CS186 Discussion #6

(Relational Algebra, ER Diagrams, FDs)

Relational Algebra

- Input and output: Relation instances (tables)
- Has set semantics
  - **No** duplicate tuples in a relation
- Useful for representing execution plans in a DBMS (more later!)

Relational Algebra

Operation	Symbol	Explanation
Selection	σ	Selects rows
Projection	π	Selects columns
Union	$\cup$	Tuples in r1 or r2
Intersection	$\cap$	Tuples in r1 and r2
Cross-product	$\times$	Combines two relations
Join	$\bowtie$	Conditional cross-product
Difference	$-$	Tuples in r1 not in r2

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Selection & Projection

- Example: $\pi_{\text{name, sid}}(\sigma_{\text{gpa} > 3.5}(R))$

name	sid	gpa
Bob	1	3.7
Sue	3	2.9
Ron	2	1.2
Al	4	4.0
Sally	5	3.6

Difference

- Takes rows in A that are not in B
- Example: $\sigma_{gpa > 3.5}(R) - \sigma_{sid\%2==0}(R)$

name	sid	gpa
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Worksheet: Relational Algebra #3,5

Consider the schema:

```
Songs  (song_id,   song_name,   album_id,  weeks_in_top_40)
Artists(artist_id, artist_name, first_year_active)
Albums  (album_id,  album_name,  artist_id, year_released, genre)
```

Write relational algebra expressions for the following query:

- Find the id of the artists who have albums of genre ‘pop’ or have spent over 10 weeks in the top 40.

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Write relational algebra expressions for the following query:

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$$\pi_{\text{Artists.artist\_id}} (\text{Artists} \bowtie (\sigma_{\text{Albums.genre = 'pop'}} \text{Albums}))$$

$$\cup$$

$$\pi_{\text{Albums.artist\_id}} (\text{Albums} \bowtie (\sigma_{\text{Songs.weeks\_in\_top\_40} > 10} \text{Songs}))$$

Consider the schema:

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Artists(artist_id, artist_name, first_year_active)
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Write relational algebra expressions for the following query:

- Find the names of all artists who do not have any albums.

Consider the schema:

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Songs   (song_id,   song_name,   album_id,  weeks_in_top_40)
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Write relational algebra expressions for the following query:

- Find the names of all artists who do not have any albums.

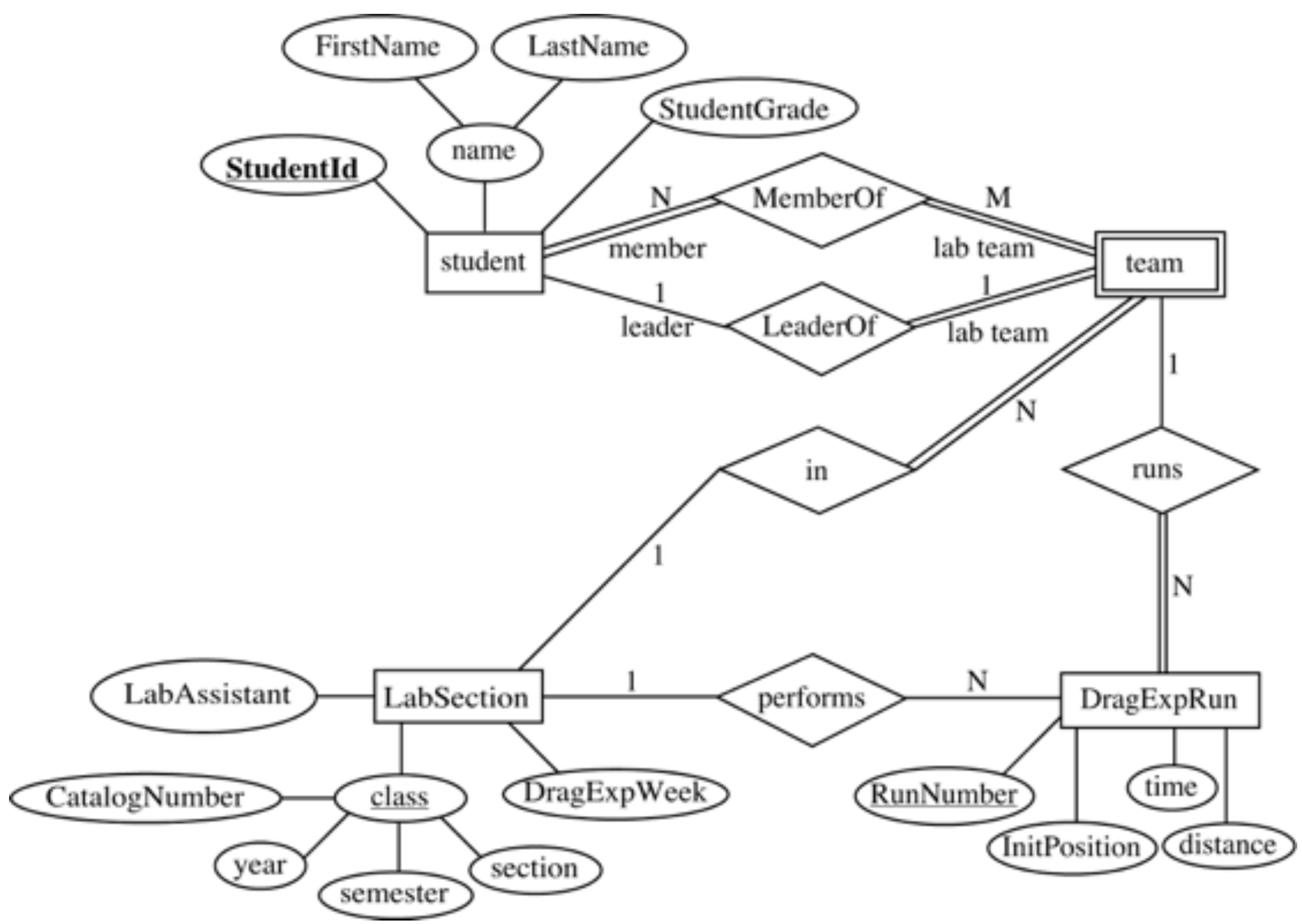
$$\pi_{\text{Artists.artist\_name}} (\text{Artists} \bowtie ($$

$$(\pi_{\text{Artists.artist\_id}} \text{Artists})-(\pi_{\text{Albums.artist\_id}} \text{Albums}))$$

