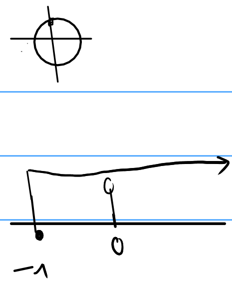
(P2₁) $D(f)$



$\sin(2x) \neq 0 \Rightarrow \sin x = 0 \Rightarrow x \neq 0$

a) $f(x) = \frac{\sqrt{x+1}}{\sin(2x)}$ $\quad \sqrt{x+1} \geq 0 \quad x+1 \geq 0$

$D(f) = \langle -1, 0) \cup (0, \infty)$ ✓ $\quad x \geq -1$

b) $\sqrt{3 - \log_2 x} \Rightarrow 3 - \log_2 x \geq 0 \wedge x > 0$ $\quad 3 \geq \log_2 x$

$2^3 \geq x$

$D(f) = (0, 8\rangle$ $\quad x \leq 8$ ✓

c) $\sqrt{-2 + \log_3(x-1)}$ $\quad x-1 > 0 \wedge -2 + \log_3(x-1) \geq 0$ $\quad x-1 \geq 3^2$

$x > 1 \wedge \log_3(x-1) \geq 2$ $\quad x \geq 10$ $\quad D(f) = \langle 10, \infty)$ ✓

d) $\sqrt{-2 + \log_{\frac{1}{3}}(x-1)}$ $\quad x > 1 \wedge -2 + \log_{\frac{1}{3}}(x-1) \geq 0$ $\quad (x-1) \geq \frac{1}{9}$

$D(f) = \langle \frac{10}{9}, \infty)$ ✗ $\quad \log_{\frac{1}{3}}(x-1) \geq 2$ $\quad x \geq \frac{10}{9}$

e) $\frac{x+1}{\sqrt{x - x^2 + 6}}$ $\quad x - x^2 + 6 \geq 0$

$(x+2)(-x+3) \geq 0$

$D(f) = (\langle -2, 3\rangle)$ ✗

	$(-\infty, -2\rangle$	$\langle -2, 3\rangle$	$\langle 3, \infty)$
$x+2$	$-$	$+$	$+$
$3-x$	$+$	$+$	$-$

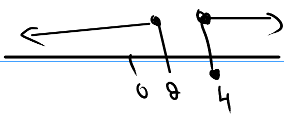
f) $\frac{\sqrt{x^2 - 4x + 3}}{x}$ $\quad x \neq 0 \wedge x^2 - 4x + 3 \geq 0$

$(x-1)(x-3)$

$D(f) = (-\infty, 0) \cup (0, 1\rangle \cup \langle 3, \infty)$ ✓

	$(-\infty, 1\rangle$	$\langle 1, 3\rangle$	$\langle 3, \infty)$
$x-1$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$

g) $\sqrt{|x-3| - 1}$ $\quad -x+3-1 \geq 0 \wedge x-3-1 \geq 0$



$D(f) = (-\infty, 2\rangle \cup \langle 4, \infty)$ $\quad x \leq 2$ $\quad x \geq 4$ ✓

h) $\sqrt{3 - \log_2(5-x)} \Rightarrow 3 - \log_2(5-x) \geq 0$ $\quad 2^3 \geq (5-x)$ $\quad x \geq -3$

$3 \geq \log_2(5-x)$ $\quad 8 \geq 5-x$ $\quad D(f) = \langle -3, \infty)$ ✗

i) $\sqrt{1 - \log_{\frac{1}{2}}(x-3)} \Rightarrow 1 \geq \log_{\frac{1}{2}}(x-3)$ $\quad x \leq \frac{7}{2}$

$\frac{1}{2} \geq (x-3)$ $\quad D(f) = (-\infty, \frac{7}{2}\rangle$ ✗

2e) $\log_5\left(\frac{1+\sqrt{x}}{2-\sqrt{x}}\right)$ $\frac{1+\sqrt{x}}{2-\sqrt{x}} > 0 \wedge x \neq 0$

$2-\sqrt{x} > 0$
$\sqrt{x} < 2$
$x < 4$

	$(0,4)$	$(4,\infty)$
$1+\sqrt{x}$	+	+
$2-\sqrt{x}$	+	−
	⊕	

$D(f) = (0,4)$ ✓

k) $\log_3\left(\frac{2+\sqrt{x}}{2+x-x^2}\right)$ $2+x-x^2 \neq 0 \wedge x \geq 0 \wedge \frac{2+\sqrt{x}}{2+x-x^2} > 0$

