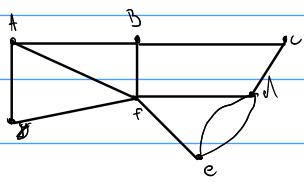
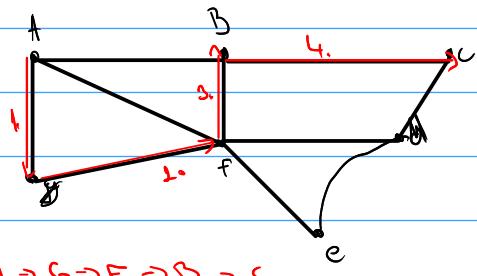


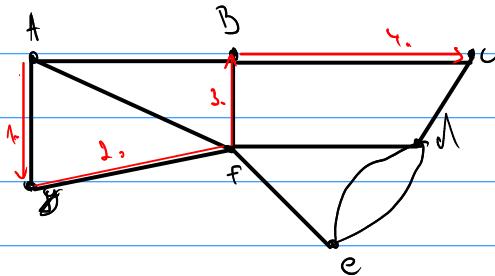
ADM 2. Übung 9.



Trail $\geq A \rightarrow C$

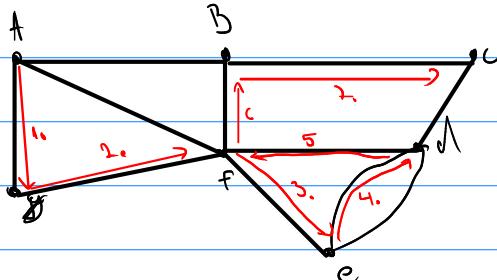


Path $\geq A \rightarrow C$



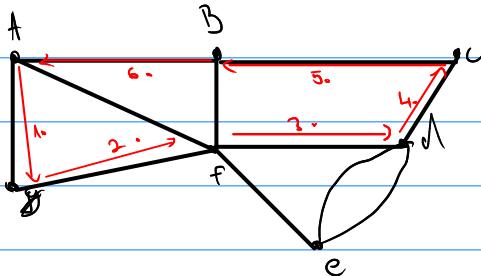
$A \rightarrow G \rightarrow F \rightarrow B \rightarrow C$

Walk $\geq A \rightarrow C$

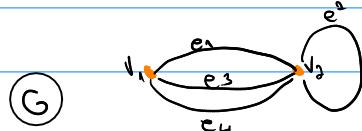


$A \rightarrow G \rightarrow F \rightarrow E \rightarrow D \rightarrow F \rightarrow B \rightarrow C$

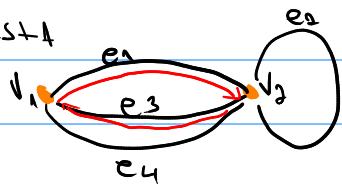
cycle $\geq A \rightarrow C$



$A \rightarrow G \rightarrow F \rightarrow D \rightarrow C \rightarrow B \rightarrow A$

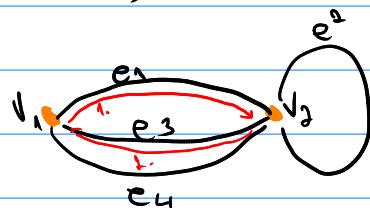


Circuit



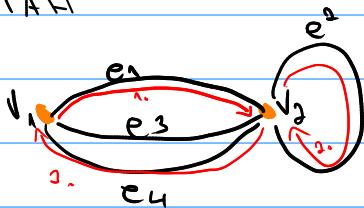
$v_1 \rightarrow e_1 \rightarrow v_2 \rightarrow e_2 \rightarrow v_1$

Circuit \Rightarrow (VZURRTEK PATH)



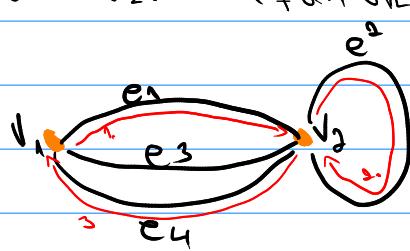
$v_1 \rightarrow e_1 \rightarrow v_2 \rightarrow e_2 \rightarrow v_1$

Trail

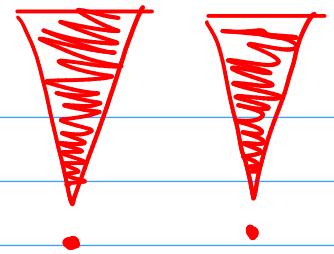
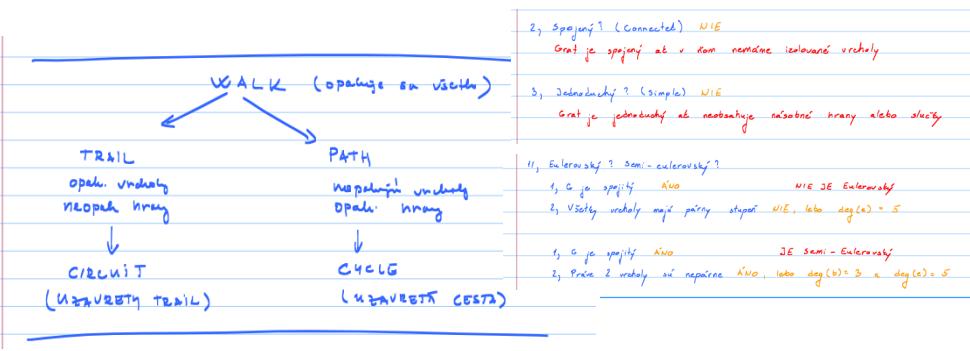


$v_1 \rightarrow e_1 \rightarrow v_2 \rightarrow e_2 \rightarrow v_1$

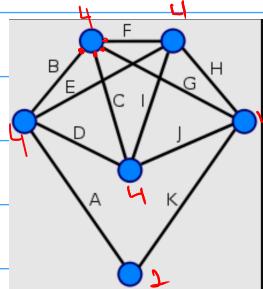
Circuit - VZURRTEK fahrt ohne



$v_1 \rightarrow e_1 \rightarrow v_2 \rightarrow e_2 \rightarrow v_1$



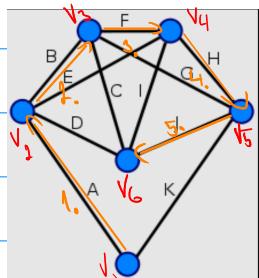
Eulerovské grafovy



→ graf je spojiteľný

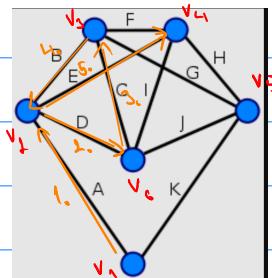
→ všetky vrcholy majú parne stupne } eulerovský graf

Path (cesta) dĺžka 5



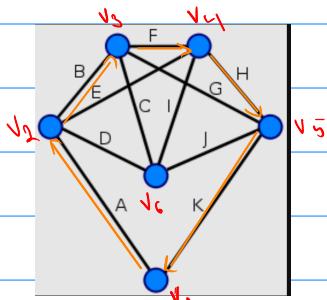
$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6$

Trail (čah) dĺžka 5



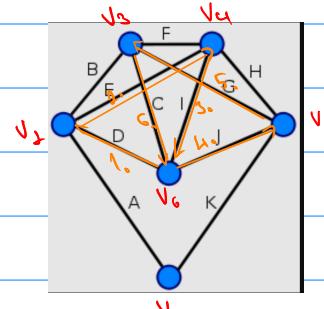
$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$

Cycle - dĺžka 5



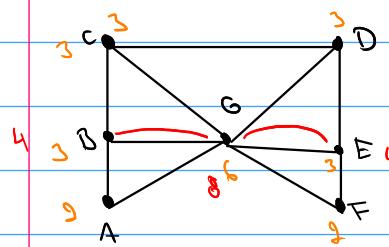
$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_1$

Circuit dĺžka 6

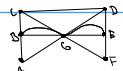


$v_1 \rightarrow v_2 \rightarrow v_4 \rightarrow v_5 \rightarrow v_3 \rightarrow v_6 \rightarrow v_1$

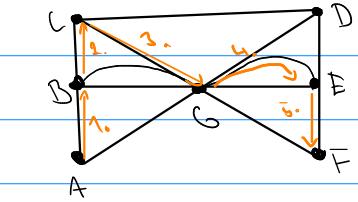
Semi-eulerovský graf



→ Po 2-mene príme z vrcholu majú neprím stupeň
→ Čiara je spojiteľná

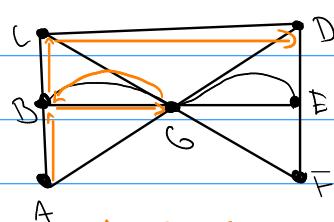


Path (cesta) - 5



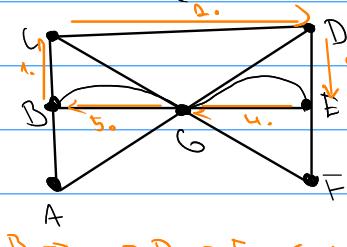
$A \rightarrow B \rightarrow C \rightarrow G \rightarrow E \rightarrow F$

Trail (čiaru) - 5



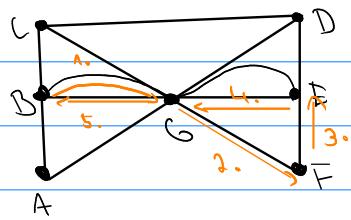
$A \rightarrow B \rightarrow G \rightarrow B \rightarrow C \rightarrow D$

Cycle - 5



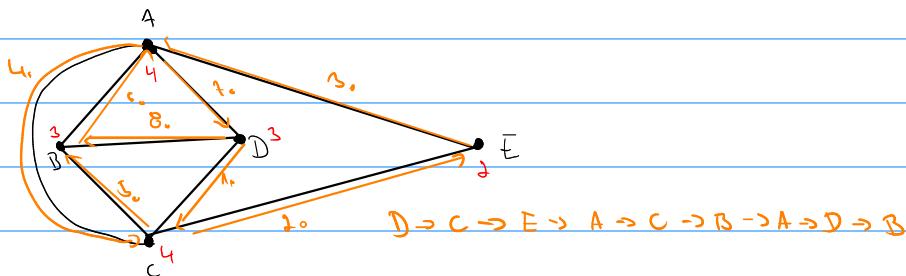
$B \rightarrow C \rightarrow D \rightarrow E \rightarrow G \rightarrow B$

