ADM Exercise

PR 1. AB ∥ AC

$A = (3,5)$, $B = (2,y)$, $C = (2,8)$

$\vec{w}_1 = B - A = (-1; y-5)$

$\vec{w}_2 = C - A = (-1; 3)$

Rovnobežnosť: $w_{1x} \cdot w_{2y} - w_{1y} \cdot w_{2x} = 0$

$-1 \cdot 3 - (y-5) \cdot (-1) = 0$

$-3 + y - 5 = 0 \qquad y = 8 \qquad B[2,8]$

Kolmosť $\Rightarrow w_{1x} \cdot w_{2x} + w_{1y} \cdot w_{2y} = 0$

$-1 \cdot (-1) + (y-5) \cdot 3 = 0$

$1 + 3y - 15 = 0$

$3y = 14 \qquad y = \frac{14}{3} \qquad B[2, \frac{14}{3}]$

b) $A(-2,5)$, $B(1,y)$, $C(4,-3)$

$\vec{w}_1 = B - A = (3, y-5)$ Rovnobežnosť: $3 \cdot 6 + (y-5) \cdot (-8) =$

$\vec{w}_2 = C - A = (6, -8)$ $= 18 + 40 - 8y \Rightarrow 8y = 58 \qquad y = \frac{58}{8} \qquad B[1, \frac{58}{8}]$

Kolmosť: $3 \cdot (-8) - (y-5) \cdot 6 = -18 - 6y + 30$

$12 = 6y \Rightarrow y = 2 \qquad B[1,2]$

c) $A = [1,5]$, $B = [-1,y]$, $C = [2,-3]$

$\vec{w}_1 = B - A = (-2, y-5)$

$\vec{w}_2 = C - A = (1, -8)$

Rovnobežnosť: $-2 + (y-5) \cdot (-8)$

$-2 + 40 - 8y = 0$

$38 = 8y \Rightarrow y = \frac{38}{8} \qquad B[-1, \frac{38}{8}]$

Kolmosť: $(y-5) \cdot 1 - (-2) \cdot (-8) =$

$y - 5 - 16 = 0 \Rightarrow y = 21 \qquad B[-1, 21]$

d) $A = [2,1]$, $B = [x,-2]$, $C = [1,3]$

$\vec{w}_1 = B - A = (x-2, -3)$

$\vec{w}_2 = C - A = (-1, 2)$

Rovnobežnosť: $2 - x - 6 = 0$

$x = -4 \qquad B[-4; -2]$

Kolmosť: $2x - 4 - 3 = 0$

$2x = 7$

$x = \frac{7}{2} \qquad B = [\frac{7}{2}; -2]$

PR 3. $A = [2,-3]$, $B = [x;0]$, $d = 5$

a) $B - A = (x-2, 3)$

$|AB| = \sqrt{(x-2)^2 + 3^2} = 5$

$(x-2)^2 + 9 = 25$

$(x-2)^2 = 16$

$x^2 - 4x + 4 - 16 = 0$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x_{1,2} = 6; -2$

$B = [6; 0]$

$B = [-2, 0]$

b) $A=[1,4]$ $B=[x,1]$ $d=4$

$B-A=(x-1,-3)$

$|AB|=\sqrt{(x-1)^2+(-3)^2}=4$

$(x-1)^2+9=16$

$x^2-2x+1+9-16=0$

$x^2-2x-6=0$

$4-4\cdot(-6)=\sqrt{28}$

$\frac{2\pm\sqrt{28}}{2}=1\pm\sqrt{7}$

c) $A=[1,5]$, $B=[1,x]$ $d=6$

$B-A=(0,x-5)$

$|AB|=\sqrt{(x-5)^2}=6$

$x^2-10x+25=36$

$x^2-10x-11=0$

$100-4\cdot(-11)=144$

$\frac{10\pm12}{2}$ $\begin{cases}11 & B[1,11]\\ -1 & B[1,-1]\end{cases}$

PR 4. a) $A=[-3,-2]$, $B=[1,4]$, $C=[-5,0]$

$\cos\varphi=\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$

$\vec{u}_1=B-A=(4,6)\Rightarrow|\vec{u}_1|=\sqrt{52}$

$\cos\varphi_1=\frac{4\cdot(-2)+6\cdot2}{\sqrt{52}\cdot\sqrt{8}}=\frac{-8+12}{\sqrt{416}}=\frac{4}{\sqrt{416}}=78°41'$

$\vec{u}_2=C-A=(-2,2)\Rightarrow|\vec{u}_2|=\sqrt{8}$

$\vec{u}_3=C-B=(-6,-4)\Rightarrow|\vec{u}_3|=\sqrt{52}$ $\cos\varphi_2=\frac{-2\cdot(-6)+2\cdot(-4)}{\sqrt{8}\cdot\sqrt{52}}=\frac{12-8}{\sqrt{416}}=\frac{4}{\sqrt{416}}=78°41'$

$\cos\varphi_3=\frac{4\cdot(-6)+6\cdot(-4)}{52}=\frac{-48}{52}=157°22'$

b) $A=[7,-3,6]$, $B=[11,-5,3]$, $C=[10,-7,8]$

$\vec{u}_1=B-A=(4,-2,-3)=|BA|=\sqrt{29}$ $\cos\varphi_1=\frac{12+8-6}{29}=\frac{14}{29}=61°8'$

$\vec{u}_2=C-A=(3,-4,2)=|AC|=\sqrt{29}$ $\cos\varphi_2=\frac{-3+8+10}{\sqrt{29}\cdot\sqrt{30}}=\frac{15}{\sqrt{870}}=59°25'$

$\vec{u}_3=C-B=(-1,-2,5)=|BC|=\sqrt{30}$ $\cos\varphi_3=\frac{-4+4-15}{\sqrt{870}}=-\frac{15}{\sqrt{870}}=120°34'$

PR 5. $\vec{a}=[3,-2]$, $\vec{b}=[-1,5]$, $\vec{c}=[x,y]$

$a\cdot c=17$

$b\cdot c=3$

$3x-2y=17\Rightarrow$

$5y-x=3\Rightarrow x=5y-3$

$x=5\cdot2-3$

$x=10-3$ $\quad x=7$

$3(5y-3)-2y=17$

$15y-9-2y=17$

$13y=26$

$y=2$

PR 6. $\vec{u}=[3,4]$, $d=15\Rightarrow\vec{n}=[4,-3]$ $\vec{u}_{x_1}=\left[3+\frac{1\pm\sqrt{401}}{2},4+\frac{1\pm\sqrt{401}}{2}\right]$

$\sqrt{(4+x)^2+(x-3)^2}=15$

$(4+x)^2+(x-3)^2=225$

$x^2+8x+16+x^2-6x+9=225$

$2x^2+2x-200=0$

$x^2+x-100=0$

$1-4\cdot(-100)=\sqrt{401}$

$\frac{-1\pm\sqrt{401}}{2}$ $\begin{cases}\frac{-1+\sqrt{401}}{2}\\ \frac{-1-\sqrt{401}}{2}\end{cases}$