$(x+1)(x+2) \neq 0$

$x_{1,2} \neq -1, 2$

	$(-\infty,-1)$	$(-1,2)$	$(2,\infty)$
$(x+1)(2-x)$	x	+	−
$2+\sqrt{x}$	x	+	+
		⊕	

$D(f) = \langle(0,2)$

l) $\arcsin(3x-5)$ $-1 \leq 3x-5 \leq 1$

$4 \leq 3x$ $3x \leq 6$

$x \geq \frac{4}{3}$ $x \leq 2$

$D(f) = (-\infty, 2\rangle \cup \langle \frac{4}{3}, \infty)$ ✗

m) $\arcsin\left(\frac{3}{x-2}\right) \Rightarrow -1 \leq \frac{3}{x-2} \leq 1 \wedge x \neq 2$

$2-x \leq 3$ $3 \leq x-2$

$-1 \leq x$ $x \geq 5$ $D(f) = \langle 5, \infty)$ ✗

h) $\arccos(x^2-2x)$ $-1 \leq x^2-2x \leq 1$ $x\cdot(x-2)$ $x_{1,2} = 0,2$

$x^2-2x+1 \geq 0$ $x^2-2x-1 \leq 0$

$(x-1)^2$ $4-4\cdot(-1) = \sqrt{8}$

$x_{1,2} = \pm 1$ $\frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

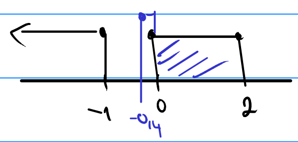
	$(-\infty,-1)$	$\langle -1,0\rangle$	$\langle 0,1\rangle$	$\langle 1,2)$	$\langle 2,\infty)$
x^2-2x	+	x	✓	✓	x

	$(-\infty, 1-\sqrt{2})$	$\langle 1-\sqrt{2}, 1+\sqrt{2}\rangle$		$\langle 1+\sqrt{2}, 2\rangle$	$(2,0)$
x^2-2x-1	✓	✓	✓ +	✓	x

$x \in (-\infty, -1\rangle \cup (0, 2\rangle$

$x \in \langle 1-\sqrt{2}, 0\rangle \cup \langle 1+\sqrt{2}, 2\rangle$

$D(f) = \langle 0, 2\rangle$ x ✗

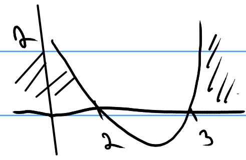


o) $\operatorname{arctg}\left(\sqrt{\frac{x^2+2x+3}{x-5}}\right)$ $\quad x^2+2x+3 \geq 0 \wedge x \neq 5 \quad D(f)=\mathbb{R}-\{5\}$

$4-4\cdot3\cdot1 = 4-12 = -8$ ✓

p) $\operatorname{arctg}\sqrt{\frac{x^2-5x+6}{x^2+x+1}}$ $\quad \frac{x^2-5x+6}{x^2+x+1} \geq 0 \wedge x^2+x+1 \neq 0 \quad (x-3)(x-2) \geq 0$

x^2+x+1 bude vždy kladné $\quad x_1 = 3, 2$



	$(-\infty,2)$	$\langle 2,3\rangle$	$\langle 3,\infty)$
$(x-3)$	$-$	$-$	$+$
$(x-2)$	$-$	$+$	$+$

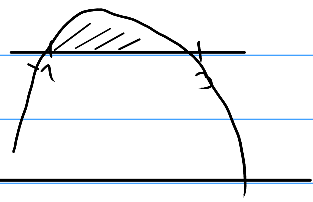
$D(f) = (-\infty,2\rangle \cup \langle 3,\infty)$ ✓

r) $\operatorname{arccotg}\frac{\sqrt{9-x^2}}{|x-1|}$ $\quad x \neq 1 \wedge 9-x^2 \geq 0 \quad D(f) = \langle -3,3\rangle - \{1\}$ ✓

$x^2 \leq 9$

$x \leq \pm 3$

s) $\operatorname{arctg}\frac{\sqrt{3+2x-x^2}}{x}$ $\quad x \neq 0 \wedge -x^2+2x+3 \geq 0$



$D(f) = \langle -1,3\rangle - \{0\}$ ✓ $\quad (x+1)(-x+3)$

$x_{1,2} = -1, 3$

PR 2.

a) $\frac{x-4}{\sqrt{x^2-x-2}}$ $\quad$ OR: $x^2-x-2 \neq 0 \quad D(f) = \mathbb{R}-\{-1,2\}$ ✗

