ADM Příprava na skúška

(1.) $\overline{AB}$ $\overline{AC}$ $\perp$
$\overline{AB}$ $\overline{AC}$ $\parallel$

a) $A=(3,5)$, $B=(2,y)$, $C=(2,8)$

$\vec{n}_1 = B-A = (-1, y-5)$

$\vec{n}_2 = C-A = (-1,3)$

$\parallel$ $\overline{n}_1x \cdot \overline{n}_2y - \overline{n}_1y \cdot \overline{n}_2x$

$-1 \cdot 3 + (y-5)(-\cancel{1}) = -3+y-5$

$y=8$ $B=(2,8)$

$\perp$ $n_1x \cdot n_2x + n_1y \cdot n_2y =$

$= 1+3y-15 \Rightarrow 3y=14 \Rightarrow y=\frac{14}{3}$

$B(2,\frac{14}{3})$

b) $A=(-2,5)$, $B=(1,y)$, $C=(4;-3)$

$\overline{n}_1 = B-A = (3, y-5)$

$\overline{n}_2 = C-A = (6,-8)$

$\perp$ $\overline{n}_1x \cdot \overline{n}_2x + \overline{n}_1y \cdot \overline{n}_2y$

$18+(40-8y)=0$

$y=\frac{58}{8}$ $B=(1,\frac{58}{8})$

$\parallel$ $\overline{n}_1x \cdot \overline{n}_2y - \overline{n}_2x \cdot \overline{n}_1y$

$= -24 - (6y-30) = -24+30-6y = 0$

$y=1$ $B=(1,1)$

c) $A=(1,5)$, $B=(-1,y)$, $C=(2,-3)$

$\overline{n}_1 = B-A = (-2,y-5)$

$\overline{n}_2 = C-A = (1,-8)$

$\perp \Rightarrow$ $\overline{n}_1x \cdot \overline{n}_2x + \overline{n}_1y \cdot \overline{n}_2y =$

$= -2+40-8y=0$

$y=\frac{38}{8}$ $B=(-1,\frac{38}{8})$

$\parallel$ $\overline{n}_1x \cdot \overline{n}_2y - \overline{n}_1y \cdot \overline{n}_2x$

$16 - y+5 \Rightarrow y=21$

$B=(-1,21)$

d) $A=(2,1)$, $B=(x,-2)$, $C=(1,3)$

$\overline{n}_1 = B-A = (x-2;-3)$

$\overline{n}_2 = C-A = (-1,2)$

$\perp \Rightarrow$ $\overline{n}_1x \cdot \overline{n}_2x + \overline{n}_1y \cdot \overline{n}_2y$

$2-x-6=0 \Rightarrow x=-4$ $B(-4,-2)$

$\parallel \Rightarrow$ $\overline{n}_1x \cdot \overline{n}_2y - \overline{n}_1y \cdot \overline{n}_2x$

$= 2x-4-3=0 \Rightarrow x=\frac{7}{2}$ $B(\frac{7}{2};-2)$

3. a) $5=\sqrt{(x-2)^2+9}$ sk: $5=\sqrt{(6-2)^2+9}$

$25=x^2-4x+4+9$ $5=\sqrt{16+9}$

$x^2-4x-12=0$ $5=\sqrt{25}$ ✓

$(x-6)(x+2)$ $x_{1,2}=6,-2$

b) $A=(1,4)$, $B=(x,1)$ $d=4$

$4=\sqrt{(x-1)^2+9}$

$16=x^2-2x+1+9$

$x^2-2x-6=0$

$4-4\cdot(-6)=\sqrt{28}$

$1\pm\sqrt{7}$

c) $A=(1,5)$, $B=(1,y)$, $d=6$ sk: $6=\cancel{\sqrt{6^2}}$ /

$6=\cancel{\sqrt{(y-5)^2}}$

$y=11$

5.) $\vec{a}=[3,-2]$, $\vec{b}=[-1,5]$ $a\cdot c=17$, $b\cdot c=3$, $\bar{c}[x_1,x_2]$

$3x_1-2x_2=17$ $x_1=5x_2-3$

$-x_1+5x_2=3$ $x_1=10-3 \Rightarrow \underline{x_1=7}$

$\cancel{3x_1}-2x_2=17$

$-\cancel{3x_1}+15x_2=9$

$13x_2=26 \Rightarrow x_2=2$

Treba si pozriet 2,5,6,7

Matice opakovanie

$$\mathbf{A} = \begin{bmatrix} -4 & 2 & 3 \\ 0 & 5 & -1 \\ 6 & 1 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 6 & -1 & 0 \\ 2 & 2 & -4 \\ 3 & -1 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 5 & -1 \\ -3 & 4 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -7 & 1 & -4 \\ 3 & -2 & 8 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 3 & -3 & 5 \\ 1 & 0 & -2 \\ 6 & 7 & -2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 8 & -1 \\ 2 & 0 \\ 5 & -3 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -4 & 2 & 3 \\ 0 & 5 & -1 \\ 6 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 6 & -1 & 0 \\ 2 & 2 & -4 \\ 3 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 7 & -5 \\ 9 & 0 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -4 & 0 & 6 \\ 2 & 5 & 1 \\ 3 & -1 & -2 \end{bmatrix} \quad E^T = \begin{bmatrix} 3 & 1 & 6 \\ -3 & 0 & 7 \\ 5 & -2 & -2 \end{bmatrix}$$

$$A^T + E^T = \begin{bmatrix} -1 & 1 & 12 \\ -1 & 5 & 8 \\ 8 & -3 & -4 \end{bmatrix} \qquad A+E = \begin{bmatrix} -1 & -1 & 8 \\ 1 & 5 & -3 \\ 12 & 8 & -4 \end{bmatrix}$$

$$Tr(D) = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} = 1+0+4 = 5$$

$$G.C + H.C = (G+H)C$$

$$\mathbf{G} = \begin{bmatrix} 5 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 3 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 11 & -2 \\ -4 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} (55-4+0) & (-10-2+0) \\ (0+8-3) & (4+1) \\ (11+0+9) & (-2-3) \end{bmatrix} = \begin{bmatrix} 51 & -12 \\ 5 & 5 \\ 20 & -5 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 6 & 3 & 1 \\ 1 & -15 & -5 \\ -2 & -1 & 10 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 11 & -2 \\ -4 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} (66-12+3) & (-12-6-1) \\ (11+60-15) & (-2+30+5) \\ (-22+4+30) & (4+2-10) \end{bmatrix} = \begin{bmatrix} 57 & -19 \\ 56 & 33 \\ 12 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 51 & -12 \\ 5 & 5 \\ 20 & -5 \end{bmatrix} + \begin{bmatrix} 57 & -19 \\ 56 & 33 \\ 12 & -4 \end{bmatrix} = \begin{bmatrix} 108 & -31 \\ 61 & 38 \\ 32 & -9 \end{bmatrix} \checkmark$$

$$G=\begin{bmatrix}5&1&0\\0&-2&-1\\1&0&3\end{bmatrix}+H=\begin{bmatrix}6&3&1\\1&-15&-5\\-2&-1&10\end{bmatrix}=\begin{bmatrix}11&4&1\\1&-17&-6\\-1&-1&13\end{bmatrix}\cdot C=\begin{bmatrix}11&-2\\-4&-2\\3&-1\end{bmatrix}=\begin{bmatrix}(121-16+3)&(-22-8-1)\\(11+68-18)&(-2+34+6)\\(-11+4+39)&(2+2-13)\end{bmatrix}$$

$$=\begin{bmatrix}108&-31\\61&38\\32&-9\end{bmatrix}\ \checkmark$$

- Riešenie neznámych v augmentových maticiach

a) $$\begin{bmatrix}2&0&-1\\2&-2&0\\0&1&-2\end{bmatrix}R_2-R_1=\begin{bmatrix}2&0&-1\\1&-2&1\\0&1&-2\end{bmatrix}{-2R_2+R_1}=\begin{bmatrix}0&4&-3\\1&-2&1\\0&1&-2\end{bmatrix}\begin{matrix}1.)\ R_1\Leftrightarrow R_2\\2.)\ -4R_3+R_2\end{matrix}=$$