$|\vec{u}_{x_1}|=$

PR7. a) $2\bar{x}+3a=\bar{b}$ $\bar{a}=[-1,2], \bar{b}=[0,-2]$ $\bar{x}=[x,y]$

$2x+3\cdot(-1)=0 \Rightarrow 2x=3 \quad x=\frac{3}{2}$

$2y+3\cdot(2)=-2 \Rightarrow 2y=-8 \Rightarrow y=-4$

$\bar{x}=[\frac{3}{2},-4]$

b) $3\bar{x}-a=2\bar{b}$ $\bar{a}=[3,-1]$, $\bar{b}=[0,2]$, $\bar{x}=[x,y]$

$3x-3=0 \Rightarrow \underline{x=1}$

$3y+1=4 \Rightarrow \underline{y=1}$

$\bar{x}=[1,1]$

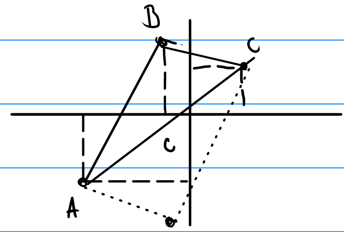
d) $3\bar{x}+5\bar{v}=4\bar{r}$, $\bar{r}=[1,-2]$, $\bar{v}=[2,4]$, $\bar{x}=[x,y]$

$3x+10=4 \Rightarrow x=-2$

$3y+20=-8 \Rightarrow y=-\frac{28}{3}$

$\bar{x}=[-2,-\frac{28}{3}]$

PR 8.



a) $A=(-4,-2)$, $B=(-1,4)$, $C=(2,2)$

$B-A=((-1+4),(4+2))=(3,6) \Rightarrow$

$|AB|=\sqrt{9+36}=\sqrt{45}$

$C-B=((2+1),(2-4))=(3,-2)$

$|BC|=\sqrt{9+4}=\sqrt{13}$

$C-A=((2+4),(2+2))=(6,4)$

$|AC|=\sqrt{36+16}=\sqrt{52}$

$BC\ \vec{n}=(2,3)$

$A \to 2x+3y+C=0$

$2\cdot(-4)+3\cdot(-2)+C=0$

$-8-6+C=0$

$C=+14$

$2x+3y-14=0$

$|AB|\ \vec{n}=(6,-3)$

$C \to 6x-3y+C=0$

$6\cdot(2)-3\cdot(2)+C=0$

$12-6+C=0$

$C=-6$

$6x-3y-6=0$

$2x-y-2=0$

$2x+3y+14=0$

$2x-y-2=0 \Rightarrow y=2x-2$

$2x+3\cdot(2x-2)+14=0$

$2x+6x-6+14=0$

$8x+8=0$

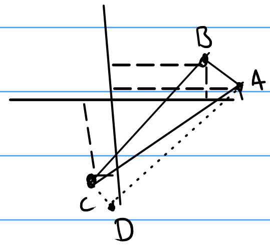
$x=-1$

$D[-1,-3]$

$y=2\cdot(-1)-2$

$y=-4$

b) $A=[5,1]$ $B[4,2]$ $C[-1,-4]$



$B-A=(-1,1)$ $\vec{n}=(1,1)=\sqrt{2}$

$C-B=(-5,-6) \Rightarrow |CB|=\sqrt{61}$ $\vec{n}(6,-5)$

$C-A=(-6,-5) \Rightarrow |CA|=\sqrt{36+25}=\sqrt{61}$

$C \to x+y+c=0$ $A \to 6x-5y+c=0$

$-1-4+c=0$ $30-5+c=0$ $c=-25$

$c=5$ $6x-5y-25=0$

$x+y+5=0$

$x+y+5=0 \Rightarrow x=-y-5$

$6x-5y-25=0$

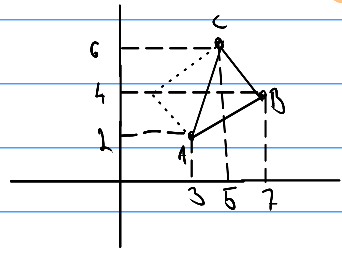
$6\cdot(-y-5)-5y-25=0$

$-11y=55$ $x=5-5=0$

$y=-5$

$D[0,-5]$

c) $A[3,2]$, $B[7,4]$, $C[5,6]$



$B-A=(4,2) \Rightarrow |AB|=\sqrt{16+4}=\sqrt{20}$ $\vec{n}(2,-4)$

$C-B=(-2,+2) \Rightarrow |BC|=\sqrt{4+4}=\sqrt{8}$ $\vec{n}=(2,+2)$

$C-A=(2,4) \Rightarrow |AC|=\sqrt{4+16}=\sqrt{20}$

$2x-4y+c=0$ $2x+2y+c=0 \leftarrow A$

$10-24+c=0$ $c=14$ $6+4+c=0$ $c=-10$

$x-2y+7=0$ $x+y-5=0$

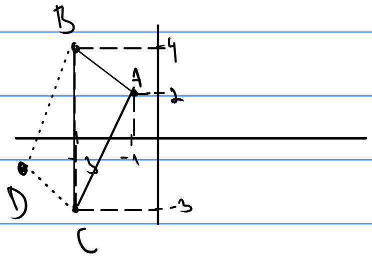
$2x+\cancel{2y}-10=0$ $1-2y+7=0$

$x\cancel{-2y}+7=0$ $y=4$

$3x-3$ $D[1,4]$

$x=1$

d) $A[-1,2]$ $B[-3,4]$ $C[-3,-3]$



$B-A=(-2,2)$ $|AB|=\sqrt{4+4}=\sqrt{8}$ $\vec{n}=(2,2)$

$C-A=(-2,-5)$ $|AC|=\sqrt{25+4}=\sqrt{29}$ $\vec{n}(5,-2)$

$C-B=(0,-7)$ $|BC|=\sqrt{49}=7$

$2x+2y+c=0 \leftarrow C$ $5x-2y+c=0 \leftarrow B$ $2x+\cancel{2y}+12=0$

$-6-6+c=0$ $c=12$ $-15-8+c=0$ $c=23$ $5x\cancel{-2y}+23=0$

$x+y+6=0$ $5x-2y+23=0$ $7x=-35$

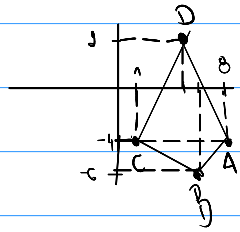
$-10+2y+12=0$ $x=-5$

$y=-1$

$D[-5,-1]$

PR 9.

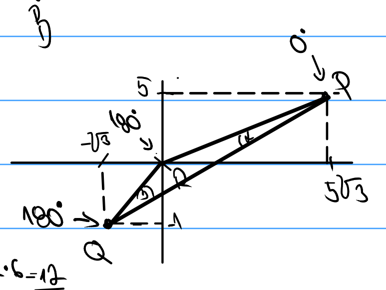
a) $A[8,-4]$ $B[5,-6]$ $C[1,-4]$ $D[4,2]$



$C-A=(-7,0)$ $|AC|=\underline{\underline{7}}$

$D-B=(-1,8)$ $|BD|=\sqrt{65}$

PR 10. $P[5\sqrt{3},5]$ $Q[-\sqrt{3},-1]$ $R[0,0]$



$\vec{u}_1=R-P=(-5\sqrt{3},-5)$ $\vec{n}_1(5,-5\sqrt{3})$ $|PR|=\sqrt{100}=10$

$\vec{u}_2=R-Q=(\sqrt{3},1)$ $\vec{n}_2(1,-\sqrt{3})$ $|QR|=\sqrt{1+3}=2$

$\vec{u}_3=Q-P=(-6\sqrt{3},-6)$ $\vec{n}_3(6,-6\sqrt{3})$ $|PQ|=\sqrt{36\cdot3+36}=\sqrt{4\cdot36}=2\cdot6=\underline{\underline{12}}$