$(x-2)(x+1) \quad x_{1,2} = 2,-1$

Parita: $f(-x) = \frac{-x-4}{\sqrt{x^2+x-2}} \neq f(x)$ ✗ Párna

Nepárna $-f(x) = f(-x)$ $\quad$ ✗ Nepárna ✓

$\frac{4-x}{\sqrt{x^2-x-2}} \neq \frac{-x-4}{\sqrt{x^2+x-2}}$

b) $= \frac{a^x+1}{a^x-1}$ $\quad a^x-1 \neq 0 \quad$ Parita: F-cia je Nepárna ✓

$a^x \neq 1 \quad$ Párna $\rightarrow f(x) = f(-x)$

$D(f)=\mathbb{R}-\{0\}$ ✓ $\quad x \neq 0 \quad \frac{a^x+1}{a^x-1} \neq \frac{\frac{1}{a^x}+1}{\frac{1}{a^x}-1} \Rightarrow \frac{\frac{1+a^x}{\cancel{a^x}}}{\frac{1-a^x}{\cancel{a^x}}} \Rightarrow \frac{1+a^x}{1-a^x}$

$-f(x) = f(-x)$

$\frac{a^x+1}{1-a^x} = \frac{1+a^x}{1-a^x}$

c) $1-\sqrt{2\cos(2x)}$ OR $2\cos(2x) \geq 0$ $D(f) = \langle -\frac{\pi}{4} + k\cdot\pi, \frac{\pi}{4} + k\cdot\pi, k \in \mathbb{Z} \rangle$

Parita: Párna ✓ $2\cos(2x) \geq \pi$

$f(x) = f(-x)$ $\cos(2x) \geq \frac{\pi}{2}$

$1-\sqrt{2\cos(2x)} = 1-\sqrt{2\cos(-2x)}$ ✓ $x \geq \frac{\pi}{4}$? X

d) $\ln\left(\frac{3+x}{3-x}\right)$ $\frac{3+x}{3-x} > 0$ $x_{1,2} = -3, 3$

	$(-\infty,-3)$	$(-3,3)$	$(3,\infty)$
$3+x$	$-$	$+$	$+$
$3-x$	$+$	$+$	$-$
		$\oplus$	

$D(f) = (-3,3)$ ✓

Parita: Nepárna ✓

Párna $f(x) = f(-x)$

$\ln\left(\frac{3+x}{3-x}\right) \neq \ln\left(\frac{3-x}{3+x}\right)^{-1}$

Nepárna $-\ln\left(\frac{3+x}{3-x}\right) = +\log_e\left(\frac{3+x}{3-x}\right)^{-1} \Rightarrow -\log_e\left(\frac{3+x}{3-x}\right)$ ✓

e) $f(x) = \log\left(\frac{x^2-2}{x}\right)$ $\frac{x^2-2}{x} > 0 \wedge x \neq 0$ Parita: X X Párna $f(x) = f(-x)$

$x\cdot(x-2) > 0 \Rightarrow x_{1,2} = 0, 2$

$\log\left(\frac{x^2-2}{x}\right) \neq \log\left(\frac{x^2-2}{-x}\right)$

$D(f) = (-\infty,-2) \cup (2,\infty)$ X

	$(-\infty,-2)$	$(-2,$	$2)$	$(2,\infty)$
x^2-2	$+$	$-$	$-$	$+$

NePá: $-\log\left(\frac{x^2-2}{x}\right) \neq +\log\left(\frac{x^2-2}{-x}\right)$

f) $\frac{x^3-x}{\sqrt{x^2-1}}$ OR: $x^2-1 \geq 0$ $D(f) = (-\infty,-1\rangle \cup \langle 1,\infty)$ Parita: Nepárna ✓

$x > \pm 1$

Párna $f(x) = f(-x)$

$\frac{x^3-x}{\sqrt{x^2-1}} \neq \frac{-x^3+x}{\sqrt{x^2-1}}$

Nepárnosť $-f(x) = +f(-x)$

$\frac{x-x^3}{\sqrt{x^2-1}} = \frac{x-x^3}{\sqrt{x^2-1}}$

g) $x\sqrt{6-2|x|}$ $6-2x \geq 0 \wedge 6+2x \geq 0$ $D(f) = (-\infty,-3\rangle \cup \langle 3,\infty)$ X