Circuit - 5

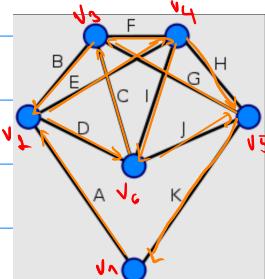
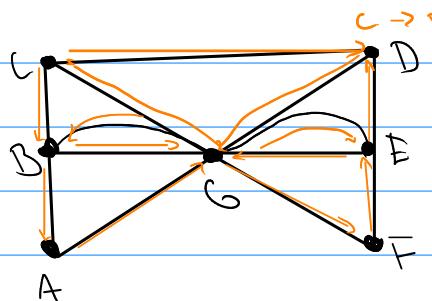


$B \rightarrow G \rightarrow F \rightarrow E \rightarrow G \rightarrow B$

Fleuryho algoritmus \Rightarrow (circuit so všetkimi bodmi)



$D \rightarrow C \rightarrow E \rightarrow A \rightarrow C \rightarrow B \rightarrow A \rightarrow D \rightarrow B$

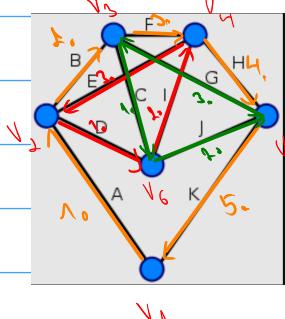


$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6 \rightarrow v_5 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_1$

Hierholzov algoritmus

graf je eukleovský takže ho bude fungovať

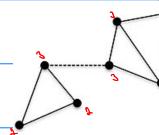
G



$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5 \rightarrow V_1$
 $(V_2 \rightarrow V_6 \rightarrow V_4 \rightarrow V_2)$
 $(V_3 \rightarrow V_6 \rightarrow V_5 \rightarrow V_3)$

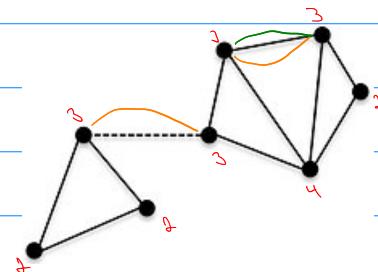
$V_1 \rightarrow V_2 \rightarrow V_6 \rightarrow V_4 \rightarrow V_2 \rightarrow V_3 \rightarrow V_6 \rightarrow V_5 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5 \rightarrow V_1$

Eulerizácia, Semieulerizácia



Eulerizácia \Rightarrow

Semi-eulerizácia



Hamiltonovský maticu

Pravda: Ak graf neni Hamilton cyclus tzn. neni ej Hamilton path (cesta) cyclus \Rightarrow cesta
 ! Napadne to neplatí cesta $\not\Rightarrow$ cyclus

POSTAEDKOVÁ PODMIEKA - DIRAC'S THEOREM

(DIRACOVÁ VETVA)

Veta: Nechť graf G má viac ako 3 vrcholy ($n \geq 3$).
 Ak každý vrchol grafu späť podmienku $|V(C_G)| = n$
 $\deg(v) \geq \frac{n}{2}$

potom G má Hamiltonovsky cycle.

n -počet vrcholov
 grafu
 $\deg(v)$ - stupeň vrcholov

Hamilton is possible

NUTNÁ PODMIEKA (NECESSARY CONDITION)

1. G je spojiteľný (connected)

2. Štandard z vrcholov nemá stupeň menší ako 2

3. G neobsahuje CUT-VERTEX (ODSTRANITEĽNÉ VRCHOLY)

- to sú tiež ktoré kedy odvádzajú množstvo sa spojlosťou grafu

Ak však sú iba 3 body (1-5) sú splnené tieto Hamilton PATH/CYCLE MÔŽU (ALE NEHUSI)
 EXISTOVAT.

Ak ktorakékoľvek z týchto podmienok nie je splnené
 tieto Hamilton PATH/CYCLE NEMÔŽE NEBUDÉ EXISTOVAT

Kompletnejši graf \Rightarrow každý vrchol je spojený s každým vrcholom

Vlastnosti kompletnejšiho grafu

1. Každý vrchol v grafu K_m má stupeň $m-1$

2. K_m má $\frac{m \cdot (m-1)}{2}$ hranič $C(m, 2) = \frac{m \cdot (m-1)}{2}$
 $\binom{m}{2}$

Travelling Salesman Problem

↳ Brute Force

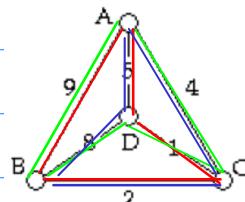
↳ Nearest Neighbour (Najbližší soused)

↳ Repetitive Nearest Neighbour (Repetitivní nejbližší soused)

↳ Cheapest Link Algorithm (Nejlevnější spojení)

↳ Nearest Insertion (

Brute force :



$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$

$$9 + 2 + 1 + 5 = 17$$

$A \rightarrow B \rightarrow D \rightarrow C \rightarrow A$

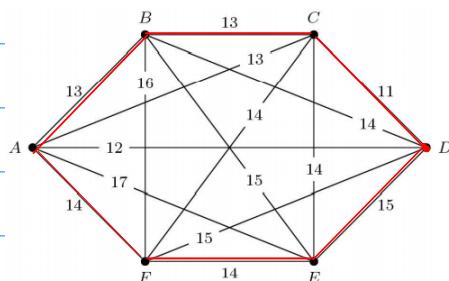
$$9 + 8 + 1 + 4 = 22$$

$A \rightarrow D \rightarrow B \rightarrow C \rightarrow A$

$$5 + 8 + 2 + 4 = 19$$

Pomocou BruteForce je
Příklad hledat možnosti

Nearest Neighbour \Rightarrow

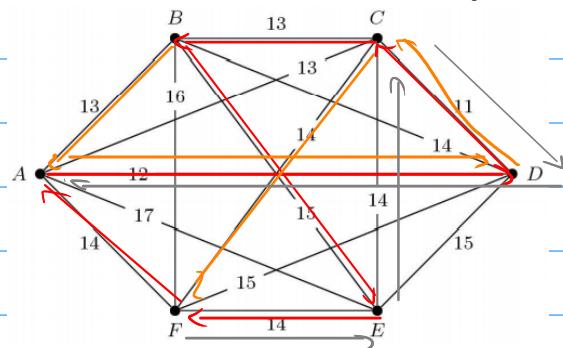


$D \rightarrow C \rightarrow B \rightarrow A \rightarrow F \rightarrow E$

$$11 + 13 + 13 + 14 + 14 + 15 = 76 + 28 + 26 =$$

$$52 + 28 = \underline{\underline{80}}$$

Repetitive Nearest Neighbour



$A \rightarrow D \rightarrow C \rightarrow B \rightarrow E \rightarrow F \rightarrow A$

$$11 + 13 + 13 + 15 + 14 + 14 = 79$$

$B \rightarrow A \rightarrow D \rightarrow C \rightarrow F \rightarrow E \rightarrow B$

$$13 + 12 + 11 + 14 + 14 + 15 = 79$$

$F \rightarrow E \rightarrow C \rightarrow D \rightarrow A \rightarrow B \rightarrow F$

$$14 + 14 + 11 + 12 + 13 + 11 = 80$$

$C \rightarrow D \rightarrow A \rightarrow B \rightarrow E \rightarrow F \rightarrow C$

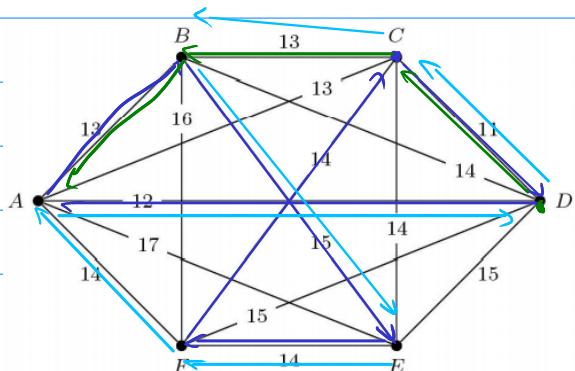
$$11 + 12 + 13 + 15 + 14 + 14 = 79$$

$D \rightarrow C \rightarrow B \rightarrow A \rightarrow F \rightarrow E \rightarrow D$

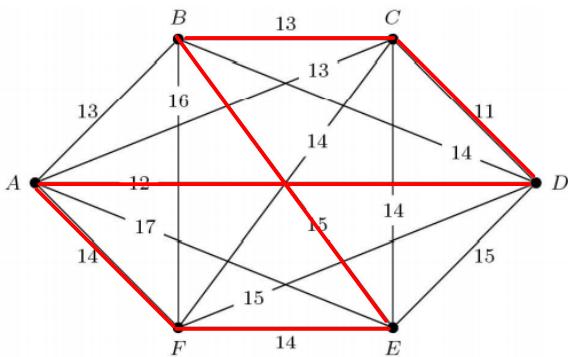
$$11 + 13 + 13 + 14 + 14 + 15 = 80$$

$E \rightarrow F \rightarrow A \rightarrow D \rightarrow C \rightarrow B \rightarrow E$

$$14 + 14 + 12 + 11 + 13 + 15 = 79$$



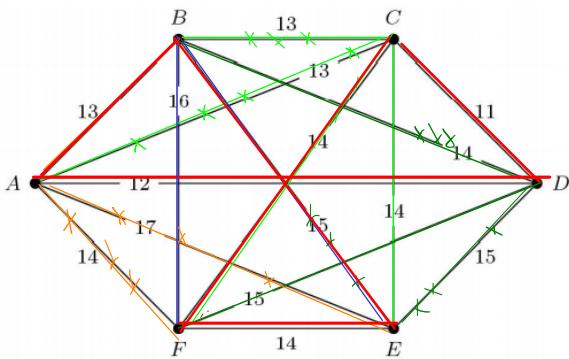
Cheapest Link algorithm



$D \rightarrow C \rightarrow B \rightarrow E \rightarrow F \rightarrow A \rightarrow D$