ER Diagrams

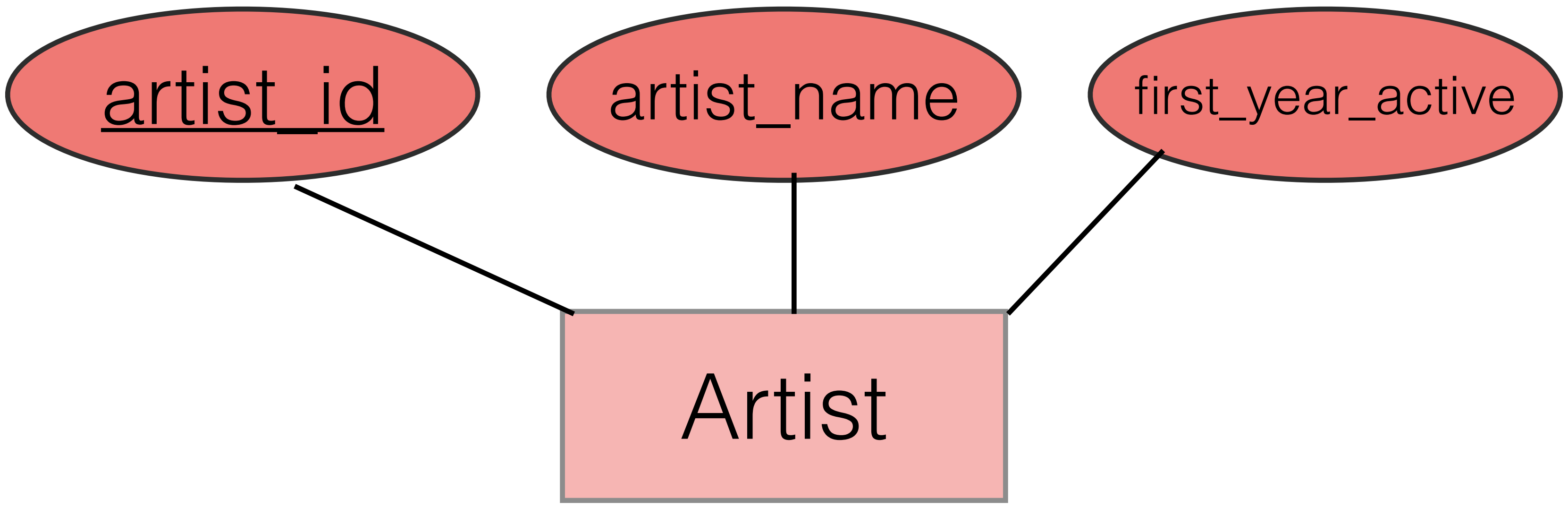
Motivation

- Visualize data schema



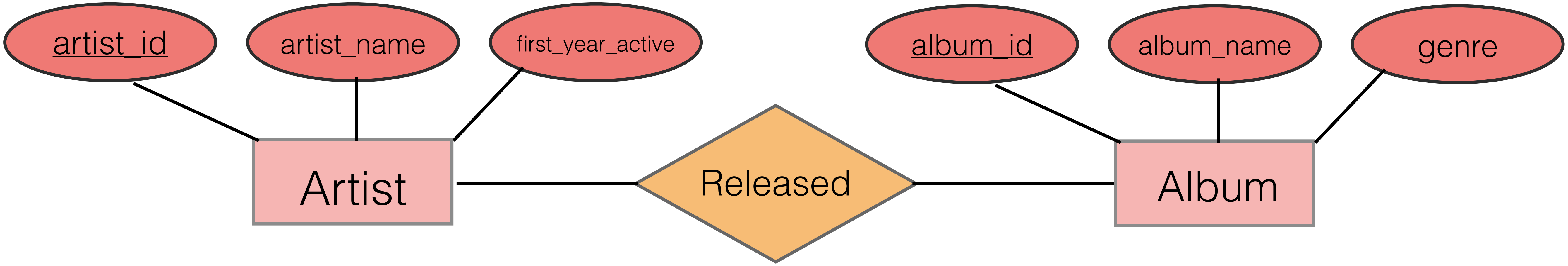
Entities

- Entity: “thing”
- Attribute: Property of the entity
  - Primary key underlined



Relationships

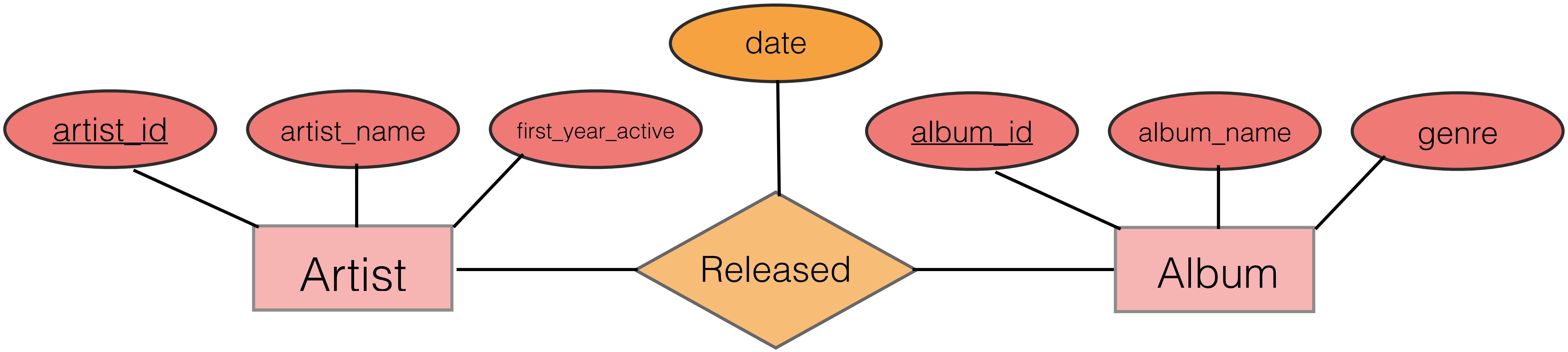
- How two entities interact



Artist 4 released album 2.

Relationships

- How two entities interact
  - Interactions can have attributes



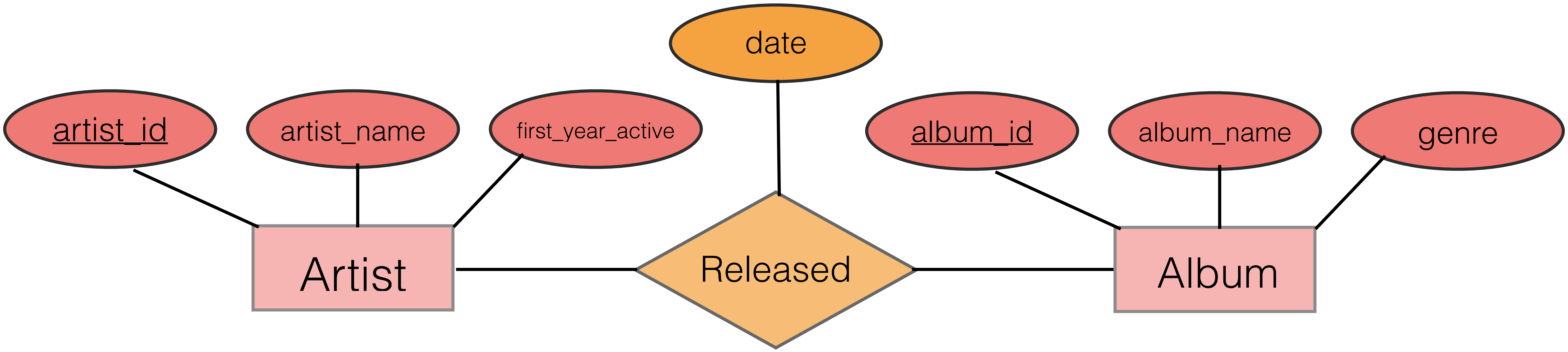
Artist 4 released album 2 on February 27, 2015.