Parita: Nepárna ✓ $2x \leq 6$ $2x \geq -6$

Pár $\Rightarrow f(x) = f(-x)$ $x \leq 3$ $x \geq -3$

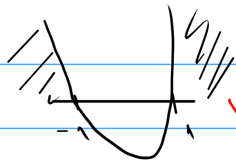
$x\sqrt{6-2|x|} \neq -x\sqrt{6-2|x|}$ X

NePár $\rightarrow -f(x) = f(-x)$

$-x\sqrt{6-2|x|} = -x\sqrt{6-2|x|}$ ✓

h) $\frac{\sqrt{x^2-1}}{|3x|}$ $\quad x \neq 0 \wedge x^2-1 \geq 0$

$x \geq \pm 1$



$\checkmark$ $D(f) = (-\infty,-1\rangle \cup \langle 1,\infty)$

Parita: Párna $\checkmark$

Pár $\rightarrow f(x) = f(-x)$

$\frac{\sqrt{x^2-1}}{|3x|} = \frac{\sqrt{x^2-1}}{|3x|}$ $\checkmark$

i) $\frac{|x|}{4-\sqrt{x^2-9}}$ $\quad x^2-9 \geq 0 \quad 4-\sqrt{x^2-9} \neq 0$ $\checkmark$ $D(f) = (-\infty,-3\rangle \cup \langle 3,\infty) - \{\pm 5\}$

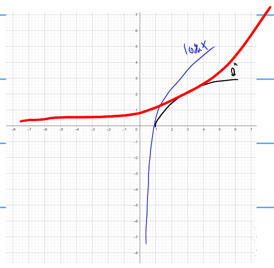
$x \geq \pm 3 \quad 16 \neq x^2-9$ Parita: Párna $\checkmark$

$x \neq \pm 5$

PR 3.

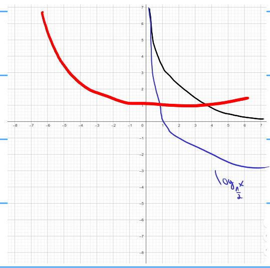
a) $y = 2^x$ ✗

$y = \log_2 x$



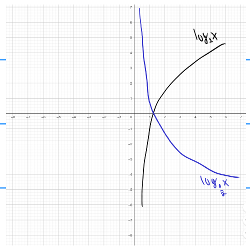
$y = \left(\frac{1}{2}\right)^x$ ✗

b) $y = \log_{\frac{1}{2}} x$



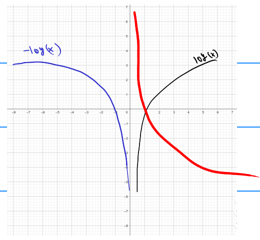
c) $y = \log_2 x$

$y = \log_{\frac{1}{2}} x$



d) $y = \log x$

$y = -\log x$ ✗

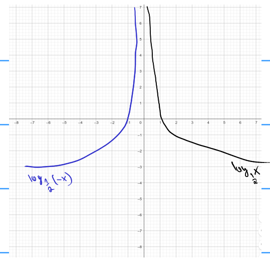


e) $y = \log(x-1) \Rightarrow x = +1$

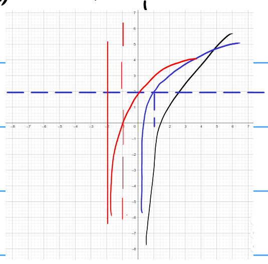
$y = |\log(x-1)|$

g) $y = -\log_{\frac{1}{2}} x$

$y = \log_{\frac{1}{2}}(-x)$



i) $y = 2\ln(x)$, $y = \ln(x)+2$, $y = \ln(x+2)$



PR 4.

a) $y = \sqrt{1-\log_2(x-1)}$ $\quad x-1>0 \wedge 1-\log_2(x-1)\geq 0 \quad 2 \geq x-1$

$x>1 \quad 1 \geq \log_2(x-1) \quad x\leq 3 \quad D(f) = (1,3\rangle$ ✓

$y^{-1} \Rightarrow x = \sqrt{1-\log_2(y-1)}$ ✓

$x^2 = 1-\log_2(y-1)$

$x^2-1 = -\log_2(y-1)$

$1-x^2 = \log_2(y-1)$

$2^{1-x^2} = y-1$

$2^{1-x^2}+1 = y$

b) $y = 3\sqrt{x}-5 \quad D(f) = \langle 0,\infty)$ ✓

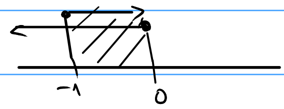
$y^{-1} \Rightarrow x = 3\sqrt{y}-5$

$x+5 = 3\sqrt{y}$

$x^2+10x+25 = 9y$

$\frac{x^2+10x+25}{9} = y$ ✓

c) $3+\arcsin(2x+1) \quad -1 \leq 2x+1 \leq 1$



$2x+1\geq -1 \quad 2x+1 \leq 1$

$2x \geq -2 \quad 2x\leq 0 \quad D(f)=\langle -1,0\rangle$ ✓

$x\geq -1 \quad x \leq 0$

$x = 3+\arcsin(2y+1)$

$x-3 = \frac{1}{\sqrt{1-(2y+1)^2}}$

$(x-3)^2 = \frac{1}{1-(4y^2+4y+1)}$

$(x-3)^2 = \frac{1}{-4y^2+4y}$

$-4y^2+4y = \frac{1}{(x-3)^2}$

$4y(-y+1) = \frac{1}{(x-3)^2}$

toto NWM ?