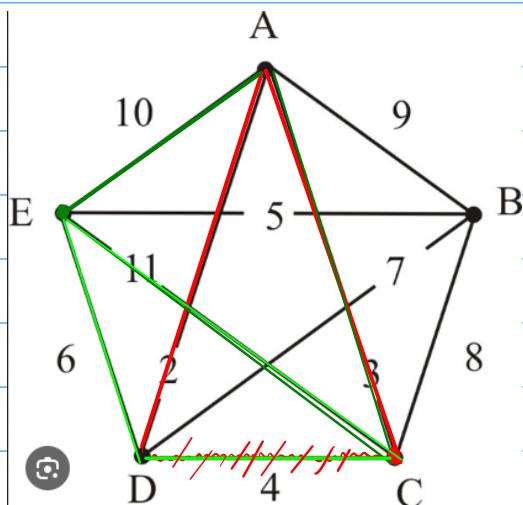
$$11 + 13 + 15 + 14 + 14 + 12 = 79$$

Nearest Insertion



$C \rightarrow D \rightarrow A \rightarrow B \rightarrow E \rightarrow F \rightarrow C$

$$11 + 12 + 13 + 15 + 14 + 12 = 79$$



$$E \quad \Delta EAD = 10 + 6 + 2 = 18$$

$$B \quad \Delta BAD = 9 + 7 + 2 = 18$$

$$C \quad \Delta CAD = 8 + 4 + 2 = 14$$

$$E_A = 10$$

$$B_A = 9$$

$$E_C = 11$$

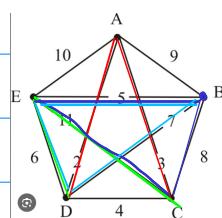
$$B_C = 8$$

$$E_D = 6$$

$$B_D = 7$$

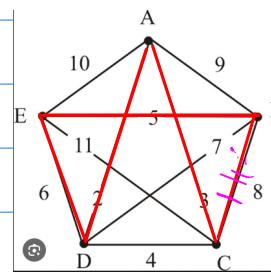
$$EO + EC - OC = 6 + 11 - 4 = 13$$

$$EA + EC - AC = 10 + 11 - 3 = 18$$



$$BE + BC - EC = 5 + 8 - 11 = 2$$

$$BE + BD - DE = 5 + 7 - 6 = 6$$

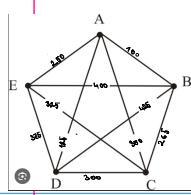


~ Najlepsza hamiltonowska cyfra

$$2 + 6 + 5 + 8 + 3 = 24$$

~ Najlepsza hamiltonowska cyfra

$$2 + 6 + 5 + 3 = 16$$



$$\triangle EAB = 250 + 160 + 100 = 750$$

$$\triangle DAB = 125 + 100 + 425 = 650$$

$$\triangle CAD = 350 + 100 + 265 = 725$$

$$EA = 250$$

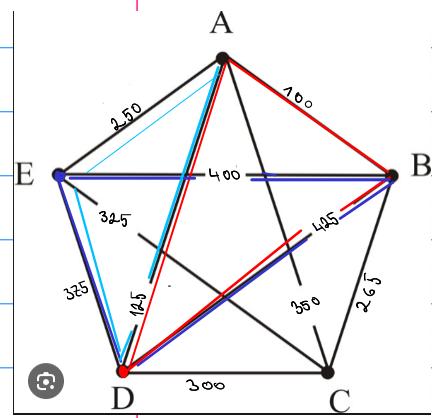
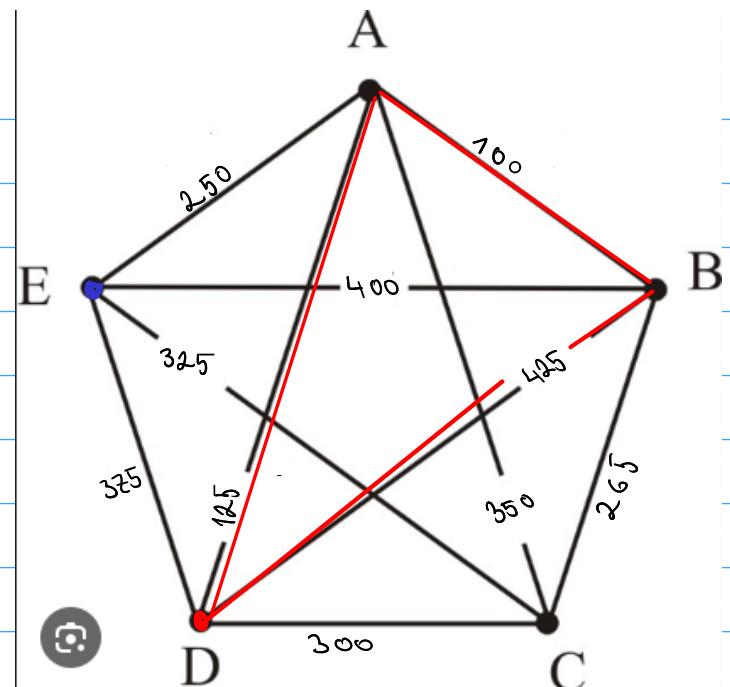
$$CA = 350$$

$$EB = 400$$

$$CB = 265$$

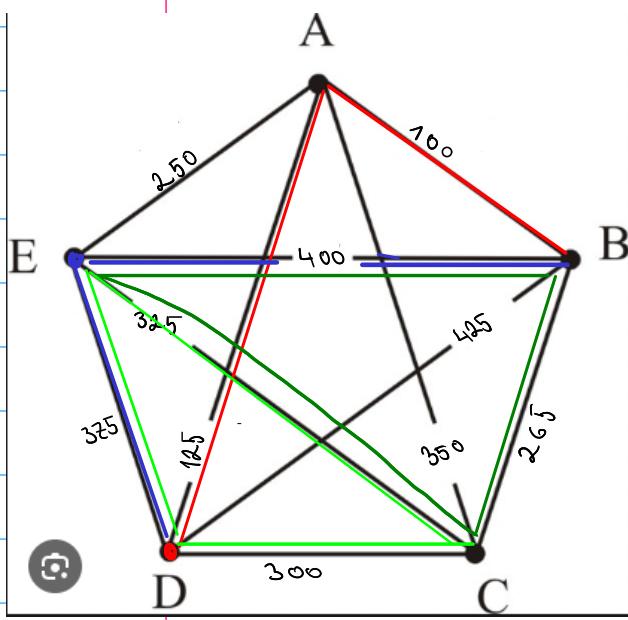
$$ED = 375$$

$$CD = 30$$



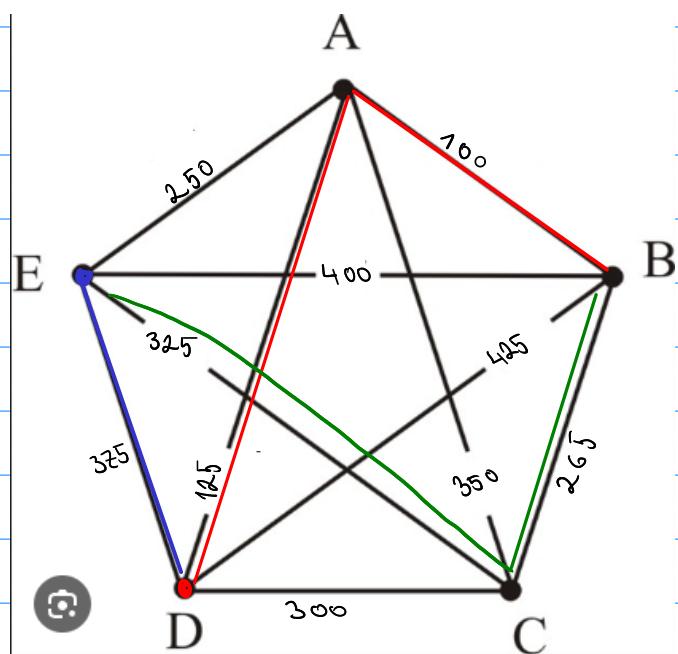
$$\triangle EBD = EB + ED - BD = 400 + 375 - 425 = 350$$

$$\triangle EAD = EA + ED - AD = 250 + 375 - 125 = 500$$



$$\triangle BCE = CB + CE - EB = 265 + 325 - 400 = 190$$

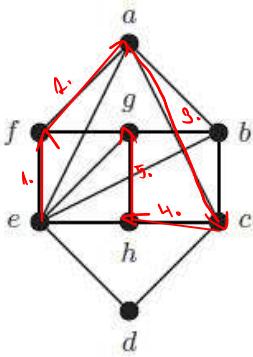
$$\triangle CDE = CD + CE - DE = 300 + 325 - 375 = 250$$



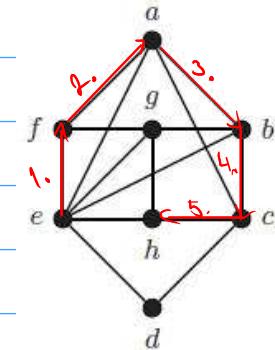
Opatkovanie:

Džížku - 5

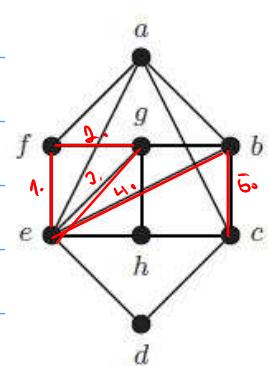
Walk



Path

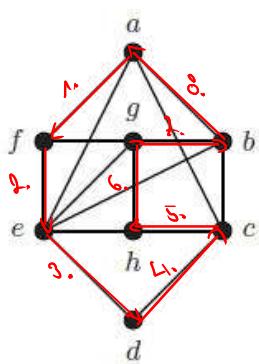


Trail



$e \rightarrow f \rightarrow a \rightarrow c \rightarrow h \rightarrow g$

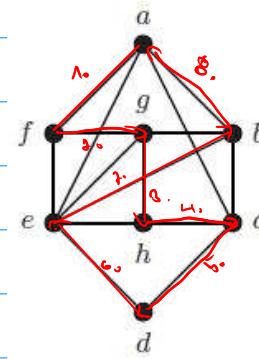
cycle (Path)



$a \rightarrow f \rightarrow e \rightarrow h \rightarrow c \rightarrow b \rightarrow g \rightarrow d \rightarrow e \rightarrow b \rightarrow a$

