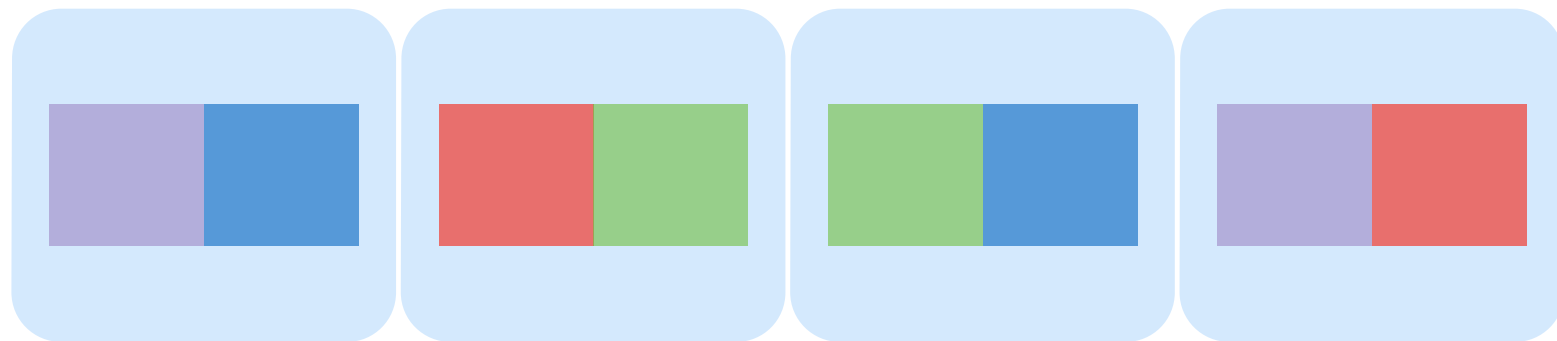
Our hash function:

$\{G, P\} \rightarrow 1$

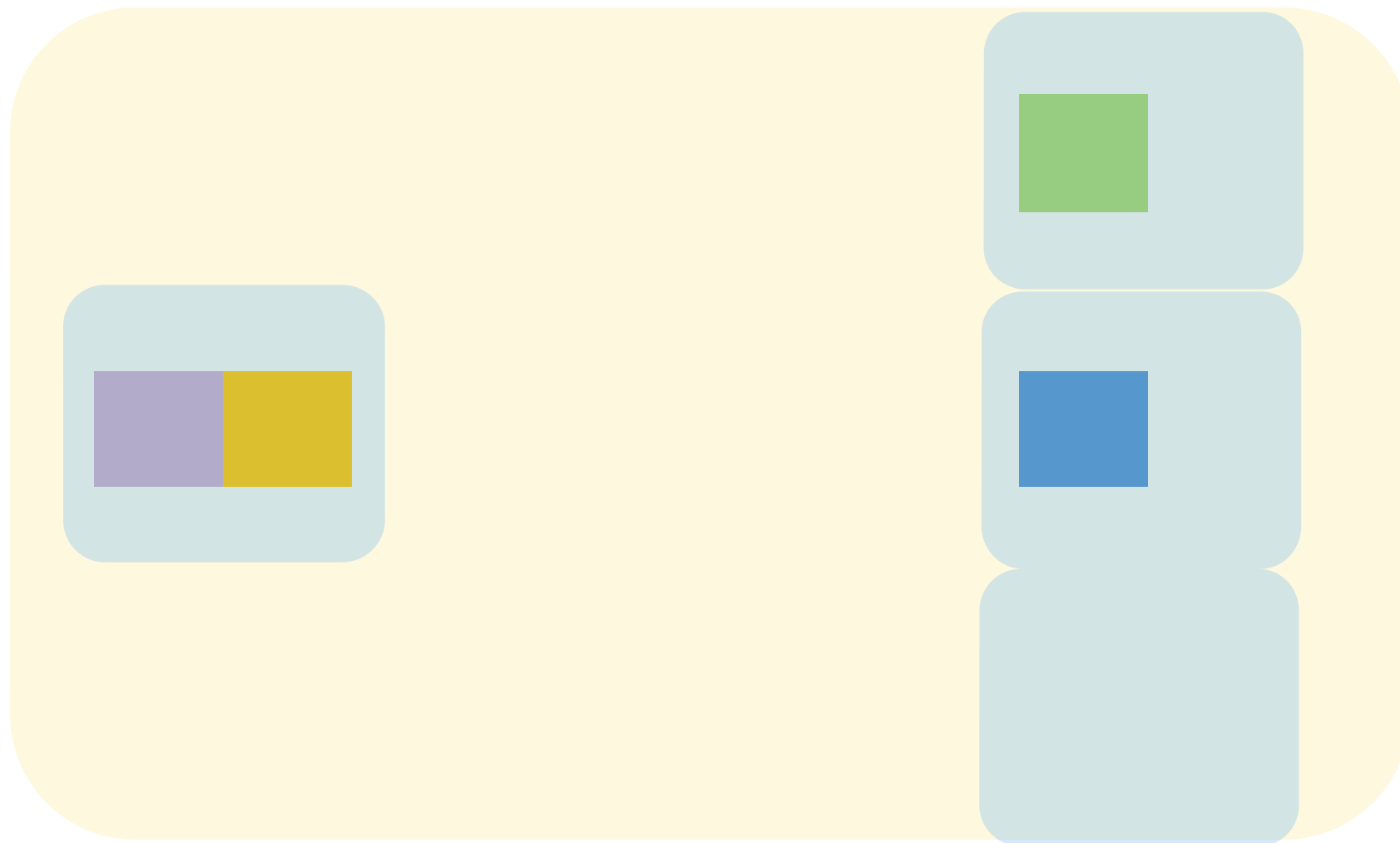
$\{B\} \rightarrow 2$

$\{R, Y\} \rightarrow 3$

Pass 1: Divide



$N=6$, $B=4$



Assign colors to 3 partitions
using hash function.

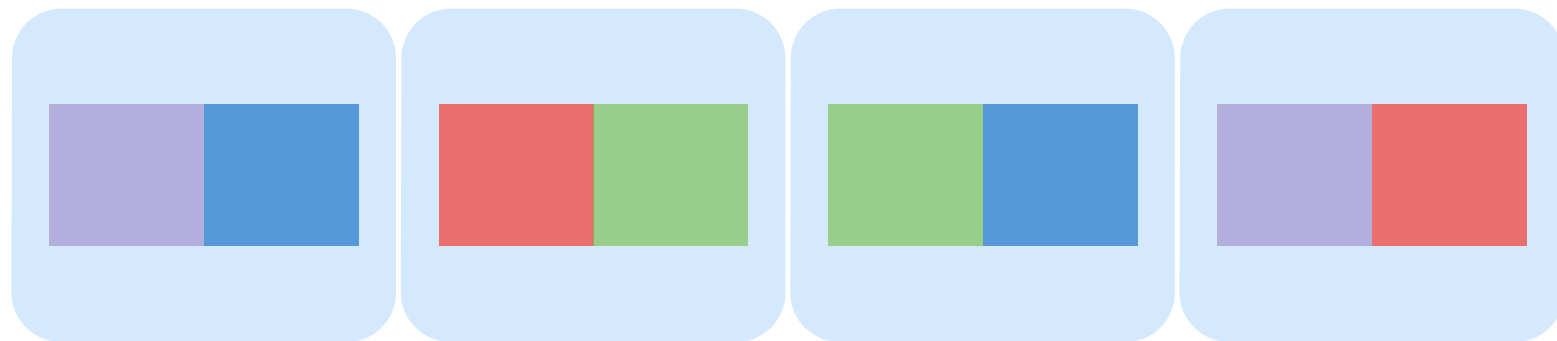
Our hash function:

$\{G, P\} \rightarrow 1$

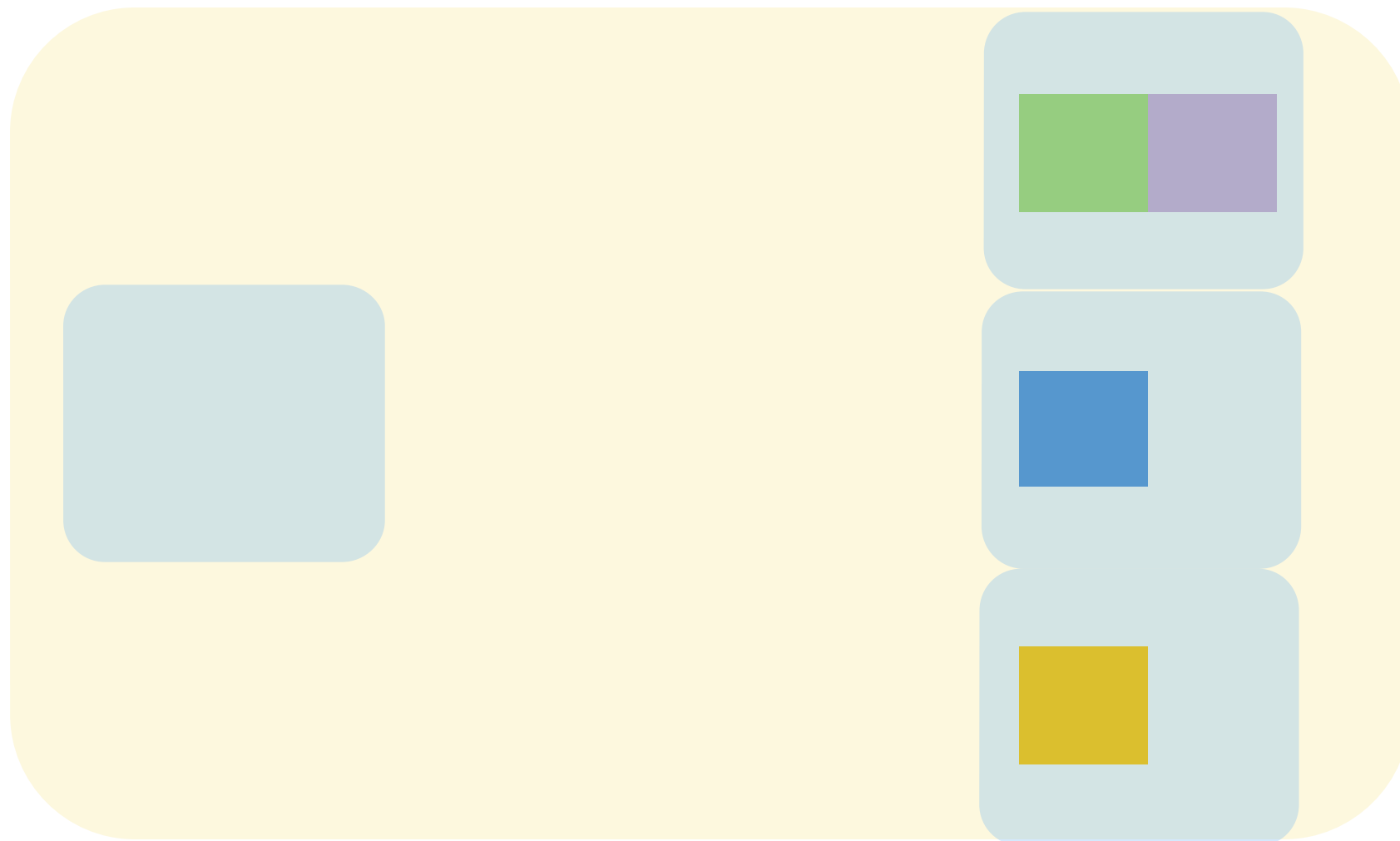
$\{B\} \rightarrow 2$

$\{R, Y\} \rightarrow 3$

Pass 1: Divide



N=6, B=4



Assign colors to 3 partitions
using hash function.

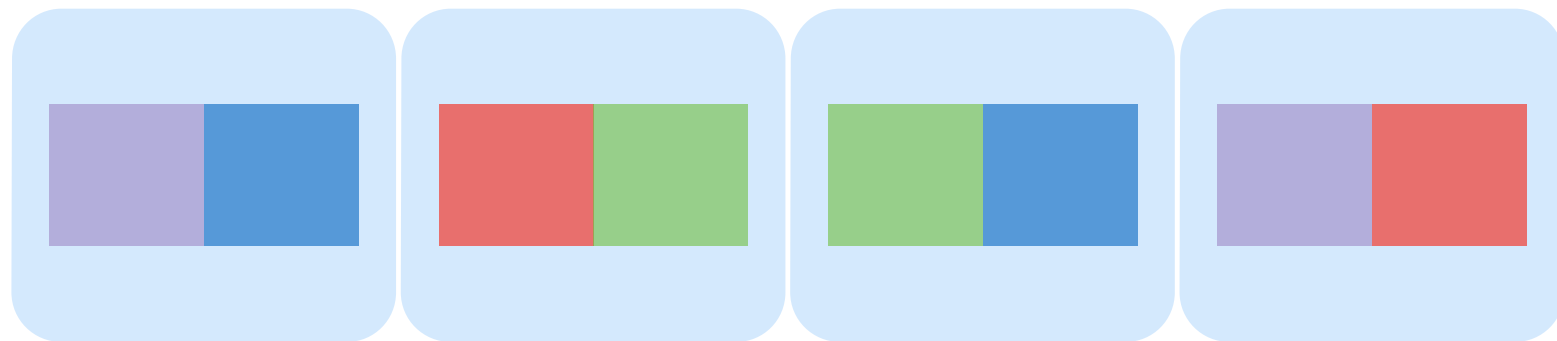
Our hash function:

