CPSC 3400 - Homework 6
DFAs, Regular Expressions and Turing machines

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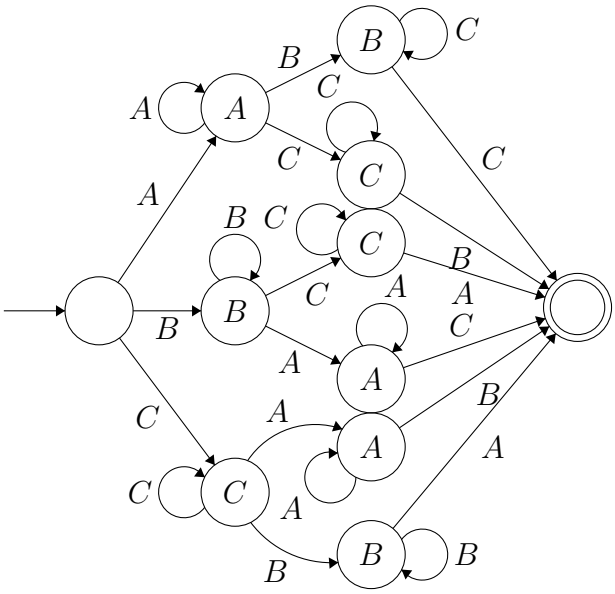
Part I. DFAs

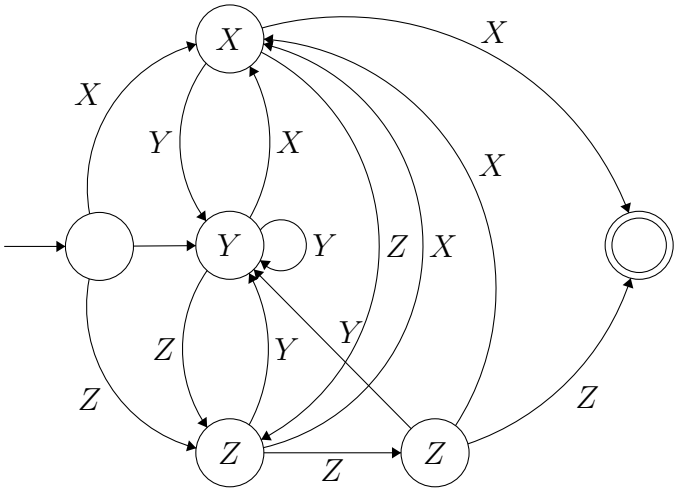
Draw DFAs for the following language specifications.

1. All strings on $\sum = \{A, B, C\}$ that contain each letter (A, B, and C) at least once.



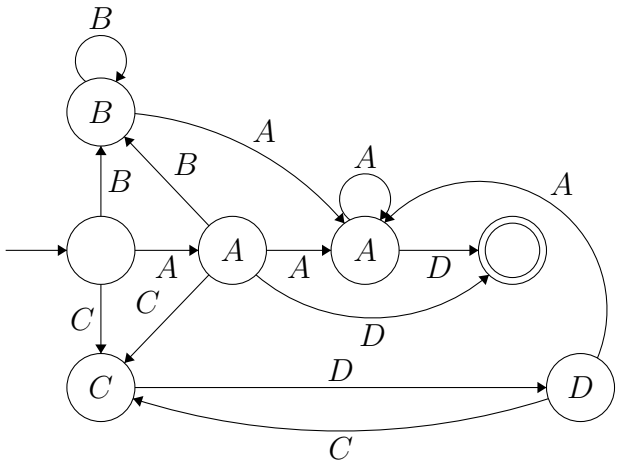
2. All strings on $\sum = \{X, Y, Z\}$ that contain two secutive Xs or three consecutive Zs (or both).

Solution



3. All strings on $\sum = \{A, B, C, D\}$ that match the Python regular expression `^(A?(B|CD)*A+D)$`

Solution



Part II. Regular Expressions

For each item, write a ***single*** regular expression that mathces that item. Note that in ALL cases, the entire string must match without additional characters.

1. A string of digits that contains only digits and contains exactly two fives. Examples of acceptable strings include: "15445", "55", "05563"

   The string should be rejected if it contains anything other than a digits.

   Solution

   - Answer: `^[0-4,6-9]*5[0-4,6-9]*5[0-4,6-9]*$`
   - Explanation:
     - `[0,4,6-9]*`: Accept all strings that contain digits in range from 0 to 9 except 5. It also accept empty strings. (1)
     - `5{1}`: Accept only a digit-5 string. (2)

2. A regular expression that matches a time expressed in the form "1:45 PM".

   The hours part must be a number from 1 to 12, the minutes range from 00 to 59, and the time must indicate either AM or PM (uppercase only and preceded by exactly one space).

   Solution

   - Answer: `^((1[0-2]|[1-9]))(:)([0-5][0-9]) ([AP]M)$`
   - Explanation:
     - `(1[0-2]|[1-9])`: Accept all digit strings that are either in range from 10 to 12 or in range from 0 to 9 without leading zeroes.
     - `(:)`: Accept the separator, ":", between hours and minutes.
     - `([0-5][0-9])`: Accept all digit strings from 00 to 59 (with leading zeroes).
     - `([AP]M)`: Accept "AM" or "PM" with one preceding space to indicate the time meridiem.

3. A regular expression that matches a string representing a comma separated list of variable names such as: `hello, get_max, sum3`

- A variable name consists of letters, digits, and underscores but cannot start with a digit.
- There is exactly one space after every comma. No other spaces are allowed.
- Commas and spaces are not allowed before the first name and after the last name.
- An empty string is considered a match.

Solution

- Answer: `^$|^(([a-zA-Z_]\w*)(, ))*([a-zA-Z_]\w*)$`
- Explanation:
  - `^$`: Accept empty string.
  - `([a-zA-Z_]\w*)`: Accept variable names that match with the requirements. `\w` is equivalent to `[a-zA-Z0-9_]`.
  - `(([a-zA-Z_]\w*)(, ))*`: As the previous one, but addtionally accept previous variable names followed by exactly one comma and one space, if there are more than one variable name.

Part III. Turing Machines

Design a Turing machine on the input alphabet {`x,y,z`} that removes all `z` characters from the input such there are no gaps. If the input string is `xzzyxzy`, the output should be `xyxy`.

- Hint: The final string does not need to reside on the same part of the tape where it started.

Solution