Shadows of Colliding Black Holes

Richard A. Mann, Jake O. Shipley and Sam R. Dolan

School of Mathematics and Statistics

Introduction

Kastor-Traschen Solution

The Kastor-Traschen (KT) solution is an exact solution to Einstein's field equations, it describes extremally charged black holes in a contracting or expanding spacetime. Although the model is an unrealistic representation of black holes, it can still be very helpful to simulate the shadows of colliding black holes. For the purposes of this investigation a two black hole system was considered. The metric for this spacetime is given by [1],

$$ds^2 = -\frac{a^2}{\Omega^2}d\tau^2 + a^2\Omega^2(dx^2 + dy^2 + dz^2),$$

$$a = e^{Ht} = -\frac{1}{H\tau},$$

$$H = \pm\sqrt{\frac{\Lambda}{3}},$$

$$\Omega = 1 + \sum_i \frac{m_i}{ar_i},$$

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2},$$

where τ and t are conformal time and physical time respectively. Λ is the cosmological constant, which is the density of energy in the vacuum of space. H is Hubbles constant, this is the rate at which the universe is expanding or contracting. For this system the two black holes are colliding, this means H is set to be a negative value. m_i is the mass of the black holes. The Hamiltonian which governs the motion of photons in the KT spacetime is,

$$\mathcal{H} = \frac{1}{2}\left[-\frac{\Omega^2}{a^2}p_\tau^2 + \frac{1}{a^2\Omega^2}\left(p_\rho^2 + p_z^2 + \frac{p_\phi^2}{\rho^2}\right)\right] = 0.$$

Due to the symmetry of the system the Hamiltonian is in cylindrical coordinates where ρ is the radius, z is the position along the z axis and ϕ is the angle. p_τ, p_ρ, p_z and p_ϕ are the momentum in the directions of the of τ, ρ, z and ϕ.

Black Hole Shadows

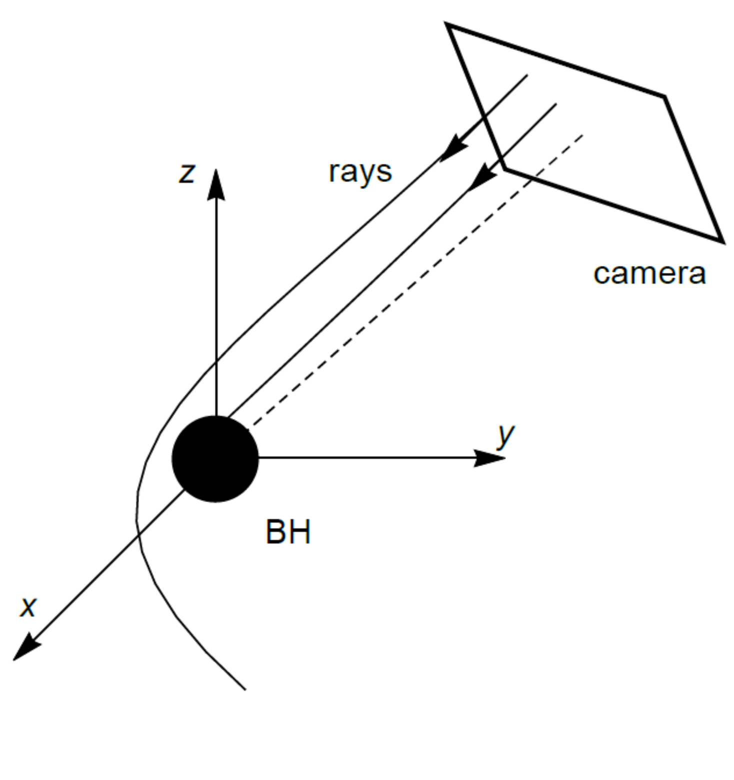

Figure 1: Tracing the trajectory of a photon backwards in time, backwards ray-tracing, to define a black hole shadow.

The shadow of a black hole is found by tracing the trajectory of a photon, backwards in time from the camera. The photon can fall into one of the black holes or it can escape the system. Thinking about what this means, moving forwards in time. If the photon falls into a black hole going backwards in time, this would look like a photon escaping a black hole going forwards in time. This is not possible due to the large gravitational field of the black hole. These trajectories define a black hole shadow [3].

Method

Using the above Hamiltonian and the initial conditions from [2], Hamilton's equations were numerically solved. Solving these equations in this way is a step by step process mapping out the trajectory of a light ray (Photon) around the binary black hole system.