Constraints

- Make relationship lines meaningful
  - Participation constraint (Partial/Total)
    - Total participation: participates at least once
  - Key/Non-key constraint
    - Key: Participates at most once

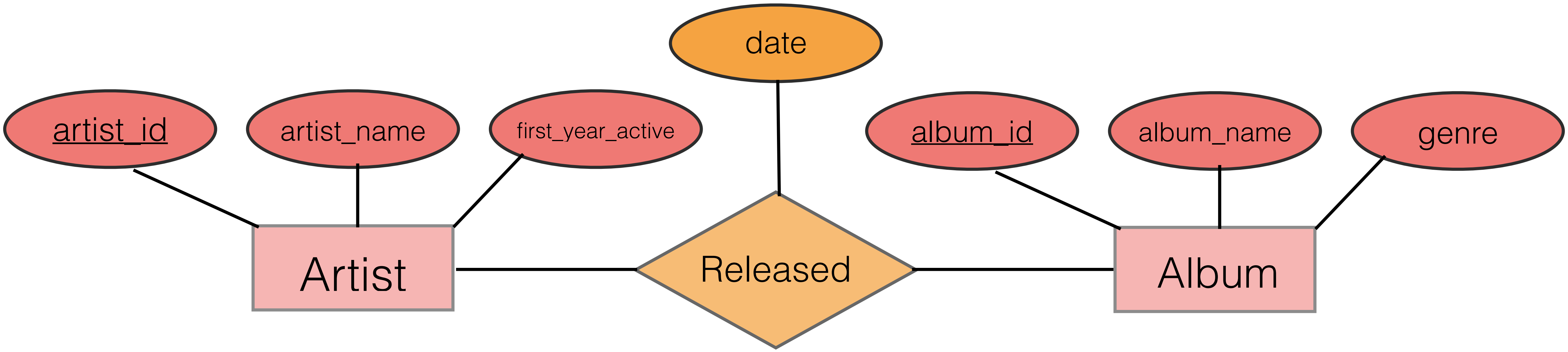
	Partial Participation	Total Participation
Non-Key	0 or More	1 or More
Key	0 or 1	Exactly 1

Constraints



Non-Key constraint with partial participation

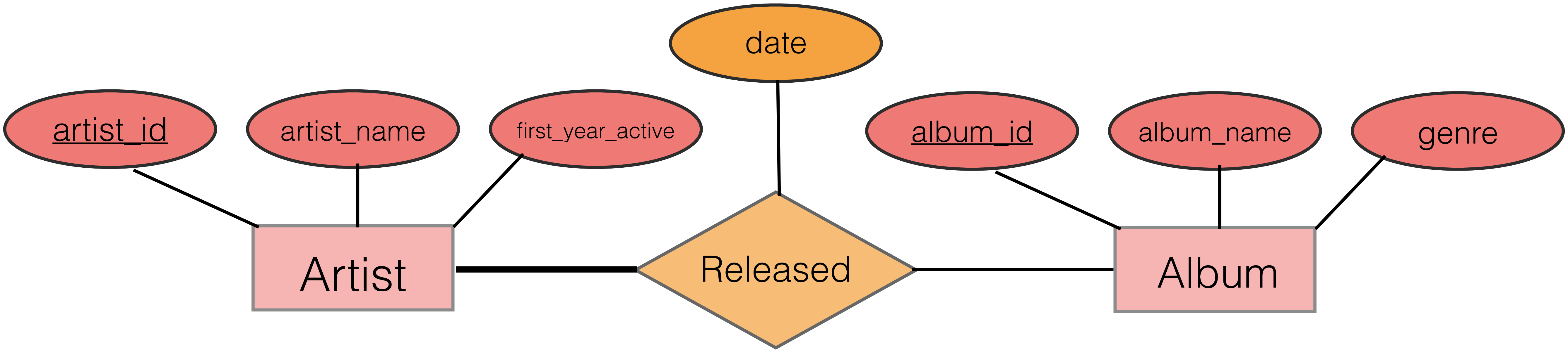
Constraints



Non-Key constraint with partial participation

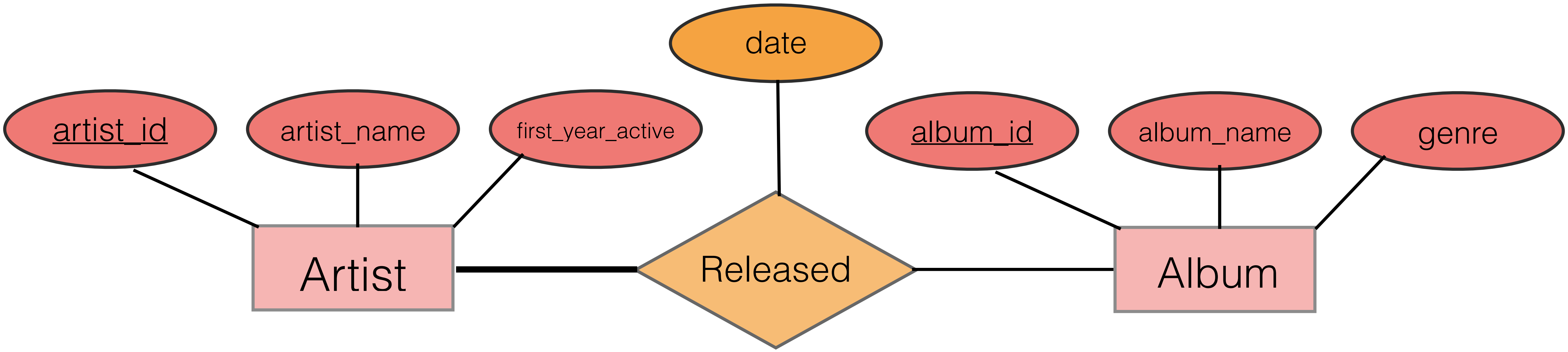
An artist can release an album zero or more times.