{G,P} -> 1

{B} -> 2

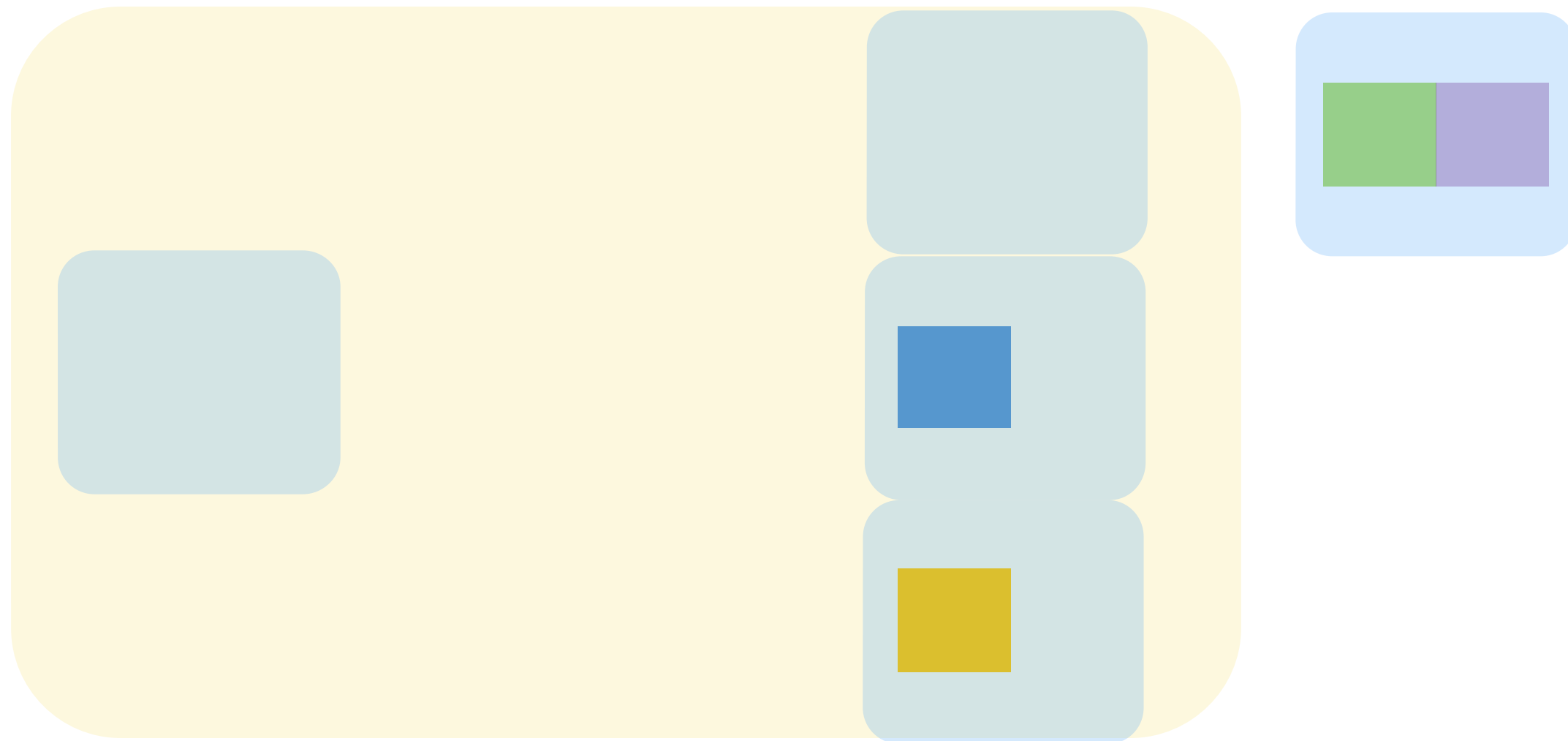
{R, Y} -> 3

Pass 1: Divide

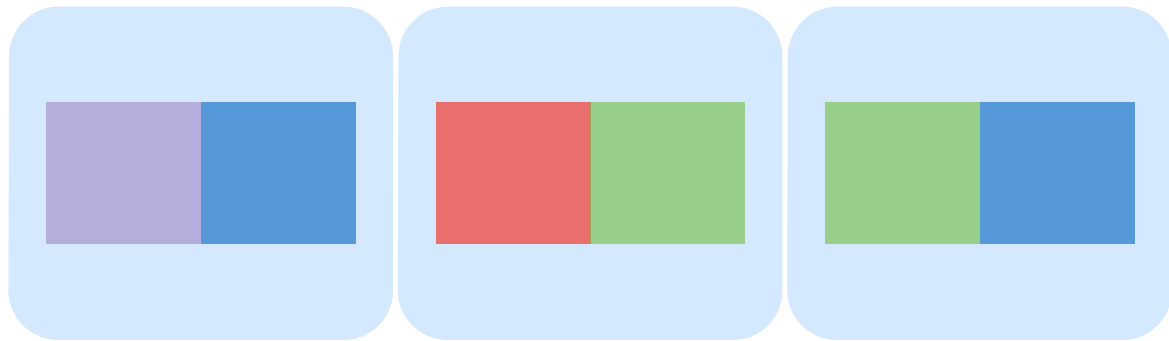


N=6, B=4

Our hash function: {G,P} \rightarrow 1, {B} \rightarrow 2, {R, Y} \rightarrow 3

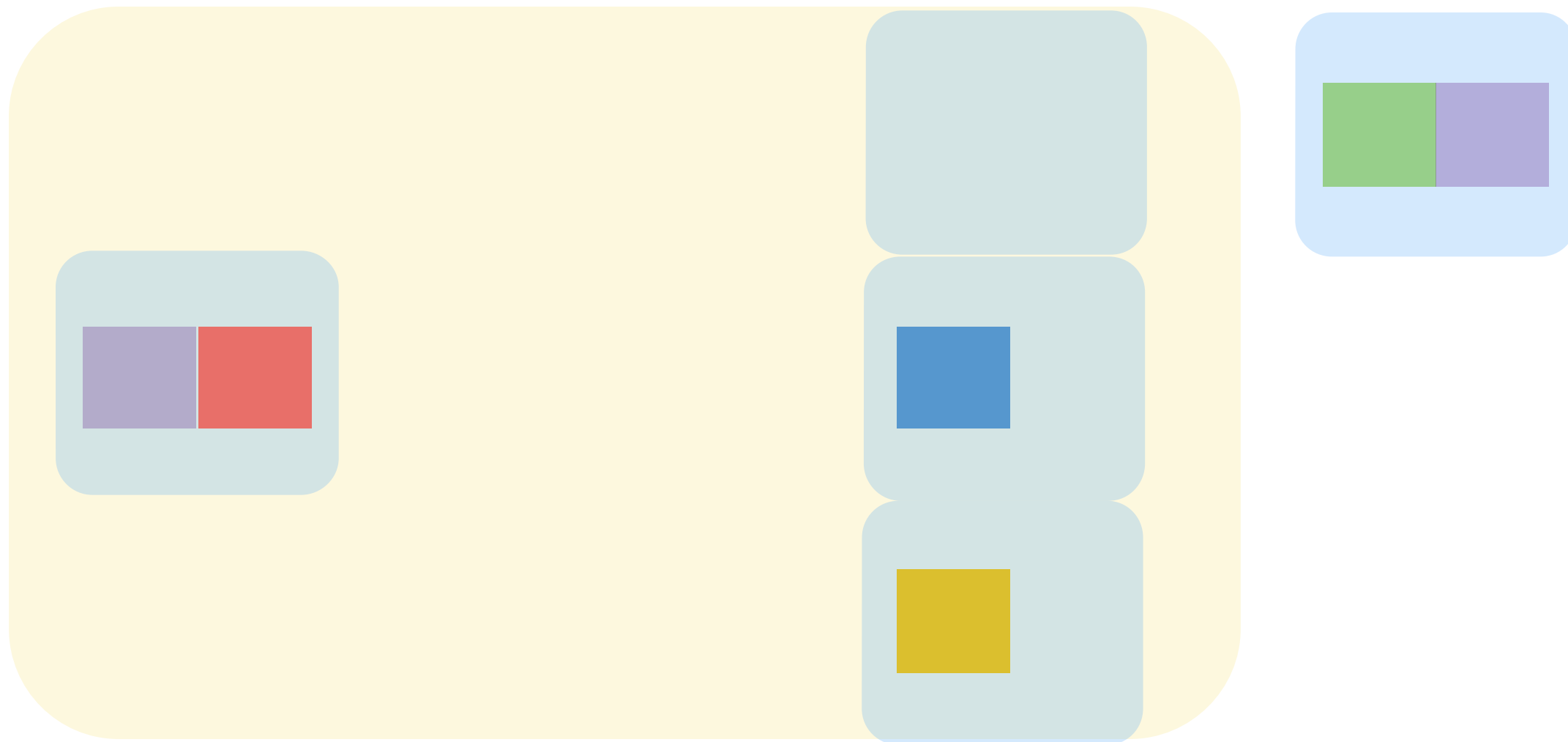


Pass 1: Divide

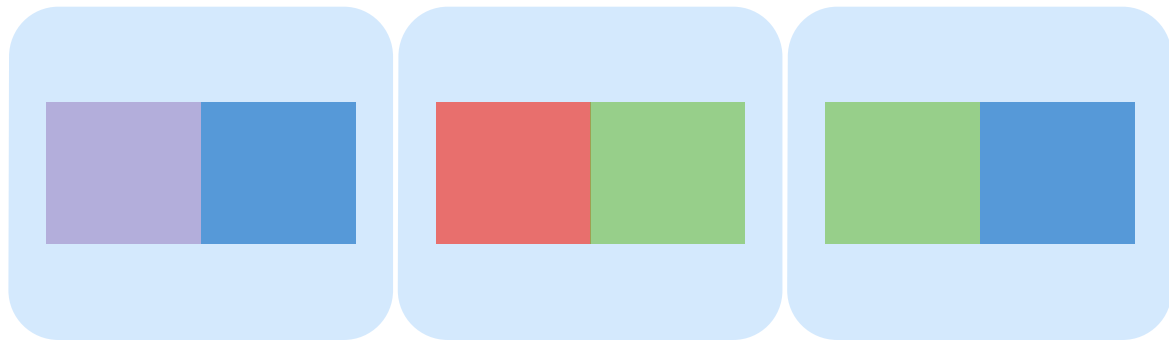


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

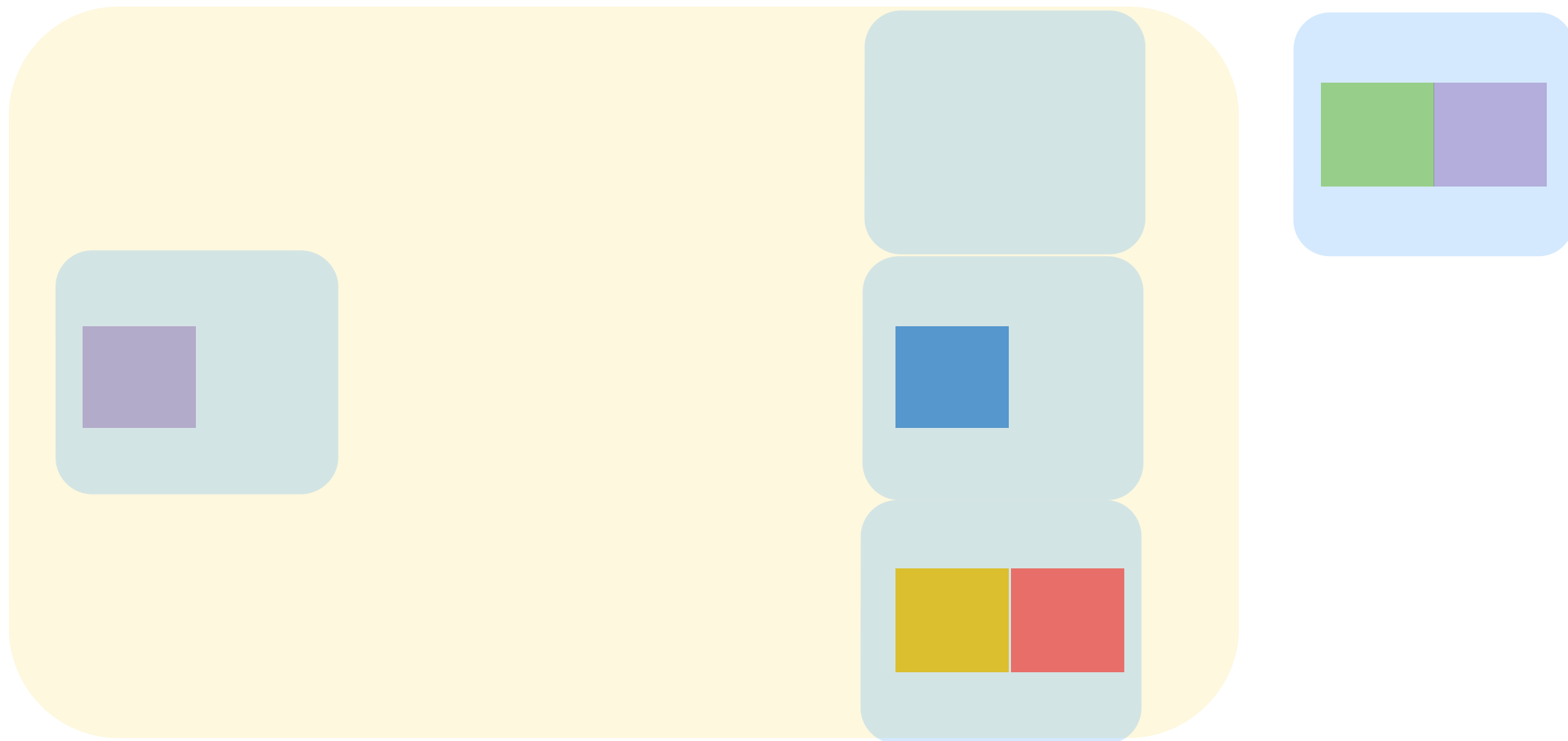


Pass 1: Divide

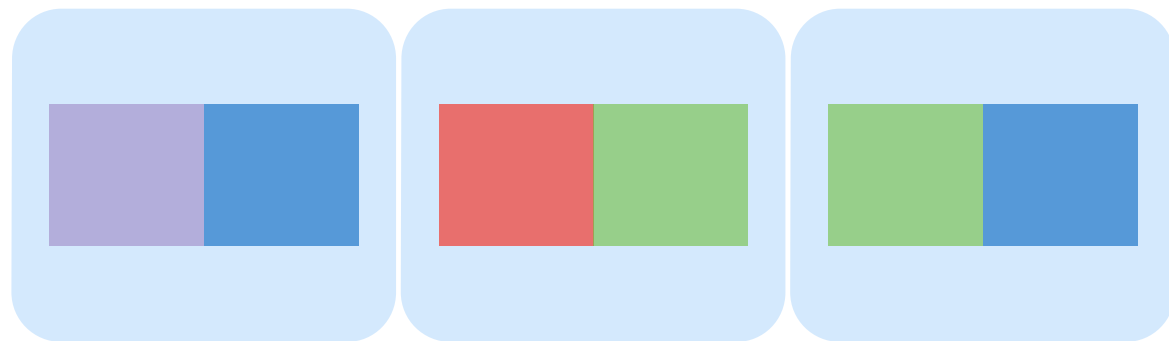


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

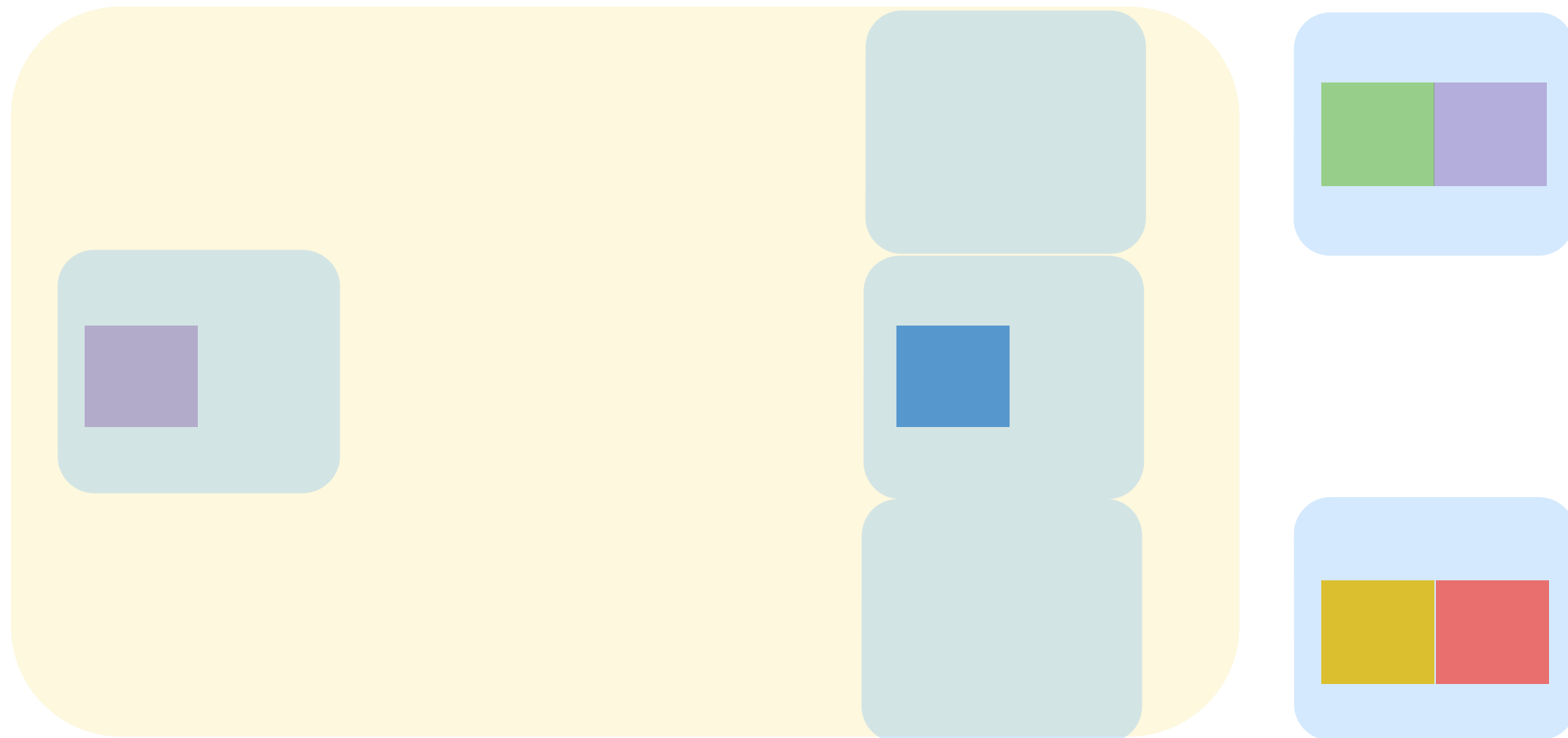