$e \rightarrow f \rightarrow a \rightarrow b \rightarrow c \rightarrow h$

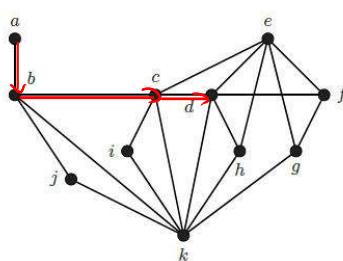
circuit (trail)



$a \rightarrow f \rightarrow g \rightarrow h \rightarrow c \rightarrow d \rightarrow e \rightarrow b \rightarrow a$

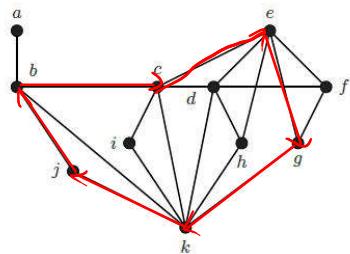
Trail / Path \neq A do G

Trail



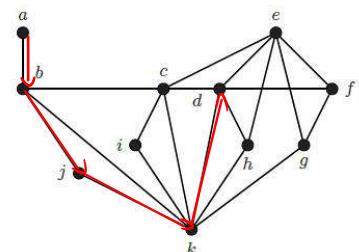
$a \rightarrow b \rightarrow c \rightarrow d$

cycle (Path)



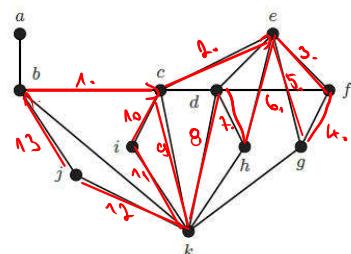
$b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g \rightarrow h \rightarrow i \rightarrow j \rightarrow k \rightarrow a$

Path



$a \rightarrow b \rightarrow j \rightarrow k \rightarrow d$

circuit (Trail)

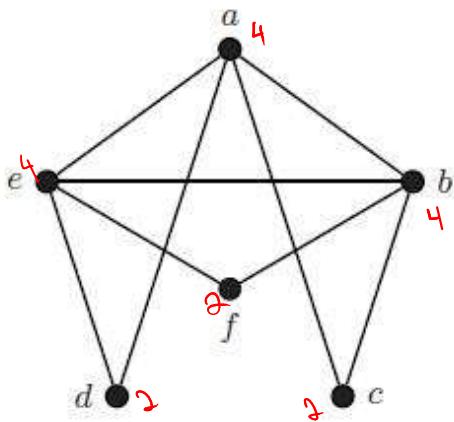


$b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g \rightarrow h \rightarrow i \rightarrow j \rightarrow k \rightarrow a$

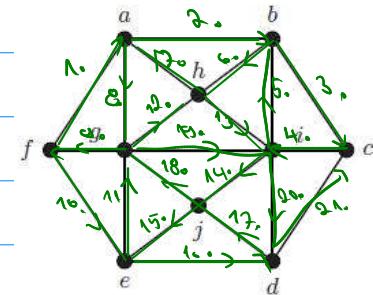
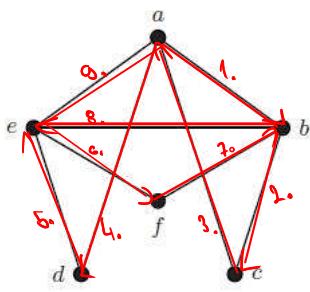
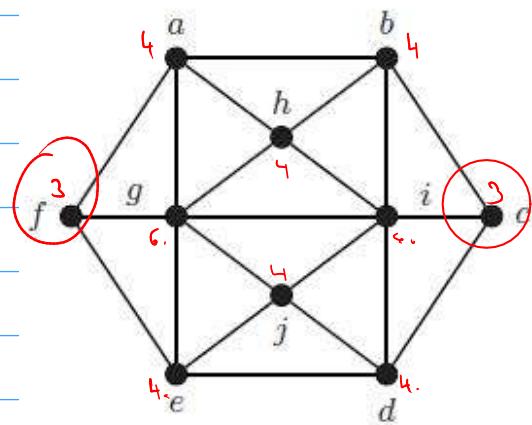
Flueruk's algorithm:

Podmienky:

- eulerovský / semi-eulerovský ✓
- súvisiací graf ✓ nemôže byť



- semi-eulerovský graf ✓
- súvisiací graf ✓



$a \rightarrow b \rightarrow c \rightarrow a \rightarrow d \rightarrow e \rightarrow f \rightarrow b \rightarrow e \rightarrow a$

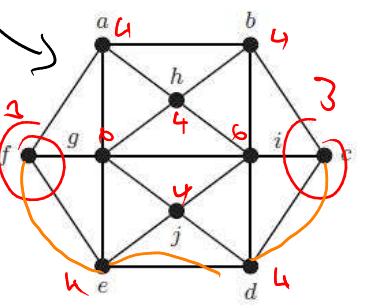
$f \rightarrow a \rightarrow b \rightarrow c \rightarrow i \rightarrow l \rightarrow h \rightarrow a \rightarrow g \rightarrow f \rightarrow e \rightarrow g \rightarrow h \rightarrow i \rightarrow j \rightarrow e \rightarrow d \rightarrow j \rightarrow g \rightarrow i \rightarrow d \rightarrow a$

Hierholzerov algoritmus

Podmienky:

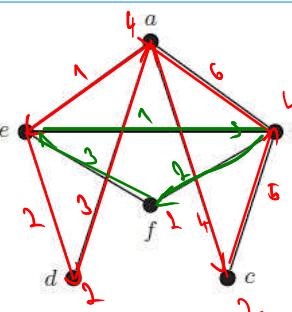
musi byť eulerovský graf X
connected → ✓

eulerizacia



Podmienky:

musi byť eulerovský
connected

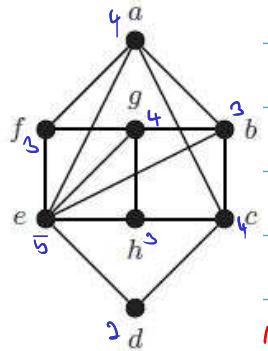


$a \rightarrow e \rightarrow d \rightarrow a \rightarrow c \rightarrow b \rightarrow a$

$c \rightarrow b \rightarrow f \rightarrow e$

$a \rightarrow c \rightarrow b \rightarrow f \rightarrow e \rightarrow d \rightarrow a \rightarrow c \rightarrow b \rightarrow a$

Podmienky:
eulerovská ✓
connected ✓



$1 \rightarrow 2 \rightarrow 4 \rightarrow c \rightarrow 1$

$2 \rightarrow 3 \rightarrow 5 \rightarrow c \rightarrow 1$

$3 \rightarrow 4 \rightarrow 5 \rightarrow 3$

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow 5 \rightarrow c \rightarrow 1 \rightarrow 4 \rightarrow c \rightarrow 1$

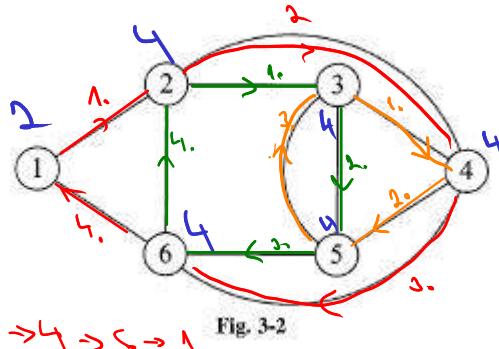


Fig. 3-2

Eulerová cesta → Chinese Postman Problem

Podmienky:

spojenie ✓

Musi mať neštvrte vrcholy ✓

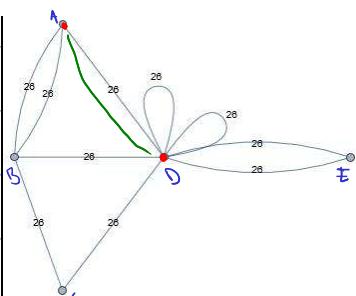
$$\deg(A) = 3$$

$$\deg(B) = 4$$

$$\deg(C) = 7$$

$$\deg(D) = 9$$

$$\deg(E) = 2$$



eulerizácia \Rightarrow najlacnejšie je duplikácia

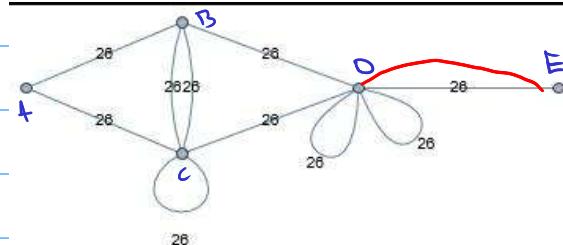
krum AD alebo je to priame spojenie

a väčšiu kruhu majú hodnotu 26

celková cena cestn : 26 AD

Podmienky

Nepárnna kruha ✓
connected ✓



$$\deg(A) = 2 \quad \deg(D) = 7$$

$$\deg(B) = 4 \quad \deg(E) = 1$$

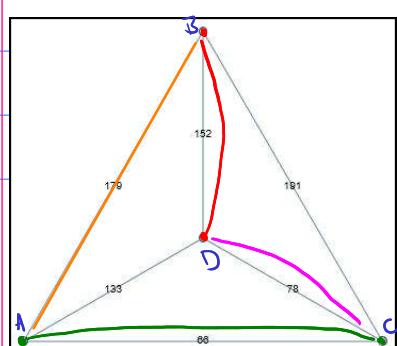
$$\deg(C) = 6$$

eulerizácia DE

celková cena cestn : 26

Podmienky:

connected
Nepárnne vrcholy



$$\deg(A) = 3$$

$$\deg(B) = 3$$

$$\deg(C) = 3$$

$$\deg(D) = 3$$

$$AC + DB = 66 + 152 = 218$$

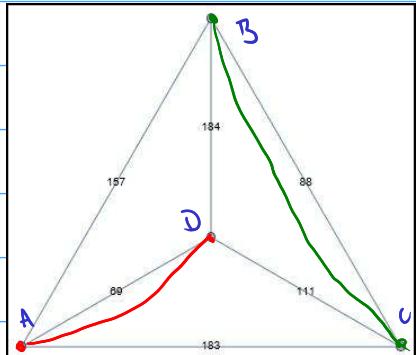
$$DC + AB = 78 + 175 = 257$$

Po spojení 2 najkratších kruhov som došiel k záveru že najlacnejšia eulerova cesta má kruhmi: AC, DB
Celková cena cestn : 218