$$\begin{bmatrix}1&-2&1\\0&0&5\\0&1&-2\end{bmatrix}\begin{matrix}1.)\ R_2\Leftrightarrow R_3\\2.)\ \frac{1}{5}R_3\end{matrix}=\begin{bmatrix}1&-2&1\\0&1&-2\\0&0&1\end{bmatrix}\begin{matrix}-R_3+R_1\\2R_3+R_2\end{matrix}=\begin{bmatrix}1&-2&0\\0&1&0\\0&0&1\end{bmatrix}2R_2+R_1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

c) $$\begin{bmatrix}0&3&-1&-1&-1\\2&3&0&4&-5\end{bmatrix}=\left[\begin{array}{cccc|c}2&3&0&4&-5\\0&3&-1&-1&-1\end{array}\right]\begin{matrix}\cdot\frac{1}{2}\\\cdot\frac{1}{3}\end{matrix}=\begin{array}{c}\begin{matrix}x_1&&&&\end{matrix}\\\begin{bmatrix}1&\frac{3}{2}&0&2&-\frac{5}{2}\\0&1&-\frac{1}{3}&-\frac{1}{3}&-\frac{1}{3}\end{bmatrix}\\\begin{matrix}&x_2&R&S&t\end{matrix}\end{array}$$

$$x_2=-\frac{1}{3}(R+S+t)$$
$$x_1=\frac{3}{2}x_2+2S-\frac{5}{2}t$$
$$x_1=\frac{-(R+S+t)}{2}+2S-\frac{5}{2}t$$

Výsledok: $(x_1,x_2,R,S,t)=\left(\frac{-(R+S+t)}{2}+2S-\frac{5}{2}t,\ -\frac{1}{3}(R+S+t),R,S,t\right)$

a) $-2x_1=6$, $3x_1=8$, $9x_1=-3$

b) $6x_1-x_2+3x_3=4$, $5x_2-x_3=1$

c) $2x_2-3x_4+x_5=0$, $-3x_1-x_2+x_3=-1$, $6x_1+2x_2-x_3+2x_4-3x_5=6$

d) $3x_1-2x_2=-1$, $4x_1+5x_2=3$, $7x_1+3x_2=2$

b) $$\begin{array}{cccccc}x_1&x_2&x_3&x_4&x_5&v\end{array}$$
$$\left[\begin{array}{ccccc|c}6&-1&3&0&0&4\\0&5&-1&0&0&1\\0&2&-3&0&1&0\end{array}\right]-2R_3+R_2$$

$$=\left[\begin{array}{ccccc|c}6&-1&3&0&0&4\\0&1&5&0&-2&1\\0&2&-3&0&1&0\end{array}\right]-2R_2+R_3\Rightarrow\left[\begin{array}{ccccc|c}6&-1&3&0&0&4\\0&1&5&0&-2&1\\0&0&-13&0&5&-2\end{array}\right]\begin{matrix}\cdot\frac{1}{6}\\\\\cdot-\frac{1}{13}\end{matrix}$$

$$\begin{array}{cccccc}x_1&x_2&x_3&x_4&x_5&v\end{array}$$
$$\left[\begin{array}{ccccc|c}1&-\frac{1}{6}&\frac{1}{2}&0&0&\frac{2}{3}\\0&1&5&0&-2&1\\0&0&1&0&-\frac{5}{13}&\frac{2}{13}\end{array}\right]$$

$$x_3-\frac{5}{13}x_5=\frac{2}{13}$$
$$x_2+5x_3-2x_5=1$$
$$x_1-\frac{1}{6}x_2+\frac{1}{2}x_3=\frac{2}{3}$$

$$\begin{array}{ccc} x_1 & x_2 & v \end{array}$$

$$\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{array}\right] \begin{array}{l} \\ -R_1+R_2 \\ -2R_1+R_3 \end{array} \Rightarrow \left[\begin{array}{cc|c} 3 & -2 & -1 \\ 1 & 7 & 4 \\ 1 & 7 & 4 \end{array}\right]$$

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

a) $(3,1,1)$
b) $(3,-1,1)$
c) $(13,5,2)$
d) $\left(\frac{13}{2},\frac{5}{2},2\right)$
e) $(17,7,5)$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 2 & -4 & -1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 4 & -6 & -2 \end{array}\right] -2R_2+R_3 = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right] \cdot \frac{1}{2}R_2 = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \end{array}\right] 3R_2+R_1$$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \end{array}\right] \end{array}$$

$x_1 - \frac{7}{2}x_3 = -\frac{1}{2}$

$x_2 - \frac{3}{2}x_3 = -\frac{1}{2}$

$3 - \frac{7}{2} = -\frac{1}{2} \wedge 1 - \frac{3}{2} = -\frac{1}{2}$

$\frac{6}{2} - \frac{7}{2} = -\frac{1}{2} \wedge -\frac{1}{2} = -\frac{1}{2}$ ✓

$-\frac{1}{2} = -\frac{1}{2}$ ✓ $\wedge$

Možnosť a)

a) $\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$ R_2+R_1

b) $\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -6 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$ $R_1 \Leftrightarrow R_3$

c) $\begin{bmatrix} 2 & 4 & -6 & 8 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$ $\cdot \frac{1}{2}R_1$

d) $\begin{bmatrix} 7 & -4 & -2 & 2 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$ $-2R_2+R_1$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 2 & 0 & -1 \\ 3 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix} \end{array} \begin{array}{l} -R_1+R_2 \\ R_1 \Leftrightarrow R_2 \\ R_2 \Leftrightarrow R_3 \end{array} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix} -2R_1+R_3 = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 4 & -3 \end{bmatrix} -4R_2+R_3 =$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$x_1 - 2x_2 + t = 0 \Rightarrow x_1 - 4t + t = 0 \Rightarrow x_1 = 3t$

$x_2 - 2t = 0 \Rightarrow x_2 = 2t$

$x_3 = t$, t je parameter, $t \in \mathbb{R}$

Výsledok: $(x_1, x_2, x_3) = (3t, 2t, t)$

a) $\begin{bmatrix} 2 & 0 & -1 \\ 3 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 0 & 2 \\ 1 & -5 & 0 \\ 0 & 1 & 2 \\ 2 & -4 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 2 & 3 & 0 & 4 & -5 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 0 & 0 & 1 & -4 \\ 3 & 0 & 2 & 1 & 7 \\ -1 & 3 & 0 & -2 & 4 \\ 0 & 0 & -1 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 0 & 2 \\ 1 & -5 & 0 \\ 0 & 1 & 2 \\ 2 & -4 & 3 \end{bmatrix} \begin{matrix} -3R_2+R_1 \\ \\ \\ -2R_2+R_4 \end{matrix} = \begin{bmatrix} 0 & 15 & 2 \\ 1 & -5 & 0 \\ 0 & 1 & 2 \\ 0 & 6 & 3 \end{bmatrix} \begin{matrix} -15R_3+R_1 \\ \\ \\ -6R_3+R_4 \end{matrix} \quad \begin{bmatrix} \cancel{0} & \cancel{0} & \cancel{-28} \\ 1 & -5 & 0 \\ 0 & 1 & 2 \\ \cancel{0} & \cancel{0} & \cancel{-9} \end{bmatrix}$$

Last: $R_1 \Leftrightarrow R_2$

$x_3 = t,\ t \in R$

$x_2 + 2t = 0$

$x_2 = -2t$

$x_1 - 5(-2t) = 0$

$x_1 = -10t$

Výsledok: $(x_1, x_2, x_3) = (-10t, -2t, t)$

PR 5)

a) $\begin{matrix} 3x-2y=4 \\ 6x-4y=9 \end{matrix}$ b) $\begin{matrix} 2x-4y=1 \\ 4x-8y=2 \end{matrix}$ c) $\begin{matrix} x-2y=0 \\ x-4y=8 \end{matrix}$

a) sú rovnobežné priamky takže $\nexists$ riešenie

b) Nekonečne veľa riešení, totožné priamky

c) $Ax \cdot b$

$A = \begin{bmatrix} 1 & -2 \\ 1 & -4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 8 \end{bmatrix} \quad x_n = \dfrac{\det(A_n)}{\det(A)}$

$\mathrm{Det}(A) = -4 - -2 = -2$

$\mathrm{Det}\left(\begin{bmatrix} 0 & -2 \\ 8 & -4 \end{bmatrix}\right) = 16 \quad x_1 = \frac{16}{-2} = \underline{\underline{-8}}$ Sk: $-8 - 2\cdot(-4) = -8+8 = 0$ ✓