Pass 1: Divide

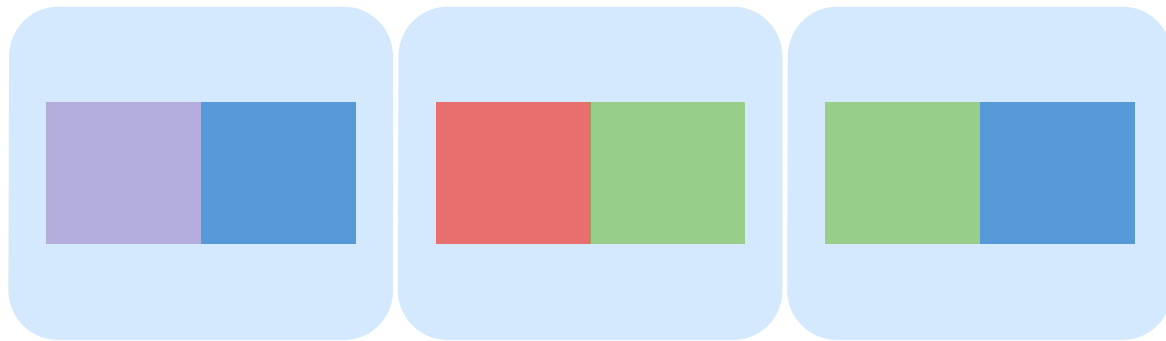


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

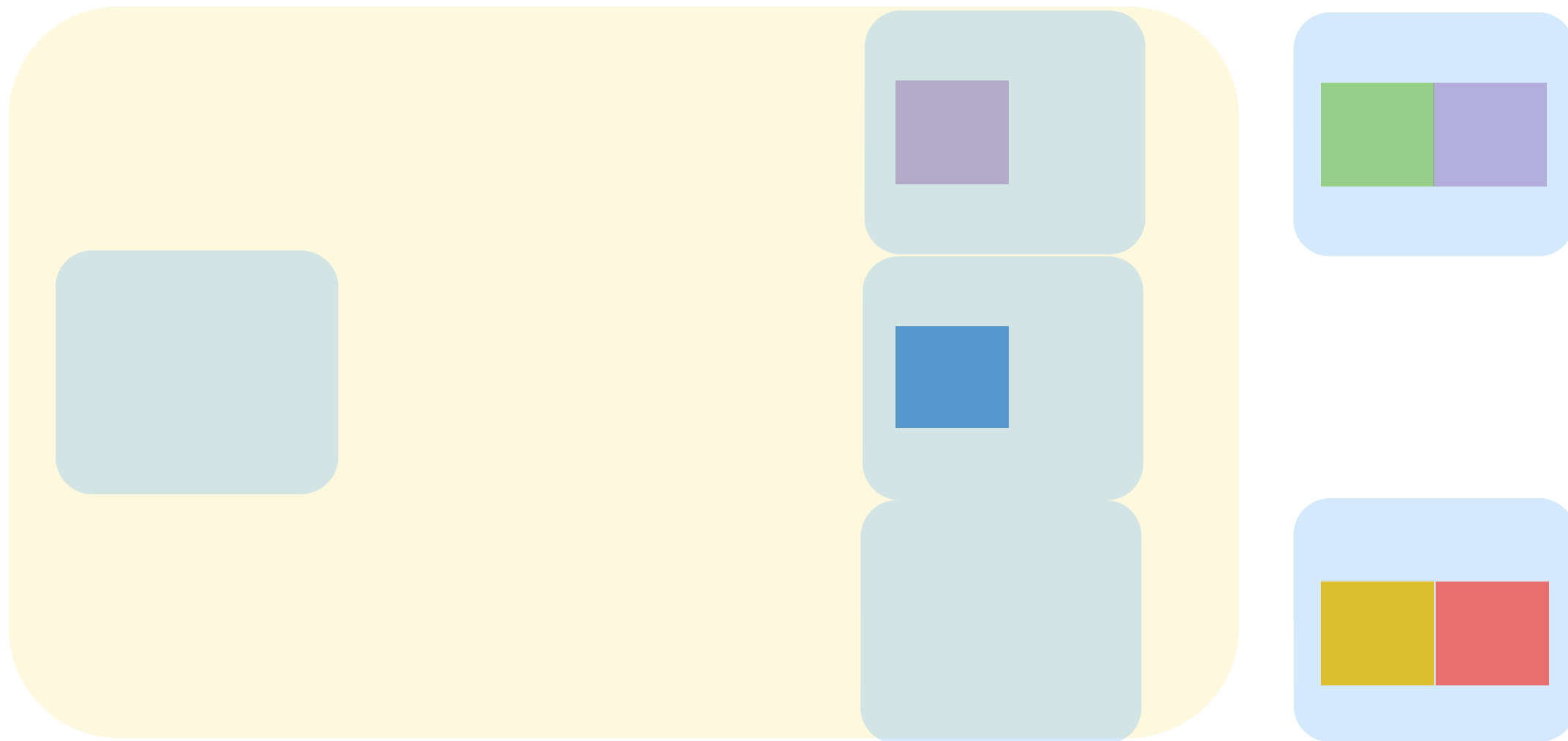


Pass 1: Divide

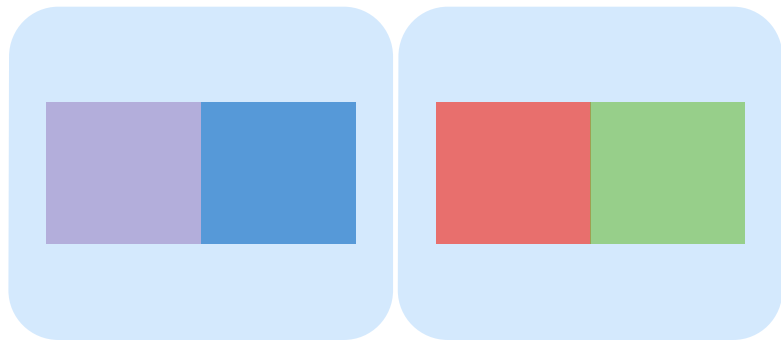


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

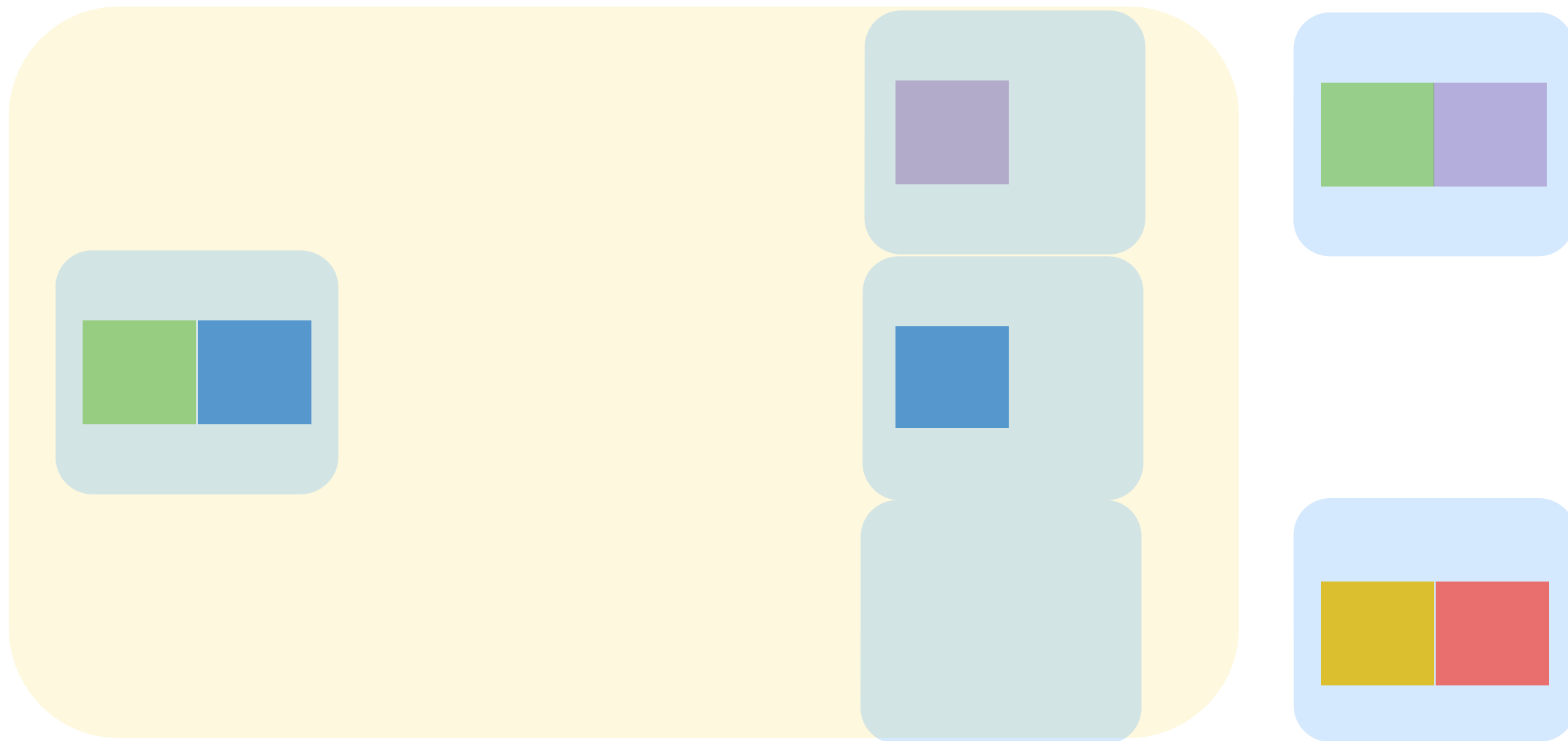


Pass 1: Divide

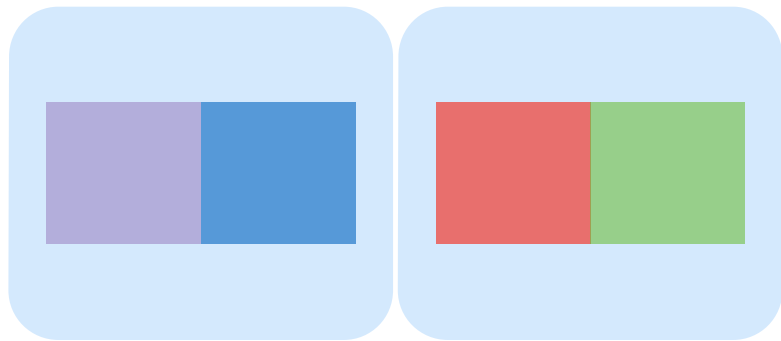


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

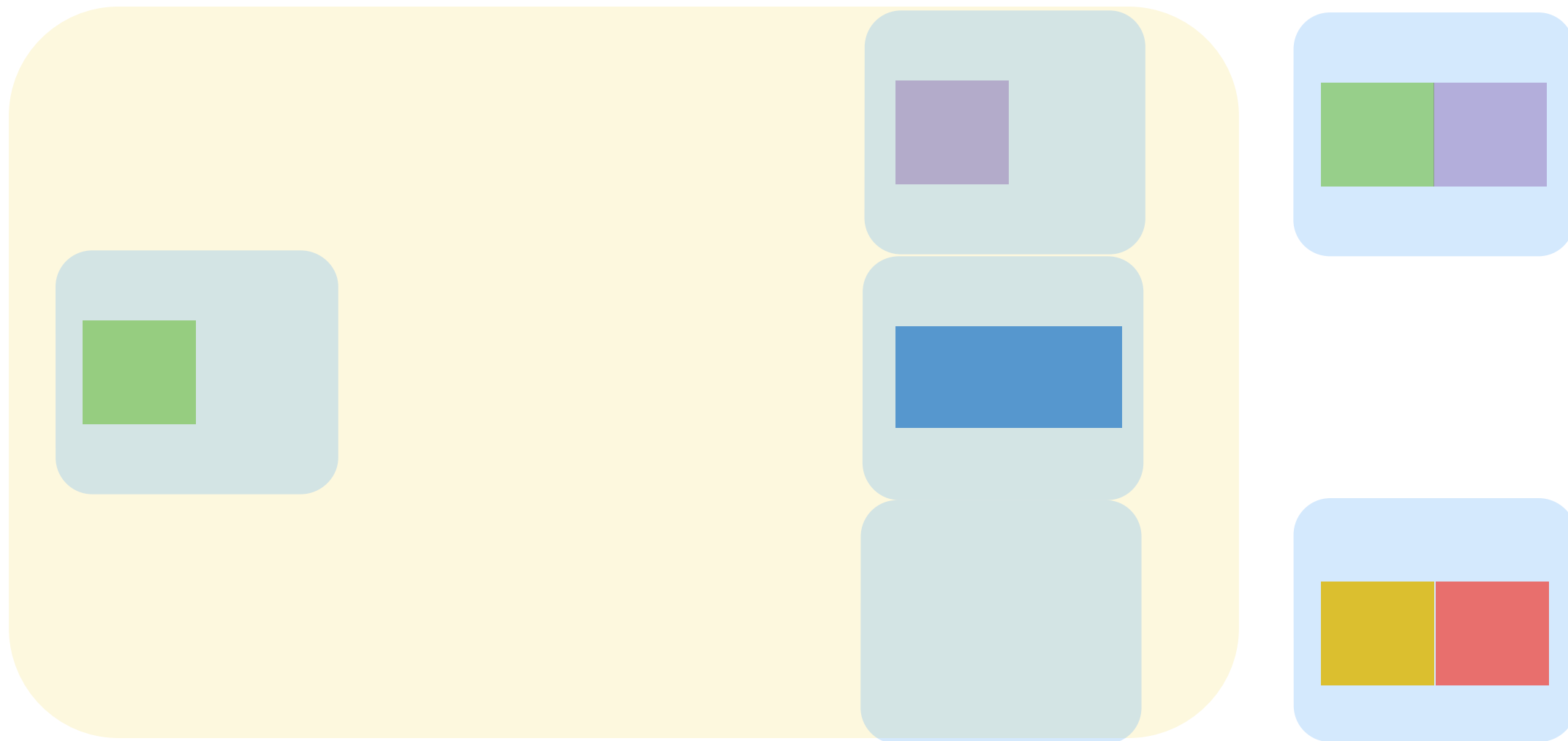


Pass 1: Divide

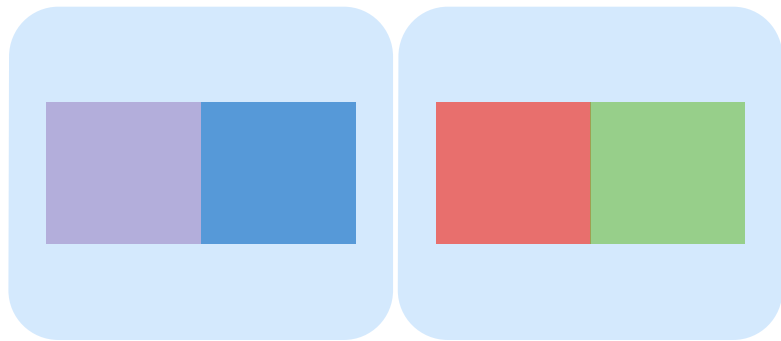


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