Podmienky:

connected

Nepárný vrchol



$$\deg(A, B, C, D) = (3, 3, 3, 3)$$

$$AD + BC = 69 + 88 = 157$$

eulerizácia so súvahom podľa 2

majhatejších hran v grafu, t. e. to je najefektívnejší spôsob.

eulerizácia: AD, BC

celková cena cestn: 157

Hamiltonovský cyklus a Path

Podmienky

I. Nutné podmienky ✓

II. Postačujúca Podmienka

I. • connected ✓

• neobsahuje cut vertex ✓

• $\# V(G) \geq \deg(v)$ ✓

II. Dirac's theorem

$$V(G) = n \wedge n \geq 3 \wedge \deg(v) \geq \frac{n}{2}$$

$$8 \geq 3$$

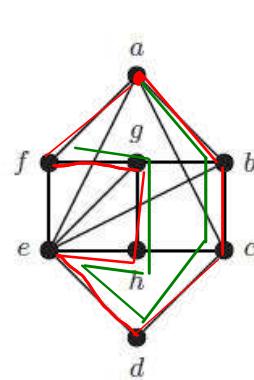
hamilton cycle $\Rightarrow a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow h \rightarrow g \rightarrow f \rightarrow a$

nejmenší $\deg(v) = 3$

$$f < \frac{8}{2}$$

hamilton path $\Rightarrow a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow h \rightarrow g \rightarrow f$

Dirac theorem hľadáva, aké hranami má toto Hamilton - c, p flexibilite



2) Podmienky:

I. Nutné podmienky ✓

II. Postačujúca Podmienka \Rightarrow hamilton cycle, path možné ak nemá \exists

I. \Rightarrow • connected ✓

• neobsahuje cut vertex ✓

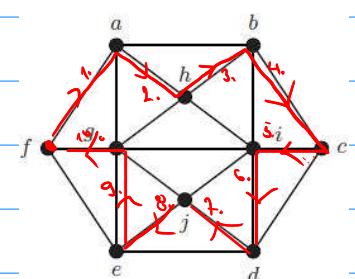
• $\# V(G) \geq 2$ ✓

II. \Rightarrow $V(G) = n \wedge n \geq 3 \wedge \deg(v) \geq \frac{n}{2}$

$$n = 10$$

$$\text{nejmenší stupeň} = 3$$

$$3 < 5$$



cycle $\Rightarrow f \rightarrow a \rightarrow h \rightarrow b \rightarrow c \rightarrow i \rightarrow d \rightarrow j \rightarrow e \rightarrow g \rightarrow f$

path $\Rightarrow f \rightarrow a \rightarrow h \rightarrow b \rightarrow c \rightarrow i \rightarrow d \rightarrow j \rightarrow g \rightarrow e \rightarrow g$

3) Podmienky:

I. Nutná podmienka ✓

II. Postačujúca podmienka ✗ \Rightarrow hamilton možné/memuje

I. \Rightarrow coh. hrad

medzisúvisie všetkých vertex

$$\# \deg(V(G)) \geq 2$$

II. \Rightarrow Dirac's theorem

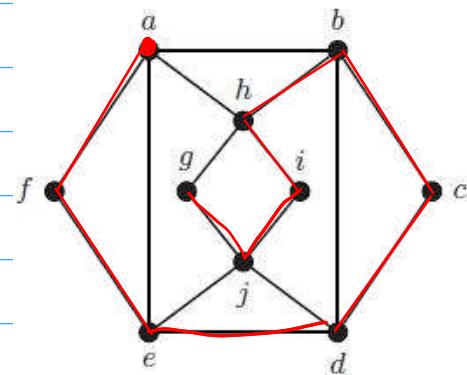
$$V(G) = n \wedge n \geq 3 \wedge \deg(V(G)) \geq \frac{n}{2}$$

$n = 10$, najmenší stupen $\Rightarrow 2$

$$2 < \frac{10}{2}$$

hamil. Path: $a \rightarrow f \rightarrow e \rightarrow d \rightarrow c \rightarrow b \rightarrow h \rightarrow i \rightarrow j \rightarrow g$

hamil. cycle ✗



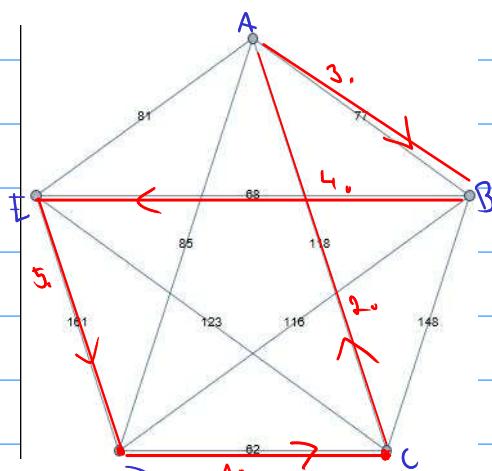
Kompletnejšia graf (kn) - Travelling Salesman Prob.

Vlastnosti: 1.) každú vrchol v grafe má stupen $n-1$

2.) kn má $\frac{n(n-1)}{2}$ hrán

Algo Brute Force ...

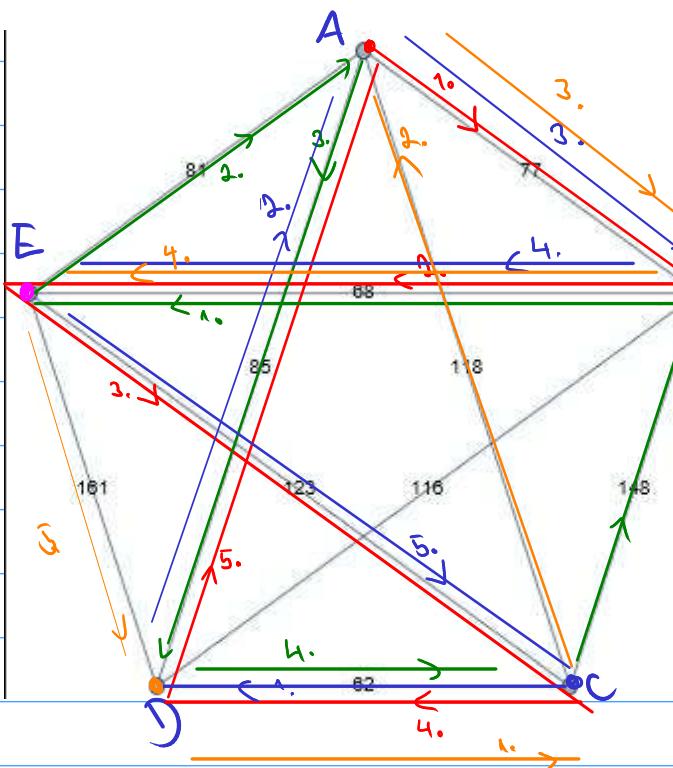
Nearest Neighbour



D \rightarrow C \rightarrow A \rightarrow B \rightarrow E \rightarrow D

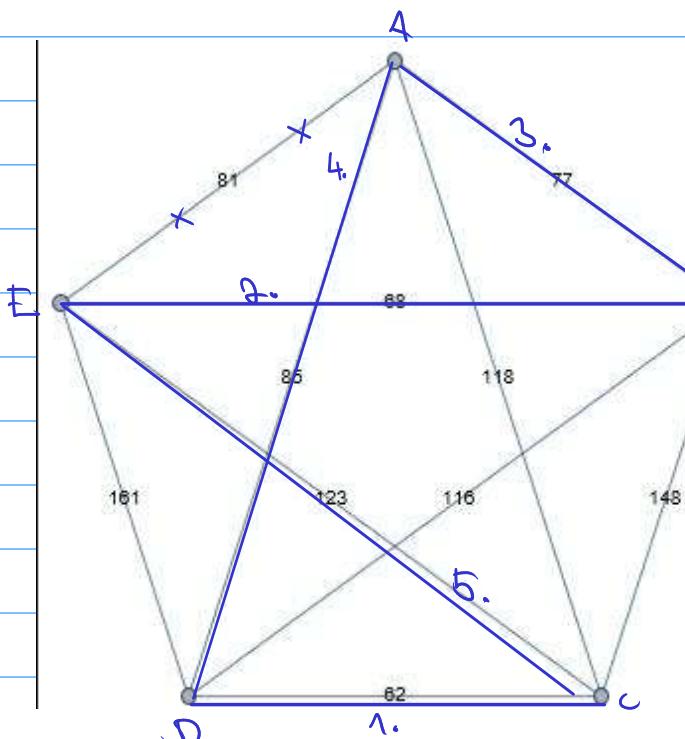
$$101 + 81 + 68 + 118 + 148 = 506$$

Repetitive NN



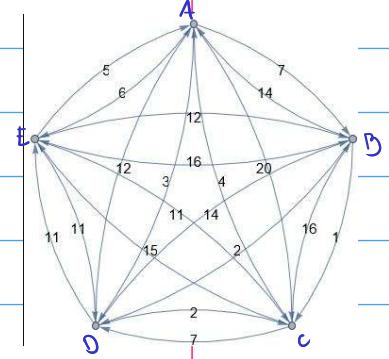
- $A \rightarrow B \rightarrow E \rightarrow C \rightarrow D \rightarrow A$
 $77 + 68 + 123 + 62 + 85 = 415$
- $B \rightarrow E \rightarrow A \rightarrow D \rightarrow C \rightarrow B$
 $68 + 81 + 85 + 62 + 148 = 444$
- $C \rightarrow D \rightarrow A \rightarrow B \rightarrow E \rightarrow C$
 $62 + 85 + 77 + 68 + 123 = 415$
- $D \rightarrow C \rightarrow A \rightarrow B \rightarrow E \rightarrow D$
 $52 + 118 + 77 + 68 + 161 = 486$
- $E \rightarrow B \rightarrow A \rightarrow D \rightarrow C \rightarrow E$
 $68 + 77 + 85 + 62 + 123 = 415$