PR 5. $y = |x|\cdot\sqrt{\frac{x^2-4}{4-x^2}}$ $\quad x \neq \pm 2$

$\frac{x^2-4}{4-x^2}\geq 0 \wedge \overbrace{\frac{x^2-4}{x^2-4}}^{1} \geq 0$ ✓ ?

	$(-\infty,-2)$	$\langle -2,2\rangle$	$(2,\infty)$
x^2-4	+	−	+
$4-x^2$	−	+	−

DÚ 2.

PR 1.

a) $(x^3-1):(x+1) = x^2 - x + 1 - \frac{2}{x+1}$

$-(x^3+x^2)$

$\cancel{x^2}-1$

$-(\cancel{x^2}-x)$

$x-1$

$-(x+1)$

-2

$-(-2)$

b) $(x^3-1):(x-1) = x^2+x+1$

$-(x^3-x^2)$

x^2-1

$-(x^2-x)$

$x-1$

c) $(3x^3+5x^2+7x+5):(x+1) = 3x^2+2x+5$

$-(3x^3+3x^2)$

$2x^2+7x$

$-(2x^2+2x)$

$5x+5$

$-(5x+5)$

0

d) $(x^5+x^4+2x^3-3x^2-7):(x+4) = x^4 - 3x^3 + 14x^2 - 59x + 236 - \frac{951}{x+4}$

$-(x^5+4x^4)$

$-3x^4+2x^3$

$-(-3x^4-12x^3)$

$14x^3-3x^2$

$-(14x^3+56x^2)$

$-59x^2-7$

$-(-59x^2-236x)$

$236x-7$

$-(236x+944)$

-951

PR 2.

a) $x^3 - x^2 - 8x + 12 = 0$

	1	-1	-8	12
-3	1	-4	4	0

$(x+3)(x^2-4x+4) \Rightarrow (x+3)(x-2)^2$

b) $x^3 - 6x^2 + 8x - 4 = 0$

	1	-5	8	-4
2	1	-3	2	0

$(x-2)(x^2-3x+2) \Rightarrow (x-2)^2(x-1)$

c) $x^4 - 4x^3 + 16x - 16 = 0$ $\quad 16 \to 2, 4, 8$

1	-4	16	-16
1			

Vysledi X

d) $x^4 + 6x^3 + 14x^2 + 18x + 9 = 0$ $\quad (x+1)(x^3 + 5x^2 + 9x + 9) = 0$

	1	6	14	18	9
-1	1	5	9	9	0

	1	5	9	9
-3	1	2	3	0

$(x+1)(x+3)(x^2+2x+3)$ ✓

PR 3.

a) $\dfrac{2x-5}{x^2-5x+6} = \quad \Rightarrow$

$\dfrac{2x-5}{\cancel{(x-2)(x-3)}} = \dfrac{A(x-3)-B(x-2)}{\cancel{(x-2)(x-3)}}$

$2x - 5 = Ax - 5A - Bx + 2B$ $\quad 2 = A - B \Rightarrow B = A - 2$

$2x - 5 = x(A-B) + 2B - 5A$ $\quad -5 = 2B - 3A$

$2x - 5 = Ax - 3A - Bx + 2B$ $\quad -5 = 2A - 4 - 3A$

$2x - 5 = x(A-B) + 2B - 3A$ $\quad +1 = +A$

$\left[\dfrac{1}{x-2} + \dfrac{1}{x-3}\right]$ ✓ $\quad B = 1 - 2 = 1$

b) $\dfrac{0x + 5}{(x-2)(x-7)}$

$0x + 5 = A(x-7) + B(x-2)$ $\quad 0 = A + B \Rightarrow -A = B$

$0x + 5 = Ax - 7A + Bx - 2B$ $\quad 5 = -2B - 7A$

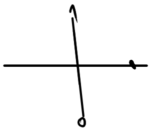
$0x + 5 = x(A+B) - 2B - 7A$ $\quad 5 = -2\cdot(-A) - 7A = -5A$