$\overrightarrow{PR}=\underline{\underline{5x-5\sqrt{3}y=0}}$

$\overrightarrow{QR}=\underline{\underline{x-\sqrt{3}y=0}}$

$\overrightarrow{PQ}=6x-6\sqrt{3}y+c=0 \leftarrow Q$

$-6\sqrt{3}+6\sqrt{3}+c=0$ $\underline{\underline{c=0}}$

$\underline{\underline{6x-6\sqrt{3}y=0}}$

$\cos\alpha=\frac{-5\sqrt{3}\cdot(-6\sqrt{3})+30}{120}$

$\cos\alpha=\frac{120}{120}=1=0°$

$\cos\gamma=\frac{-15-5}{2\cdot10}=-1=180°$

$\cos\beta=\frac{-18-6}{2\cdot12}=-1=180°$

PR 11.

a) $u(3,4)$ $v(v_1,v_2)$

$3v_1+4v_2=0$

$\sqrt{v_1^2+v_2^2}=15$

$v_2=-\frac{3}{4}v_1$

$\sqrt{v_1^2+\frac{9}{16}v_1^2}=15$

$\frac{25}{16}v_1^2=225$

$v_1^2=144=\underline{\underline{12}}$

$v_2=-\frac{3}{4}12=-9$

$v(12,-9)$

b) $u(-1,2)$ $v(v_1,v_2)$ $|v|=7$

$-v_1+2v_2=0$ | $\frac{1}{2}v_1=v_2$

$\sqrt{v_1^2+\frac{1}{4}v_1^2}=7$

$\frac{5}{4}v_1^2=49$

$v_1=\sqrt{\frac{196}{5}}=\frac{14}{\sqrt{5}}$ Sk: $\sqrt{\frac{196}{4\cdot5}+\frac{196}{5}}=\sqrt{49}=\underline{\underline{7}}$ ✓

$v_2=\frac{14}{2\sqrt{5}}$

c) $u(2,-5)$ $|v|=21$

$2v_1-5v_2=0$ $5v_2=2v_1$, $v_2=\frac{2}{5}v_1$

$\sqrt{v_1^2+\frac{4}{25}v_1^2}=21$

$\frac{29}{25}v_1^2=441$ $v_2=\frac{210}{\sqrt{25\cdot29}}=\frac{210}{\sqrt{725}}$

$v_1=\frac{105}{\sqrt{29}}$ Sk: $\sqrt{\frac{105^2}{29}+\frac{210^2}{725}}=21$ ✓

PR 12. a) $-2\cdot6+3\cdot(-4)-3=-12-12-3=-27$ ×

b) $\frac{50}{6}+\frac{16}{3}+0-\frac{135}{2}=\frac{82}{6}-\frac{405}{6}$ ×

c) $\frac{8\cdot-37}{3}-48-108-\frac{8\cdot37}{3}-\frac{16}{3}$ × Žiadne vektory niesú rovnobežné

d) $0+0+\frac{8}{3}+6+\frac{50}{3}$

PR. 13. $\mathrm{Proj}_a b$ & $b - \mathrm{Proj}_a b$ $\qquad \mathrm{Proj}_a b = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\cdot\vec{a}$

$\mathrm{Proj}_b a$ & $a - \mathrm{Proj}_b a$

a) $a = (2,1,5)$ $b = (1,4,-5)$

$\vec{a}\cdot\vec{b} = 2 + 4 - 25 = -19$

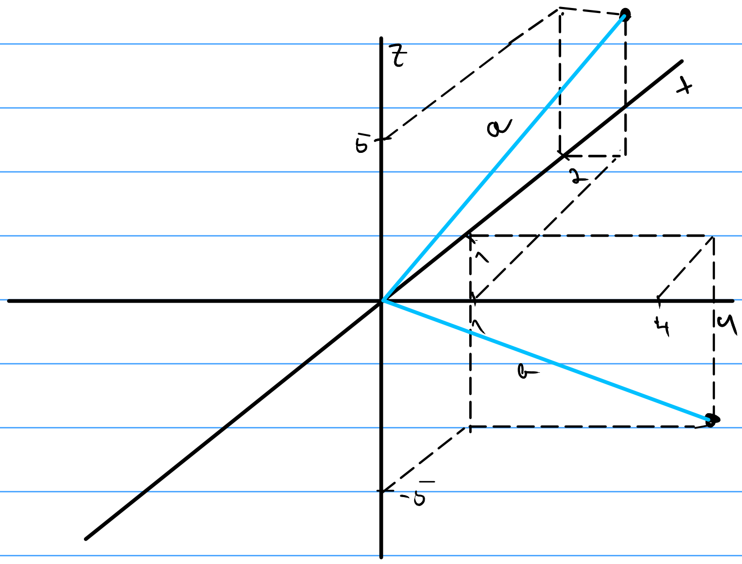
$\|\vec{a}\|^2 = 2^2 + 1 + 25 = 30$

$\mathrm{Proj}_a b = -\frac{19}{30}(2,1,5) = \left(\frac{-38}{30}, \frac{-19}{30}, \frac{-95}{30}\right) = \left(\frac{-19}{15}, \frac{-19}{30}, \frac{-19}{6}\right)$

$b - \mathrm{Proj}_a b = (1,4,-5) - \left(-\frac{19}{15}, \frac{-19}{30}, \frac{-19}{6}\right) = \left(\frac{34}{15}, \frac{139}{30}, \frac{-11}{6}\right) = \frac{68}{30} + \frac{139}{30} - \frac{55}{30} = \frac{152}{30}$

$\mathrm{Proj}_b a = \frac{-19}{42}(1,4,-5) = \left(\frac{-19}{42}, \frac{-76}{42}, \frac{95}{42}\right)$

$a - \mathrm{Proj}_b a = (2,1,5) - \left(\frac{-19}{42}, \frac{-76}{42}, \frac{95}{42}\right) = \left(\frac{103}{42}, \frac{108}{42}, \frac{115}{42}\right) = \underline{\underline{\frac{326}{42}}}$



b) $a = (6,2,4)$ $b = (1,-5,4)$

$\mathrm{Proj}_a b = \frac{\vec{a}\cdot\vec{b}}{|a|}\cdot\vec{a} = \frac{6-10+16}{36+4+16}\cdot(6,2,4) = \frac{6}{28}(6,2,4) = \left(\frac{36}{28}, \frac{3}{7}, \frac{6}{7}\right)$

$b - \mathrm{Proj}_a b = (1,-5,4) - \left(\frac{9}{7}, \frac{3}{7}, \frac{6}{7}\right) = -\frac{2}{7} - \frac{38}{7} + \frac{24}{7} = -\frac{16}{7}$

$\mathrm{Proj}_b a = \frac{12}{30}(1,-5,4) = \left(\frac{2}{5}, -2, \frac{8}{5}\right)$

$a - \mathrm{Proj}_b a = (6,2,4) - \left(\frac{2}{5}, -2, \frac{8}{5}\right) = \frac{28}{5} + 0 + \frac{12}{5} = \underline{\underline{8}}$

DÚ 1. část 2

$A = \begin{bmatrix} -4 & 2 & 3 \\ 0 & 5 & -1 \\ 6 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 6 & -1 & 0 \\ 2 & 2 & -4 \\ 3 & -1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 5 & -1 \\ -3 & 4 \end{bmatrix}$ $D = \begin{bmatrix} -7 & 1 & -4 \\ 3 & -2 & 8 \end{bmatrix}$ $E = \begin{bmatrix} 3 & -9 & 5 \\ 1 & 0 & -2 \\ 6 & 7 & -7 \end{bmatrix}$ $F = \begin{bmatrix} 8 & -1 \\ 2 & 0 \\ 5 & -3 \end{bmatrix}$

a) $= A + B = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 7 & -5 \\ 9 & 0 & -1 \end{bmatrix}$ c) $4A = \begin{bmatrix} -16 & 8 & 12 \\ 0 & 20 & -4 \\ 24 & 4 & -8 \end{bmatrix}$ d) $2A - 3B = \begin{bmatrix} -8 & 4 & 6 \\ 0 & 10 & -2 \\ 12 & 2 & -4 \end{bmatrix} - \begin{bmatrix} 18 & -3 & 0 \\ 6 & 6 & -12 \\ 9 & -3 & 3 \end{bmatrix} =$