$\mathrm{Det}\left(\begin{bmatrix} 1 & 0 \\ 1 & 8 \end{bmatrix}\right) = 8 \quad x_2 = \frac{8}{-2} = \underline{\underline{-4}}$ $\quad -8 - 4\cdot(-4) = -8+16 = 8$ ✓

a) $\left[\begin{array}{cc|c} 2 & -3 & 1 \\ 6 & -9 & 3 \end{array}\right] -3R_1+R_2 = \left[\begin{array}{cc|c} 2 & -3 & 1 \\ 0 & 0 & 0 \end{array}\right]$ $x_2 = t,\ t \in R$

$2x_1 - 3t = 1 \quad x_1 = \frac{1+3t}{2}$

Výsledok: $(x_1, x_2) = \left(\frac{1+3t}{2} ; t\right)$

b) $\left[\begin{array}{ccc|c} 1 & 3 & -1 & -4 \\ 3 & 9 & -3 & -12 \\ -1 & -3 & 1 & 4 \end{array}\right] \begin{matrix} -3R_1+R_2 \\ R_1+R_3 \end{matrix} = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 3 & -1 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

$x_3 = t,\ x_2 = s,\ s, t \in R$

$x_1 + 3s - t = -4$

$x_1 = t - 3s - 4$

Výsledok: $(x_1, x_2, x_3) = (t - 3s - 4, s, t)$

c) $\left[\begin{array}{cc|c} 6 & 2 & -8 \\ 3 & 1 & -4 \end{array}\right] \Rightarrow$ $x_2 = t$ $\quad 3x_1 + t = -4$ $\quad x_1 = \dfrac{-4-t}{3}$

$(x_1, x_2) = \left(\frac{-4-t}{3}, t\right)$

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & -4 \\ 6 & -3 & 6 & -12 \\ -4 & 2 & -4 & 8 \end{array}\right] \begin{matrix} -3R_1+R_2 \\ 2R_1+R_3 \end{matrix} \Rightarrow \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \left[\begin{array}{ccc|c} 2 & -1 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_3 = t$

$x_2 = s$

$2x_1 - s + 2t = 4$

$x_1 = \dfrac{4+s-2t}{2}$

Výsledok: $(x_1, x_2, x_3) = \left(\dfrac{4+s-2t}{2}, s, t\right)$

9. Určte, či nasledovné matice sú v riadkovom echelonovom tvare, alebo v redukovanom riadkovom echelonovom tvare, alebo v oboch

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10. V nasledujúcich úlohách je uvedená augmentovaná matica lineárnych systémov. Upravte ju do riadkovo redukovaného echolonovho tvaru a následne nájdite riešenie lineárneho systému.

a) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

c) $\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

b) $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

a) $$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \begin{matrix} -4R_3+R_1 \\ -2R_3+R_2 \end{matrix} = \left[\begin{array}{ccc|c} 1 & -3 & 0 & -13 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{array}\right] 3R_2+R_1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

c) $x_5 = t$; $x_4 + 3t = 9 \Rightarrow x_4 = 9 - 3t$

$x_3 + (9-3t) + 6t = 5$; $x_3 = -4 - 3t$

$x_2 = s$; $x_1 + 7s - 2(-4-3t) - 8t = -3$

$x_1 + 7s + 8 + 6t - 8t = -3$

$x_1 = 2t - 7s - 11$

Výsledok: $(x_1, x_2, x_3, x_4, x_5) = (2t-7s-11, s, -4-3t, 9-3t, t)$

Gauss-JL

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \begin{matrix} R_2+R_1 \\ \\ 3R_2+R_3 \end{matrix} = \left[\begin{array}{ccc|c} 0 & -1 & 5 & 9 \\ -1 & -2 & 3 & 1 \\ 0 & -13 & 13 & 13 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -1 & 5 & 9 \\ 0 & -1 & 1 & 1 \end{array}\right] -R_2+R_3 = \left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{matrix} 3R_3+R_1 \\ -5R_3+R_2 \end{matrix} = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & +1 & 0 & +1 \\ 0 & 0 & 1 & 2 \end{array}\right] -2R_2+R_1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x_1 = 3$ $\quad x_3 = 2$

$x_2 = 1$

a) $\begin{aligned} x_1+x_2+2x_3&=8\\ -x_1-2x_2+3x_3&=1\\ 3x_1-7x_2+4x_3&=10 \end{aligned}$

b) $\begin{aligned} 2x_1+2x_2+2x_3&=0\\ -2x_1+5x_2+2x_3&=1\\ 8x_1+x_2+4x_3&=-1 \end{aligned}$

c) $\begin{aligned} x-y+2z-w&=-1\\ 2x+y-2z-2w&=-2\\ -x+2y-4z-2w&=1\\ 3x-3w&=-3 \end{aligned}$

d) $\begin{aligned} -2b+3c&=1\\ 3a+6b-3c&=-2\\ 6a+6b+3c&=5 \end{aligned}$

Gauss-Forw

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \begin{array}{l} \\ R_1+R_2 \\ -3R_1+R_3 \end{array} = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] -10R_2+R_3 = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right]$$

$x_1+1+4=8$ $\quad$ $x_3=2$

$x_1=3$ $\quad$ $x_2-5\cdot(2)=-9$

$x_2=1$

Gauss-Backw

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} -2R_3+R_1 \\ 5R_3+R_2 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] -R_2+R_1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\Rightarrow$ $x_1=3$ $\quad x_3=2$
$x_2=1$

b) GF

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \begin{array}{l} \\ R_1+R_2 \\ -4R_1+R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3=t,\ t\in\mathbb{R}$

$7x_2+4t=1$ $\quad$ $2x_1+\frac{2-8t}{7}+2t=0$

$x_2=\frac{1-4t}{7}$ $\quad$ $x_1=\dfrac{-2t-\frac{2-8t}{7}}{2}$

Rješenje: $(x_1,x_2,x_3)=\left(-t-\frac{1-4t}{7};\ \frac{1-4t}{7};\ t\right)$

Gauss Forw

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & -2 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \\ R_1+R_3 \\ -3R_1+R_4 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right] -3R_3+R_4 \Rightarrow$$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{array}\right] \end{array}$$

$x_4=0$, $x_3=t,\ t\in\mathbb{R}$ $\quad$ $x_1-\cancel{2t}+3x_4+\cancel{2t}-x_4=-1$

$x_2-2t-3x_4=0$ $\quad$ $x_1=x_4-1$

$x_2=2t+3x_4$

Rješenje: $(x_1,x_2,x_3,x_4)=(x_4-1,\ 2t+3x_4,\ t,\ 0)$

$$\left[\begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 7 & 1 & -8 & 9 & 0 \\ 2 & 8 & 1 & -1 & 0 \end{array}\right] \begin{array}{l} -3R_1+R_2 \\ -R_1+R_3 \end{array} = \left[\begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 1 & 10 & -20 & 12 & 0 \\ 0 & 11 & -3 & 0 & 0 \end{array}\right] -2R_2+R_1 \Rightarrow$$

$$\left[\begin{array}{cccc|c} 0 & -23 & 44 & -25 & 0 \\ 1 & 10 & -20 & 12 & 0 \\ 0 & 11 & -3 & 0 & 0 \end{array}\right] \begin{array}{l} 1)\ 2R_3+R_1 \\ 2.)\ R_1 \Leftrightarrow R_2 \end{array} = \left[\begin{array}{cccc|c} 1 & 10 & -20 & 12 & 0 \\ 0 & -1 & 38 & -25 & 0 \\ 0 & 11 & -3 & 0 & 0 \end{array}\right] 11R_2+R_3 =$$

$$\left[\begin{array}{cccc|c} 1 & 10 & -20 & 12 & 0 \\ 0 & -1 & 38 & -25 & 0 \\ 0 & 0 & 83 & -55 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 10 & -20 & 12 & 0 \\ 0 & 1 & -38 & 25 & 0 \\ 0 & 0 & 1 & -\frac{55}{83} & 0 \end{array}\right]$$

$\Rightarrow x_4 = t,\ t \in R \ \Big|\ x_3 = \frac{55}{83}t \ \Big|\ x_1 + 380\left(\frac{55}{83}t\right) - 20\left(\frac{55}{83}t\right) + 12t = 0$

$x_2 - 38\left(\frac{55}{83}t\right) + 25t = 0 \qquad x_1 = -360\left(\frac{55}{83}t\right) - 12t$