Cheapest link algorithm

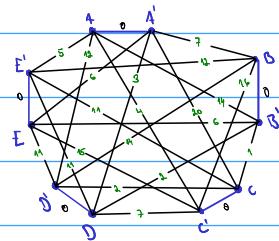


- 1.) DC
 - 2.) EB
 - 3.) BA
 - 4.) EA ~~(incorrect)~~
 - 5.) AD
- Q) Dokončení → 1 možnost
 $\hookrightarrow EC$
- $A \rightarrow B \rightarrow E \rightarrow C \rightarrow D \rightarrow A$
- $77 + 68 + 123 + 62 + 85 = 415$
- Cheap Algorithm → 415

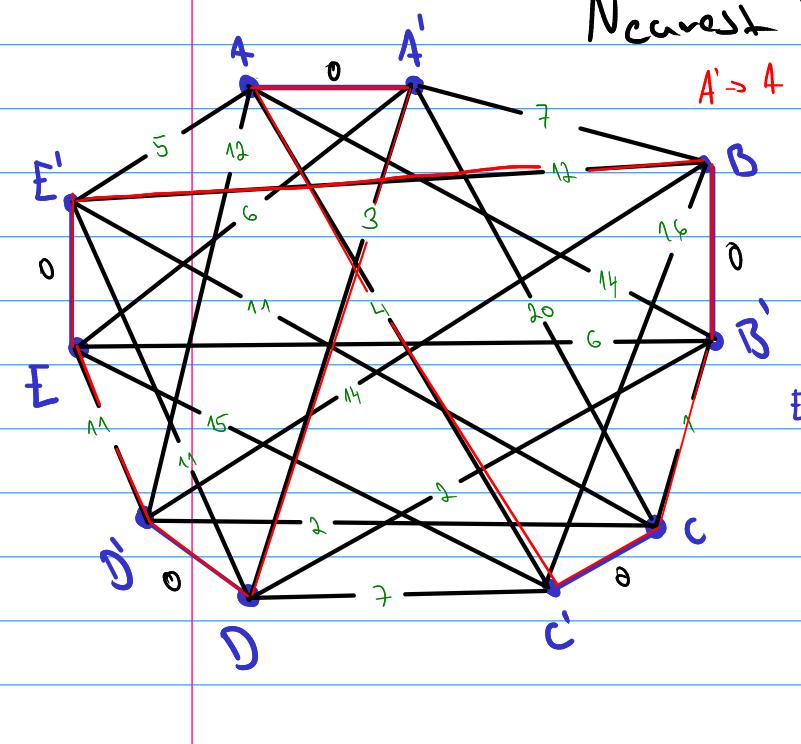
Asymmetric travelling Salesman Problem



Undirected graph

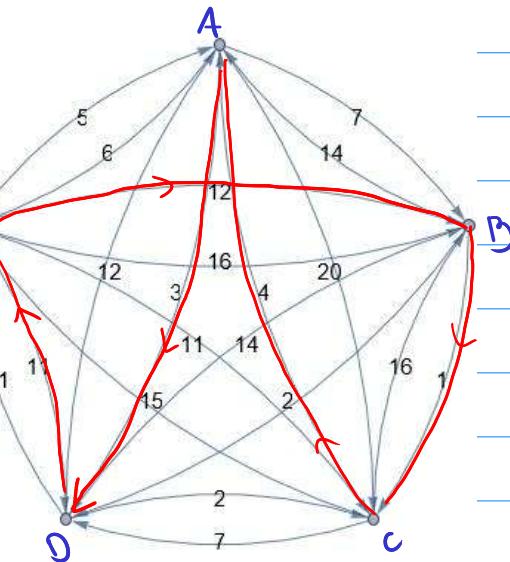


Nearest Neighbour algorithm

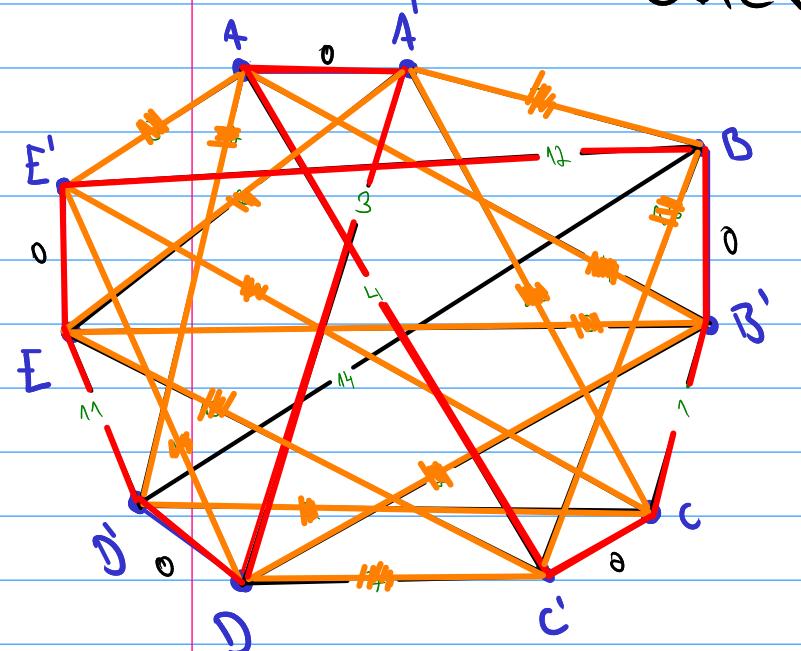


$A \rightarrow A' \rightarrow C' \rightarrow C \rightarrow B' \rightarrow B \rightarrow E' \rightarrow E \rightarrow D' \rightarrow D \rightarrow A$

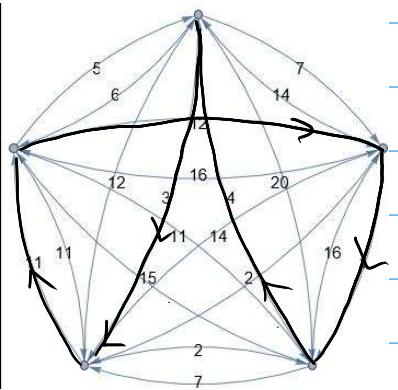
Path cost : $4 + 1 + 12 + 11 + 3 + 0 + 0 + 1 + 12 + 4 = 31$



cheapest Link algorithm

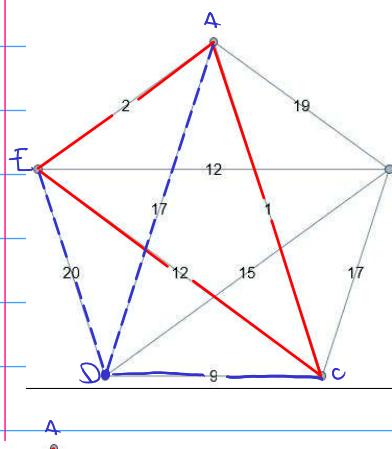


- 1.) $A \rightarrow A'$ $C \rightarrow C'$ $E \rightarrow E'$
 $B \rightarrow B'$ $D \rightarrow D'$
- 2.) $C \rightarrow B'$
- 3.) $A' \rightarrow D$
- 4.) $E \rightarrow D'$
- 5.) $E' \rightarrow B$



$A' \rightarrow A \rightarrow C \rightarrow C' \rightarrow B' \rightarrow B \rightarrow E' \rightarrow E \rightarrow D' \rightarrow D \rightarrow A'$

Nearest Insertion



1) najkratšia hranu $\Rightarrow AC$

2) najblíži bod ku Ac je E

B $W(BA) = 19$

D $W(DA) = 17$

$W(BC) = 17$

W(DC) = 9

$W(BE) = 12$

$W(CD) = 2$

3) Ďalšia hranu DC

4) Mazanie hranu:

$$\hookrightarrow W(DE) + W(DC) - W(CE) = 20 + 9 - 12 = 17$$

$$\hookrightarrow W(CD) + W(DA) - W(AC) = 9 + 17 - 1 = 25$$

5.) Zostáva Bod B

$W(AB) = 19$

$W(DB) = 15$

$W(CB) = 17$

W(BE) = 12

najblíži bod $\Rightarrow E$

6.) Mazanie:

$$\hookrightarrow W(BE) + W(BA) - W(AE) = 12 + 19 - 2$$

$$\hookrightarrow W(BE) + W(BD) - W(DE) = 12 + 15 - 20 = 7$$

mazeme ED

ham. cycle: $A \rightarrow C \rightarrow D \rightarrow B \rightarrow E \rightarrow A$

cena cesta: $1 + 9 + 15 + 12 + 2 = 39$

ham. Path: $B \rightarrow E \rightarrow A \rightarrow C \rightarrow D$

Cena ham. Path = $12 + 12 + 1 + 9 = 24$

1.) Najkratšia hranu $\Rightarrow AC$

2.) Najblíži bod ku $Ac \Rightarrow D$

3) ostat bod B

$W(BA) = 19$, **$W(BD) = 8$** , $W(BC) = 10$

4.) Pridame hranu BD 5.) Mazanie hranu

$$W(BD) + W(BA) - W(DA) = 8 + 19 - 12 = 15$$

$$W(BD) + W(BC) - W(CD) = 8 + 10 - 16 = 11$$

Mazeme CD , Pridame BC

cycle: $B \rightarrow D \rightarrow A \rightarrow C \rightarrow B$

Path: $B \rightarrow D \rightarrow A \rightarrow C$

Dikstrov algoritmus

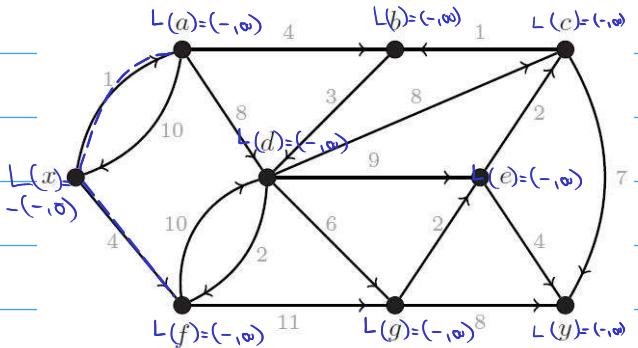
Podmienky:

connected ✓ Väčšin G ✓
Simple ✓

$$1.) F = \{a, \emptyset\}$$

$$W(x) + W(xa) = 0 + 1 = 1 < W(a) \checkmark$$

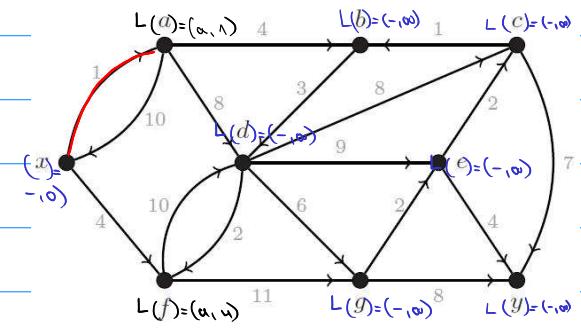
$$W(x) + W(xd) = 0 + 4 = 4 < W(d) \checkmark$$



$$2.) F = \{b, d\}$$

$$W(a) + W(af) = 1 + 4 = 5 < W(f) \checkmark$$

$$W(a) + W(ad) = 1 + 8 = 9 < W(d) \checkmark$$



$$3.) F = \{d\}$$

$$W(b) + W(bd) = 5 + 3 = 8 < W(d) \checkmark$$

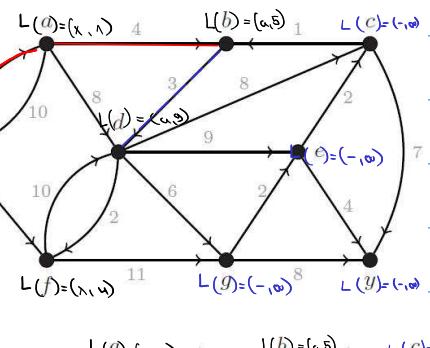
$$4.) F = \{c, e, g, f\}$$

$$W(d) + W(cd) = 8 + 8 = 16 < W(c)$$

$$W(d) + W(ed) = 8 + 9 = 17 < W(e)$$

$$W(d) + W(dy) = 8 + 6 = 14 < W(y) \checkmark$$

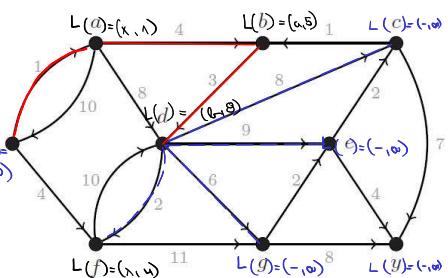
$$W(d) + W(df) = 8 + 2 = 10 > W(f)$$



$$5.) F = \{e, g\}$$

$$W(g) + W(eg) = 14 + 2 = 16 < W(g)$$

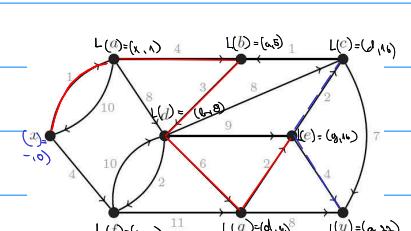
$$W(g) + W(hg) = 14 + 8 = 22 > W(h)$$



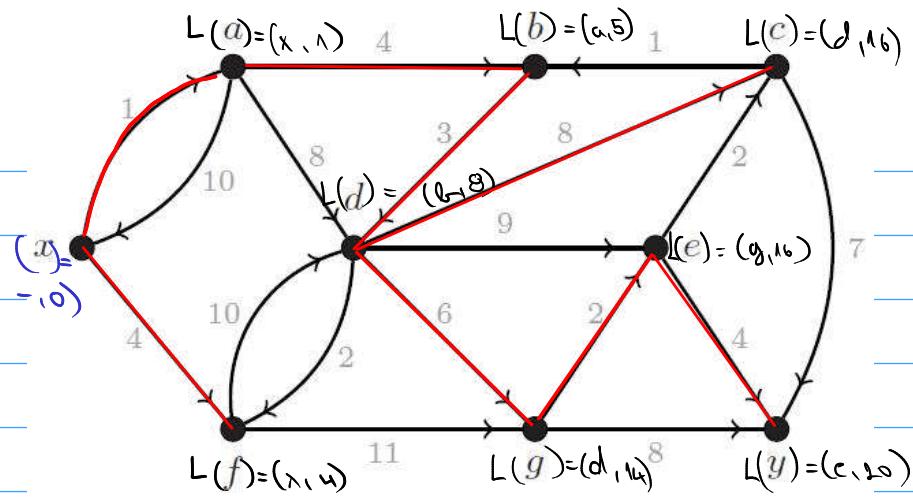
$$6.) F = \{c, u\}$$

$$W(e) + W(ce) = 16 + 2 = 18 = W(c)$$

$$W(e) + W(ue) = 16 + 4 = 20 < W(u) \checkmark$$



x	0	-
a	1	x
b	5	9
c	14	1
d	8	b
e	16	g
f	4	x
g	14	d
y	20	e

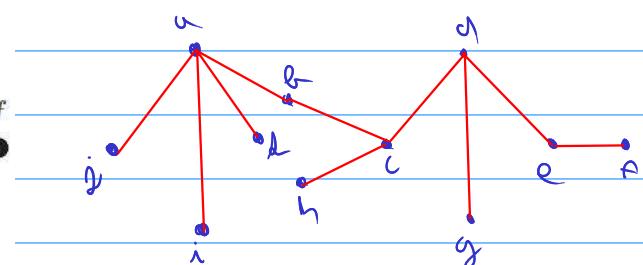
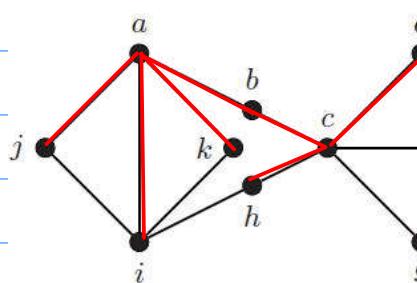


Spanning tree (kostka)

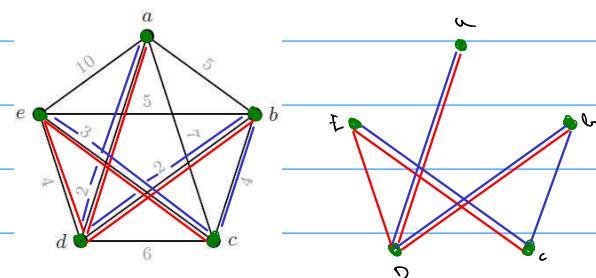
- Strom (tree) \Rightarrow graf G musi byt
- acyklicky
 - konecne
- Les (Forest) \Rightarrow graf G musi byt
- acyklicky
 - nemusi, but connected

Vlastnosti Stromov:

- $V(G) = n \wedge E(G) = n - 1$
- $\deg(V(G)) = 2n - 2$
- If connected $G \wedge V(G) = n \wedge E(G) = n - 1 \Rightarrow$ Strom



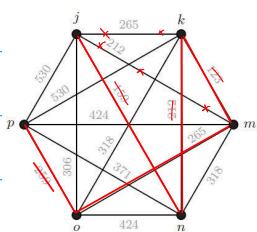
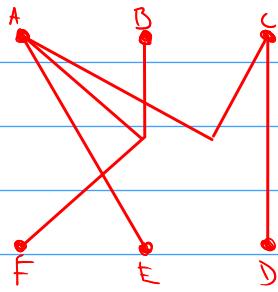
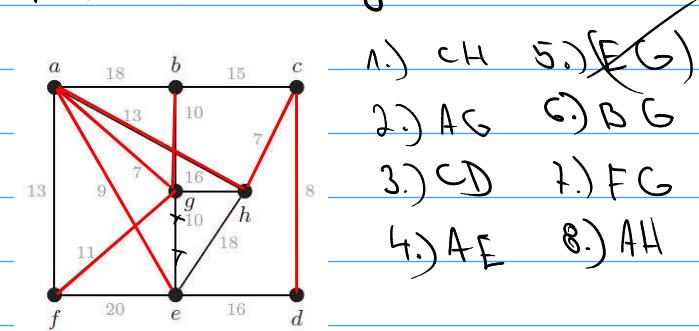
Kruskal's Algorithm



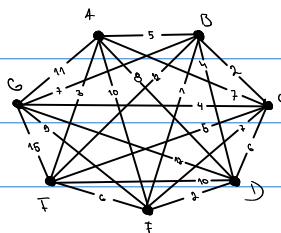
- 1.) AD
- 2.) DB
- 3.) CE
- 4.) DE

- 1.) AD
- 2.) DB
- 3.) CE
- 4.) BC

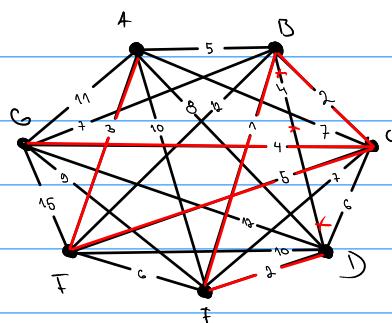
Kruskal's Algorithm



- 1.) k_m 3.) k_h 5.) P_0 7.) M_0
2.) j_h 4.) (j_m) 6.) (j_k)



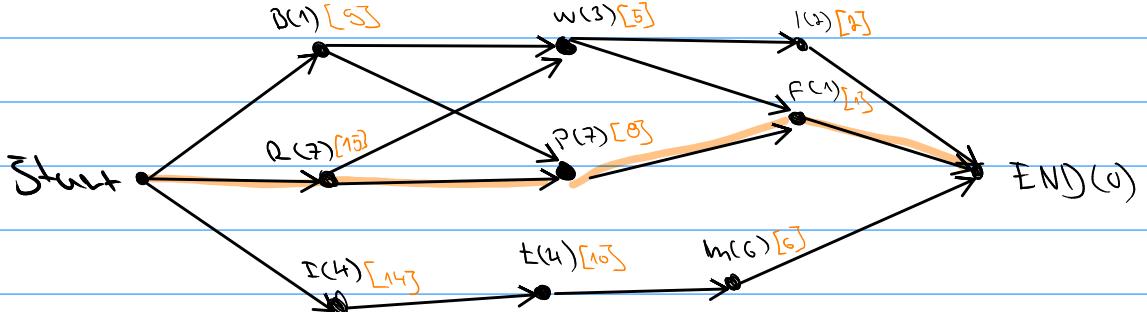
	a	b	c	d	e	f	g
a	.	5	7	8	10	3	11
b	5	.	2	7	4	12	7
c	7	2	.	6	7	5	4
d	8	4	6	.	2	10	12
e	10	1	7	2	.	6	9
f	3	12	5	10	6	.	15
g	11	7	4	12	9	15	.