$= \begin{bmatrix} -26 & 7 & 6 \\ -6 & 4 & 10 \\ 3 & 5 & -7 \end{bmatrix}$

$L)\ 2C^T - 3F = \begin{bmatrix}10&-6\\-2&8\end{bmatrix} - \begin{bmatrix}24&-3\\6&0\\15&-9\end{bmatrix} = NR$

$M)\ 5\cdot(F^T - D^T) = 5\cdot\left(\begin{bmatrix}8&2&5\\-1&0&-3\end{bmatrix} - \begin{bmatrix}-7&3\\1&-2\\-4&8\end{bmatrix}\right) = NR.$

$A = \begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix} \quad B = \begin{bmatrix}4&-1\\0&2\end{bmatrix} \quad C = \begin{bmatrix}1&4&2\\3&1&5\end{bmatrix} \quad D = \begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix} \quad E = \begin{bmatrix}6&1&3\\-1&1&2\\4&1&3\end{bmatrix}$

PR2. $tr(D) = tr\left(\begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix}\right) = 1+0+4 = \underline{5}$

PR3. $(CD)\cdot E = \begin{bmatrix}1&4&2\\3&1&5\end{bmatrix}\cdot\begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix} = \begin{bmatrix}1-4+6&5+0+4&2+4+8\\3-1+15&15+0+10&6+1+20\end{bmatrix} =$

$\begin{bmatrix}3&9&14\\17&25&27\end{bmatrix}\cdot\begin{bmatrix}6&1&3\\-1&1&2\\4&1&3\end{bmatrix} = \begin{bmatrix}18-9+56&3+9+14&9+18+42\\6\cdot17-25+4\cdot27&17+25+27&3\cdot17+50+81\end{bmatrix} = \begin{bmatrix}65&26&69\\185&69&182\end{bmatrix}$

$C(BA) = \begin{bmatrix}4&-1\\0&2\end{bmatrix}\cdot\begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix} = NR$

$tr(DE^T) = \begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix}\cdot\begin{bmatrix}6&-1&4\\1&1&1\\3&2&3\end{bmatrix} = \begin{bmatrix}6+5+6&-1+5+4&4+5+6\\-6+0+3&1+0+2&-4+0+3\\18+2+12&-3+2+8&12+2+12\end{bmatrix} = \begin{bmatrix}17&8&15\\-3&3&-1\\32&7&26\end{bmatrix}$

$- 17\cdot3\cdot26 = 17+3+26 = 46$

$3\times2 \cdot 3\times3$

$tr(BC) \qquad 2\times2 \cdot 3\times2$ ✓

$\begin{bmatrix}4&-1\\0&2\end{bmatrix}\cdot\begin{bmatrix}1&4&2\\3&1&5\end{bmatrix} = \begin{bmatrix}4-3&16-1&8-5\\6&2&10\end{bmatrix} = tr\left(\begin{bmatrix}1&15&3\\6&2&10\end{bmatrix}\right) = \underline{\underline{NR}}$

PR6.

$A(4\times5) \quad B(4\times5) \quad C(5\times2) \quad D(4\times2) \quad E(5\times4)$

a) $BA \quad 4\times5\cdot4\times5$ x

$A-3E^T \Rightarrow 4\times5-3\cdot(4\times5)$ ✓

$BC-3D \Rightarrow$ x

$AB^T \quad 4\times5\cdot5\times4$ ✓

$E(5B+A) \Rightarrow 5\times4\cdot4\times5$ ✓

$D^T(BE)\ (2\times4)\cdot((4\times5)\cdot(5\times4))$ ✓

$(2\times4)(4\times5)$ x

$AC+D \Rightarrow$ x

$CD^T \Rightarrow 5\times2\cdot2\times4$ x

$E(AC)$ x

$DC \Rightarrow 4\times2\cdot5\times2$ x

$B^TD + ED \Rightarrow (5\times4)(4\times2)$ x

$BA^T + D \Rightarrow ((4\times5)(5\times4)) + (4\times2)$ x

PR 7.   A, B, C, D, E, F, G, H, I, J, K, L, M, N, P, Q, R

Štvorcové ⇒ B, C, Q   Horní trojuholn ⇒ L

Identická ⇒ G, N, P

Obyčajná ⇒ A, D, R

Diagon ⇒ E, N

Binárna ⇒ F, J, M

Anti symetrická ⇒ K, H

2x3: $\mathbf{A}=\begin{bmatrix}-2 & 3\\ 6 & 5\\ 1 & -4\end{bmatrix}$ 3x3: $\mathbf{B}=\begin{bmatrix}-5 & 3 & 6\\ 3 & 8 & 0\\ -2 & 0 & 4\end{bmatrix}$ 2x3: $\mathbf{C}=\begin{bmatrix}11 & -2\\ -4 & -2\\ 3 & -1\end{bmatrix}$

4x3: $\mathbf{D}=\begin{bmatrix}-1 & 4 & 3 & 7\\ 2 & 1 & 7 & 5\\ 0 & 5 & 5 & -2\end{bmatrix}$ 4x4: $\mathbf{E}=\begin{bmatrix}1 & 1 & 0 & 1\\ 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 1 & 0\end{bmatrix}$ 2x4: $\mathbf{F}=\begin{bmatrix}9 & -3\\ 5 & -4\\ 2 & 0\\ 8 & -3\end{bmatrix}$

3x3: $\mathbf{G}=\begin{bmatrix}5 & 1 & 0\\ 0 & -2 & -1\\ 1 & 0 & 3\end{bmatrix}$ 3x3: $\mathbf{H}=\begin{bmatrix}6 & 3 & 1\\ 1 & -15 & -5\\ -2 & -1 & 10\end{bmatrix}$ 1x3: $\mathbf{J}=\begin{bmatrix}8\\ -1\\ 4\end{bmatrix}$

3x2: $\mathbf{K}=\begin{bmatrix}2 & 1 & -5\\ 0 & 2 & 7\end{bmatrix}$ 2x2: $\mathbf{L}=\begin{bmatrix}10 & 9\\ 8 & 7\end{bmatrix}$ 2x2: $\mathbf{M}=\begin{bmatrix}7 & -1\\ 11 & 3\end{bmatrix}$

2x2: $\mathbf{N}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ 2x2: $\mathbf{P}=\begin{bmatrix}3 & -1\\ 4 & 7\end{bmatrix}$ 4x2: $\mathbf{Q}=\begin{bmatrix}1 & 4 & -1 & 6\\ 8 & 7 & -3 & 3\end{bmatrix}$

3x1: $\mathbf{R}=\begin{bmatrix}-3 & 6 & -2\end{bmatrix}$ 4x1: $\mathbf{S}=\begin{bmatrix}6 & -4 & 3 & 2\end{bmatrix}$ 3x1: $\mathbf{T}=\begin{bmatrix}4 & -1 & 7\end{bmatrix}$

a) **AB** ✗
b) **BA** ✓
c) **JM** ✗
d) **DF** ✓
e) **RJ** ✓
f) **JR** ✓
g) **RT** ✗
h) **SF** ✓
i) **KN** ✗
j) $\mathbf{F}^2$ ✗
k) $\mathbf{B}^2$ ✓
l) $\mathbf{E}^3$ ✓
m) $(\mathbf{TJ})^3$ ✗
n) $\mathbf{D}(\mathbf{FK})$ ✓
o) $(\mathbf{CL})\mathbf{G}$ ✗