$x_2 = 38\left(\frac{55}{83}t\right) - 25t$

Riešenie: $(x_1, x_2, x_3, x_4) = \left(-360\left(\frac{55}{83}t\right) - 12t,\ 38\left(\frac{55}{83}t\right) - 25t,\ \frac{55}{83}t,\ t\right)$

DÚ 4.

a) $\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = 2 \cdot 1 - 5 \cdot 3 = 2 - 15 = -13$

b) $\begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix} = -6$ | c) $\begin{bmatrix} 6 & -12 \\ -4 & 8 \end{bmatrix} = 24 + 12 \cdot 4 = 72$

d) $\begin{bmatrix} \cos 0 & \sin 0 \\ -\sin 0 & \cos 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$

e) $\left[\begin{array}{ccc|cc} 2 & 0 & 5 & 2 & 0 \\ -4 & 1 & 7 & -4 & 1 \\ 0 & 3 & -3 & 0 & 3 \end{array}\right] = (-6 + 0 - 60) - (-42 + 0) = -66 + 42 = -24$

f) $\left[\begin{array}{ccc|cc} 3 & -2 & 4 & 3 & -2 \\ 5 & 1 & -2 & 5 & 1 \\ -1 & 3 & 6 & -1 & 3 \end{array}\right] = (18 - 4 + 60) - (-4 - 18 - 60) = 74 + 82 = 156$

$$\begin{bmatrix} 5 & 0 & 0 \\ 3 & -2 & 0 \\ -1 & 8 & 4 \end{bmatrix}\begin{matrix} 5 & 0 \\ 3 & -2 \\ -1 & 8 \end{matrix} = (-40+0+0)-(0+0+0) = -40$$

$$\begin{bmatrix} -6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{matrix} -6 & 0 \\ 0 & 2 \\ 0 & 0 \end{matrix} = -12 \cdot 5 = \underline{\underline{-60}}$$

$$\begin{bmatrix} 3 & 1 & -2 \\ -1 & 4 & 5 \\ 3 & 1 & -2 \end{bmatrix}\begin{matrix} 3 & 1 \\ -1 & 4 \\ 3 & 1 \end{matrix} = (-24+15+2)-(-24+15+2) = -7-(-7) = 0$$

$$\begin{bmatrix} -2 & 4 & 3 \\ 3 & -1 & 6 \\ 5 & -2 & 4 \end{bmatrix}\begin{matrix} -2 & 4 \\ 3 & -1 \\ 5 & -2 \end{matrix} = (8+120-18)-(-15+24+51) = 110-60 = \underline{\underline{50}}$$

4. V nasledujúcich príkladoch vypočítajte determinant daných matíc. Ak je matica invertibilná, použite vzťah z prednášky na nájdenie inverznej matice.

a) $\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix}$ d) $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$

a) $\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} = 12 + 10 = 22$ ✓ $A^{-1} = \begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix} = 8 - 8 = 0$ $\nexists A^{-1}$

c) $\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix} = 10 + 14 = 24$ $A^{-1} = \begin{bmatrix} -2 & -7 \\ 7 & -5 \end{bmatrix}$

d) $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix} = \sqrt{6} - 4\sqrt{6} = -3\sqrt{6}$ $A^{-1} = \begin{bmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{bmatrix}$

6. Vypočítajte hodnotu determinantu matíc **A**, **B**, ak

$$\mathbf{A} = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix}$$

rozvojom (kofaktory) podľa

a) prvého riadku
b) druhého riadku
c) tretieho riadku

d) prvého stĺpca
e) druhého stĺpca
f) tretieho stĺpca

a) $\begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix} = C_{11} \cdot M_{11} + C_{12} \cdot M_{12} + C_{13} \cdot M_{13} = 3\begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix} + 2\begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix} + 4\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$

$= 3 \cdot 12 + 2 \cdot 28 + 4 \cdot 16 = 36 + 64 + 56 = \underline{\underline{156}}$

b) $\begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix} = C_{21} \cdot M_{21} + C_{22} \cdot M_{22} + C_{23} \cdot M_{23} = -5\begin{bmatrix} -2 & 4 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix} + 2\begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix}$

$-5 \cdot (-24) + 22 + 2 \cdot 7 = 142 + 14 = 156$

c) $\begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix} = C_{31} \cdot M_{31} + C_{32} \cdot M_{32} + C_{33} \cdot M_{33} = -\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} - 3\begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix} + 6\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$

$= 0 - 3(-26) + 6 \cdot 13 = 78 + 78 = \underline{\underline{156}}$ ✓

7. Vypočítajte hodnotu determinantu matíc rozvojom podľa vhodne vybratého riadku alebo stĺpca

a) $\begin{bmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix}$

c) $\begin{bmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{bmatrix}$

d) $\begin{bmatrix} 5 & 2 & 1 & 0 \\ -1 & 3 & 5 & 2 \\ 4 & 1 & 0 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 0 & 5 & 4 & 0 \\ 4 & 1 & -2 & 7 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{bmatrix}$

f) $\begin{bmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

g) $\begin{bmatrix} 0 & 4 & 1 & 3 & -2 \\ 2 & 2 & 3 & -1 & 0 \\ 3 & 1 & 2 & -5 & 1 \\ 1 & 0 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \end{bmatrix}$

$$\begin{bmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{bmatrix} = C_{12}\cdot M_{12} + C_{22}\cdot M_{22} + C_{32}\cdot M_{32} = 5\begin{bmatrix} -3 & 7 \\ -1 & 5 \end{bmatrix} = 5\cdot(-8) = \underline{\underline{-40}}$$

$$\begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix} = C_{21}\cdot M_{21} + C_{23}\cdot M_{23} = -\begin{bmatrix} 3 & 1 \\ -3 & 5 \end{bmatrix} + 4\begin{bmatrix} 3 & 3 \\ 1 & -3 \end{bmatrix} = -18 + 4\cdot(-12) = -18-48 = -66$$

$$\begin{bmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} \Rightarrow C_{11}\cdot M_{11} = 2\left[\begin{array}{ccc|cc} -1 & 3 & 8 & -1 & 3 \\ 0 & 5 & 2 & 0 & 5 \\ 0 & 0 & 6 & 0 & 0 \end{array}\right] = \underline{\underline{-30}}$$

$$\begin{bmatrix} 5 & 2 & 1 & 0 \\ -1 & 3 & 5 & 2 \\ 4 & 1 & 0 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix} \Rightarrow C_{24}\cdot M_{24} + C_{34}\cdot M_{34} = 2\begin{bmatrix} 5 & 2 & 1 \\ 4 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 5 & 2 & 1 \\ -1 & 3 & 5 \\ 0 & 2 & 3 \end{bmatrix} =$$

$$= 2\big((15+8)-(24)\big) - 2\big((45-2)-(50-6)\big) =$$

$$= -2 - 2 = \underline{\underline{-4}}$$

$$\begin{bmatrix} 0 & 4 & 1 & 3 & -2 \\ 2 & 2 & 3 & -1 & 0 \\ 3 & 1 & 2 & -5 & 1 \\ 1 & 0 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \end{bmatrix} = C_{52}\cdot M_{52} + C_{55}\cdot M_{55} = -3\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 3 & -1 & 0 \\ 3 & 2 & -5 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} + 2\begin{bmatrix} 0 & 4 & 1 & 3 \\ 2 & 2 & 3 & -1 \\ 3 & 1 & 2 & -5 \\ 1 & 0 & -4 & 0 \end{bmatrix}$$

$$-3\left[\begin{array}{cccc|ccc} 0 & 1 & 3 & -2 & 0 & 1 & 3 \\ 2 & 3 & -1 & 0 & 2 & 3 & -1 \\ 3 & 2 & -5 & 1 & 3 & 2 & -5 \\ 1 & -4 & 0 & 0 & 1 & -4 & 0 \end{array}\right] + 2\left[\begin{array}{cccc|ccc} 0 & 4 & 1 & 3 & 0 & 4 & 1 \\ 2 & 2 & 3 & -1 & 2 & 2 & 3 \\ 3 & 1 & 2 & -5 & 3 & 1 & 2 \\ 1 & 0 & -4 & 0 & 1 & 0 & -4 \end{array}\right]$$

$$-3\big((-1)-(4)\big) + 2\big((-60-24)-(9+160)\big) = 15 - 426$$

$$2(-253)+15$$

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

$$\left[\begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] R_1 \Leftrightarrow R_2 = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 2 & -4 & -1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ -3R_1+R_3 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 4 & -6 & -2 \end{array}\right] -2R_2+R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot \frac{1}{2} R_2 \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right] 3R_2+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right] \quad x_3 \in \mathbb{R}$$

$$x_2 - \frac{3}{2}x_3 = -\frac{1}{2}$$
$$x_1 - \frac{7}{2}x_3 = -\frac{1}{2}$$
$$x_2 - \frac{3}{2}x_3 = x_1 - \frac{7}{2}x_3 \quad \cdot 2$$
$$2x_2 - 3x_3 = 2x_1 - 7x_3$$
$$(2x_2 - 2x_1 = -4x_3)!$$

a) $2 - 6 = -4$ ✓
b) $-2 - 6 - -4$ X
c) $10 - 26 = -8$ X
d) $5 - 13 = -8$ ✓
e) $14 - 34 = -20$ ✓

a, d, e

(11.)