Constraints



Non-Key constraint with total participation

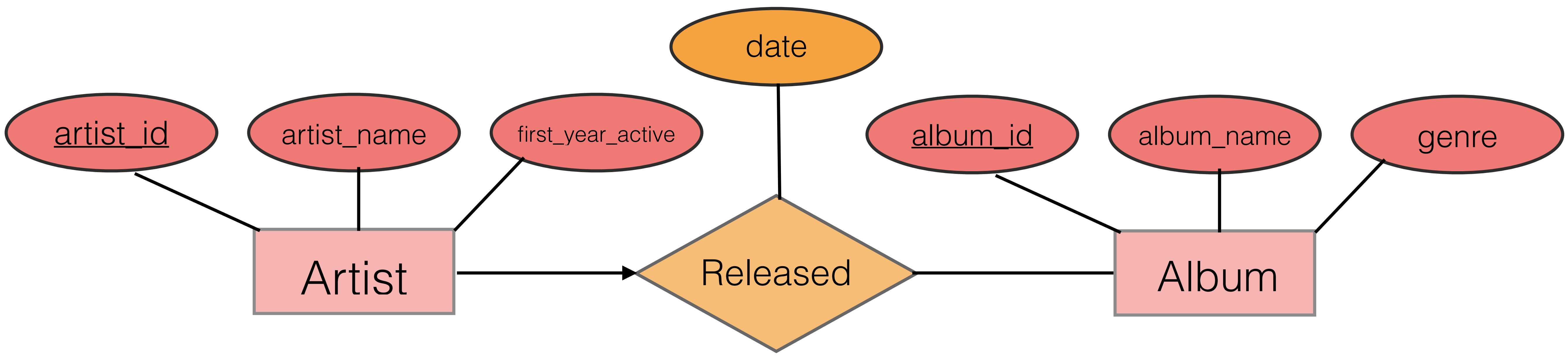
Constraints



Non-Key constraint with total participation

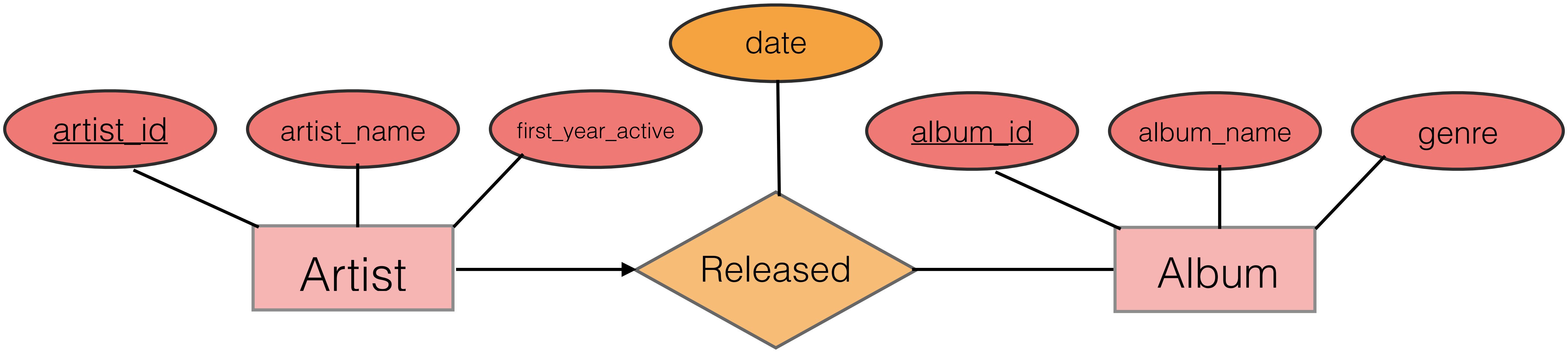
An artist can release an album one or more times.

Constraints



Key constraint with partial participation

Constraints



Key constraint with partial participation

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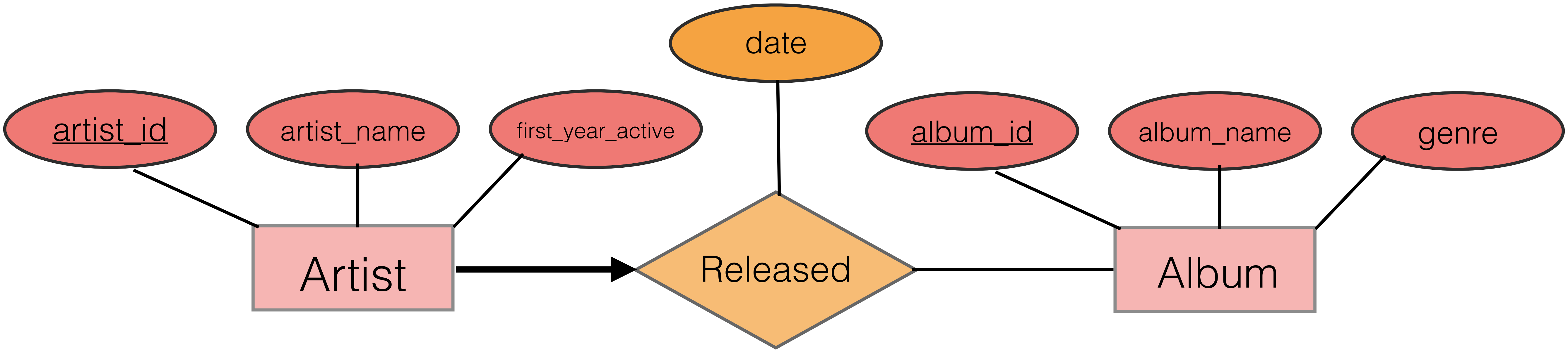
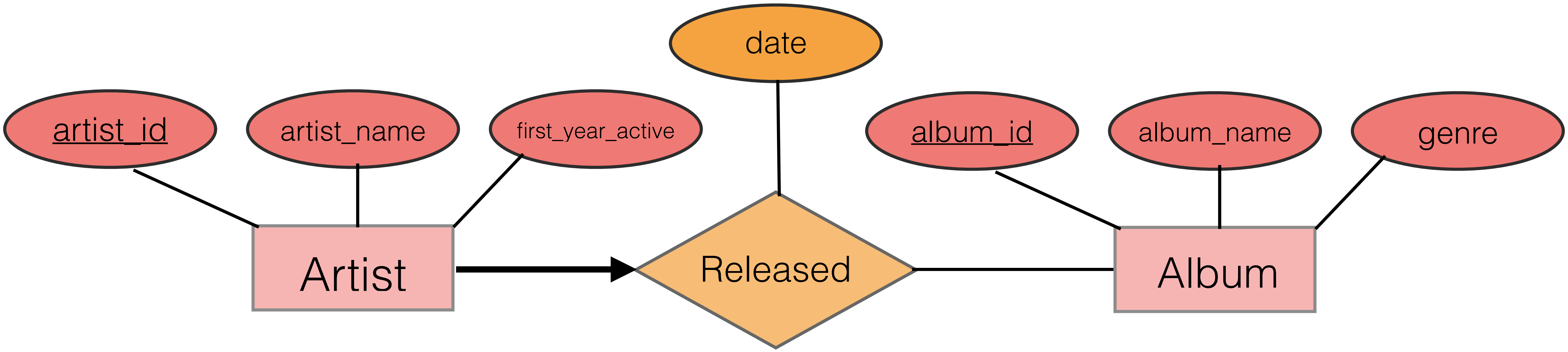
Constraints