8. Ktoré z týchto súčinov matíc je možné vykonať? Zdôvodnite. Ak je to možné, vypočítajte ich súčin.

a) **AB** ✗
b) **BA**
c) **JM** ✗
d) **DF**
e) **RJ**
f) **JR**
g) **RT** ✗
h) **SF**
i) **KN**
j) $\mathbf{F}^2$
k) $\mathbf{B}^2$
l) $\mathbf{E}^3$
m) $(\mathbf{TJ})^3$
n) $\mathbf{D}(\mathbf{FK})$
o) $(\mathbf{CL})\mathbf{G}$

b) $$\begin{bmatrix}-5 & 3 & 6\\ 3 & 8 & 0\\ -2 & 0 & 4\end{bmatrix}\cdot\begin{bmatrix}-2 & 3\\ 6 & 5\\ 1 & -4\end{bmatrix}=\begin{bmatrix}10+18+6 & -15+15-24\\ -6+48 & 9+40\\ 4+4 & -6-16\end{bmatrix}=\begin{bmatrix}34 & -24\\ 42 & 49\\ 8 & -22\end{bmatrix}$$

d) $$\begin{bmatrix}-1 & 4 & 3 & 7\\ 2 & 1 & 7 & 5\\ 0 & 5 & 5 & -2\end{bmatrix}\cdot\begin{bmatrix}9 & -3\\ 5 & -4\\ 2 & 0\\ 8 & -3\end{bmatrix}=\begin{bmatrix}-9+20+6+56 & 3-16+0-21\\ 18+5+14+40 & -6-4-15\\ 25+10-16 & -20+0+6\end{bmatrix}=\begin{bmatrix}73 & -34\\ 77 & -25\\ 19 & -14\end{bmatrix}$$

e) $$\begin{bmatrix}-3 & 6 & -2\end{bmatrix}\cdot\begin{bmatrix}8\\ -1\\ 4\end{bmatrix}=-24-6-8=\underline{-38}\ ✓$$

f) $$\begin{bmatrix}8\\ -1\\ 4\end{bmatrix}\cdot\begin{bmatrix}-3 & 6 & -2\end{bmatrix}=\begin{bmatrix}-24 & 48 & -16\\ 3 & -6 & 2\\ -12 & 24 & -8\end{bmatrix}$$

1x3 · 3x1

h) $$\begin{bmatrix}6 & -4 & 3 & 2\end{bmatrix}\cdot\begin{bmatrix}9 & -3\\ 5 & -4\\ 2 & 0\\ 8 & -3\end{bmatrix}=\begin{bmatrix}54-20+6+16 & -18+16+0-6\end{bmatrix}=\begin{bmatrix}56 & -8\end{bmatrix}$$

4x1 · 2x4

DÚ 2. Propičítanie

3.) a)
$$\begin{aligned} -2x_1 &= -6 \\ 3x_1 &= 8 \\ 9x_1 &= -3 \end{aligned} \qquad \left[\begin{array}{c|c} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{array}\right]$$

e)
$$\begin{aligned} 2x_1 + 2x_3 &= 1 \\ 3x_1 - x_2 + 4x_3 &= 7 \\ 6x_1 - x_2 - x_3 &= 0 \end{aligned} \qquad \left[\begin{array}{ccc|c} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & -1 & -1 & 0 \end{array}\right]$$

b)
$$\begin{aligned} 6x_1 - x_2 + 3x_3 &= 4 \\ 5x_2 - x_3 &= 1 \\ 2x_2 - 3x_4 + x_5 &= 0 \end{aligned} \qquad \left[\begin{array}{ccccc|c} 6 & -1 & 3 & 0 & 0 & 4 \\ 0 & 5 & -1 & 0 & 0 & 1 \\ 0 & 2 & 0 & -3 & 1 & 0 \end{array}\right]$$

c)
$$\begin{aligned} -3x_1 - x_2 + x_3 &= -1 \\ 6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 &= 6 \end{aligned} \qquad \left[\begin{array}{ccccc|c} -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{array}\right]$$

d)
$$\begin{aligned} 3x_1 - 2x_2 &= -1 \\ 4x_1 + 5x_2 &= 3 \\ 7x_1 + 3x_2 &= 2 \end{aligned} \qquad \left[\begin{array}{cc|c} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{array}\right]$$

f)
$$\left[\begin{array}{c|c} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{array}\right]$$

$$\begin{aligned} 2x_1 - 4x_2 - x_3 &= 1 \\ x_1 - 3x_2 + x_3 &= 1 \\ 3x_1 - 5x_2 - 3x_3 &= 1 \end{aligned} \qquad \left[\begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 2 & -4 & -1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right] \begin{array}{l} \\ -2R_1 \\ -3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 1 & -5 & -1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -5 & -1 \\ 0 & 2 & -3 & -1 \end{array}\right] -2R_2+R_3 \qquad \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 7 & 1 \end{array}\right] \cdot \frac{1}{7}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right] 5R_3+R_2 \qquad \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & 0 & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right] 3R_2+R_1,\ -R_3+R_1$$

$\Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1-\frac{6}{7}-\frac{1}{7} \\ 0 & 1 & 0 & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right] \Rightarrow \begin{array}{l} x=0 \\ y=-\frac{2}{7} \\ z=\frac{1}{7} \end{array}$$

Opýtať sa ako to treba riešiť ?

5.

a) $3x-2y=4$, $6x-4y=9$ $\left[\begin{array}{cc|c} 3 & -2 & 4 \\ 6 & -4 & 9 \end{array}\right] -2R_1+R_2 \left[\begin{array}{cc|c} 3 & -2 & 4 \\ 0 & 0 & 1 \end{array}\right]$ N/R.

b) Nekonečne veľa

c) $\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & -4 & 8 \end{array}\right] -R_1+R_2 \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & -2 & 8 \end{array}\right] \cdot -\frac{1}{2} \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 1 & -4 \end{array}\right] 2R_2+R_1 \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -4 \end{array}\right]$

$x=-8$ $y=-4$ sk $x-2y=0$ $\underline{\underline{0=0}}$ ✓

$-8-2\cdot(-4)=0$

6.