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \begin{array}{l} R_1+R_2 \\ -3R_1+R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -5 & -1 & -7 \end{array}\right] -5R_2+R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} -2R_3+R_1 \\ -5R_3+R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & +4 \\ 0 & +1 & 0 & +1 \\ 0 & 0 & 1 & 2 \end{array}\right] -R_2+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow x_1 = 3,\ x_2 = 1,\ x_3 = 2$$

DÚ 4.

(4.) $\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} \Rightarrow \det(A) = 12 - (-10) = 22$ ✓ $\quad A^{-1} \Rightarrow \begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix} \Rightarrow \det(B) = 8 - 8 = 0 \Rightarrow B^{-1} \nexists$

c) $\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix} \quad \det(C) = 10 + 49 = 59 \quad C^{-1} = \begin{bmatrix} -2 & -7 \\ 7 & -5 \end{bmatrix}$

d) $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$ $\det(D) \Rightarrow \sqrt{6} - 4\sqrt{6} = -3\sqrt{6}$ $D^{-1} = \begin{bmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{bmatrix}$

5.) b) $\Rightarrow$ Razvoj po kofaktorima $\begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix} \Rightarrow C_{11} M_{11} \Rightarrow$

1. $\begin{bmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{bmatrix}$

$\lambda \cdot (\lambda - 1) - 6 = 0$ $\quad (\lambda-3)(\lambda+2)$

$\lambda^2 - \lambda - 6 = 0$ $\quad \lambda_{1,2} = 3, -2$

6) $\begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix}$ $\det(A) = C_{11} \cdot M_{11} + C_{12} \cdot M_{12} + C_{13} \cdot M_{13}$

$3 \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix} + 2 \begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix} + 4 \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix} = 3 \cdot 12 + 2 \cdot 28 + 4 \cdot 16$

$= 36 + 56 + 64 = 156$

b) $C_{21} \cdot M_{21} + C_{22} \cdot M_{22} + C_{23} \cdot M_{23} = -5 \begin{bmatrix} -2 & 4 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix} + 2 \begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix}$

$= -5(-24) + 22 + 2 \cdot 7 = 120 + 22 + 14 = 156$

c) $C_{31} \cdot M_{31} + C_{32} \cdot M_{32} + C_{33} \cdot M_{33} = -1 \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} - 3 \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix} + 6 \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$

$= -1 \cdot 0 - 3(-26) + 6 \cdot 13 = 78 + 78 = 156$

stupca

a) $\begin{bmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix}$

c) $\begin{bmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{bmatrix}$

d) $\begin{bmatrix} 5 & 2 & 1 & 0 \\ -1 & 3 & 5 & 2 \\ 4 & 1 & 0 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 0 & 5 & 4 & 0 \\ 4 & 1 & -2 & 7 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{bmatrix}$

f) $\begin{bmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

g) $\begin{bmatrix} 0 & 4 & 1 & 3 & -2 \\ 2 & 2 & 3 & -1 & 0 \\ 3 & 1 & 2 & -5 & 1 \\ 1 & 0 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} -3 & \boxed{0} & 7 \\ 2 & \boxed{5} & 1 \\ -1 & \boxed{0} & 5 \end{bmatrix} \Rightarrow C_{12} \cdot M_{12} + C_{22} \cdot M_{22} + C_{32} \cdot M_{32} = 0 + 5 \begin{bmatrix} -3 & 7 \\ -1 & 5 \end{bmatrix} + 0 = 5 \cdot (-8) = -40$

d) $C_{24} \cdot M_{24} + C_{34} \cdot M_{34} = 2 \left[\begin{array}{ccc|cc} 5 & 2 & 1 & 5 & 2 \\ 4 & 1 & 0 & 4 & 1 \\ 0 & 2 & 3 & 0 & 2 \end{array}\right] - 2 \left[\begin{array}{ccc|cc} 5 & 2 & 1 & 5 & 2 \\ -1 & 3 & 5 & -1 & 3 \\ 0 & 2 & 3 & 0 & 2 \end{array}\right]$

$= 2 \cdot ((15+8) - (24)) - 2((45-2) - (50-6)) = -2 + 2 = 0$

e) $C_{21} \cdot M_{21} + C_{31} \cdot M_{31} = -4 \cdot \left[\begin{array}{ccc|cc} 5 & 4 & 0 & 5 & 4 \\ 0 & 3 & 0 & 0 & 3 \\ 2 & 1 & 5 & 2 & 1 \end{array}\right] - 1 \left[\begin{array}{ccc|cc} 5 & 4 & 0 & 5 & 4 \\ 1 & -2 & 7 & 1 & -2 \\ 2 & 1 & 5 & 2 & 1 \end{array}\right]$

$= -4 \cdot 75 - 1((1-50+56) - (35+20)) = -300 + 49 = -251$

f) $C_{11}\cdot M_{11} = 2\cdot \begin{bmatrix} -1 & 3 & 8 & -1 & 3 \\ 0 & 5 & 2 & 0 & 5 \\ 0 & 0 & 6 & 0 & 0 \end{bmatrix} = 2\cdot((-30)) = -60$

g) $= C_{41}\cdot M_{41} + C_{43}\cdot M_{43} =$

$$-1\begin{bmatrix} 4 & 1 & 3 & -2 & 4 & 1 & 3 \\ 2 & 3 & -1 & 0 & 2 & 3 & -1 \\ 1 & 2 & -5 & 1 & 1 & 2 & -5 \\ 3 & 0 & 0 & 2 & 3 & 0 & 0 \end{bmatrix} + 4\begin{bmatrix} 0 & 4 & 3 & -2 & 0 & 4 & 3 \\ 2 & 2 & -1 & 0 & 2 & 2 & -1 \\ 3 & 1 & -5 & 1 & 3 & 1 & -5 \\ 0 & 3 & 0 & 2 & 0 & 3 & 0 \end{bmatrix} =$$

$-1((24\cdot(-5)-3)-(12+18))+4(-136) = 153 - 154 = -1$

(12)

$$\begin{bmatrix} 10 & 4 & 21 \\ 0 & -4 & 3 \\ -5 & -1 & -12 \end{bmatrix} \begin{matrix} 2R_3+R_1 \end{matrix} \Rightarrow \begin{bmatrix} 0 & 2 & -3 \\ 0 & -4 & 3 \\ -5 & -1 & -12 \end{bmatrix} 2R_1+R_2 \Rightarrow \begin{bmatrix} 0 & 2 & -3 \\ 0 & 0 & -3 \\ -5 & -1 & -12 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & -1 & -12 \\ 0 & 2 & -3 \\ 0 & 0 & -3 \end{bmatrix} \Rightarrow \det(A) = 30$$

$$\begin{bmatrix} 18 & -9 & -14 \\ 6 & -3 & -5 \\ -3 & 1 & 2 \end{bmatrix} \begin{matrix} 6R_3+R_1 \\ 2R_3+R_2 \end{matrix} \Rightarrow \begin{bmatrix} 0 & -7 & -10 \\ 0 & -1 & -1 \\ -3 & 1 & 2 \end{bmatrix} -7R_2+R_1 \Rightarrow \begin{bmatrix} 0 & 0 & -3 \\ 0 & -1 & -1 \\ -3 & 1 & 2 \end{bmatrix} \quad \det(A) = -9$$