- 1.) BE 4.) AF
2.) BC 5.) ~~BD~~
3.) DE C.) CG
7.) sfinjenie AF > zvukovim
↳ hajhratje sfinjenie FC/AB
FC

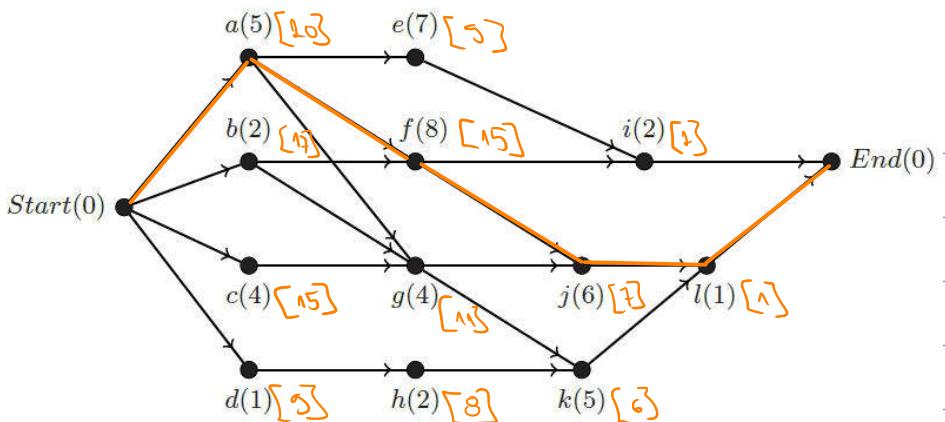
Kvitková Česta

Task	Vertex Name	Processing Time	Precedence Relationships
Buy plants	b	1	
Remove bushes	r	7	
Remove ivy	i	4	
Weed flower beds	w	3	b, r
Plant bushes	p	7	b, r
Plant flowers	f	1	w, p
Trim trees	t	4	t
Mow and rake lawn	m	6	t
Install lighting	l	2	w



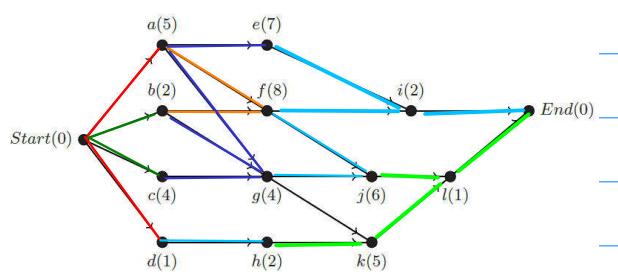
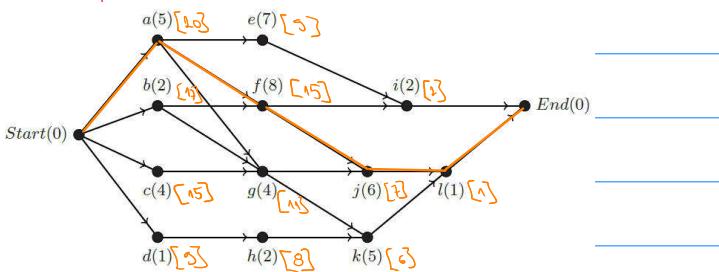
Najdlängsia celler \Rightarrow Stunt \rightarrow Q \rightarrow P \rightarrow f

Zurdeutung: R → I → T → B → P → W → M → F → L → END



Najdłuższa trasa: Start \rightarrow A \rightarrow F \rightarrow J \rightarrow L \rightarrow END

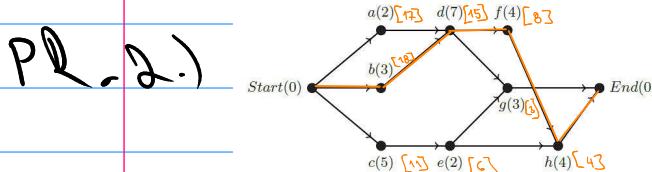
P1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
P1	A	A	A	A	A	F	F	F	F	F	F	F	J	J	J	J	J	J	J	L	*	*	*	*	*	*	*	*	*	
P2	B	B	C	C	C	C	G	G	G	D	H	H	K	K	K	K	e	e	e	e	e	e	e	i	i					



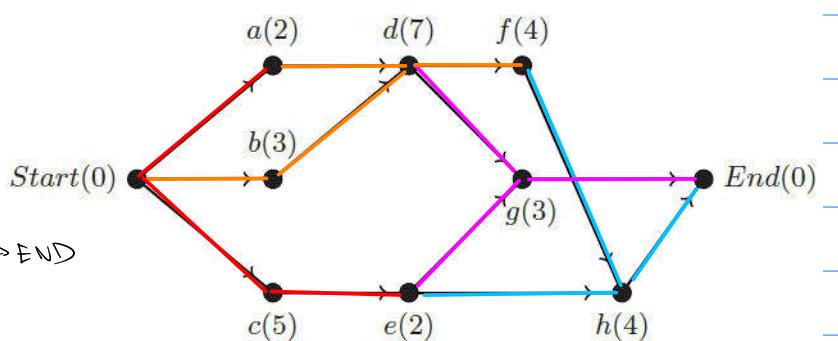
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
P1	A	A	A	A	A	D	F	F	F	F	F	F	F	H	H	J	J	J	J	J	J	J	i	i						
P2	B	B	C	C	C	C	G	G	G	G	E	E	E	E	F	F	K	K	K	K	K	L	*							

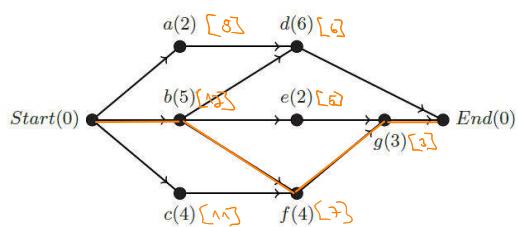
↳ Ustalil som 3 Høg obom Preparacionum Adijsic & zvjevod Jann Ra

A → P → B → C → F → G → E → H → J → K → I → END



Start $\rightarrow A \rightarrow D \rightarrow F \rightarrow H \rightarrow \text{END}$
2nd num: $B \rightarrow A \rightarrow D \rightarrow C \rightarrow E \rightarrow F \rightarrow G \rightarrow H \rightarrow \text{END}$

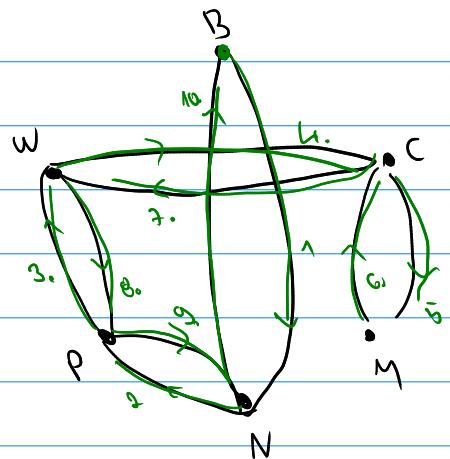
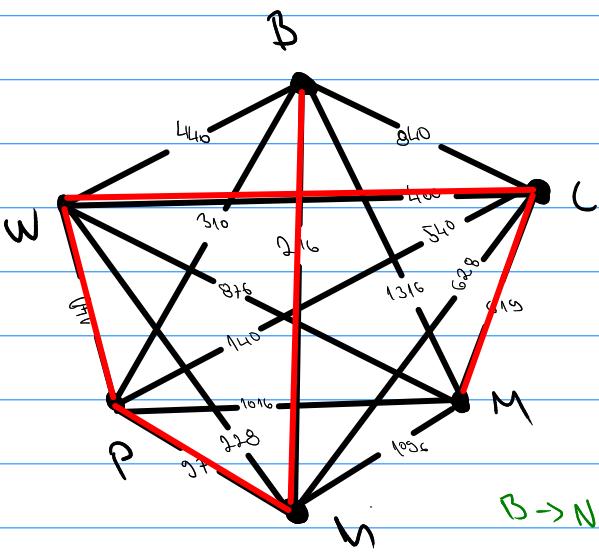
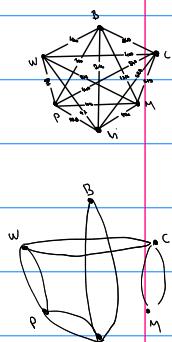




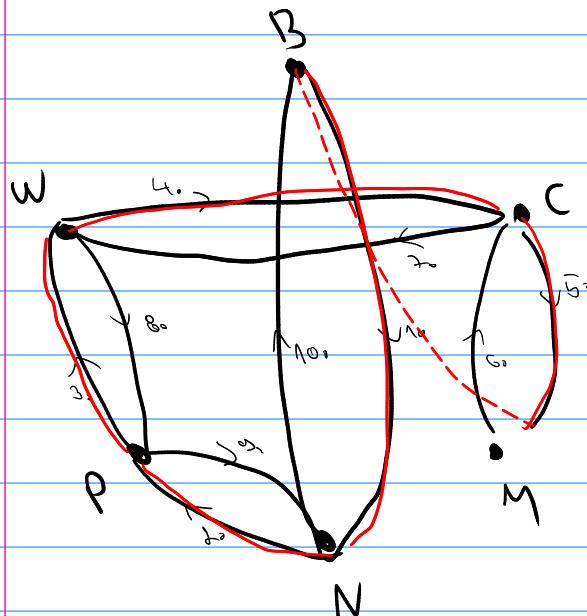
Start \rightarrow B \rightarrow F \rightarrow G \rightarrow END

Zugriffsmögl.: B \rightarrow C \rightarrow A \rightarrow F \rightarrow E \rightarrow G \rightarrow D

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	B	B	B	B	F	F	F	G	G	G																			
C	C	C	C	A	A	E	E	D	D	D	D	D	D																



B \rightarrow N \rightarrow P \rightarrow W \rightarrow L \rightarrow M \rightarrow C \rightarrow W \rightarrow P \rightarrow N \rightarrow B



B N P W L C B

$$216 + 97 \times 140 + 400 + 619 + 1316$$