Key constraint with total participation

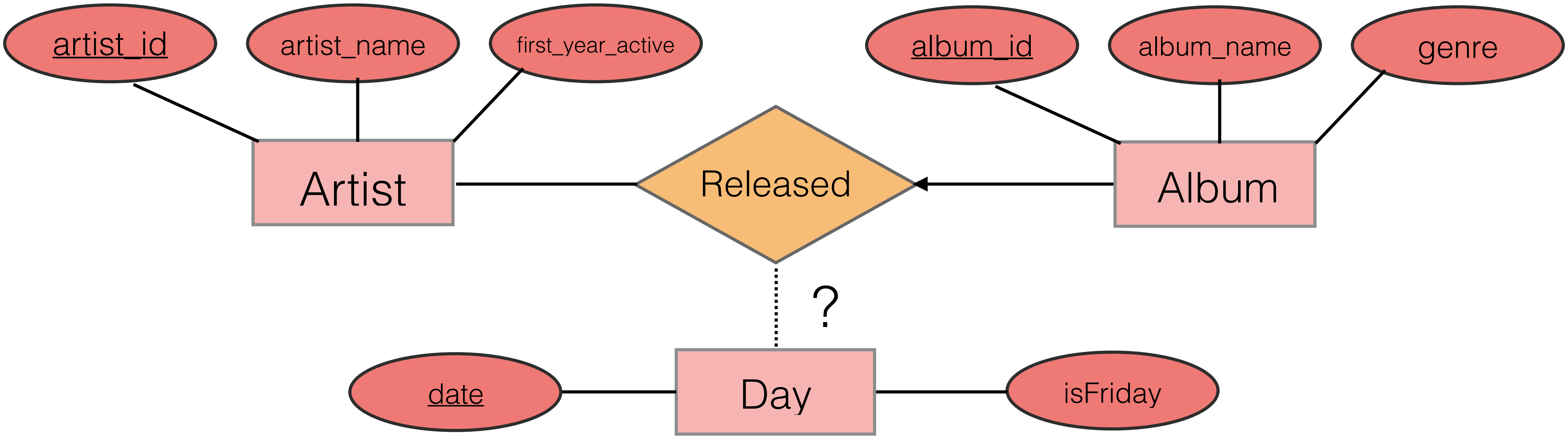
Constraints



Key constraint with total participation

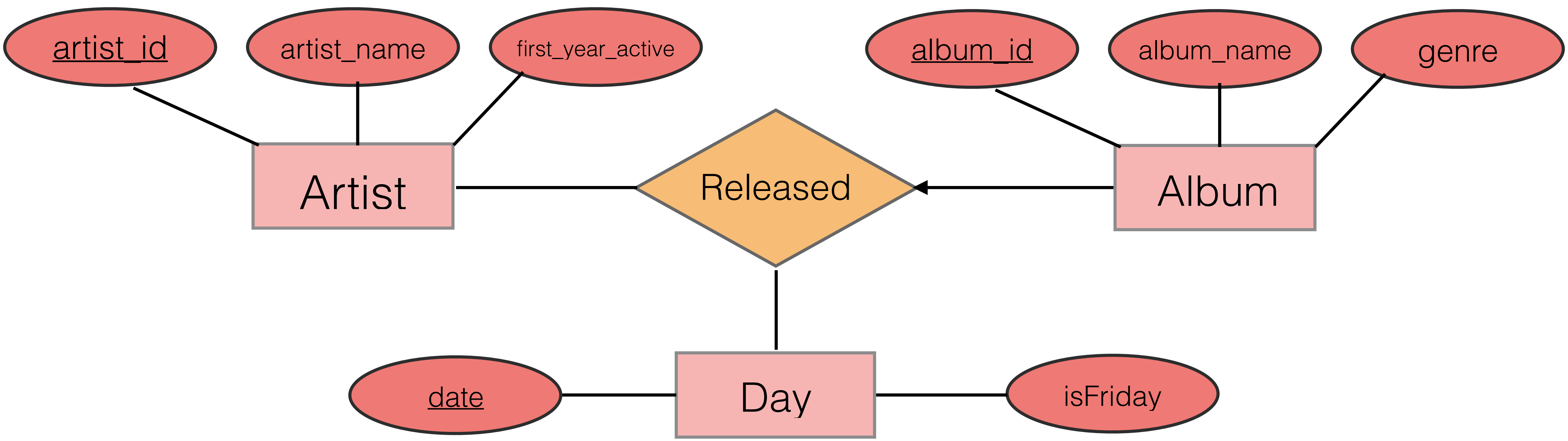
An artist can release exactly one album.

Ternary Relations



An artist may release zero or more albums.
An album may be released or unreleased.
Releasing an album can occur ??? times a day.

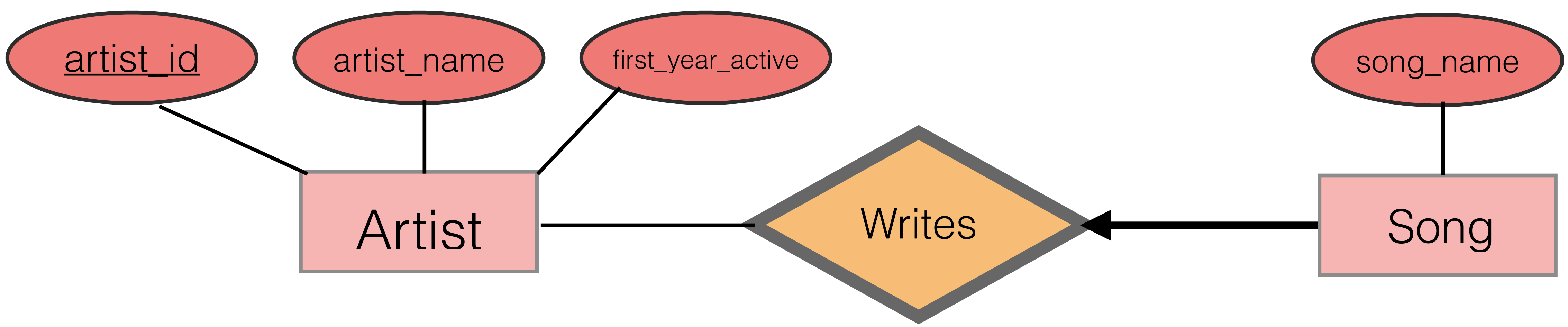
Ternary Relations



An artist may release zero or more albums.
An album may be released or unreleased.
Releasing an album can occur 0 or more times a day.

Weak Entities

- Weak entity can only be identified only when considering primary key of another (owner) entity.