c) $$\begin{bmatrix} 1 & -1 & 5 & 1 \\ -2 & 1 & -7 & 1 \\ -3 & 2 & -12 & -2 \\ 2 & -1 & 9 & 1 \end{bmatrix} \begin{matrix} 2R_1+R_2 \\ 3R_1+R_3 \\ -2R_1+R_4 \end{matrix} \Rightarrow \begin{bmatrix} 1 & -1 & 5 & 1 \\ 0 & -1 & 3 & 3 \\ 0 & -1 & 3 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \begin{matrix} -R_2+R_3 \\ R_2+R_4 \end{matrix} \Rightarrow \begin{bmatrix} 1 & -1 & 5 & 1 \\ 0 & -1 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix} \Rightarrow 4$$

d) $$\begin{bmatrix} -8 & 4 & -3 & 2 \\ 2 & 1 & -1 & -1 \\ -3 & 2 & -12 & -2 \\ 2 & -1 & 9 & 1 \end{bmatrix} \begin{matrix} 4R_4+R_1 \\ -R_4+R_2 \\ R_4+R_3 \end{matrix} \Rightarrow \begin{bmatrix} 0 & 0 & 33 & 6 \\ 0 & 2 & -10 & -2 \\ -1 & 1 & -3 & -1 \\ 2 & -1 & 9 & 1 \end{bmatrix} 2R_3+R_4 \Rightarrow \begin{bmatrix} 0 & 0 & 33 & 6 \\ 0 & 2 & -10 & -2 \\ -1 & 1 & -3 & -1 \\ 0 & 1 & 3 & -1 \end{bmatrix} \Rightarrow -2\cdot 33 = \underline{\underline{-66}}$$

e) $$\begin{bmatrix} 5 & 3 & -8 & 4 \\ \frac{15}{2} & \frac{1}{2} & -1 & -7 \\ -\frac{5}{2} & \frac{3}{2} & -4 & 1 \\ 10 & -3 & 8 & -8 \end{bmatrix} \begin{matrix} 2R_3+R_1 \\ 3R_3+R_2 \\ \\ 4R_3+R_4 \end{matrix} \Rightarrow \begin{bmatrix} 0 & 6 & -16 & 6 \\ 0 & 5 & -13 & -4 \\ -\frac{5}{2} & \frac{3}{2} & -4 & 1 \\ 0 & 3 & -8 & -4 \end{bmatrix} \begin{matrix} -2R_4+R_1 \\ -2R_4+R_2 \end{matrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & -1 & 3 & 4 \\ -\frac{5}{2} & \frac{3}{2} & -4 & 1 \\ 0 & 3 & -8 & -4 \end{bmatrix} 3R_2+R_4 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & -1 & 3 & 4 \\ -\frac{5}{2} & \frac{3}{2} & -4 & 1 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

$\Rightarrow \det(A) = \underline{\underline{-5}}$

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 2 & 4 & -3 & 1 & -4 \\ 2 & 6 & 4 & 8 & -4 \\ -3 & -8 & -1 & 1 & 0 \\ 1 & 3 & 3 & 10 & 1 \end{bmatrix} \begin{matrix} -2R_1+R_2 \\ -2R_1+R_3 \\ 3R_1+R_4 \\ -R_1+R_5 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & -1 & -5 & -4 \\ 0 & 2 & 6 & 2 & -4 \\ 0 & -2 & -4 & 10 & 0 \\ 0 & 1 & 4 & 7 & 1 \end{bmatrix} \begin{matrix} R_4+R_3 \\ 2R_5+R_4 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & -1 & -5 & -4 \\ 0 & 0 & 2 & 12 & -4 \\ 0 & 0 & 4 & 24 & 2 \\ 0 & 1 & 4 & 7 & 1 \end{bmatrix} -2R_3+R_4$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & -1 & -5 & -4 \\ 0 & 0 & 2 & 12 & -4 \\ 0 & 0 & 0 & 0 & 10 \\ 0 & 1 & 4 & 7 & 1 \end{bmatrix} 2R_2+R_3 \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & -1 & -5 & -4 \\ 0 & 0 & 0 & 2 & -12 \\ 0 & 0 & 0 & 0 & 10 \\ 0 & 1 & 4 & 7 & 1 \end{bmatrix} \Rightarrow \det(A) = \underline{\underline{-20}}$$

g) $$\begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix} \begin{matrix} 4R_5+R_1 \\ -R_5+R_2 \\ -R_5+R_3 \\ -R_5+R_4 \end{matrix} \Rightarrow \begin{bmatrix} 0 & 5 & 5 & 5 & -15 \\ 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & -5 & 0 & 5 \\ 0 & 0 & 0 & -5 & 5 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix} R_2+R_1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & -4 \\ 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & 5 & 5 & -10 \\ 0 & 0 & -5 & 0 & 5 \\ 0 & 0 & 0 & -5 & 5 \end{bmatrix} R_3+R_4, R_3+R_5$$

$\det(A) = \underline{0}$

14. $$\left[\begin{array}{ccc|cc} 2 & 5 & 5 & 2 & 5 \\ -1 & -1 & 0 & -1 & -1 \\ 2 & 4 & 3 & 2 & 4 \end{array}\right] \Rightarrow \det(A) = (-6-20)-(-10-15) = -26+25 = -1$$

$M_{11} = -3 \quad M_{12} = +3 \quad M_{13} = -2$

$M_{21} = +5 \quad M_{22} = -4 \quad M_{23} = +2$

$M_{31} = 5 \quad M_{32} = 5 \quad M_{33} = 3$

$$A^{-1} = \frac{1}{\det(A)} \cdot A^{T} = -1 \cdot \begin{bmatrix} -3 & 5 & 5 \\ 3 & -4 & -5 \\ -2 & 2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$$

b) $$\left[\begin{array}{ccc|cc} 2 & 0 & 3 & 2 & 0 \\ 0 & 3 & 2 & 0 & 3 \\ -2 & 0 & -4 & -2 & 0 \end{array}\right] \det(A) = -24+18 = -6 \qquad A^{-1} \exists$$

$M_{11} = -12 \quad M_{12} = -4 \quad M_{13} = 6$

$$A^{-1} = \frac{1}{\det(A)} \cdot A^{T} = -\frac{1}{6} \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 0 & \frac{3}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -1 & 0 & -1 \end{bmatrix}$$

$M_{21} = 0 \quad M_{22} = -2 \quad M_{23} = 0$

$M_{31} = -9 \quad M_{32} = -4 \quad M_{33} = 6$

c) $$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow \det(A) = 4 \quad A^{-1} \exists$$

$M_{11} = 2 \quad M_{12} = 0 \quad M_{13} = 0$

$M_{21} = 6 \quad M_{22} = 4 \quad M_{23} = 0$

$$A^{-1} = \frac{1}{\det(A)} \cdot \bar{A} = \frac{1}{4} \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$M_{31} = 4 \quad M_{32} = 6 \quad M_{33} = 2$

$$\mathbf{A}=\begin{bmatrix}4&2&8\\-2&1&-4\\3&1&6\end{bmatrix}\qquad \text{g)}\ \mathbf{A}=\begin{bmatrix}2&0&0\\8&1&0\\-5&3&6\end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix}1&0&-1\\9&-1&4\\8&9&-1\end{bmatrix}\qquad \text{h)}\ \mathbf{A}=\begin{bmatrix}\sqrt{2}&-\sqrt{7}&0\\3\sqrt{2}&-3\sqrt{7}&0\\5&-9&0\end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix}-3&0&1\\5&0&6\\8&0&3\end{bmatrix}\qquad \text{i)}\ \mathbf{A}=\begin{bmatrix}2&0&3\\0&3&2\\-1&0&-4\end{bmatrix}$$

$$\left[\begin{array}{ccc|cc}4&2&8&4&2\\-2&1&-4&-2&1\\3&1&6&3&1\end{array}\right]\Rightarrow \det(A)=(\cancel{24}-\cancel{24}-16)-(\cancel{24}-16-\cancel{24})=-16+16=0$$

$A^{-1}\ \nexists$

$$\left[\begin{array}{ccc|cc}1&0&-1&1&0\\9&-1&4&9&-1\\8&0&3&8&0\end{array}\right]\Rightarrow \det(A)=-3-8=-11\qquad A^{-1}=\frac{1}{\det(A)}\cdot A^T$$