Results

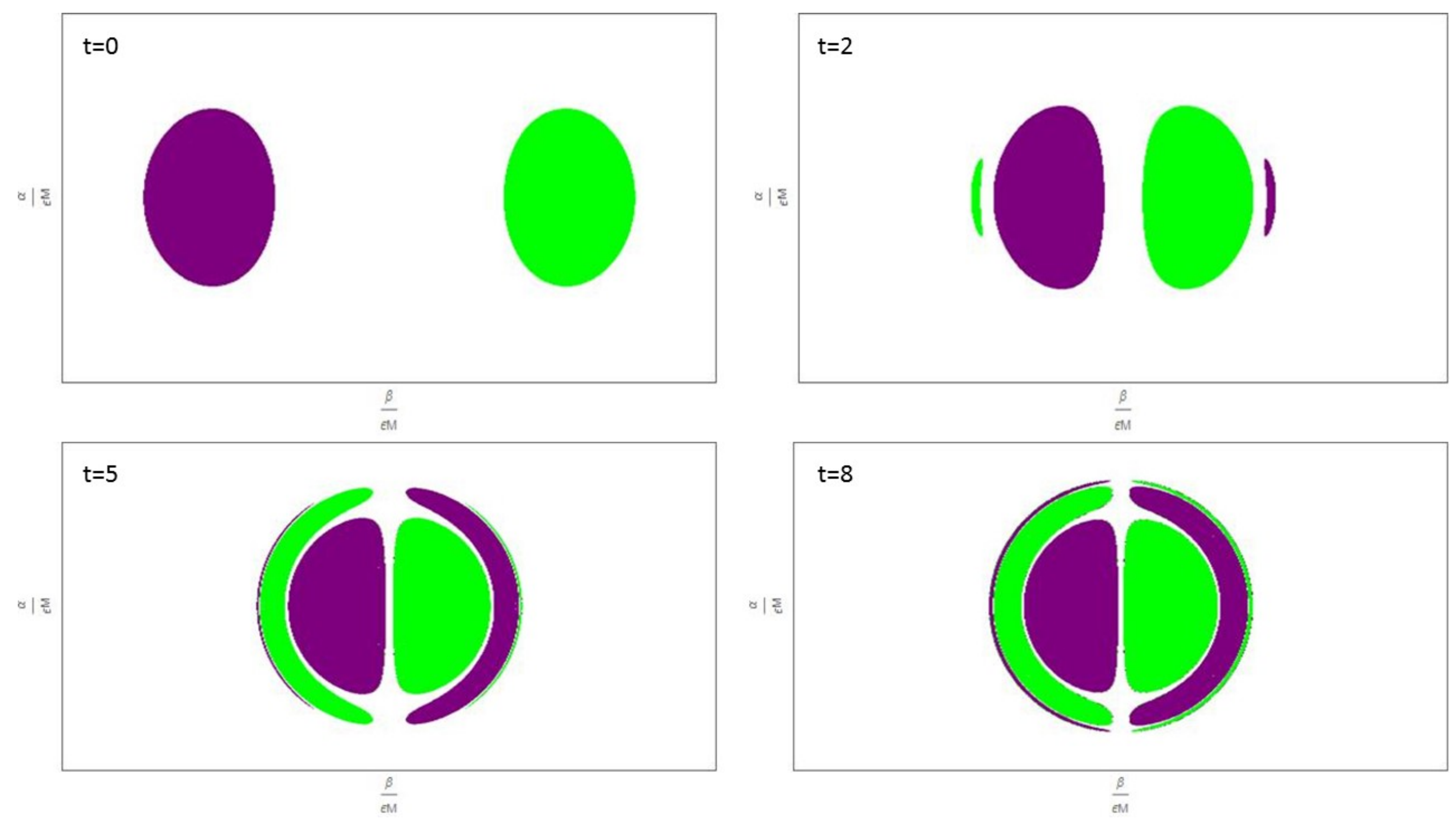

Figure 2: The black hole shadows of two colliding black holes in a contracting spacetime.

As the simulation moves through time we can see the black holes coming together and the effect that one black hole has on the other black holes shadow. This is the eyebrow structure that was seen in the research done in [2].

Zooming into these structures it becomes evident that rather than the eyebrow just being one structure it is in fact a layered structure of many eyebrows all side by side and decreasing in length. This is in fact a fractal. The reason for this is an effect discussed in [3], in the space between these eyebrows there are what is known as perpetual orbits. These are special trajectories where a photon can orbit round the black hole indefinitely, meaning they fall into neither of the black holes nor escape off to infinity.