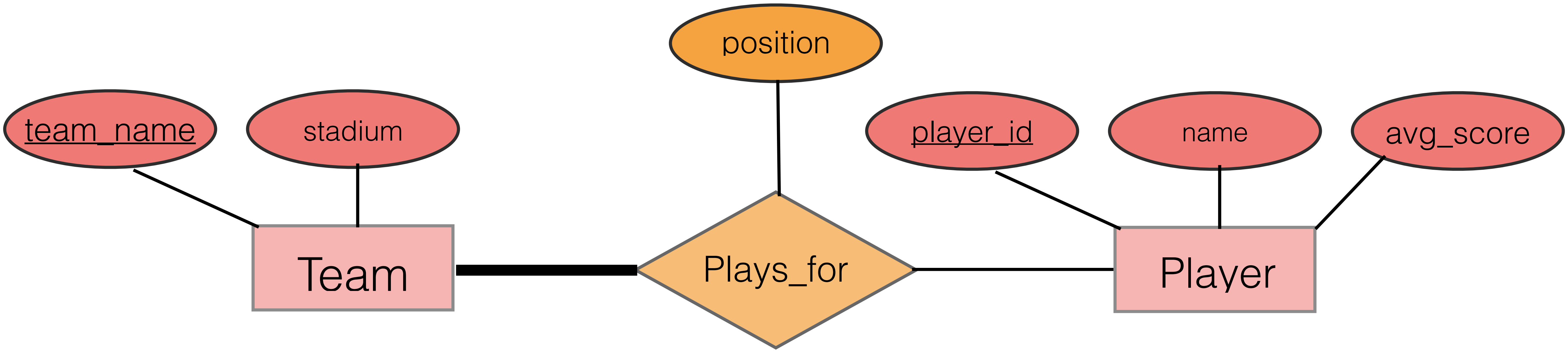


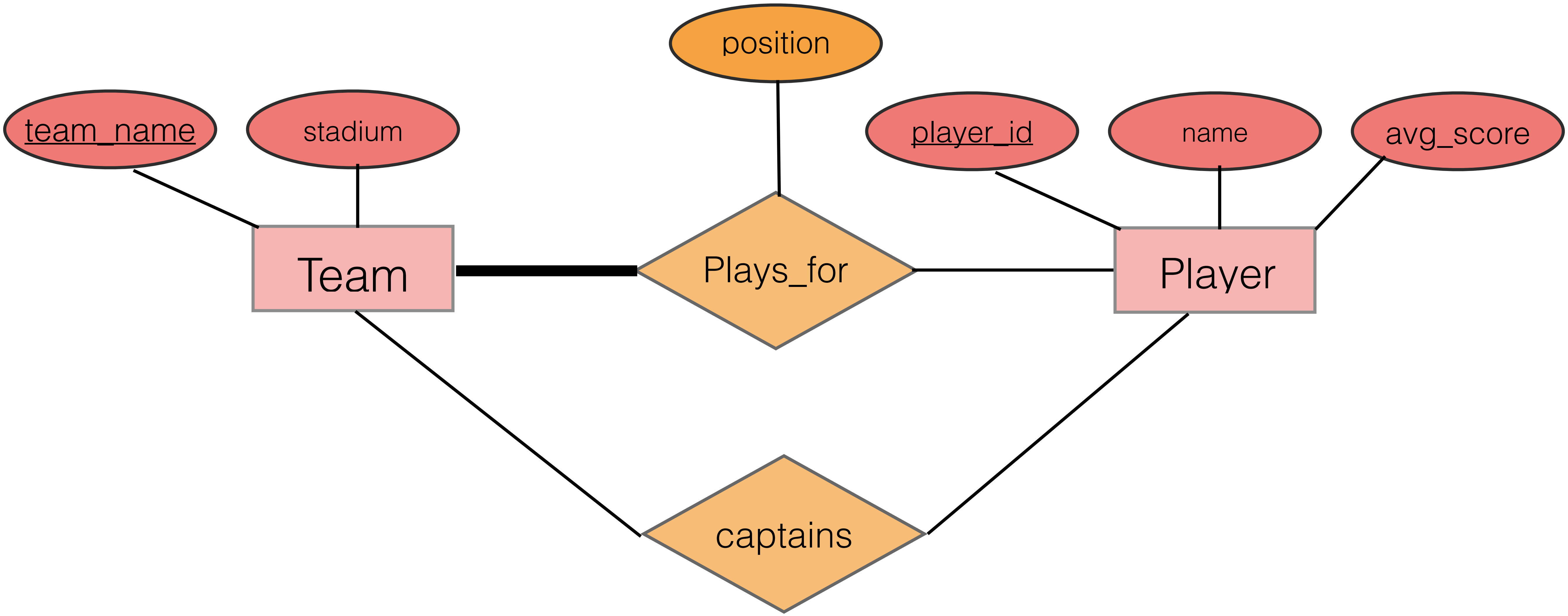
- Song's key is actually (Artist.artist_id, Song.song_name)
- Can there be two songs with the same name?
- Can a song exist without an artist?