a) $7x-5y=3$

$x=t$ $-5y=3-7t \;/\cdot\frac{1}{-5}$

$\underline{\underline{y=-\frac{1}{5}(3-7t)}}$

Riešenie: $\left(t, \frac{1}{5}(7t-3)\right)$

b) $3x_1-5x_2+4x_3=7$

$x_1=t$, $x_2=r$

$4x_3=7-3t+5r$

$x_3=\frac{1}{4}(7-3t+5r)$

riešenie: $\left(t, r, \frac{1}{4}(7-3t+5r)\right)$

c) $-8x_1+2x_2-5x_3+6x_4=1$

$x_1=r$, $x_2=S$, $x_3=t$

$x_4=\frac{1}{6}(1+8r-2s+5t)$

Riešenie $\left(r, s, t, \frac{1}{6}(1+8r-2s+5t)\right)$

d) $3v-8w+2x-y+4z=0$

$v=r$, $w=s$, $x=t$, $y=m$

$z=\frac{1}{4}(8s-3r-2t+m)$

$\left(r, s, t, m, \frac{1}{4}(8s-3r-2t+m)\right)$

e) $x+10y=3$

$x=t$

$y=\frac{1}{10}(3-t)$

$\left(t, \frac{1}{10}(3-t)\right)$

f) $x_1-5x_2+2x_3=-1$

$x_1=r$, $x_2=s$

$x_3=\frac{1}{2}(5s-r-1)$

$\left(r, S, \frac{1}{2}(5s-r-1)\right)$

g) $4x_1+2x_2-3x_3-x_4=2$

$x_4=(4r+2s-3t-2)$

$(r, S, t, 4r+2s-3t-2)$

h) $v+w+x-5y+7z=0$

$z=\frac{1}{7}(5Q-r-s-t)$

$\left(r, s, t, Q, \frac{1}{7}(5Q-r-s-t)\right)$

7. a) $2x - 3y = 1$

$x = r$

$y = \frac{1}{3}(2r - 1)$

$(r, (2r-1))$

b) $-x_1 - 3x_2 + x_3 = 4$

$x_3 = 4 + r + 3t$

$(r, t, (4+r+3t))$

c) $3x_1 + x_2 = -4$

$x_2 = -4 - 3t$

$(t, (-3t-4))$

d) $2\overset{r}{x} - \overset{s}{y} + 2z = -6$

$z = \frac{1}{2}(s - 6 - 2r)$

$(r, s, \frac{1}{2}(s-6-2r))$

8. a) $2R_2 + R_1$ c) $\frac{1}{2}R_1$

b) $R_1 \leftrightarrow R_3$ d) $R_3 + R_1$

10. a)

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} 3R_2+R_1 \\ -R_3+R_2 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 13 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] -10R_3+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$x_1 = -37$

$x_2 = -3$

$x_3 = 5$

b)

$$\left[\begin{array}{cccc|c} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} -8R_3+R_1 \\ -4R_3+R_2 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -13 & 6 \\ 0 & 1 & 0 & -13 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$$

$x_1 - 13x_4 = 6$ $x_1 = r, x_2 = s, x_3 = t$

$x_2 - 13x_4 = 3$ $(r, s, t, (2+3t))$

$x_3 + x_4 = 2$

$x_4 = 2 + 3t$

d)

$$\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} 3R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 19 & 1 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

$x_1 + 19x_3 = 1$

$x_2 + 4x_3 = 0$

$x_1 = r, x_2 = s$

$(r, s, (1-19r))$

c)

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix} \\ \begin{array}{cccccc} r & s & t & u & v & \end{array} \end{array}$$

(12.)

a)
$$2x_1 - 3x_2 + 4x_3 - x_4 = 0$$
$$7x_1 + x_2 - 8x_3 + 9x_4 = 0$$
$$2x_1 + 8x_2 + x_3 - x_4 = 0$$

$$x_4 = (2r - 3s + 4t)$$

$$(r, s, t, (2r - 3s + 4t))$$

OPýtať sa na Ácho.

b)
$$\begin{array}{c}\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 1 & -8 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \\ \begin{array}{ccc} r & s & t \end{array}\end{array}$$

$$t = 0$$

$$s - 8t = 0$$

$$x_1 + 3s - t = 0 \Rightarrow x_1 = t - 3s$$

$$(t, s, (t - 3s))$$

(13.)
$$\left[\begin{array}{cc|c} 2 & -3 & 5 \\ -1 & 1 & -3 \end{array}\right] 2R_2 + R_1 \Rightarrow \left[\begin{array}{cc|c} 0 & -1 & -1 \\ -1 & 1 & -3 \end{array}\right] R_1 + R_2$$

$$\left[\begin{array}{cc|c} 0 & 1 & 1 \\ -1 & 0 & -4 \end{array}\right] \cdot (-1),\ R_1 \Leftrightarrow R_2 \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{l} x_1 = 4 \\ x_2 = 1 \end{array}$$

sk: $2x - 3y = 2 \cdot 4 - 3 =$
$8 - 3 = \underline{\underline{5}}$ ✓

c)
$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 4 \\ 1 & 4 & 1 & 2 \\ 4 & 1 & -1 & 1 \end{array}\right] \begin{array}{l} -2R_2 + R_1 \\ \\ -4R_2 + R_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 0 & -8 & -3 & 0 \\ 1 & 4 & 1 & 2 \\ 0 & -15 & -5 & -7 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & -8 & -3 & 0 \\ 0 & -15 & -5 & -7 \end{array}\right] -2R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & -8 & -3 & 0 \\ 0 & 1 & 1 & -7 \end{array}\right] 8R_3 + R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 0 & 5 & -56 \\ 0 & 1 & 1 & -7 \end{array}\right] \left(\cdot \frac{1}{5}\right) \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & 1 & -7 \\ 0 & 0 & 1 & -\frac{56}{5} \end{array}\right] -R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & 0 & \frac{21}{5} \\ 0 & 0 & 1 & -\frac{56}{5} \end{array}\right] -4R_2 + R_1,\ -R_3 + R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 - \frac{84}{5} + \frac{56}{5} \\ 0 & 1 & 0 & \frac{21}{5} \\ 0 & 0 & 1 & -\frac{56}{5} \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{18}{5} \\ 0 & 1 & 0 & \frac{21}{5} \\ 0 & 0 & 1 & -\frac{56}{5} \end{array}\right]$$

$$x_1 = \frac{-18}{5},\ x_2 = \frac{21}{5},\ x_3 = -\frac{56}{5}$$

MAT

d) $\begin{aligned} -3x + y + z &= 2 \\ -4z &= 0 \\ -4x + 2y - 3z &= 1 \end{aligned}$

$$\left[\begin{array}{ccc|c} -3 & 1 & 1 & 2 \\ 0 & 0 & -4 & 0 \\ -4 & 2 & -3 & 1 \end{array}\right] \left[\begin{array}{ccc|c} -4 & 2 & -3 & 1 \\ -3 & 1 & 1 & 2 \\ 0 & 0 & -4 & 0 \end{array}\right] \begin{array}{l} -R_2+R_1 \\ \\ \cdot\left(-\frac{1}{4}\right) \end{array} \Rightarrow \left[\begin{array}{ccc|c} -1 & 1 & -4 & -1 \\ -3 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \cdot(-1) \\ -3R_1+R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & -2 & 13 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} x = -\frac{3}{2} \\ y = -\frac{5}{2} \\ z = 0 \end{array}$$

e) $\begin{aligned} 2x_1 - x_3 &= 4 \\ x_1 + 4x_2 + x_3 &= 2 \end{aligned}$

Riešenie $\Rightarrow (x_1, x_2, x_3) = \left(\frac{1}{2}(4+x_3), -\frac{3}{8}x_3, x_3\right)$

$x_1 = \frac{1}{2}(4+x_3)$ $\qquad 4x_2 = -\frac{3}{2}x_3$

$\cancel{2 + \frac{1}{2}x_3 + 4x_2 + x_3 = 2}$ $\qquad x_2 = -\frac{3}{8}x_3$

f) $\begin{aligned} 4x_1 + x_2 - 4x_3 &= 1 \\ 4x_1 - 4x_2 + 2x_3 &= -2 \end{aligned}$

$$\left[\begin{array}{ccc|c} 4 & 1 & -4 & 1 \\ 4 & -4 & 2 & -2 \end{array}\right] -R_1+R_2 \Rightarrow \left[\begin{array}{ccc|c} 4 & 1 & -4 & 1 \\ 0 & -5 & 6 & -3 \end{array}\right] \begin{array}{l} \cdot\frac{1}{4} \\ \cdot -\frac{1}{5} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & -1 & \frac{1}{4} \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \end{array}\right] -\frac{1}{4}R_2+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{14}{20} & \frac{2}{20} \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \end{array}\right]$$