$M_{11}=-3\quad M_{12}=5\quad M_{13}=-8$

$M_{21}=0\quad M_{22}=11\quad M_{23}=0$

$M_{31}=-1\quad M_{32}=13\quad M_{33}=-1$

$$-\frac{1}{11}\cdot\begin{bmatrix}-3&0&-1\\5&11&-13\\-8&0&-1\end{bmatrix}=\begin{bmatrix}\frac{3}{11}&0&\frac{1}{11}\\-\frac{5}{11}&-1&\frac{13}{11}\\\frac{8}{11}&0&\frac{1}{11}\end{bmatrix}$$

$$\begin{bmatrix}2&0&0\\8&1&0\\-5&3&6\end{bmatrix}\ \det(A)=2\cdot1\cdot6=12\qquad A^{-1}\ \exists$$

$M_{11}=6\quad M_{12}=-48\quad M_{13}=29$

$M_{21}=0\quad M_{22}=12\quad M_{23}=-6$

$M_{31}=0\quad M_{32}=0\quad M_{33}=2$

$$\frac{1}{12}\cdot\begin{bmatrix}6&0&0\\-48&12&0\\29&-6&2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0\\-4&1&0\\\frac{29}{12}&-\frac{1}{2}&\frac{1}{6}\end{bmatrix}$$

$$\begin{bmatrix}\sqrt{2}&-\sqrt{7}&0\\3\sqrt{2}&-3\sqrt{7}&0\\5&-9&0\end{bmatrix}\Rightarrow \det(A)=0\qquad A^{-1}\ \nexists$$

$$\left[\begin{array}{ccc|cc}2&0&3&2&0\\0&3&2&0&3\\-1&0&-4&-1&0\end{array}\right]\Rightarrow \det(A)=-24+9=-15$$

$M_{11}=-12\quad M_{12}=2\quad M_{13}=3$

$M_{21}=0\quad M_{22}=-5\quad M_{23}=0$

$M_{31}=-9\quad M_{32}=-4\quad M_{33}=6$

$$-\frac{1}{15}\begin{bmatrix}-12&0&-9\\-2&-5&-4\\3&0&6\end{bmatrix}=\begin{bmatrix}\frac{12}{15}&0&\frac{9}{15}\\\frac{2}{15}&\frac{1}{3}&\frac{4}{15}\\-\frac{3}{15}&0&-\frac{6}{15}\end{bmatrix}$$

16. Nasledujúce sústavy rovníc riešte pomocou výpočtu inverznej matice (cez adjugovanú m
a pomocou Cramerovho pravidla

a) $7x_1-2x_2=3$, $3x_1+x_2=5$

b) $2x+3y=4$, $2x+2y=4$

c) $5x-5y=7$, $2x-3y=6$

d) $2x+5y=4$, $4x+y=3$

e) $-9x-4y=3$, $-7x+5y=-10$

f) $-10x-7y=-12$, $12x-11y=5$

g) $-x-3y=4$, $-8x+4y=3$

h) $-2x+y-4z=-8$, $-4y+z=3$, $4x-z=-8$

k) $x-4y+z=6$, $4x-y+2z=-1$, $2x+2y-3z=-20$

l) $x_1-3x_2+x_3=4$, $2x_1-x_2=-2$, $4x_1-3x_3=0$

m) $-x_1-4x_2+2x_3+x_4=-32$, $2x_1-x_2+7x_3+9x_4=14$, $-x_1+x_2+3x_3+x_4=11$, $x_1-2x_2+x_3-4x_4=-4$

n) $3x_1-x_2+x_3=4$, $-x_1+7x_2-3x_3=1$, $2x_1+6x_2-x_3=5$

o) $x_1+x_2+2x_3=8$, $-x_1-2x_2+3x_3=1$, $3x_1-7x_2+4x_3=10$

p) $2x_1+2x_2+2x_3=0$, $-2x_1+5x_2+2x_3=1$, $8x_1+x_2+4x_3=-1$

$$\begin{bmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad \begin{bmatrix} 6 \\ -1 \\ -20 \end{bmatrix}$$

$$\det(A) = (3 - 16 + 8) - (-2 + 4 + 48) = -5 - 50 = -55$$

$M_{11} = -1 \qquad M_{12} = +16 \qquad M_{13} = 10$

$M_{21} = -10 \qquad M_{22} = -5 \qquad M_{23} = -10$

$M_{31} = -7 \qquad M_{32} = +2 \qquad M_{33} = 15$

$$-\frac{1}{55} \begin{bmatrix} -1 & -10 & -7 \\ 16 & -5 & 2 \\ 10 & -10 & 15 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{55} & \frac{10}{55} & \frac{7}{55} \\ -\frac{16}{55} & \frac{5}{55} & -\frac{2}{55} \\ -\frac{10}{55} & \frac{10}{55} & -\frac{15}{55} \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -1 \\ -20 \end{bmatrix} = \begin{bmatrix} \frac{6}{55} - \frac{10}{55} - \frac{140}{55} \\ -\frac{96}{55} - \frac{5}{55} + \frac{40}{55} \\ -\frac{60}{55} - \frac{10}{55} + \frac{300}{55} \end{bmatrix} \Rightarrow \begin{aligned} x_1 &= \frac{-144}{55} \\ x_2 &= -\frac{61}{55} \\ x_3 &= \frac{230}{55} \end{aligned} \quad \checkmark$$

$$\begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \Rightarrow \det(A_1) = 13 \qquad x_1 = \frac{\det(A_1)}{\det(A)} = \frac{13}{13} = 1$

$\begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow \det(A_2) = 26 \qquad x_2 = \frac{26}{13} = 2$

$\det(A) \Rightarrow 13$

$$\left[\begin{array}{ccc|cc} -2 & 1 & -4 & -2 & 1 \\ 0 & -4 & 1 & 0 & -4 \\ 4 & 0 & -1 & 4 & 0 \end{array}\right] \begin{bmatrix} -8 \\ 3 \\ -8 \end{bmatrix}$$

$$\det(A) = (-8 + 4) - (64) = -68$$

$M_{11} = 4 \qquad M_{12} = +4 \qquad M_{13} = 16$

$M_{21} = 1 \qquad M_{22} = 18 \qquad M_{23} = 4$

$M_{31} = -15 \qquad M_{32} = 2 \qquad M_{33} = 8$

$$\frac{1}{-68} \cdot \begin{bmatrix} 4 & 1 & -15 \\ 4 & 18 & 2 \\ 16 & 4 & 8 \end{bmatrix} = \begin{bmatrix} -\frac{1}{17} & -\frac{1}{68} & \frac{15}{68} \\ -\frac{1}{17} & -\frac{9}{34} & -\frac{1}{34} \\ -\frac{4}{17} & -\frac{1}{17} & -\frac{2}{17} \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 3 \\ -8 \end{bmatrix} = \begin{bmatrix} \frac{8}{17} - \frac{3}{68} - \frac{120}{68} \\ \frac{8}{17} - \frac{27}{34} + \frac{8}{34} \\ \frac{32}{17} - \frac{3}{17} + \frac{16}{17} \end{bmatrix} = \begin{bmatrix} -\frac{91}{68} \\ -\frac{3}{34} \\ \frac{45}{17} \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} 1 & -3 & 1 & 1 & -3 \\ 2 & -1 & 0 & 2 & -1 \\ 4 & 0 & -3 & 4 & 0 \end{array}\right] \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix} \Rightarrow \det(A) = 3 - (-4 + 18) = -11$$

$M_{11} = 3 \quad M_{12} = 6 \quad M_{13} = 4$

$M_{21} = -9 \quad M_{22} = -7 \quad M_{23} = -12$

$M_{31} = 1 \quad M_{32} = 2 \quad M_{33} = 5$

$$-\frac{1}{11} \cdot \begin{bmatrix} 3 & -9 & 1 \\ 6 & -7 & 2 \\ 4 & -12 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{3}{11} & \frac{9}{11} & -\frac{1}{11} \\ -\frac{6}{11} & \frac{7}{11} & -\frac{2}{11} \\ -\frac{4}{11} & \frac{12}{11} & -\frac{5}{11} \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{12}{11} - \frac{18}{11} \\ -\frac{24}{11} - \frac{14}{11} \\ -\frac{16}{11} - \frac{24}{11} \end{bmatrix} = \begin{bmatrix} -\frac{30}{11} \\ -\frac{38}{11} \\ -\frac{40}{11} \end{bmatrix}$$

Transformácia M

domain → Počet stĺpcov → d(f)

codomain → Počet riadkov → h(f)

- Reflexia → os x $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (R+Os)
  $\;\;x \quad y$
- 3D Reflexia → x y $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

- Projekcia h → os x $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (Ponecháš os x)
- 3D Projekcia → x y $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

DÚ 5.