$\left[\dfrac{-1}{(x-7)} + \dfrac{1}{(x-2)}\right]$ X $\quad A = -1$

$B = 1$

c) $\dfrac{1}{x^3 - x} \Rightarrow \dfrac{0x^2 + 0x + 1}{x(x^2-1)} \Rightarrow \dfrac{A}{x} + \dfrac{B}{(x^2-1)}$

$1 = A(x^2-1) + Bx$ Musim si pozriet na YT



	1	-2	-5	6
-2	1	-4	3	0

a) $\lim\limits_{x\to 1} \frac{x^3-2x^2-5x+6}{x^2+2x-3} = \lim\limits_{x\to 1} \frac{(x+2)(x^2-4x+3)}{(x+3)(x-1)} = \lim\limits_{x\to 1} \frac{(x+2)\cancel{(x-1)}(x-3)}{(x+3)\cancel{(x-1)}} = \frac{3\cdot(-2)}{4} = \frac{-3}{2}$ ✓

b) $\lim\limits_{x\to 0} \left[\frac{\sqrt{x+4}-2}{\sin(2x)} + \ln(1-x^2)\right] = \lim\limits_{x\to 0}\left(\frac{\sqrt{x+4}-2}{\sin(2x)}\right) + \overbrace{\lim\limits_{x\to 0}\left(\ln(1-x^2)\right)}^{0}$

$\lim\limits_{x\to 0}\left(\frac{\sqrt{x+4}-2}{2\sin(x)\cdot\cos(x)}\right) = \lim\limits_{x\to 0} \frac{\frac{d}{x}\left((x+4)^{\frac{1}{2}}-2\right)}{\frac{d}{x}\left(\sin(2x)\right)} = \lim\limits_{x\to 0} \frac{\left(\frac{1}{2\sqrt{x+4}}\right)}{\frac{\cos(2x)\cdot 2}{1}} = \lim\limits_{x\to 0} \frac{1}{2\cos(2x)\cdot 2\sqrt{x+4}}$

$= \frac{1}{2\cdot 2\sqrt{4}} = \frac{1}{2\cdot 4} = \frac{1}{8}$ ✓

c) $\lim\limits_{x\to 0} \frac{tg(5x)}{tg(6x)} = \lim\limits_{x\to 0} \frac{\frac{\sin(5x)}{\cos(5x)}}{\frac{\sin(6x)}{\cos(6x)}} = \lim\limits_{x\to 0} \frac{\sin(5x)\cdot\cos(6x)}{\cos(5x)\cdot\sin(6x)}$

$= \lim\limits_{x\to 0} \frac{5\cancel{\sin x\cdot\cos x}\cdot(\cos^6(x)-\sin^6(x))}{6\cancel{\sin x\cdot\cos x}\cdot(\cos^5(x)-\sin^5(x))} = \frac{5}{6}\cdot\frac{1^6-0^6}{1^5-0^5} = \frac{5}{6}$ ✓

d) $\lim\limits_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1} = \lim\limits_{x\to\infty}\left(\frac{x+1+2-2}{x-2}\right)^{2x-1} = \lim\limits_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x-1}$ $\quad t\to\infty \quad t = x-2 \quad x = t+2$

$\lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t+4-1} \Rightarrow \lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t+3} = \lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t}\cdot\lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{3} = e^6\cdot 1$ ✓

e) $\lim\limits_{x\to 0}\left(\frac{3-2x}{2+5x}\right)^{\frac{\sqrt{x+1}-1}{x}} = \lim\limits_{x\to 0}\left(\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right) = \lim\limits_{x\to 0}\left(\frac{x+1-1}{x\sqrt{x+1}+x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right) =$

$\lim\limits_{x\to 0}\left(\frac{1}{1\cdot\sqrt{\frac{1}{x}+1}+1}\right) = \lim\limits_{x\to 0}\left(\frac{1}{1\cdot 1+1}\right) = \frac{1}{2}$

$\left(\frac{\lim\limits_{x\to 0}(3-2x)}{\lim\limits_{x\to 0}(2+5x)}\right)^{\frac{1}{2}} = \left(\frac{3}{2}\right)^{\frac{1}{2}} = \sqrt{\frac{3}{2}}$ ✓

h) $\lim\limits_{x\to\infty}\left(\frac{x^2+3x-5}{2x^3-4x+1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right) = \lim\limits_{x\to\infty}\left(\frac{\frac{1}{x}+\frac{3}{x^2}-\frac{5}{x^3}}{2-\frac{4}{x^2}+\frac{1}{x^3}}\right) =$