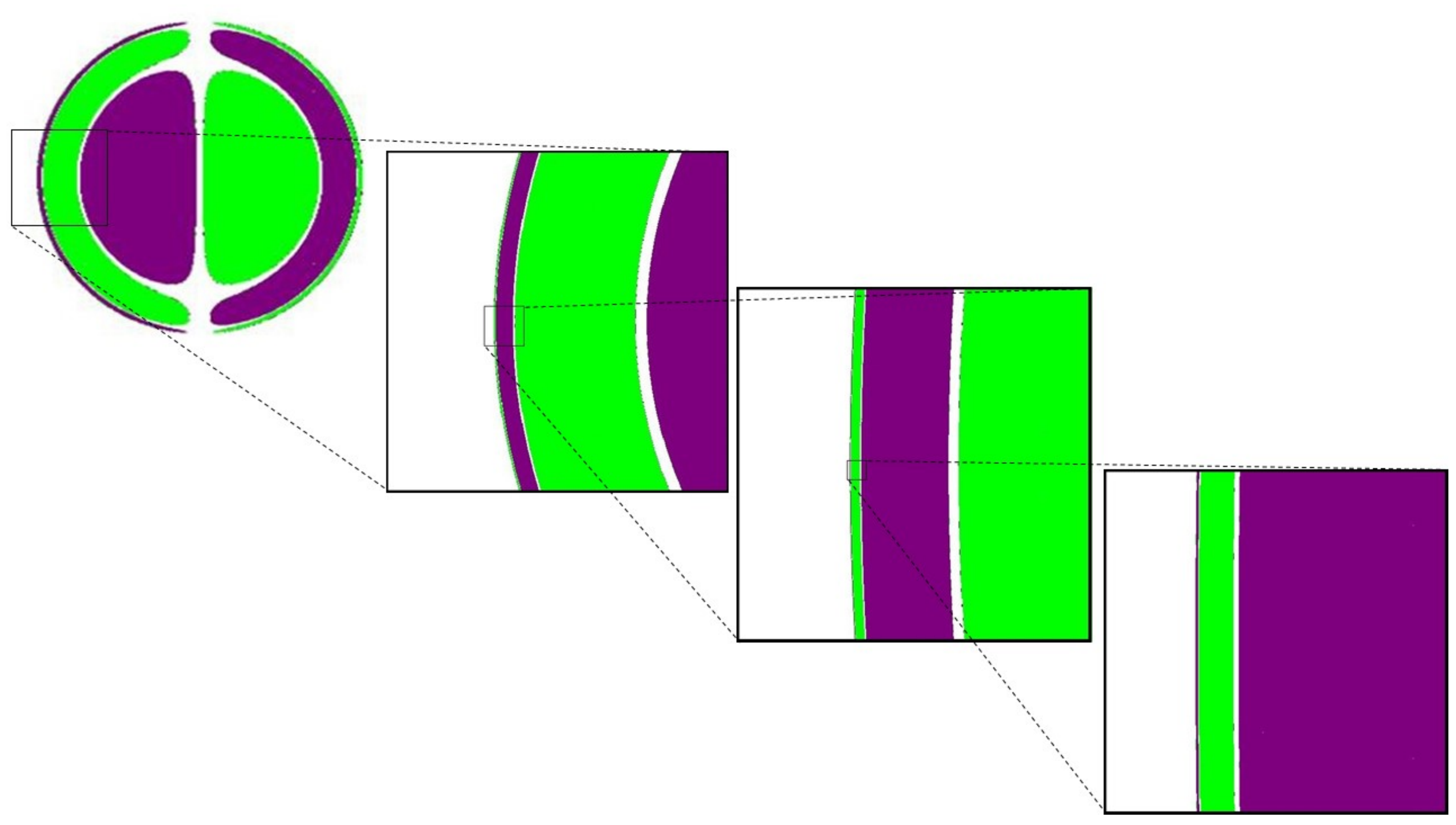

Figure 3: A closer look at the eyebrow structure created as the two black holes get closer to one another.

These orbits exist on the boundaries of the black hole shadows. This means as you go from falling into a black hole to escaping to infinity there is a small window in which these orbits exist.

References

[1] D. Kastor and J. Traschen, Cosmological Multi-Black Hole Solutions, Phys.Rev. D47 (1993) 5370-5375

[2] D. Nitta, T. Chiba and N. Sugiyama, Shadows of colliding black holes, Phys.Rev.D84:063008, 2011

[3] J. Shipley and S. Dolan, Binary black hole shadows, chaotic scattering and Cantor set, Class.Quant.Grav. 33 (2016) no.17, 175001