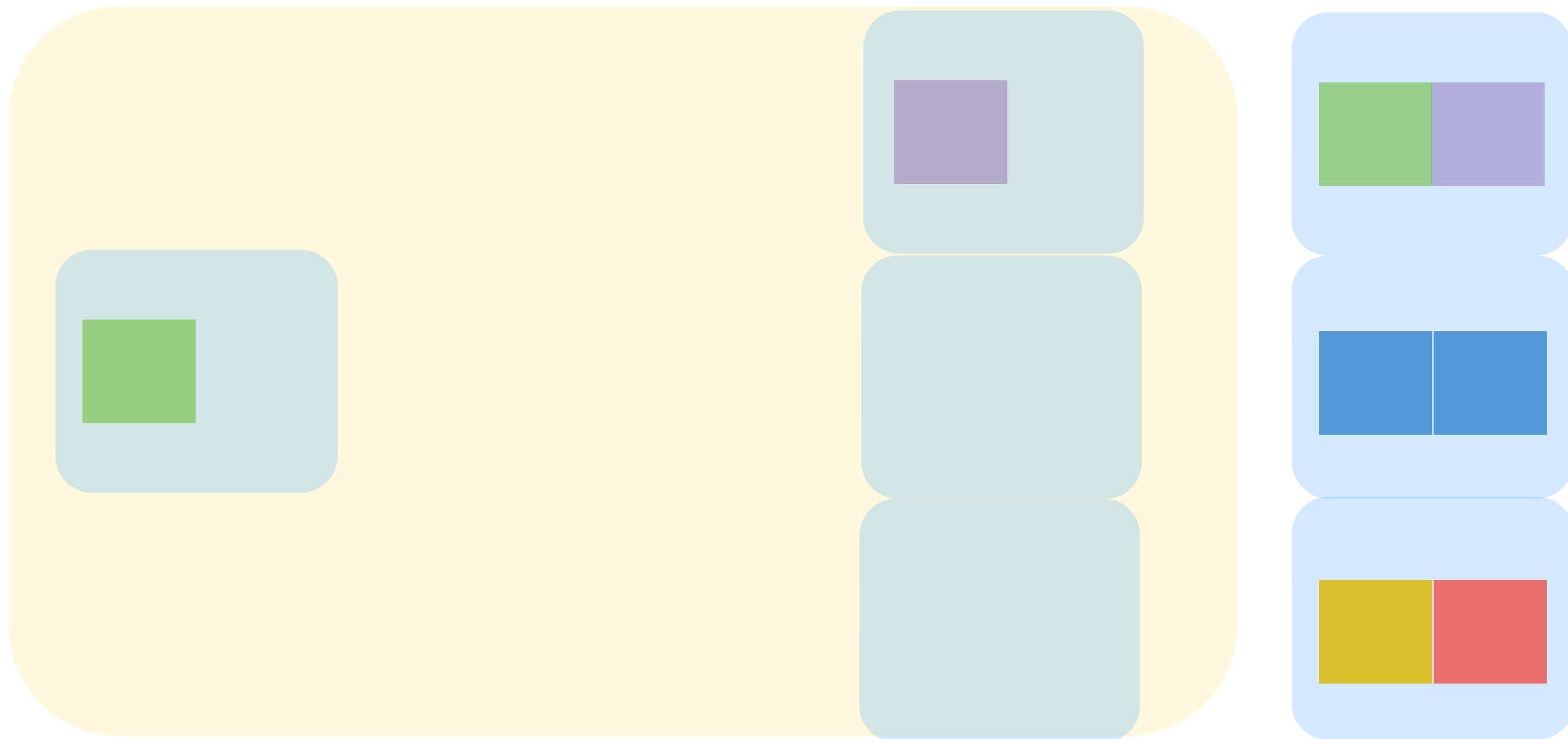


Pass 1: Divide

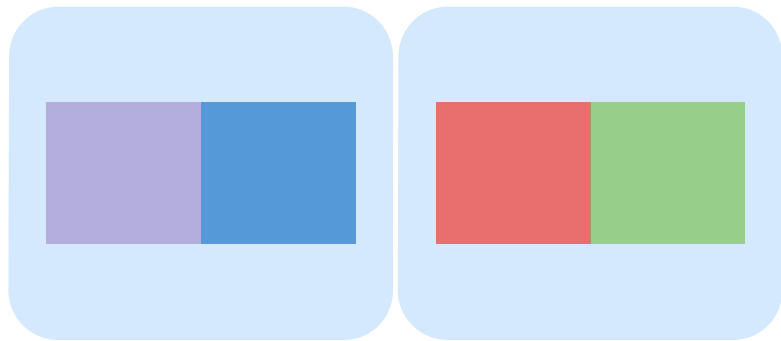


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

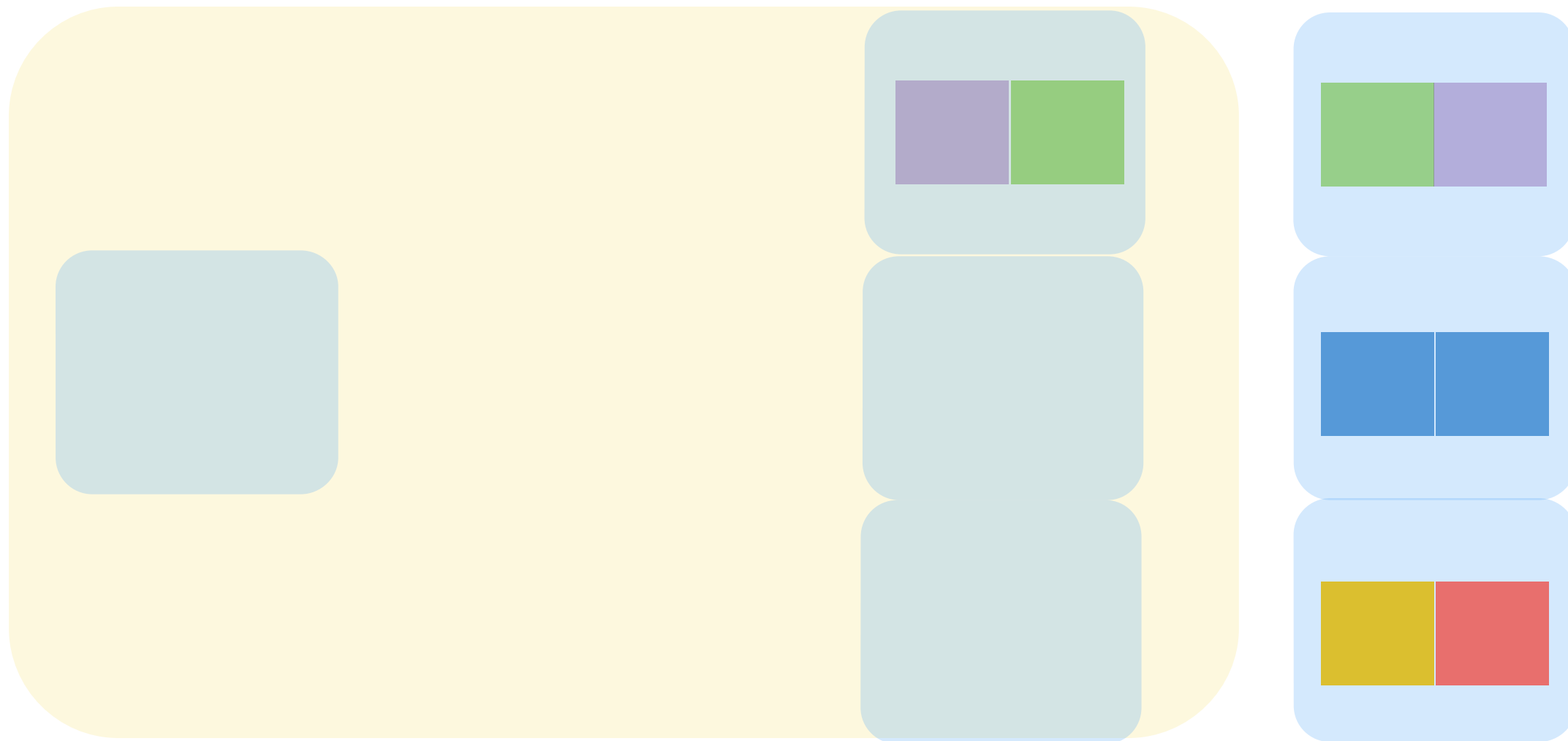


Pass 1: Divide

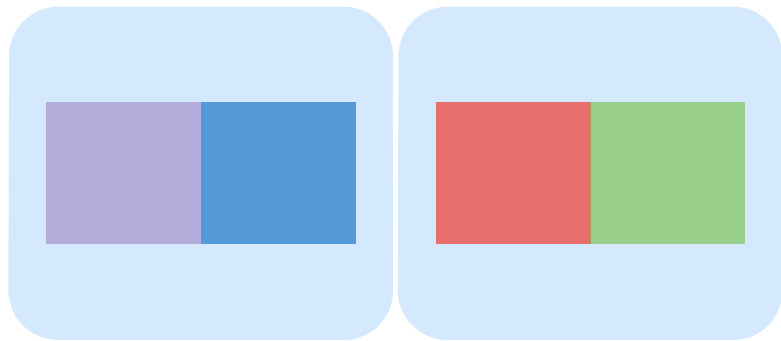


$N=6, B=4$

Our hash function: $\{G,P\} \rightarrow 1, \{B\} \rightarrow 2, \{R, Y\} \rightarrow 3$

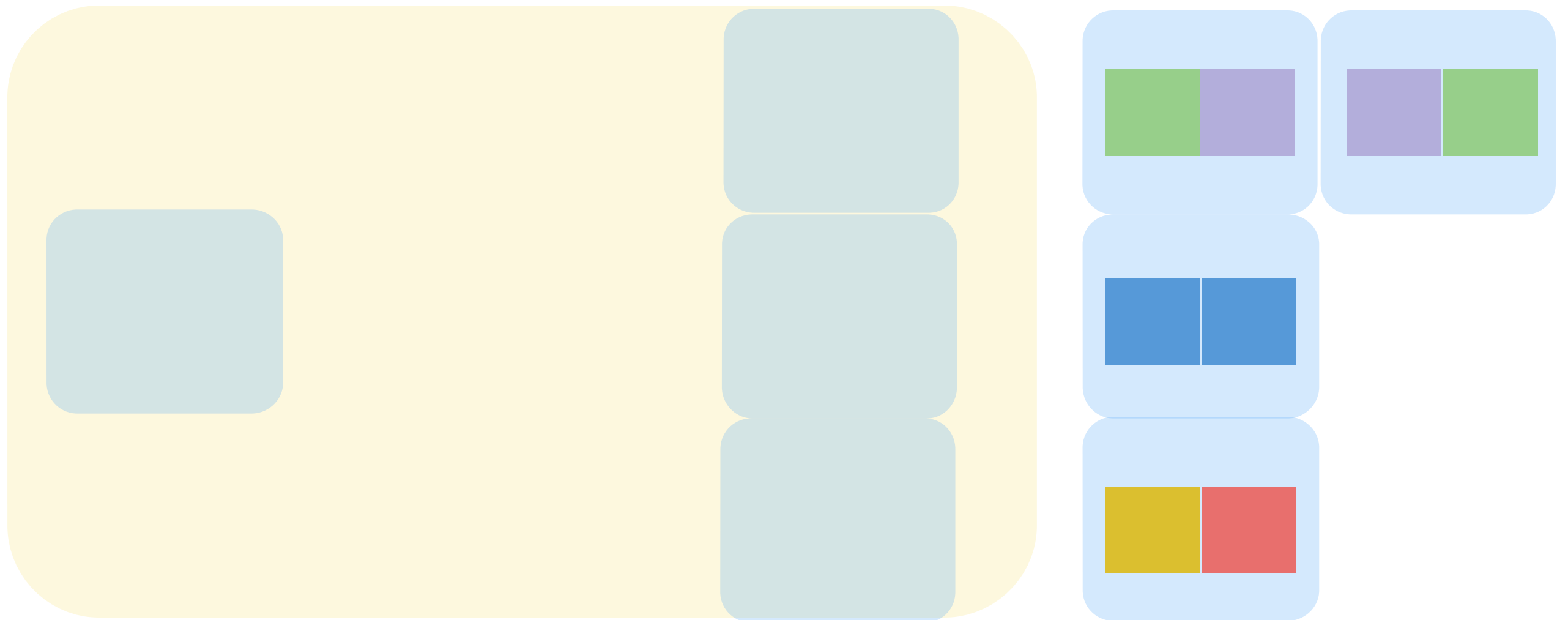


Pass 1: Divide

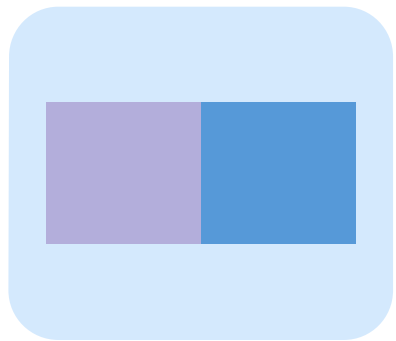


$N=6$, $B=4$

Our hash function: $\{G,P\} \rightarrow 1$, $\{B\} \rightarrow 2$, $\{R, Y\} \rightarrow 3$