$x_1 + \frac{7}{10}x_3 = \frac{1}{10} \Rightarrow x_1 = \frac{1}{10}(1 - 7x_3)$

$x_2 - \frac{6}{5}x_3 = \frac{3}{5} \Rightarrow x_2 = \frac{3}{5}(1 + 2x_3)$

Riešenie: $(x_1, x_2, x_3) = \left(\frac{1}{10}(1-7x_3), \frac{3}{5}(1+2x_3), x_3\right)$

g) $\begin{aligned} 2x_1 + 4x_2 + 2x_3 + 2x_4 &= -2 \\ 4x_1 - 2x_2 - 3x_3 - 2x_4 &= 2 \\ x_1 + 3x_2 + 3x_3 - 3x_4 &= -4 \end{aligned}$

$$\left[\begin{array}{cccc|c} 2 & 4 & 2 & 2 & -2 \\ 4 & -2 & -3 & -2 & 2 \\ 1 & 3 & 3 & -3 & -4 \end{array}\right] \begin{array}{l} \cdot\frac{1}{2} \\ -2R_1+R_2 \\ -\frac{1}{2}R_1+R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & -1 \\ 0 & -10 & 7 & -6 & 6 \\ 0 & 1 & \frac{9}{2} & -2 & -5 \end{array}\right] \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & -1 \\ 0 & 1 & \frac{9}{2} & -2 & -5 \\ 0 & -10 & -7 & -6 & 6 \end{array}\right] 10R_2+R_3 \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & -1 \\ 0 & 1 & \frac{9}{2} & -2 & -5 \\ 0 & 0 & 38 & -26 & -44 \end{array}\right] \begin{array}{l} -R_2+R_1 \\ \\ \cdot\frac{1}{38} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{7}{2} & 3 & 4 \\ 0 & 1 & \frac{9}{2} & -2 & -5 \\ 0 & 0 & 1 & -\frac{26}{38} & -\frac{44}{38} \end{array}\right] \begin{array}{l} \frac{7}{2}R_3+R_1 \\ -\frac{9}{2}R_3+R_2 \Rightarrow \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{63}{38} & -\frac{1}{19} \\ 0 & 1 & 0 & \frac{41}{38} & \frac{4}{19} \\ 0 & 0 & 1 & -\frac{26}{38} & -\frac{44}{38} \end{array}\right]$$

$x_1 + \frac{63}{38}x_4 = -\frac{1}{19}$

$x_2 + \frac{41}{38}x_4 = \frac{4}{19}$

$x_3 - \frac{26}{38}x_4 = -\frac{44}{38}$

Riešenie $(x_1; x_2, x_3, x_4) = \left(-\frac{1}{19} - \frac{63}{38}x_4;\ \frac{4}{19} - \frac{41}{38}x_4;\ \frac{26}{38}x_4 - \frac{44}{38};\ x_4\right)$ X

h) $3x_1 - 3x_2 + 4x_4 = -3$

$-4x_1 + 2x_2 - 2x_3 - 4x_4 = 4$

$4x_2 - 3x_3 + 2x_4 = -3$

$$\left[\begin{array}{cccc|c} 3 & -3 & 0 & 4 & -3 \\ -4 & 2 & -2 & -4 & 4 \\ 0 & 4 & -3 & 2 & -3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} -4 & 2 & -2 & -4 & 4 \\ 3 & -3 & 0 & 4 & -3 \\ 0 & 4 & -3 & 2 & -3 \end{array}\right] R_2 + R_1 \quad \left[\begin{array}{cccc|c} -1 & -1 & -2 & 0 & 1 \\ 3 & -3 & 0 & 4 & -3 \\ 0 & 4 & -3 & 2 & -3 \end{array}\right] 3R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & -6 & -6 & 4 & 0 \\ 0 & 4 & -3 & 2 & -3 \end{array}\right] \cdot \frac{1}{2} \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & -3 & -3 & 2 & 0 \\ 0 & 4 & -3 & 2 & -3 \end{array}\right] R_2 + R_3 \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & -3 & -3 & 2 & 0 \\ 0 & 1 & -6 & 4 & -3 \end{array}\right] 3R_3 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & 0 & -21 & 14 & -9 \\ 0 & 1 & -6 & 4 & -3 \end{array}\right] \cdot -\frac{1}{21} \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 0 & 1 & -6 & 4 & -3 \\ 0 & 0 & 1 & -\frac{14}{21} & \frac{3}{7} \end{array}\right] \begin{array}{l} -R_2 + R_1 \\ 6R_3 + R_2 \Rightarrow \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 8 & -4 & 4 \\ 0 & 1 & 0 & 0 & -\frac{3}{7} \\ 0 & 0 & 1 & -\frac{14}{21} & \frac{3}{7} \end{array}\right]$$

$x_1 + 8x_3 - 4x_4 = 4 \qquad x_1 = 4 + 4x_4 - 8x_3$

$x_2 = -\frac{3}{7} \qquad x_3 = \frac{3}{7} + \frac{14}{21}x_4$

$x_3 - \frac{14}{21}x_4 = \frac{3}{7}$ Riešenie: $(x_1, x_2, x_3, x_4) = \left(4 + 4x_4 - 8x_3, -\frac{3}{7}, \frac{3}{7} + \frac{14}{21}x_4, x_4\right)$ X

15.) $x_1 + 2x_2 = -1 \qquad x_1 = -2x_2 - 1$

$x_3 = -2$

Riešenie $(x_1, x_2, x_3) = (-2x_2 - 1 ; x_2 ; -2)$

$$Proj_a b = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\cdot\vec{a}$$

Skúšobný test Štart 20:35

1.) a) $4A^T\cdot C = \begin{bmatrix}4&0&4&0\\-8&4&0&0\\-8&-8&12&8\\4&12&4&-4\end{bmatrix}\cdot\begin{bmatrix}1&2&3&-1\\0&1&-1&3\\2&1&2&-1\\0&0&2&-4\end{bmatrix} = \begin{bmatrix}4&0&8&0\\-16&4&0&0\\-24&8&24&16\\-4&36&-4&16\end{bmatrix} - \begin{bmatrix}2&10&-4&2\\-2&2&4&-6\\2&4&4&2\\0&0&4&-8\end{bmatrix} = \begin{bmatrix}2&-10&12&-2\\-14&2&-4&6\\-26&4&20&14\\-4&36&-8&24\end{bmatrix}$

b) Platí $B(A+C) = BA+BC$ pretože keď roznásobíme ľavú stranu dostaneme $BA+BC$

2) $a=[2,-1,5]$ $b=[1,2,-5]$ $Proj_a b \quad \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\cdot\vec{a}$

$\frac{2-2-25}{30}\cdot[2,-1,5] = \frac{-25}{30}\cdot[2,-1,5] = \frac{-25}{15}+\frac{25}{30}-\frac{25}{6} = \frac{-25-125}{30} = -5$

$[2,-1,5]+\frac{25}{30} = \left[\frac{75}{30};\frac{-25}{30};\frac{125}{30}\right]$

$Proj_b a = \frac{-25}{30}$

$b - Proj_b a = [1,2,-5]+\frac{25}{30} = \left[\frac{55}{30};\frac{85}{30};-\frac{10}{3}\right]$

3.) $\begin{bmatrix}5&-5&3&-5\\-4&2&0&-1\\0&2&0&2\\1&0&4&-2\end{bmatrix}\begin{bmatrix}-9\\1\\8\\7\end{bmatrix}$ $R_1 \leftrightarrow R_4$ $\left[\begin{array}{cccc|c}1&0&4&-2&7\\-4&2&0&-1&1\\0&2&0&2&8\\5&-5&3&-5&-9\end{array}\right]\begin{array}{l}\\4R_1+R_2\\\\-5R_1+R_4\end{array} \Rightarrow$