$\left(\frac{0+0+0}{2-0+0}\right) = \underline{\underline{0}}$ ✓

i) $\lim\limits_{x\to-\infty}\left(\frac{4x^3-2x^2+7}{7x^3-3x^2-6x+9}\right) = \frac{4}{7}$ ✓

f) $\lim\limits_{x\to\infty}\left(-4x^5+5x^3-7x+10\right) \Rightarrow (-\infty)$ ✓

g) $\lim\limits_{x\to-\infty}\left(-4x^5+5x^3-7x+10\right) \Rightarrow (\infty)$ ✓

j) $\lim\limits_{x\to\infty} \frac{x^5-3x^2+2x-1}{2x^3-x^2+x-1} \Rightarrow (\infty)$ ✓

k) $\lim\limits_{x\to 0}\left(2^{\cot g(x)}-1\right) =$

↳ $\lim\limits_{x\to 0}\left(-\frac{1}{\sin^2(x)}\right)$ $\nexists$

n) $\lim\limits_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{\ln(x)}} = \left(\frac{0}{1}\right) =$?

l) $2^{-\lim\limits_{x\to\frac{\pi}{4}}\left(\frac{1}{\sin^2(x)}\right)} - 1 = 2^{1}-1 = 1$ ✓

p) $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{2x-1} = \lim\limits_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x-1}$ $\quad t\to\infty \quad t = 2x-2 \quad \frac{1}{2}(t+2) = x$

$\lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{t+1} = \lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)^{t}\cdot\underbrace{\lim\limits_{t\to\infty}\left(1+\frac{3}{t}\right)}_{1}$

$= \underline{\underline{e^3}}$ ✓

m) $\lim\limits_{x\to\frac{\pi}{2}}\left(2^{\cot g(x)}-1\right) = 2^0-1 = \underline{0}$ ✓

PQ 2.

$\lim_{x\to 0}\left(\frac{1}{x}\right) \nexists$

$\lim_{x\to 0}\frac{\frac{d}{x}(-\cos x)}{\frac{d}{x}(x)} = \frac{\sin(x)}{1} = 0$ ✗

PQ 3.

$\lim_{x\to\infty}\left(\frac{x+5}{x-3}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right) = \lim_{x\to\infty}\left(\frac{1+\frac{5}{x}}{1-\frac{3}{x}}\right) = \underline{1}$ ✓

$\lim_{x\to -\infty}\left(\frac{1+0}{1-0}\right) = \underline{1}$ ✓

$\lim_{x\to 3^+}\left(\frac{x+5}{x-3}\right) = \frac{8}{0^+} = \infty$ ✓

$\lim_{x\to 3^-}\left(\frac{x+5}{x-3}\right) = \frac{8}{0^-} = -\infty$ ✓

PQ 5.

a) $\lim_{x\to 2}\frac{x^3-4x^2+x+6}{-x^2+3x-2}$

$-x^2+3x-2 \Rightarrow (x-2)(-x+1)$ $\rightarrow$ $\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2-2x-3)}{\cancel{(x-2)}(1-x)} = \frac{-3}{-1} = \underline{3}$ ✓

	1	-4	1	6
2	1	-2	-3	0

b) $a=0$

$\lim_{x\to 0^-}\left(p\cdot\left(\frac{\sin(2x)}{x}\right)\right) = \lim_{x\to 0^+}\left(\frac{8-x}{p}\right)$ $\quad 2p = \frac{8}{p}$ $\quad 2p^2 = 8$ $\quad p^2 = 4$ $\quad \underline{\underline{p = \pm 2}}$ ✓

c) $a=1$

$\lim_{x\to 1^-}\left(2x+\frac{p}{2}\right) = \lim_{x\to 1^+}\left(-x+\frac{5p}{4}\right)$ $\quad 8+2p = 5p-4$ $\quad \underline{\underline{p=4}}$ ✓

$\frac{4+p}{2} = \frac{5p-4}{4}$ $\quad 3p = 12$

d) $a=1$ ✗ Neședi

$\lim_{x\to 1^-}(px^2) = \lim_{x\to 1^+}\left(\frac{6}{p}-\frac{px}{2}\right)$ $\quad p^2\cdot 2p = 12-p^2$ $\quad p = \sqrt[3]{3}$

$p^2 = \frac{6}{p}-\frac{p}{2} \quad /\cdot 2p$ $\quad 2p^2\cdot 2p = 12$

$2p^3 = 6$

$p^3 = 3$

PQ 4.