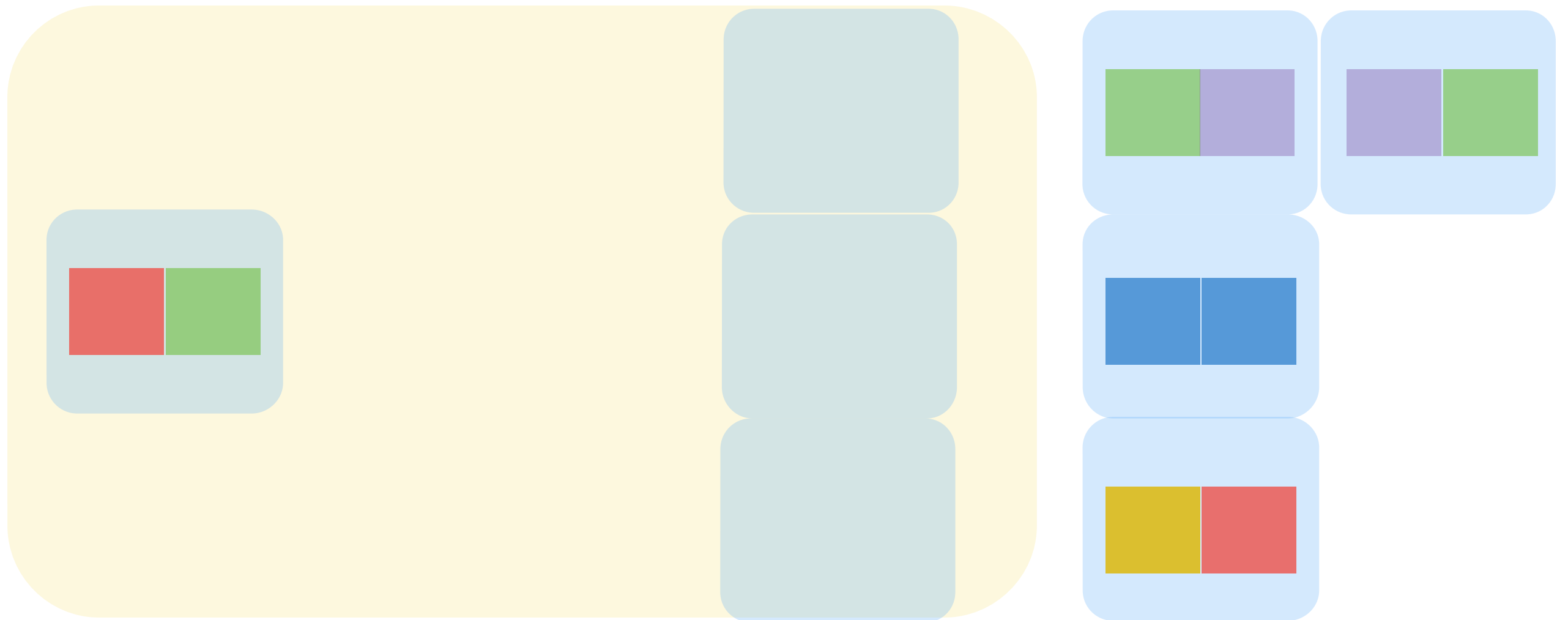


Pass 1: Divide

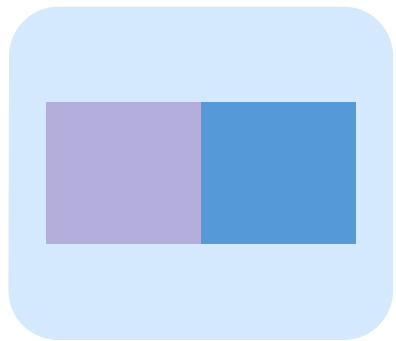


N=6, B=4

Our hash function: {G,P} -> 1, {B} -> 2, {R, Y} -> 3

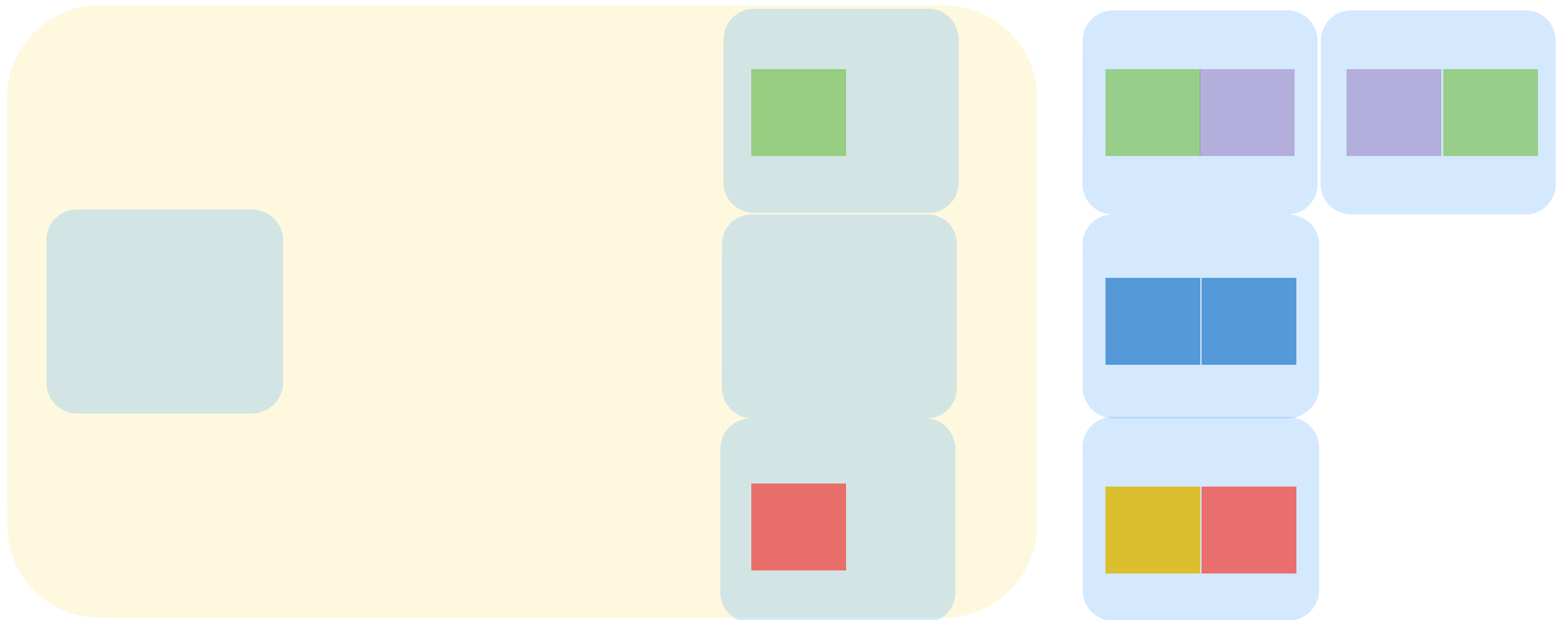


Pass 1: Divide



N=6, B=4

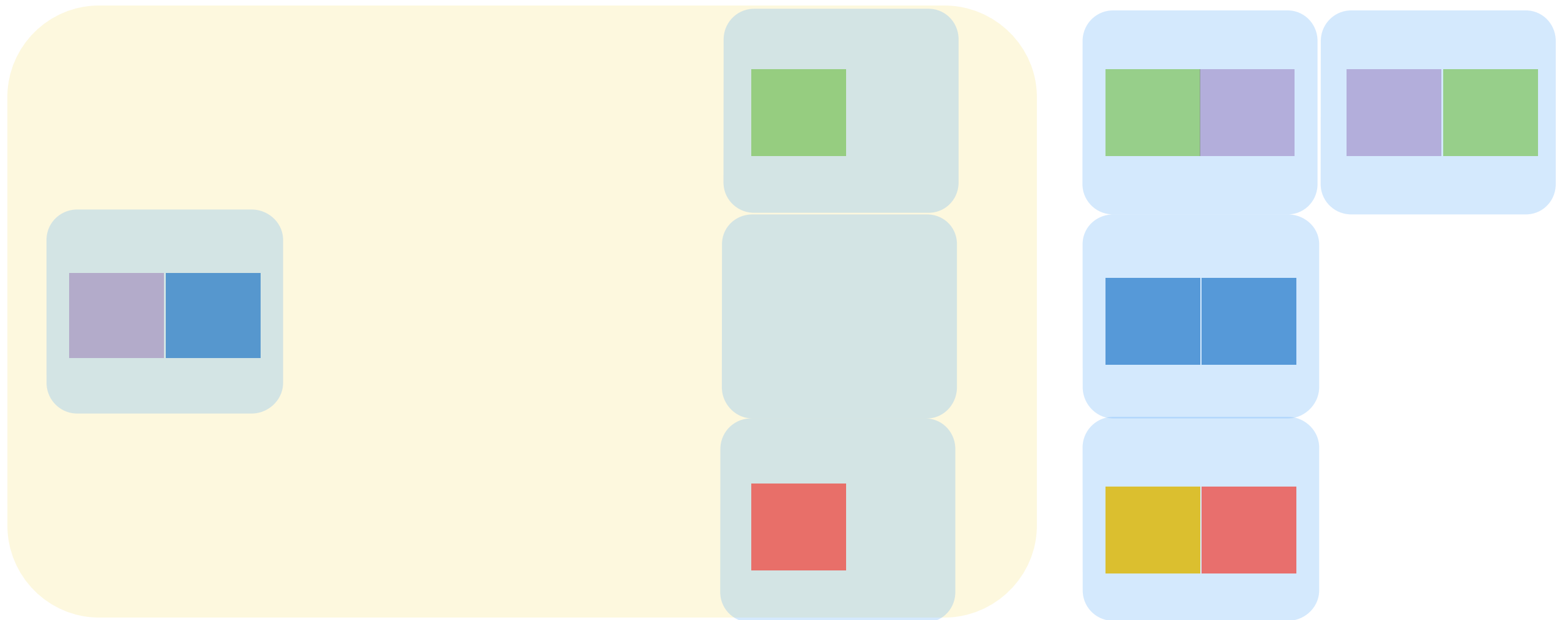
Our hash function: {G,P} -> 1, {B} -> 2, {R, Y} -> 3



Pass 1: Divide

N=6, B=4

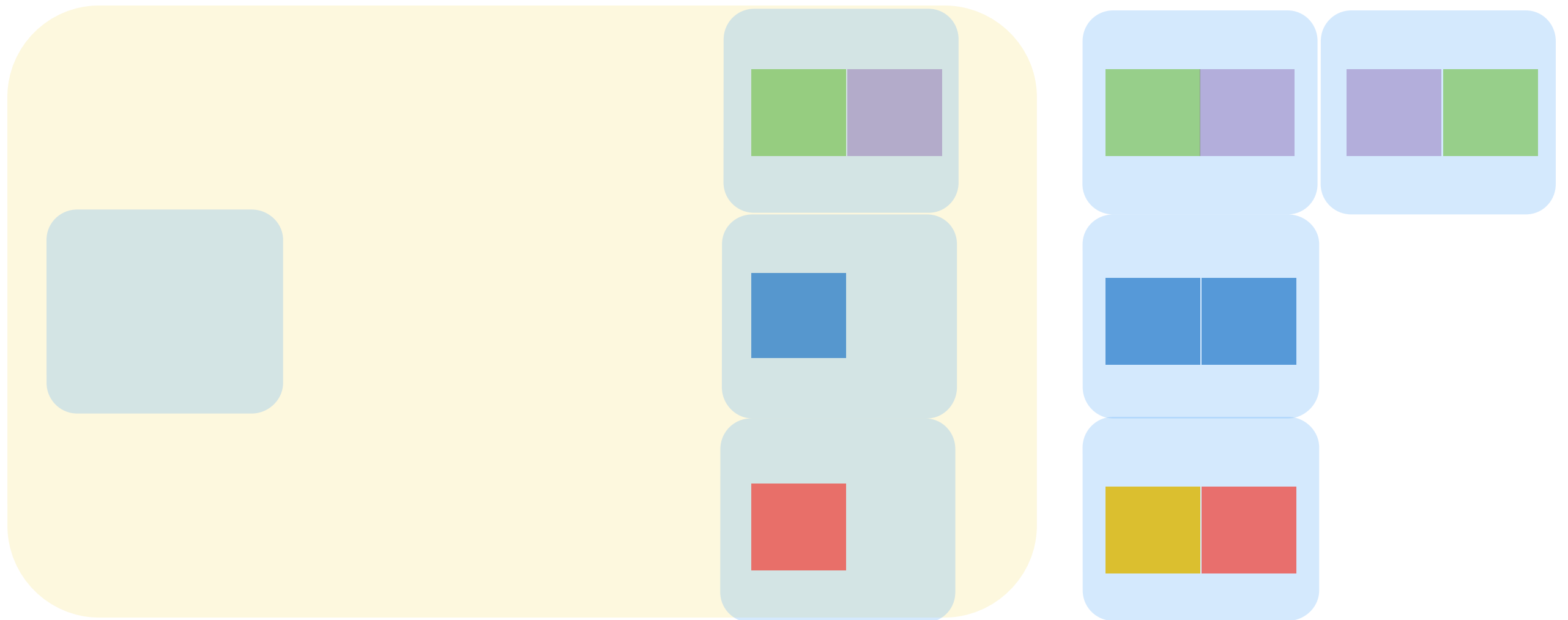
Our hash function: {G,P} -> 1, {B} -> 2, {R, Y} -> 3



Pass 1: Divide

N=6, B=4

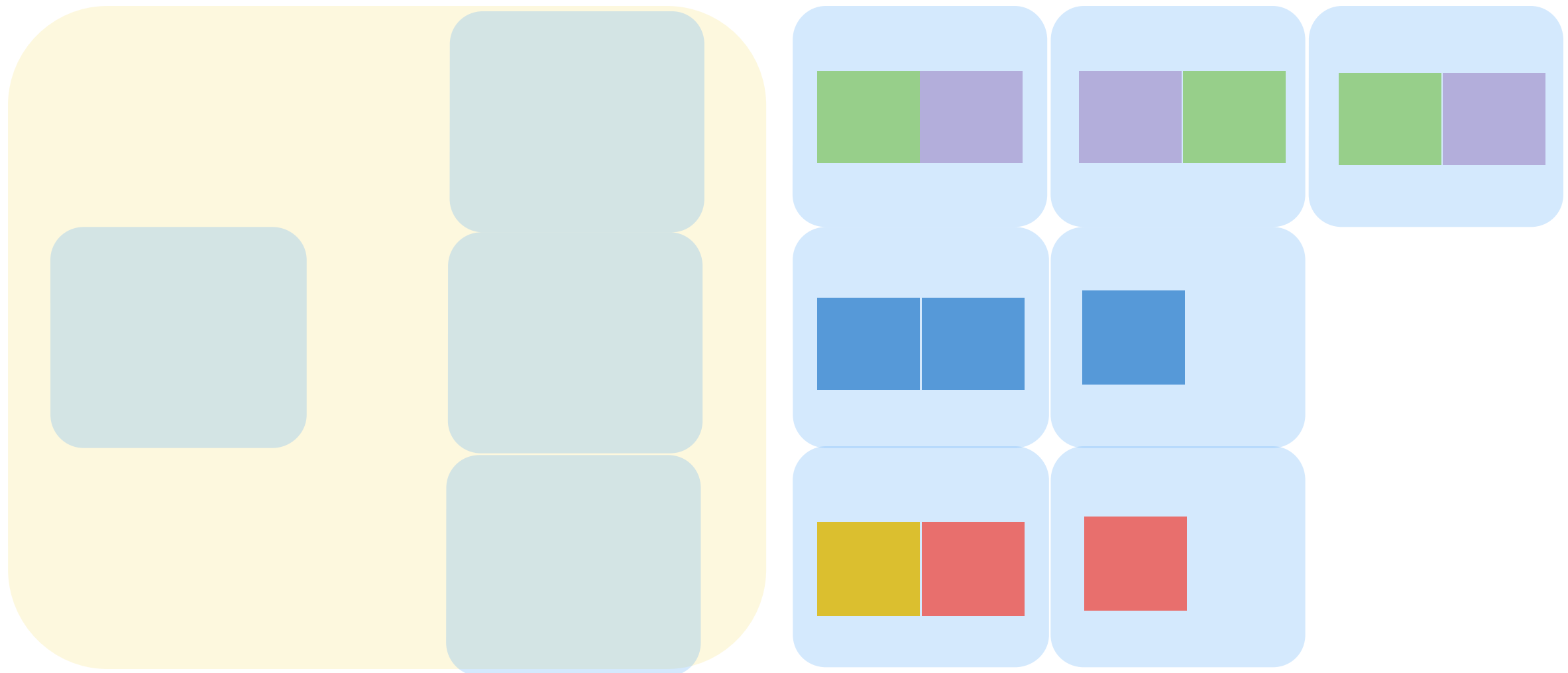
Our hash function: {G,P} -> 1, {B} -> 2, {R, Y} -> 3