$\left[\begin{array}{cccc|c}1&0&4&-2&7\\0&2&16&-9&29\\0&2&0&2&8\\0&-5&-17&5&-44\end{array}\right]\begin{array}{l}\\\\\frac{1}{2}R_3\\2R_2+R_4\end{array} \Rightarrow \left[\begin{array}{cccc|c}1&0&4&-2&7\\0&2&16&-9&29\\0&1&0&1&4\\0&-1&15&-13&14\end{array}\right]\begin{array}{l}\\-2R_3+R_2\\\\R_3+R_4\end{array} \Rightarrow$

$\left[\begin{array}{cccc|c}1&0&4&-2&7\\0&0&16&-11&21\\0&1&0&1&4\\0&0&15&-12&18\end{array}\right]\begin{array}{l}\\-R_4+R_2\\\\\cdot\frac{1}{3}\end{array} \Rightarrow \left[\begin{array}{cccc|c}1&0&4&-2&7\\0&0&1&1&3\\0&1&0&1&4\\0&0&5&-4&6\end{array}\right]\begin{array}{l}\\\\\\-5R_2+R_4\end{array} \Rightarrow \left[\begin{array}{cccc|c}1&0&4&-2&7\\0&0&1&1&3\\0&1&0&1&4\\0&0&0&-9&-9\end{array}\right]\begin{array}{l}\\R_2\leftrightarrow R_3\\\\\cdot\left(-\frac{1}{9}\right)\end{array}$

$\left[\begin{array}{cccc|c}1&0&4&-2&7\\0&1&0&1&4\\0&0&1&1&3\\0&0&0&1&1\end{array}\right]\begin{array}{l}2R_4+R_1\\-R_4+R_2\\-R_4+R_3\\\end{array} \quad \left[\begin{array}{cccc|c}1&0&4&0&9\\0&1&0&0&3\\0&0&1&0&2\\0&0&0&1&1\end{array}\right]\begin{array}{l}-4R_3+R_1\\\\\\\end{array} \quad \left[\begin{array}{cccc|c}1&0&0&0&1\\0&1&0&0&3\\0&0&1&0&2\\0&0&0&1&1\end{array}\right]$

$x_1 = 1,\ x_2 = 3,\ x_3 = 2,\ x_4 = 1$

$5x_1 - 5x_2 + 3x_3 - 5x_4 = -9$

$-4x_1 + 2x_2 - x_4 = 1$

$2x_2 + 2x_4 = 8 \quad x_2 = 4 - x_4$

$x_1 + 4x_3 - 2x_4 = 7$

$5x_1 + \cancel{5x_4} + 3x_3 - \cancel{5x_4} = 11$

$-4x_1 - 2x_4 - x_4 = -7$

$x_1 + 4x_3 - 2x_4 = 7 \Rightarrow x_1 = 7 + 2x_4 - 4x_3$

$35 + 10x_4 - 20x_3 + 3x_3 = 11$

$-28 - 8x_4 + 16x_3 - 3x_4 = -7$

$10x_4 - 17x_3 = -24 \Rightarrow x_4 = -\frac{24}{10} + \frac{17}{10}x_3$

$-11x_4 + 16x_3 = 21$

$-11\left(\frac{17}{10}x_3 - \frac{24}{10}\right) + 16x_3 = 21$

$-\frac{187}{10}x_3 + \frac{264}{10} + \frac{160}{10}x_3 = \frac{210}{10} \quad | \cdot 10$

$-27x_3 = -54$

$x_3 = 2 \qquad x_4 = -\frac{24}{10} + \frac{17 \cdot 2}{10} = \frac{10}{10} = 1$

$x_2 = 4 - 1 = 3 \qquad x_1 = 7 + 2 \cdot (1) - 4 \cdot 2 = 7 + 2 - 8 = 1$

21:15 Pauza 21:20 Počet!!

PR 4.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ p & q & r & s & & \end{array}$$

$p =$

$3x_5 = 9 - s \qquad x_5 = \frac{1}{3}(9 - s)$

$6x_5 = 5 - r - s \qquad x_5 = \frac{1}{6}(5 - r - s)$

$-8x_5 = -3 + 2r - 7q - p \qquad x_5 = -\frac{1}{8}(-3 + 2r - 7q - p)$

Rješenje: $(p, q, r, s, x_5) = \left(-3 + 2r - 7q + 8x_5,\ \frac{1}{7}(2r - 3 - p + 8x_5),\ 5 - s - 6x_5,\ 9 - 3x_5,\ x_5\right)$

6.

$$\left[\begin{array}{ccccc|cccc} -1 & 1 & 1 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 1 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$\det(A) = 1 \cdot (-1) \cdot (-1) \cdot 1 \cdot 1 = \underline{1}$?

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_3 + R_4 \\ R_1 + R_5 \end{array} \Rightarrow \begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} \begin{array}{l} R_2 + R_1 \\ R_3 + R_2 \\ \\ \\ 2R_3 + R_5 \end{array} \Rightarrow$$

$$\begin{matrix} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \end{matrix}$$

lepše minora a cofactom

$1 \cdot 1 \cdot (-1) \cdot (-1) = \underline{\underline{0}}$

$\boxed{\text{PR 8.}}$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} +\frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{2}}{2} & 0 \\ 0 & -\frac{3\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{3\sqrt{2}}{2} & 0 \\ 0 & -3\sqrt{2} \end{bmatrix}$$

$\theta\ 60$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{8} \\ 0 & \frac{1}{8} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{5}{8} \\ 0 & -\frac{1}{4} & 0 \end{bmatrix}$$

21:45 koniec   Celkovo 55 min

18.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 2 & 3 & -2 & b \\ -1 & -1 & 1 & c \end{array}\right] \begin{array}{l} 1.)\ 2R_3+R_2 \\ 2.)\ R_1+R_3 \end{array} \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 0 & 1 & 0 & b+2c \\ 0 & 1 & 0 & c+a \end{array}\right] -2R_3+R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2c-a \\ 0 & 1 & 0 & b+2c \\ 0 & 1 & 0 & c+a \end{array}\right]$$

$x_1 - x_3 = -2c - a$

$x_2 = b + 2c \Rightarrow 2c = x_2 - b$

$x_2 = c + a \Rightarrow a = x_2 - c$

$x_1 - x_3 = -x_2 + b - x_2 + c$

$x_1 + 2x_2 - x_3 = b + c$

$a = b + c$ ak $a = 1$

$b = a - c$ $\quad 1 = b + c$

$c = a - b$ $\quad b = 1 - c$

$c = 1 - b$

$2x_1 - 2x_3 = -2x_2$

$x_1 - x_3 = -x_2$

$x_2 = x_3 - x_1$

$x_1 + 2x_2 - x_3 = a$

$x_1 + 2(x_3 - x_1) \overset{-x_3}{=} a$

$\cancel{-x_1} - x_3 \cancel{+x_1} + x_3 = c$

$2x_1 + 3\cdot(x_3 - x_1) - 2x_3 = b$

$2x_1 + 3x_3 - 3x_1 - 2x_3 = b$

$-x_1 + x_3 = b$

$\left.\begin{array}{l} -x_1 + x_3 = a \\ -x_1 + x_3 = b \end{array}\right\} a = b$

$c = 0$

$x_1 + 2x_2 - x_3 = 2x_1 + 3x_2 - 2x_3$

$-x_3 - x_2 + x_3 = 0$

$a - b = c$ $\quad c = 0$ $\quad a, b = 1$

$a = b + c$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & -2 & 1 \\ -1 & -1 & 1 & 0 \end{bmatrix} \begin{array}{l} -2R_1 + R_2 \\ R_1 + R_3 \end{array} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{array}\right] 2R_2+R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

$x_1 - x_3 = -1$ ak $x_3 = 0$ tak $x_1 = -1$

$x_2 = 1$