Reflexia

6.

a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

8.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix}$

ORtogonálna projekcia

10.

a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

12.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

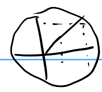
c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$

14.

$30^\circ \Rightarrow \frac{\pi}{6}$

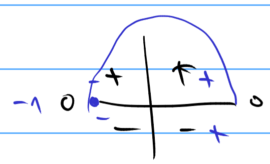
a) $R\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{-3\sqrt{3} - 2}{2} \\ \frac{-3}{2} + \sqrt{3} \end{bmatrix}$

cos  sin



b) $45^\circ \Rightarrow \frac{\pi}{4}$

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{-5\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$



c) $60^\circ \Rightarrow \frac{\pi}{3}$

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{-3-2\sqrt{3}}{2} \\ \frac{-3\sqrt{3}+2}{2} \end{bmatrix}$

d) $-60^\circ \Rightarrow \frac{\pi}{3} \Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{-3+2\sqrt{3}}{2} \\ \frac{3\sqrt{3}+2}{2} \end{bmatrix}$

e) $90^\circ \Rightarrow \pi \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

16.

$x = (2; -3, 1)$

$30^\circ \Rightarrow \frac{\pi}{6} \Rightarrow \begin{bmatrix} \frac{\sqrt{3}}{2} & -0{,}5 \\ 0{,}5 & \frac{\sqrt{3}}{2} \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ \frac{-3\sqrt{3} - 1}{2} \\ \frac{-3+\sqrt{3}}{2} \end{bmatrix}$

$-30 \Rightarrow \frac{\pi}{6} \Rightarrow \begin{bmatrix} \frac{\sqrt{3}}{2} & +\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{-3\sqrt{3}+1}{2} \\ \frac{3+\sqrt{3}}{2} \end{bmatrix}$

c) $-45° = \frac{\pi}{4}$ os Y $\oplus$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} - \frac{\sqrt{2}}{2} \\ -3 \\ \sqrt{2} + \frac{\sqrt{2}}{2} \end{bmatrix}$$

d) os z $\pm 90' = \pi$ $\quad \sin \Rightarrow 0$, $\cos \Rightarrow -1$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Kontrakcia/Dilatácia

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ a) $k = 3$ b) $k = \frac{1}{2}$

↑ Dilat   ↑ kontrak

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}$$

21.

os x; $k = 3$
$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

os Y; $k = \frac{1}{4}$
$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$$

os x; $k = \frac{1}{2}$
$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 2 \end{bmatrix}$$

os Y; $k = 2$
$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 & 2 \\ -6 & -19 & 10 \\ 3 & 9 & -5 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \qquad u = \begin{bmatrix} -1 & -3 & 2 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \cdot S_1 = -1$

$R_2 \cdot S_1 = -6 \cdot -1 + 0 + 0 = 6$

$R_3 \cdot S_1 = 3 \cdot (-1) + 0 + 0 = -3$

$R_1 \cdot S_2 = u_{12} + 0 u_{22} + 0 u_{32} = -3$

$R_2 \cdot S_2 = -18 + u_{22} = -19 \Rightarrow -1$

$R_3 \cdot S_2 = 9 \cdot L_{32} + 0 = 9 \Rightarrow 0$

$R_1 \cdot S_3 = 2$

$R_2 \cdot S_3 = 12 + u_{23} = 10 \Rightarrow -2$

$R_3 \cdot S_3 = -6 + u_{33} = -5 \Rightarrow 1$

$L \cdot Y = \dots$

$$\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -9 \\ -59 \\ 28 \end{bmatrix}$$

$y_1 = -9$

$-54 + y_2 = -59 \Rightarrow y_2 = -5$

$27 + y_3 = 28 \qquad y_3 = 1$

$U \cdot X = \dots$

$$\begin{bmatrix} -1 & -3 & 2 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -9 \\ -5 \\ 1 \end{bmatrix}$$

$x_3 = 1 \qquad +x_1 = 12$

$+x_2 = +3$

$$\begin{bmatrix} 2 & 5 & 6 \\ 4 & 13 & 19 \\ 6 & 27 & 50 \end{bmatrix} \qquad L \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 3 & 7 \\ 0 & 0 & 4 \end{bmatrix}$$

$R_1 \cdot S_1 = 2 \qquad R_1 \cdot S_2 = u_{12} = 5 \qquad R_1 \cdot S_3 = 6$

$R_2 \cdot S_1 = 2L_{21} = 4 \Rightarrow 2 \qquad R_2 \cdot S_2 = 10 + u_{22} = 13 \Rightarrow 3 \qquad R_2 \cdot S_3 = 12 + u_{23} = 19$

$R_3 \cdot S_1 = 2L_{31} = 6 \Rightarrow 3 \qquad R_3 \cdot S_2 = 15 + 3L_{32} = 27 \Rightarrow 4 \qquad R_3 \cdot S_3 = 18 + 28 = 50$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 6 \\ 0 & 3 & 7 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 6 \\ 4 & 13 & 19 \\ 6 & 27 & 50 \end{bmatrix}$$

x_0

$$x_1 = \frac{15 - 2x_2 - x_3}{11} \Rightarrow \frac{15}{11} \qquad x_1 = \frac{15 - \frac{32}{10} + \frac{1}{8}}{11}$$

$$x_2 = \frac{16 - x_1 - 2x_3}{10} \Rightarrow \frac{16}{10} \Rightarrow \qquad x_2 = \frac{16 - \frac{15}{11} + \frac{1}{4}}{10}$$

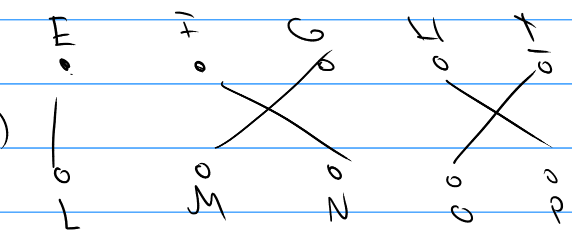
$$x_3 = \frac{1 - 2x_1 - 3x_2}{-8} \Rightarrow -\frac{1}{8} \qquad x_3 = \frac{1 - \frac{30}{11} - \frac{48}{10}}{-8} \Rightarrow$$

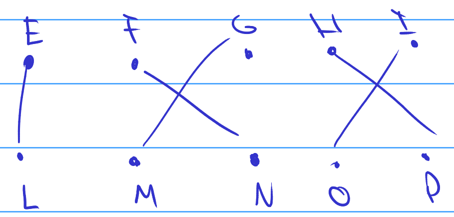
7. Použite Galey-Shaley algoritmus pre určenie najlepšieho možného párovania v prípade preferencií

Edith:	l	>	n	>	o	>	~~m~~	>	p		Liam:	~~f~~	>	e	>	h	>	g	>	i
Faye:	n	>	l	>	m	>	o	>	p		Malik:	~~f~~	>	~~i~~	>	g	>	f	>	h
Grace:	p	>	m	>	o	>	n	>	l		Nate:	f	>	g	>	i	>	h	>	e
Hanna:	p	>	n	>	o	>	l	>	m		Olaf:	i	>	e	>	f	>	g	>	h
Iris:	p	>	o	>	m	>	n	>	l		Pablo:	~~i~~	>	h	>	g	>	e	>	i
	5		4		3		2		1			5		4		3		2		1

a) V prípade, že muž si vyberá ako prvý.
b) V prípade, že žena si vyberá ako prvá.

b)





5.12 Apply the Gale-Shapley Algorithm to the set of preferences below with

(a) the men proposing
(b) the women proposing

Alice:	r	>	s	>	t	>	v		Rich:	a	>	d	>	b	>	c
Beth:	s	>	r	>	v	>	t		Stefan:	~~a~~	>	~~c~~	>	d	>	b
Cindy:	v	>	~~t~~	>	r	>	s		Tom:	~~c~~	>	b	>	d	>	a
Dahlia:	t	>	v	>	s	>	r		Victor:	c	>	d	>	b	>	a