Pass 1: Divide

N=6, B=4

Our hash function: {G,P} -> 1, {B} -> 2, {R, Y} -> 3



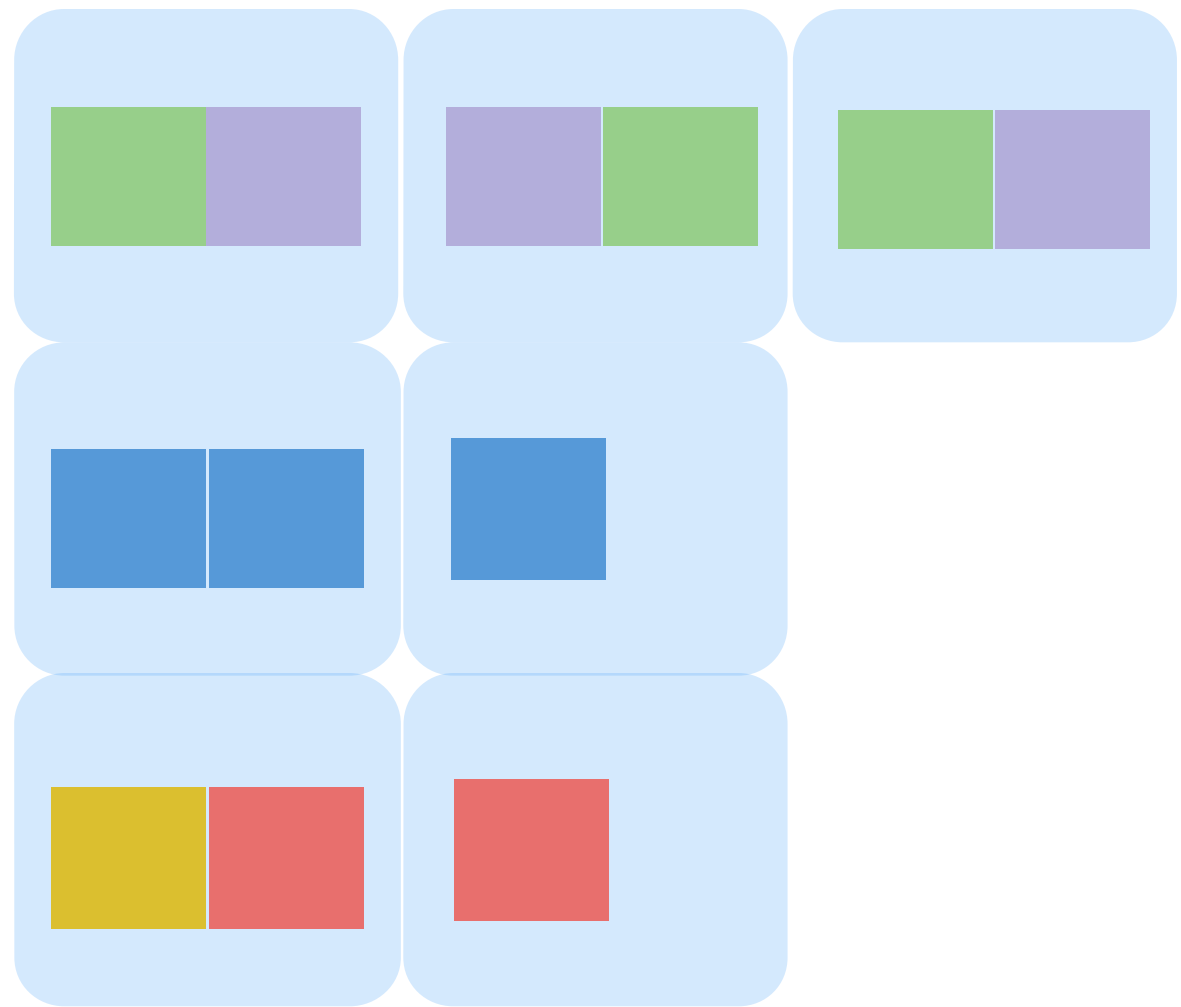
Pass 2: Conquer

- Rehash each partition.
- For a partition to fit in memory, it can only have B pages.
- To hash larger tables, use the partition algorithm recursively until the partition fits into memory
- $\# \text{ I/O's} = 2N$

Pass 2: Conquer

Create in-memory table for each partition.

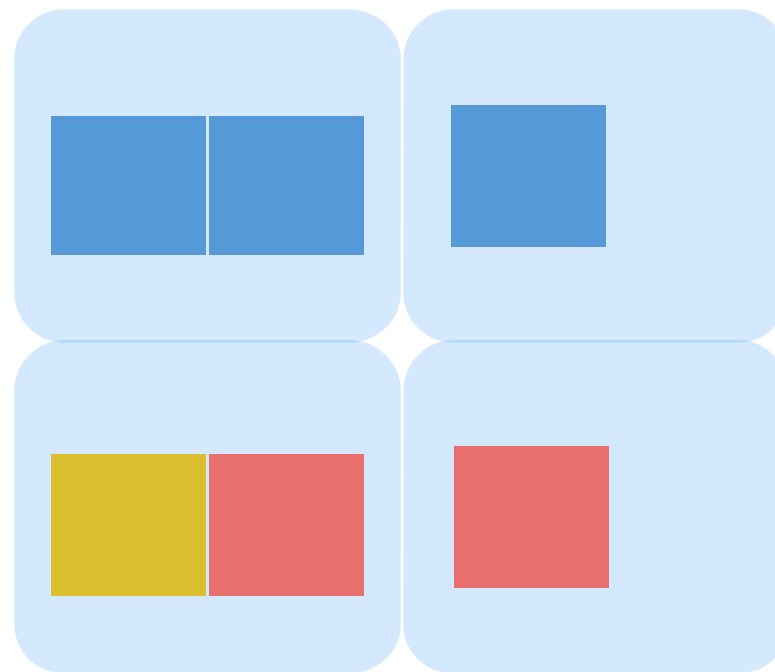
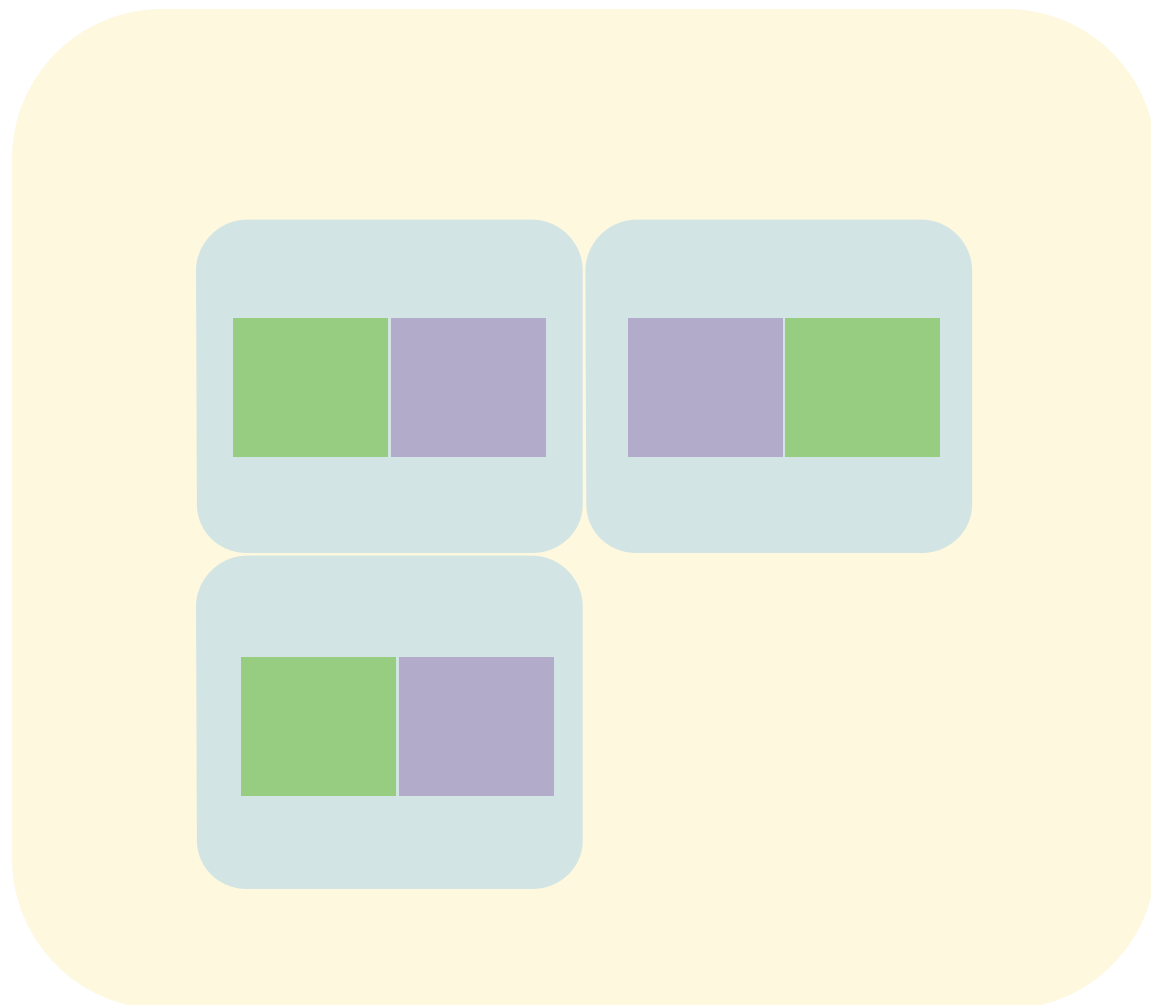
N=6, B=4



Pass 2: Conquer

Create in-memory table for each partition.

$N=6$, $B=4$



Pass 2: Conquer

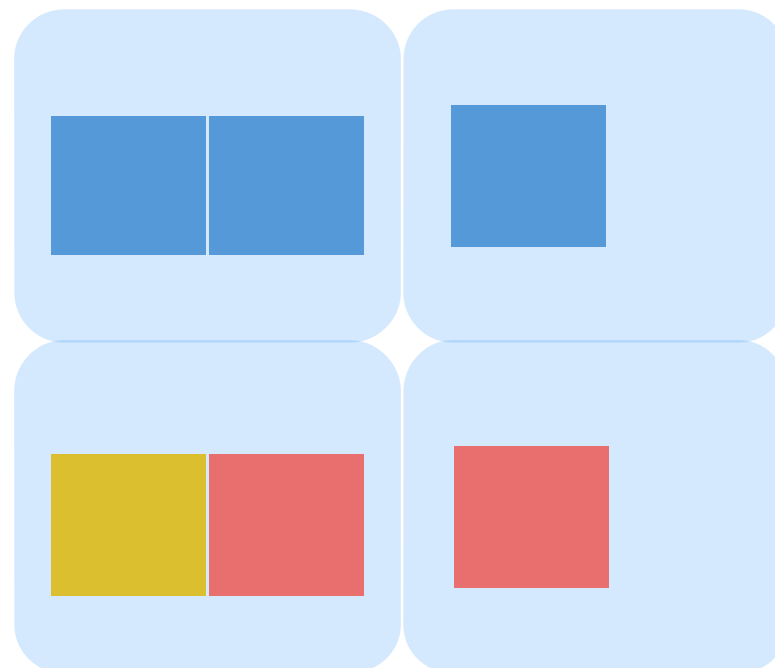
Create in-memory table for each partition.

$N=6$, $B=4$

Green



Purple



Worksheet #5, 6, 7

Why can we process $B * (B - 1)$ pages of data with external hashing in just two passes (divide and conquer phases)?

Why can we process $B * (B - 1)$ pages of data with external hashing in just two passes (divide and conquer phases)?

- Our main limitation is how big the partitions can be after the partition hashing. Since we need to be able to read in the whole partition into memory, each partition can be at most B pages big.

If you're processing exactly $B * (B - 1)$ pages of data, is it likely that you'll have to perform recursive external hashing? Why?

If you're processing exactly $B * (B - 1)$ pages of data, is it likely that you'll have to perform recursive external hashing? Why?

- You would have to have an absolutely perfect hash function that evenly distributes any record into the $B-1$ partitions. This is almost impossible in practice. Rather, we should expect that some partitions may be larger than B after partition hashing.

While you recursively perform external hashing, you reuse the same hash functions for partitioning. What's the problem with this?

While you recursively perform external hashing, you reuse the same hash functions for partitioning. What's the problem with this?

- The partition that is too big to fit in memory will still be too big to fit in memory if we maintain the same partition hashing strategy.

Tournament Sort

- Used to gather initial runs for external sorting algorithms
- Alternative to quicksort
 - Quicksort faster, but tournament sort longer runs
 - Average length of a run: $2(B-2)$

H1

H2

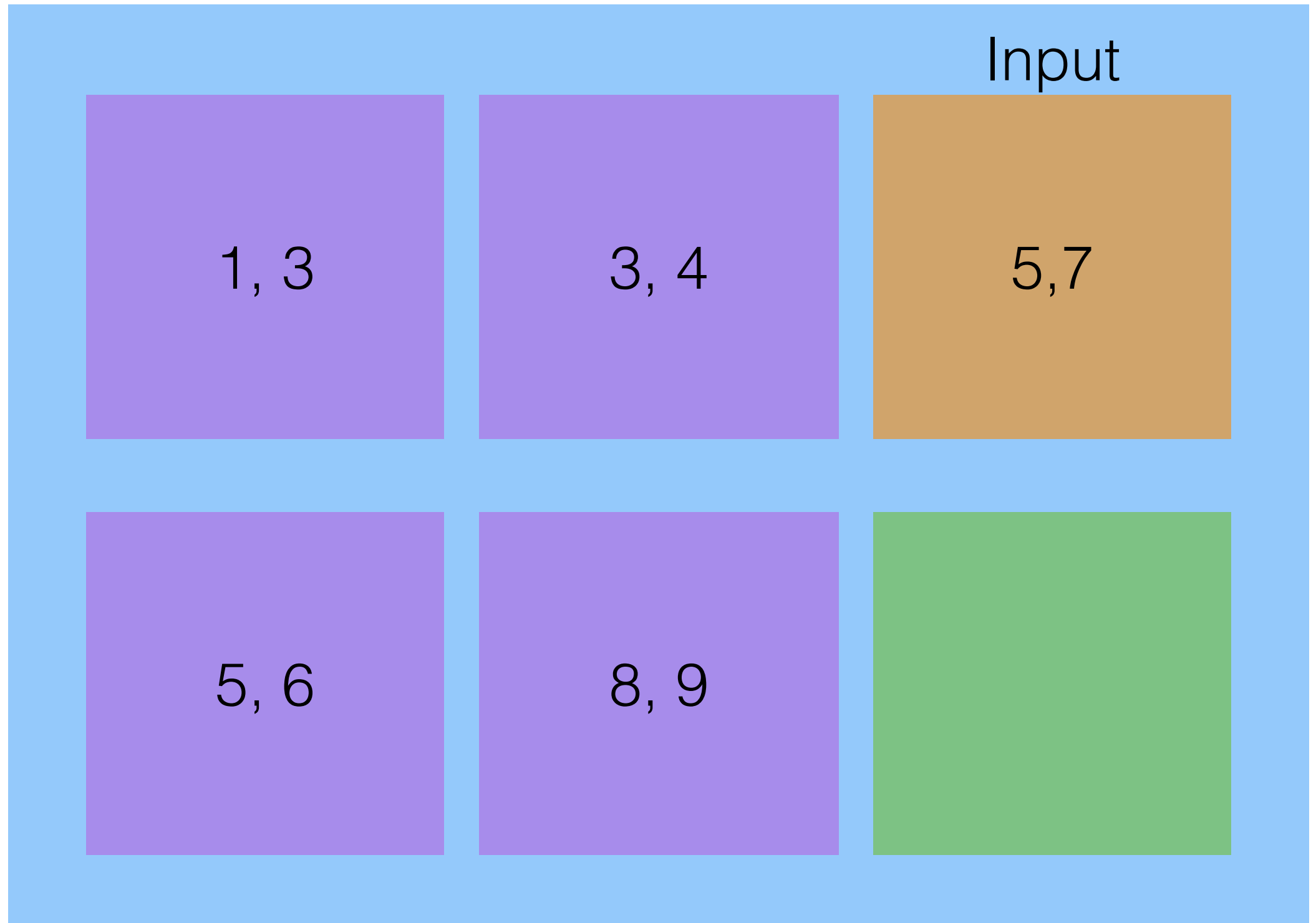
2, 1

4, 4

9, 8

Output:

Tournament Sort



H1

H2

2, 1

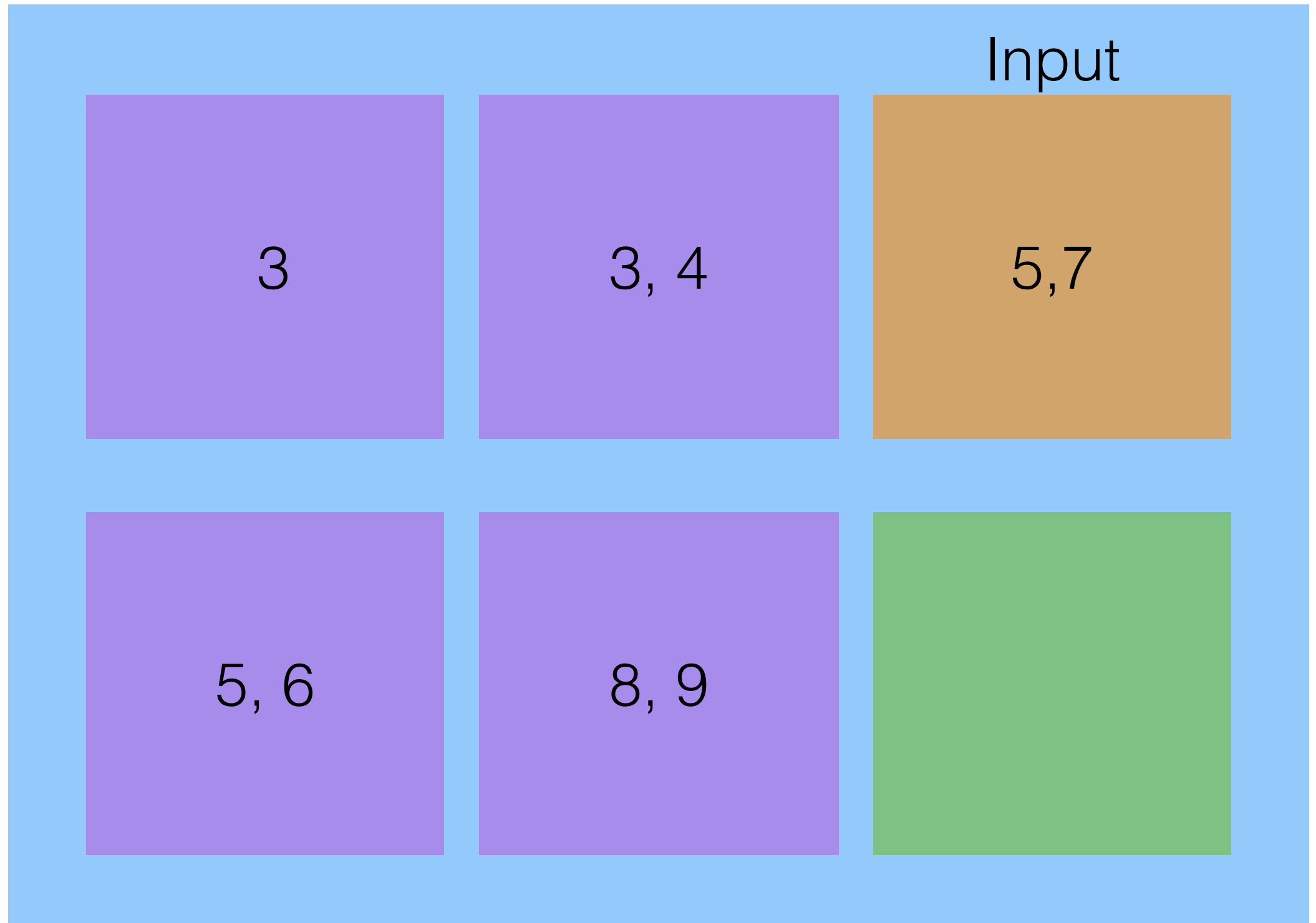
4, 4

9, 8

Output:

1

Tournament Sort



H1

H2

2, 1

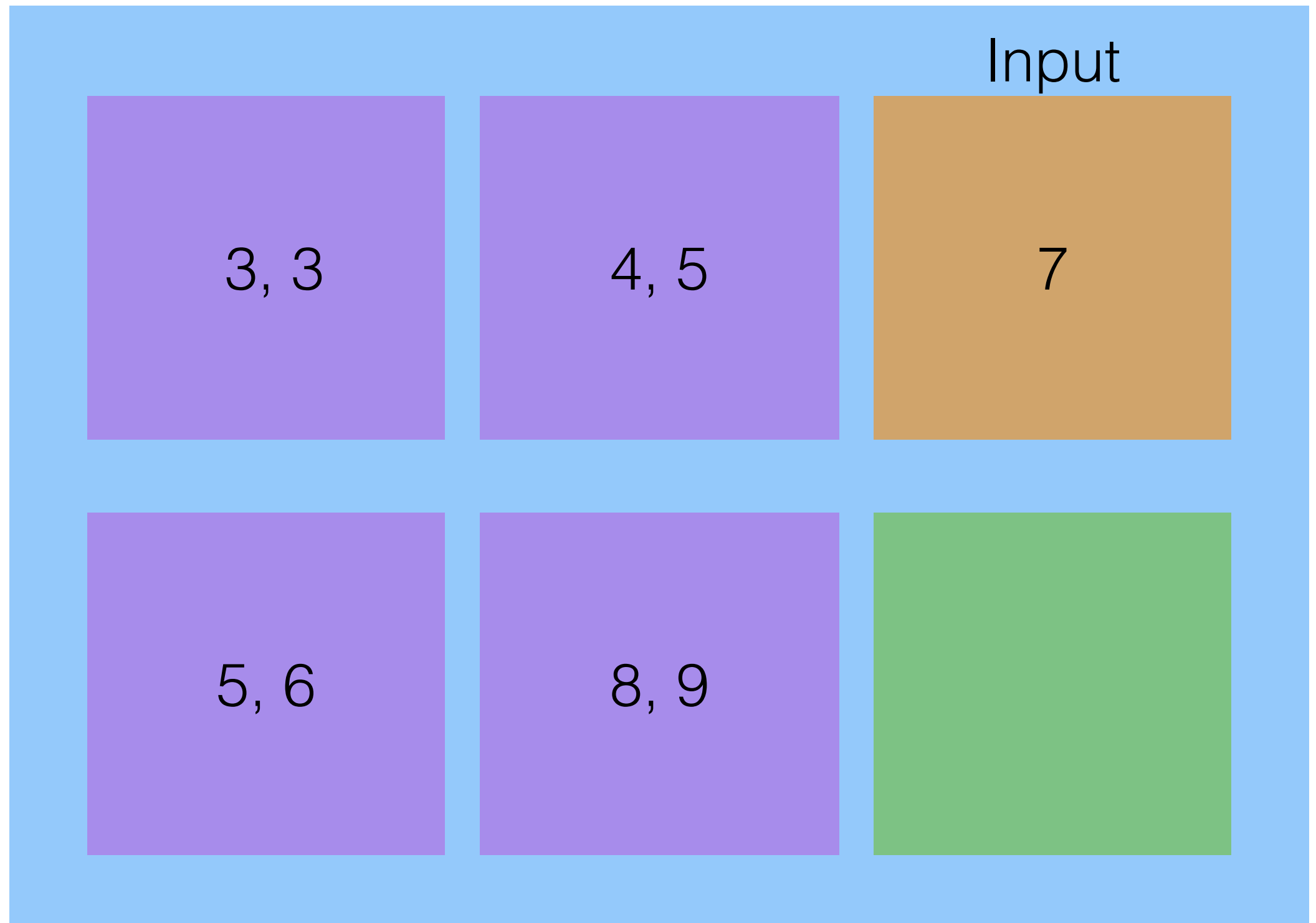
4, 4

9, 8

Output:

1

Tournament Sort



H1

H2

2, 1

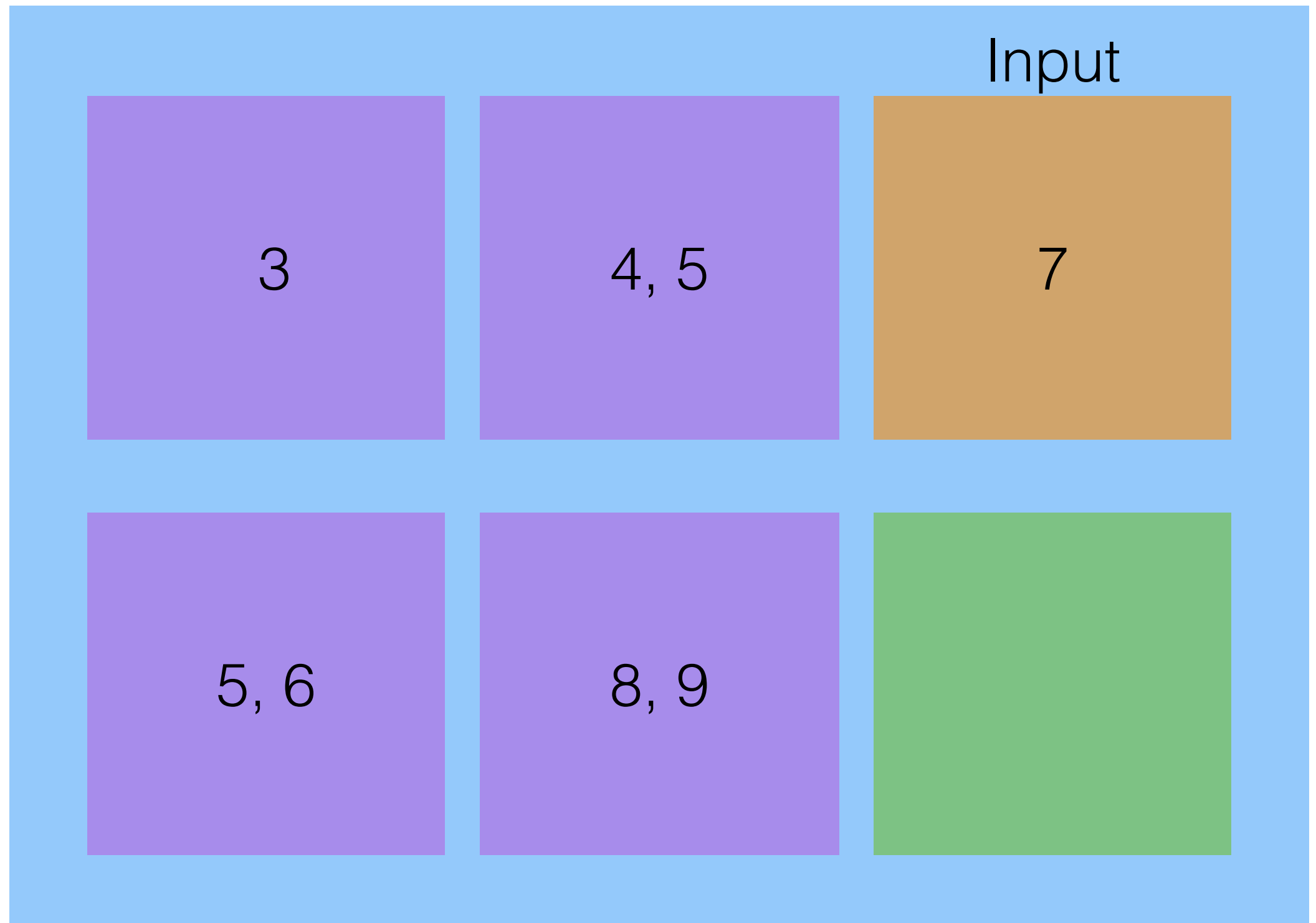
4, 4

9, 8

Output:

1, 3

Tournament Sort



H1

H2

2, 1

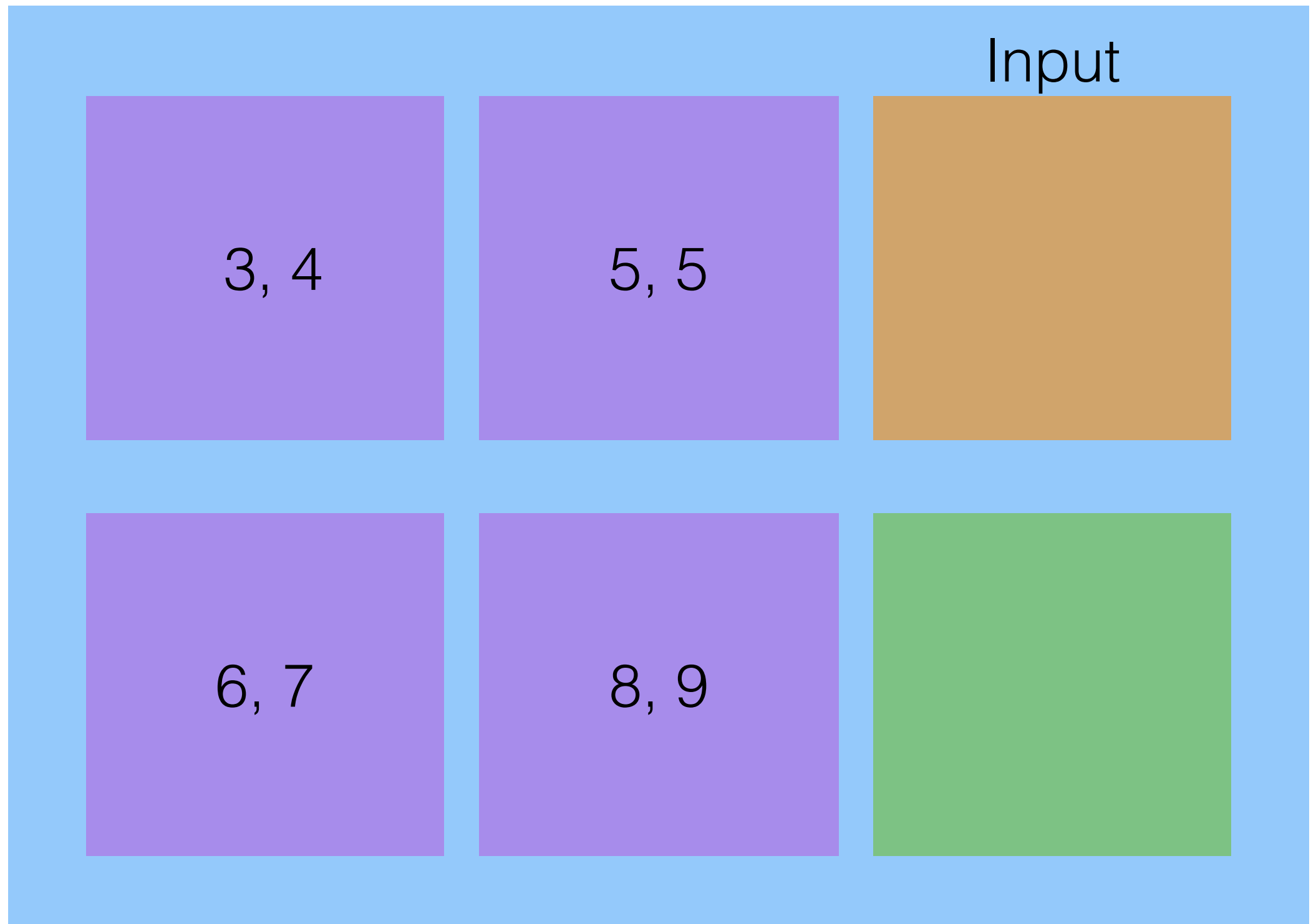
4, 4

9, 8

Output:

1, 3

Tournament Sort



H1

H2

Tournament Sort

Input

3, 4

5, 5

2, 1

4, 4

9, 8

6, 7

8, 9

Output:

1, 3

H1

H2

Tournament Sort

Input

4

5, 5

2, 1

4, 4

9, 8

6, 7

8, 9

Output:

1, 3, 3

H1

H2

Tournament Sort

Input

4

5, 5

1

4,4

9,8

6, 7

8, 9

2

Output:

1, 3, 3

H1

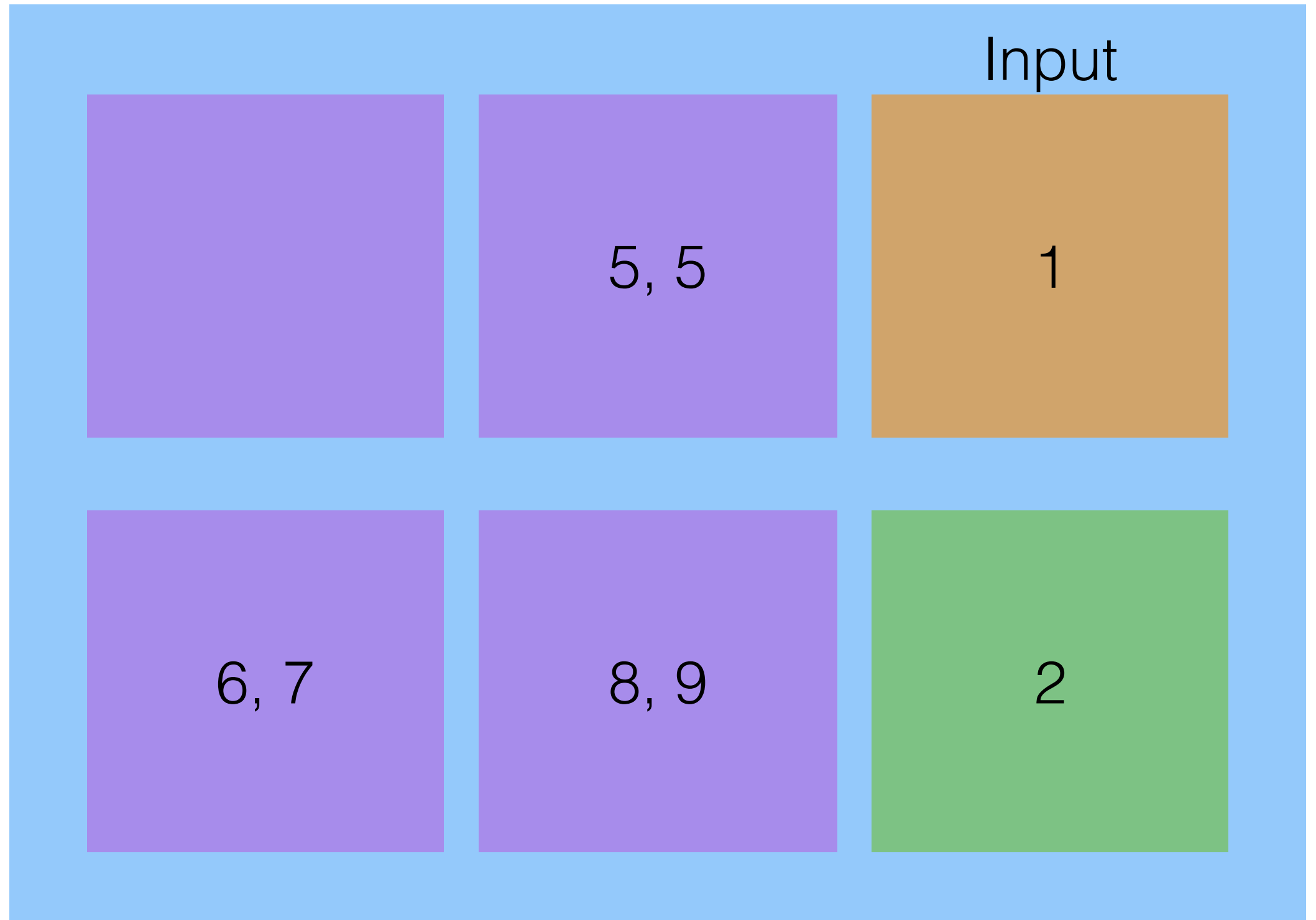
H2

4,4

9,8

Output:
1, 3, 3, 4

Tournament Sort



H1

H2

Tournament Sort

4, 4

9, 8

Output:
1, 3, 3, 4

5, 5

Input

6, 7

8, 9

1, 2

H1

H2

Tournament Sort

Input

5, 5

4, 4

9, 8

6, 7

8, 9

1, 2

Output:
1, 3, 3, 4

H1

H2

Tournament Sort

Input

4, 4

5

9, 8

1, 2

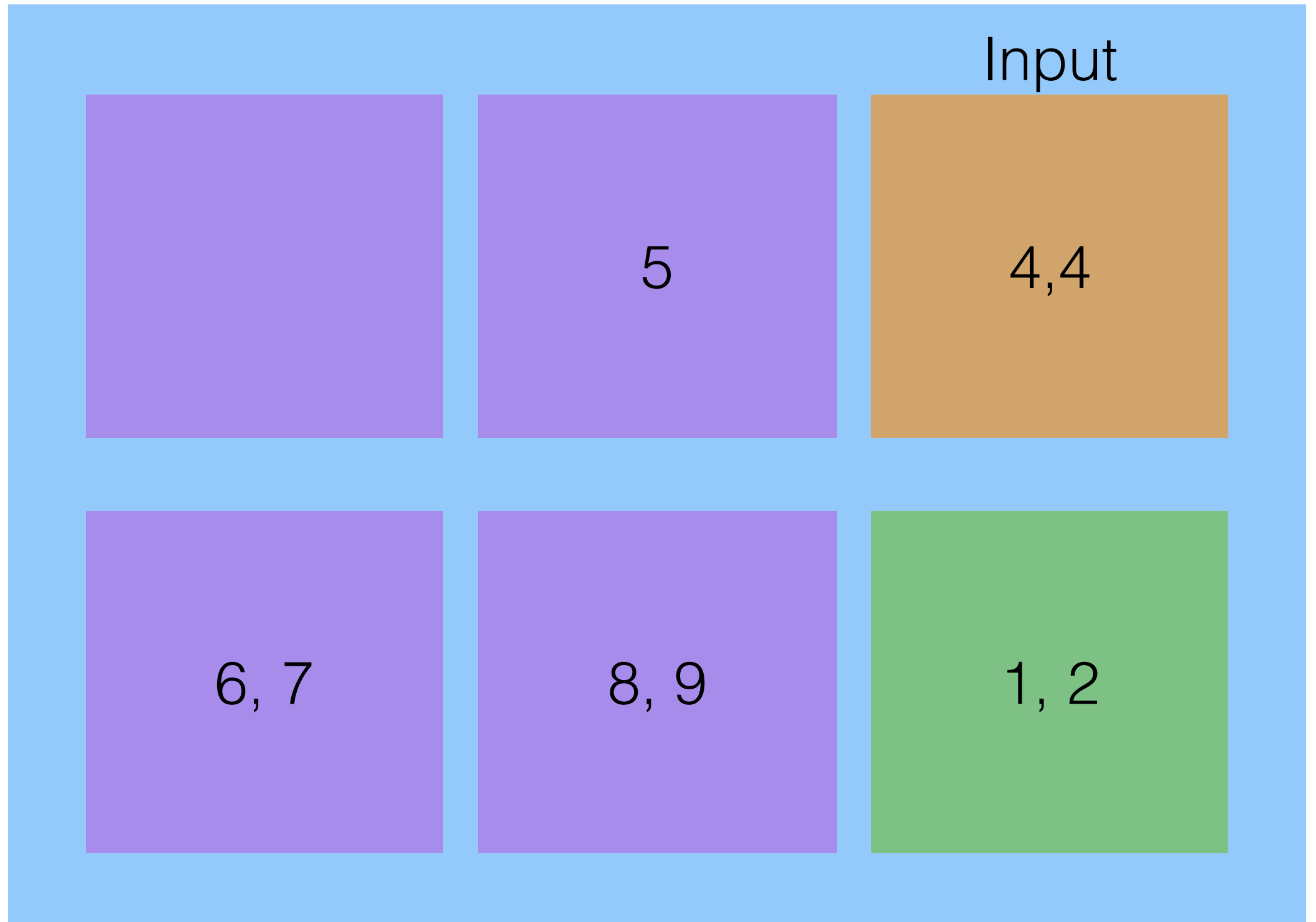
8, 9

6, 7

Output:

1, 3, 3, 4,

5



H1

H2

Tournament Sort

Input

4

5

4

9,8

6, 7

8, 9

1, 2

Output:

1, 3, 3, 4,

5

H1

H2

Tournament Sort

Input

4

4

9,8

6, 7

8, 9

1, 2

Output:

1, 3, 3, 4,
5, 5

H1

H2

Tournament Sort

Input

4, 4

9, 8

6, 7

8, 9

1, 2

Output:

1, 3, 3, 4,
5, 5

H1

H2

Tournament Sort

Input

4, 4

9, 8

6, 7

8, 9

1, 2

Output:

1, 3, 3, 4,
5, 5

H1

H2

Tournament Sort

Input

4, 4

9, 8

7

8, 9

1, 2

Output:

1, 3, 3, 4,
5, 5, 6

H1

H2

Tournament Sort

Input

4, 4

8

7, 8

9, 9

1, 2

Output:

1, 3, 3, 4,
5, 5, 6

H1

H2

Tournament Sort

Input

4, 4

8

8

9, 9

1, 2

Output:

1, 3, 3, 4,
5, 5, 6, 7

H1

H2

Tournament Sort

Input

4, 4

8, 8

9, 9

1, 2

Output:

1, 3, 3, 4,
5, 5, 6, 7

H1

H2

Tournament Sort

Input

4, 4

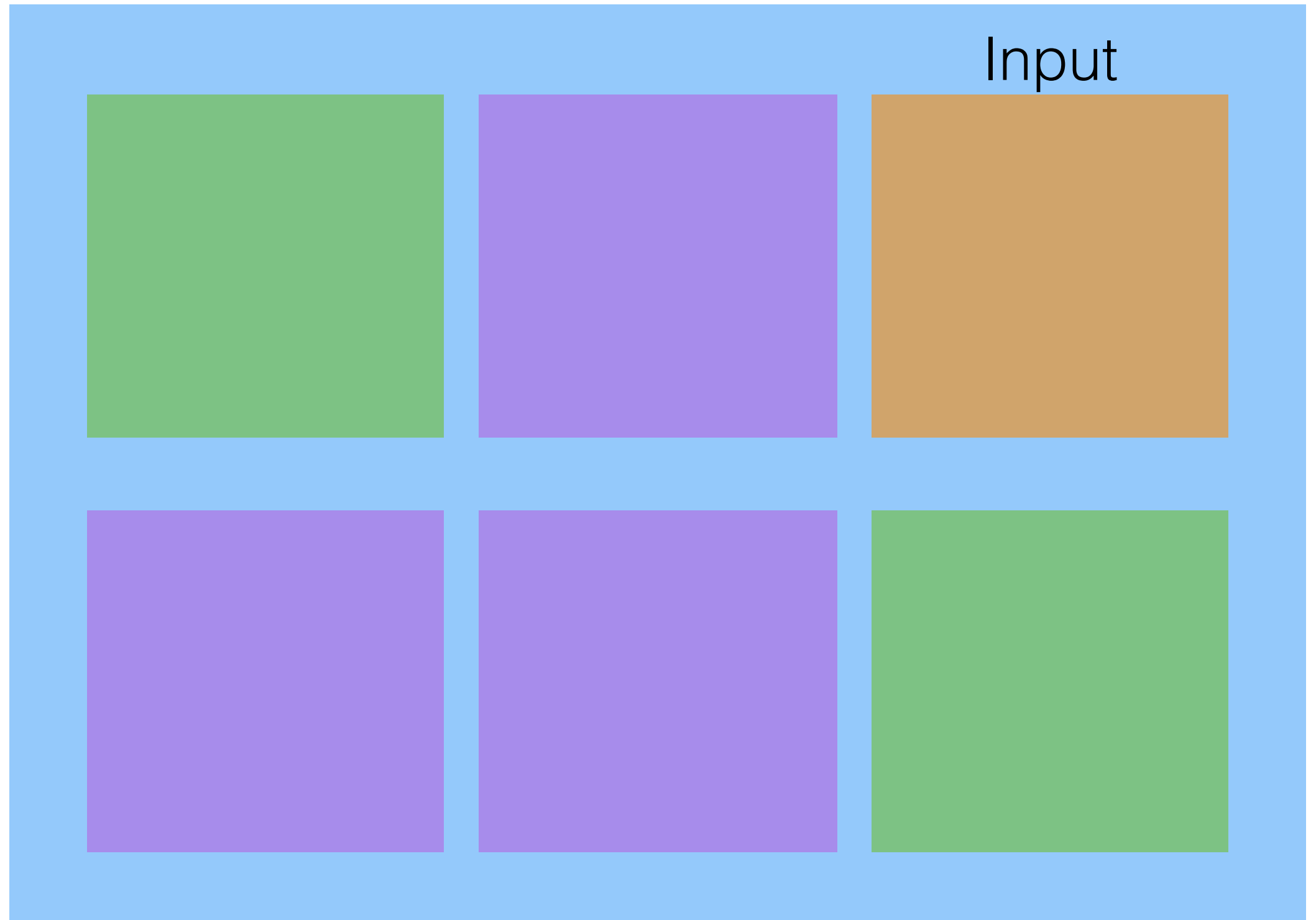
1, 2

Output:

1, 3, 3, 4,
5, 5, 6, 7,
8, 8, 9, 9



Tournament Sort



Output:

1, 3, 3, 4,

5, 5, 6, 7,

8, 8, 9, 9

1, 2, 4, 4

Hashing Midterm Example

- **Hashing Animals:**
 - 4 tuples (animals) a page
 - 101 buffer pages, 32,000 animals
 - Want to group animals by type
- How many times do we have run the Partitioning stage of hashing to hash the animals, assuming all the partitions end up being the same length?

- **Hashing Animals:**
 - 4 tuples (animals) a page
 - 101 buffer pages, 32,000 animals
 - Want to group animals by type
- How many times do we have run the Partitioning stage of hashing to hash the animals, assuming all the partitions end up being the same length?
 - $32,000/4 = 8,000$ pages of data
 - $8,000/100 = 80$ pages per partition
 - $(80 < 101) \Rightarrow$ each partition will fit in memory \Rightarrow only one partition stage required!