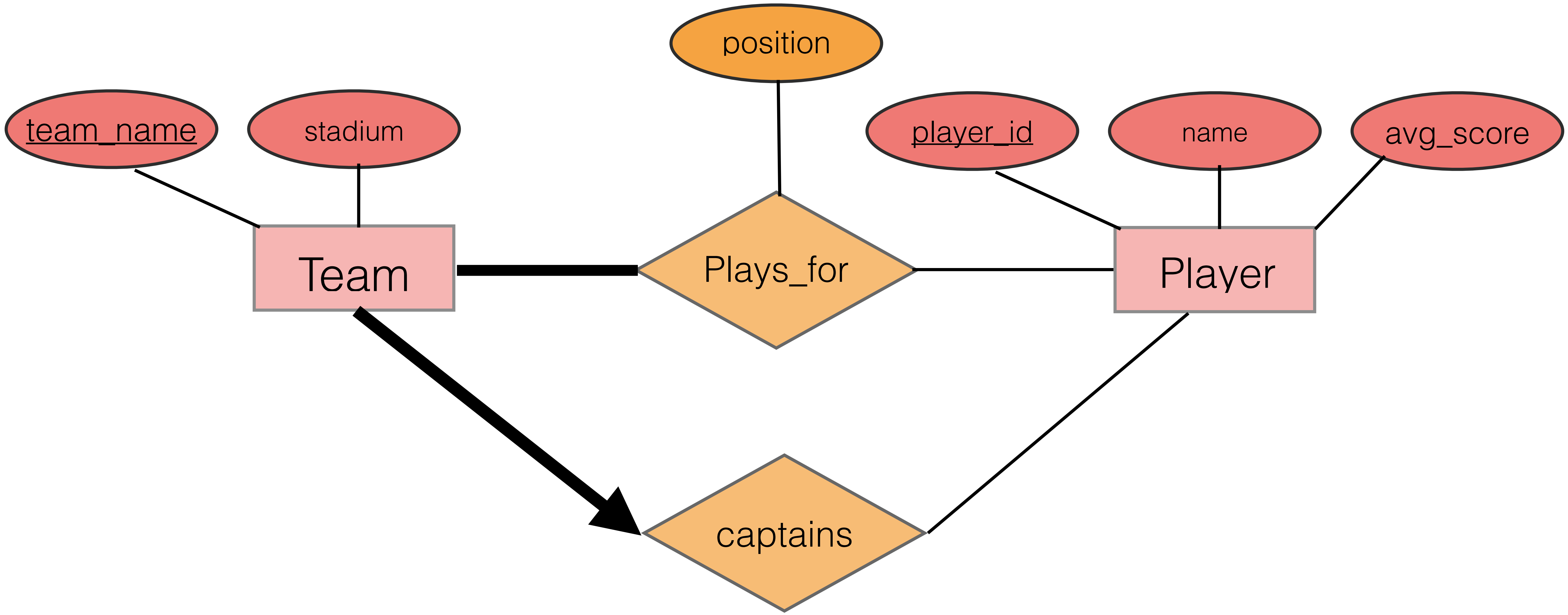
Worksheet: ER Diagrams


Team
Plays_for
Player


Team
Plays_for
Player


position
team_name
stadium
player_id
name
avg_score
Team
Plays_for
Player


position
team_name
stadium
player_id
name
avg_score
Team
Plays_for
Player
captains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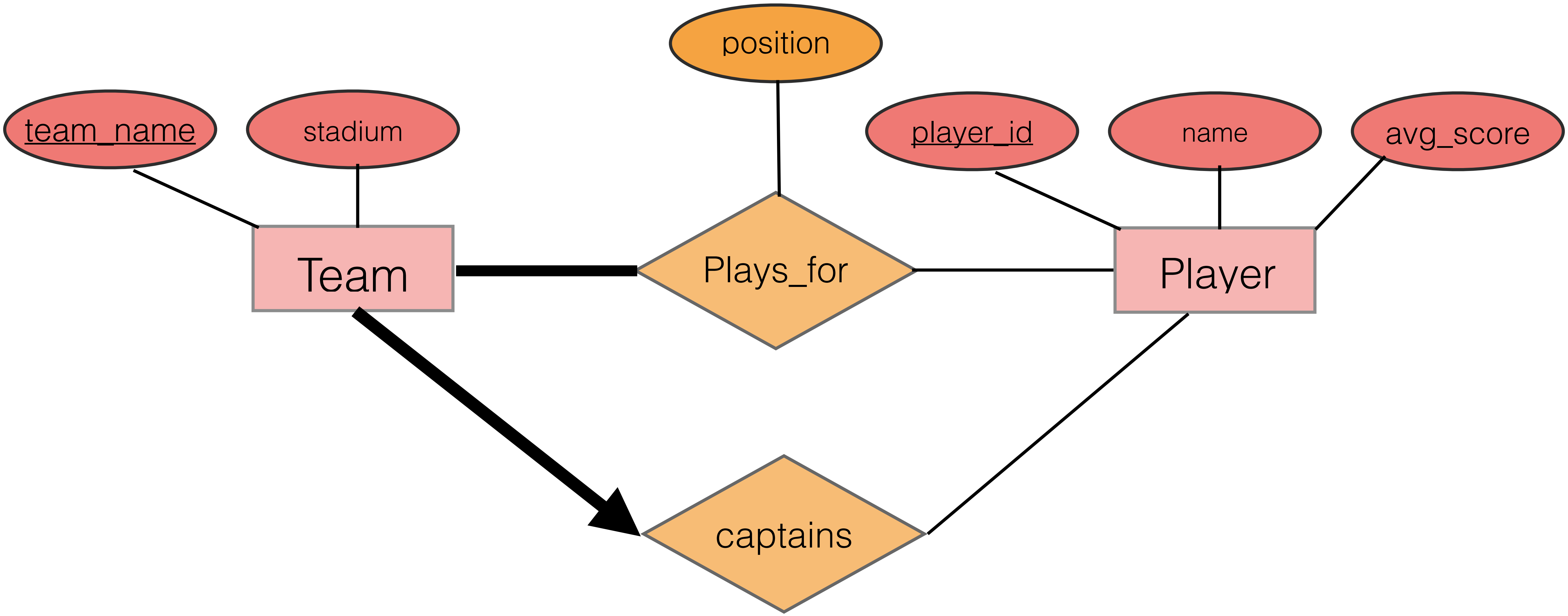

No weak entities!

Functional Dependencies

Functional Dependencies

- X → Y reads "X determines Y"
- Used to detect redundancies and refine schema

id	rating	wage
1	3	20
2	2	10
3	3	20
4	2	10

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rating → wage

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id	rating	wage
1	3	20
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id → rating, wage

Functional Dependencies

- Key → All attributes of relation

id	rating	wage
1	3	20
2	2	10
3	3	20
4	2	10

id → rating, wage

Armstrong's Axioms

- **Reflexivity**: if $X \supseteq Y$, then $X \rightarrow Y$
  - **Examples**: $A \rightarrow A$, $AB \rightarrow A$
- **Augmentation**: if $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
- **Transitivity**: if $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- Useful rules derived from Armstrong's Axioms:
  - **Union**: if $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - **Decomposition**: if $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Find the set of functional dependencies:

Flights(<u>**F**light_no, **D**ate</u>, f**R**om, **T**o, **P**lane_id),

ForeignKey(**P**lane_id)

Planes(<u>**P**lane_id</u>, t**Y**pe)

Seat(<u>**S**eat_no, **P**lane_id</u>, **L**egroom), ForeignKey(**P**lane_id)

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- FD → RTP

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- FD → RTP
- P → Y

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Seat(<u>**S**eat_no, **P**lane_id</u>, **L**egroom), ForeignKey(**P**lane_id)

- FD → RTP
- P → Y
- SP → L

Closures

- Functional dependency closure: F+
  - Set of all FDs implied by F, including trivial dependencies
  - Example: $F = \{A \to B, B \to C\}$
    - $F+ = \{A \to B, B \to C, A \to C, A \to A, A \to AB, \ldots\}$

Closures

- Attribute closure: X+
  - Given just X, what can we determine?
  - Example: F = {A → B, B → C}
    - A+ = ABC

Attribute Closures

- A methodical algorithm, given a set of FDs F:
  - Initialize X+ := X
  - Repeat until no change:
    - If U → V is in F such that U is in X+, add V to X+

- R = ABCDE
- F = {B → CD, D → E, B → A, E → C, AD → B }
- B+ = ?

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- F = {B → CD, D → E, B → A, E → C, AD → B }
- B+ = ABCDE   B is a key of R!

Now consider the attribute set R = ABCDE and the FD set $F = \{AB \rightarrow C, A \rightarrow D, D \rightarrow E, AC \rightarrow B\}$. Compute the closure for the following attributes.

- A:
- AB:
- B:
- D:

Now consider the attribute set R = ABCDE and the FD set F = {AB → C, A → D, D → E, AC → B}. Compute the closure for the following attributes.

- A: ADE
- AB: ABCDE
- B: B
- D: DE

Boyce-Codd Normal Form (BCNF)

- Motivation: Schema design is hard, want a way to ensure a reasonable design
- BCNF ≈ “reasonable schema”

Boyce-Codd Normal Form (BCNF)

- Definition: Relation R with FDs F is in BCNF if for all $X \rightarrow A$ in F+:
  - $X \rightarrow A$ is reflexive (a trivial FD) OR
  - X is a superkey for R
    - Superkey: Key that does not need to be minimal

BCNF Decomposition

- If $X \rightarrow A$ violates BCNF, decompose R into R - A and XA
  - Repeat as necessary

- $R = ABCEG$
- $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$

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- F = {$AB \rightarrow C$, $AC \rightarrow B$, $BC \rightarrow G$, $E \rightarrow G$}
- ABEG, ABC

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- ABEG, ABC
- ABE, EG, ABC

Decomposition Properties

- Lossless join: Can we reconstruct R?
  - Decomposing R into X and Y is lossless iff:
    - $X \cap Y \rightarrow X$, or
    - $X \cap Y \rightarrow Y$

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- Example:
  - ABC decomposed to AB, BC
  - FDs: $A \rightarrow B$, $C \rightarrow B$
  - This is lossy! $AB \cap BC = B \rightarrow B$

Decomposition Properties

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    - Decomposing R into X and Y is lossless iff:
        - $X \cap Y \rightarrow X$, or
        - $X \cap Y \rightarrow Y$
- Example:
    - ABC decomposed to AC, BC
    - FDs: $A \rightarrow B$, $C \rightarrow A$
    - This is lossless! $AC \cap BC = C \rightarrow AC$

Decomposition Properties

- Dependency-preserving: Can we verify all FDs without joins?

- Example: ABE, EG, ABC
    - $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$

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Decomposition Properties

- Dependency-preserving: Can we verify all FDs without joins?

- Example: ABE, EG, ABC
  - F = {AB → C, AC → B, BC → G, E → G}
  - This dependency was not preserved!
  - We can fix this by adding BCG, but this may break BCNF.

Minimal Cover

- G for a set of FDs F
  - Closure of G = closure of F
  - Right hand side of each FD is G is a single attribute
- Implies lossless join and dependency preserving decomposition

- Example: $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$

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- Example: $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$
- $A \rightarrow B$

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- Example: $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$
- $A \rightarrow B$, $ACD \rightarrow E$

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- Example: A → B, ABCD → E, EF → GH, ACDF → EG
- A → B, ACD → E, EF → G, EF → H

Minimal Cover

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  - Closure of G = closure of F
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- Example: A → B, ABCD → E, EF → GH, ACDF → EG
- A → B, ACD → E, EF → G, EF → H

Decompose R = ABCDEFG into BCNF, given the FD set:
F = {AB → CD, C → EF, G → A, G → F, CE → F}.

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- ABEFG, ABCD

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- ABEFG, ABCD
- BEFG, ABCD, AG

Decompose R = ABCDEFG into BCNF, given the FD set:
F = {AB → CD, C → EF, G → A, G → F, CE → F}.

- ABEFG, ABCD
- BEFG, ABCD, AG
- BEG, FG, ABCD, AG

Decompose R = ABCDEFG into BCNF, given the FD set:
F = {AB → CD, C → EF, G → A, G → F, CE → F}.

- ABEFG, ABCD
- BEFG, ABCD, AG
- BEG, FG, ABCD, AG

Does BEG, FG, ABCD, AG preserve dependencies?

$F = \{AB \to CD, C \to EF, G \to A, G \to F, CE \to F\}$.

Does BEG, FG, ABCD, AG preserve dependencies?
$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

- No, $C \rightarrow EF$ and $CE \rightarrow F$ are not preserved.

Give a minimal cover for:

$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

Give a minimal cover for:

$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

- $AB \rightarrow C$
- $AB \rightarrow D$

Give a minimal cover for:

F = {AB → CD, C → EF, G → A, G → F, CE → F}.

- AB → C
- AB → D
- C → F
- C → E

Give a minimal cover for:

$F = \{AB \rightarrow CD,\ C \rightarrow EF,\ G \rightarrow A,\ G \rightarrow F,\ CE \rightarrow F\}$.

- $AB \rightarrow C$
- $AB \rightarrow D$
- $C \rightarrow F$
- $C \rightarrow E$
- $G \rightarrow A$

Give a minimal cover for:

$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

- $AB \rightarrow C$
- $AB \rightarrow D$
- $C \rightarrow F$
- $C \rightarrow E$
- $G \rightarrow A$
- $G \rightarrow F$

Give a minimal cover for:

$F = \{AB \to CD, C \to EF, G \to A, G \to F, CE \to F\}$.

- $AB \to C$
- $AB \to D$
- $C \to F$
- $C \to E$
- $G \to A$
- $G \to F$

Give a minimal cover for:
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More lossless practice:

F = {AB -> CDE, BE -> X, A -> E}

Is ABC, BCDEX a lossless decomposition?

More lossless practice: $F = \{AB \rightarrow CDE, BE \rightarrow X, A \rightarrow E\}$ Is ABC, BCDEX a lossless decomposition?

- No, it is lossy. ABC ∩ BCDEX = BC, which is not a superkey of ABC nor BCDEX.

R: ABCDE

Given FD $=\{AE \rightarrow BC, AC \rightarrow D, CD \rightarrow BE, D \rightarrow E\}$

Give three candidate keys.

R: ABCDE

Given FD ={AE → BC, AC → D, CD → BE, D → E}

Give three candidate keys.

- AE, AC and AD are candidate keys, as each of their attribute closures include all attributes and no subset of them is a super key by itself.

R: ABCDE

Given FD ={AE → BC, AC → D, CD → BE, D → E}

Is R already in BCNF?

R: ABCDE

Given FD ={AE → BC, AC → D, CD → BE, D → E}

Is R already in BCNF?

- No, because both CD → BE and D → E violate BCNF.

R: ABCD

Given FD $=\{A \rightarrow B, B \rightarrow D, C \rightarrow D\}$

Decomposed to AB, CD, AC. Is this lossless?

R: ABCD
Given FD =$\{A \rightarrow B, B \rightarrow D, C \rightarrow D\}$
Decomposed to AB, CD, AC. Is this lossless?

- Yes, a lossless decomposition would be: ABC CD which is lossless because C is a key for CD and then a further decomposition of ABC into AB and AC which is lossless because A is a key for AB.