a) $\lim_{x\to 1}\left(\frac{x^3-1}{1-x}\right) = \lim_{x\to 1}\left(\frac{(x-1)(x^2-x+1)}{(1-x)}\right) = (-1)\cdot\lim_{x\to 1}\left(\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}\right)$

$= -(1+1+1) = -3$ ✓

b) $a=0$

$\lim_{x\to 0^-}\left(x\cdot\operatorname{arctg}\left(\frac{1}{x}\right)\right) \Rightarrow x\cdot\operatorname{arctg}(\infty)$

Cvika 3 derivácie

PR 1.

a) $y = 7x^4 - 12x^3 + 2\sqrt{x}$

$y' = 28x^3 - 36x^2 + \frac{1}{\sqrt{x}}$

b) $y = (x^2 - 2x + 5)(3x - 2)$

$y' = (2x - 2)(3x - 2) + 3\cdot(x^2 - 2x + 5) = 6x^2 - 10x + 4 + 3x^2 - 6x + 15 = 9x^2 - 16x + 19$

c) $y = \frac{1-x}{1+x}$ $\quad y' = \frac{-1(1+x) - (1-x)}{x^2+2x+1} = \frac{-1 \cancel{-x} - 1 \cancel{+x}}{x^2+2x+1} = \frac{-2}{(1+x)^2}$

d) $y = tg(x) - 3x\log_4 x$ $\quad y' = \frac{1}{\cos^2(x)} - 3\cdot\log_4 x - \frac{3\cancel{x}}{\cancel{x}\cdot\ln(4)}$

e) $y = 4\cdot 3^x \cdot 2^{-x}$ $\quad y' = 4\cdot\frac{3^x}{2^x} = 4\cdot\left(\frac{3}{2}\right)^x \Rightarrow 4\cdot\left(\frac{3}{2}\right)^x \cdot \ln\left(\frac{3}{2}\right)$

f) $y = e^{\sin^2(x^3)}$ $\quad y' = e^{\sin^2(x^3)}\cdot 2\sin(x^3)\cdot\cos(x^3)\cdot 3x^2 = 6x^2\cdot e^{\sin^2(x^3)}$.

g) $y = \sin(\cos(2x))$ $\quad y' = \cos(\cos(2x))\cdot(-\sin(2x))\cdot 2 = -2\cos(\cos(2x))\cdot\sin(2x)$

h) $y = \ln(\sqrt{\cos x})$ $\quad y' = \frac{1}{\cos(x)^{\frac{1}{2}}}\cdot\frac{1}{2\sqrt{\cos x}}\cdot(-\sin x) = \frac{-\sin(x)}{2\cos x} =$

i) $y = 2^{tg(x)}$ $\Rightarrow$ $y' = 2^{tg(x)}\cdot\ln(2)\cdot\frac{1}{\cos^2(x)} = \frac{2^{tg(x)}\cdot\ln(2)}{\cos^2(x)}$

X j) $y = \text{arccotg}(x^{\frac{1}{3}}) = -\frac{1}{1+x^{\frac{2}{3}}}\cdot\frac{1}{3}x^{-\frac{2}{3}} = -\frac{1}{1+x^{\frac{2}{3}}}\cdot\frac{1}{3\sqrt[3]{x^2}} = -\frac{1}{(1+\sqrt[3]{x^2})\cdot 3\sqrt[3]{x^2}}$

j) k) l)

p)

m) $y = 10^{\sqrt{x}}\cdot x$ $\quad y' = 10^{\sqrt{x}}\cdot\ln(10)\cdot\frac{1}{2\sqrt{x}}\cdot x + 10^{\sqrt{x}} = 10^{\sqrt{x}}\left(\frac{\ln(10)\sqrt{x}}{2} + 1\right)$

n) $y = 3^{\cot(x)}\cdot\arcsin$

$y' = 3^{\cot g(x)}\cdot\ln(3)\cdot\left(-\frac{1}{\sin^2(x)}\right)\cdot\arcsin + 3^{\cot g(x)}\cdot\frac{1}{\sqrt{1-x^2}} = 3^{\cot g(x)}\left(-\frac{\ln(3)\cdot\arcsin}{\sin^2(x)} + \frac{1}{\sqrt{1-x^2}}\right)$

o) $y = \log_5(tg\,x^3)$ $\quad y' = \frac{1}{tg(x^3)\cdot\ln(5)}\cdot\frac{1}{\cos^2(x^3)}\cdot 3x^2 = \frac{3x^2}{tg(x^3)\cdot\ln(5)\cdot\cos^2(x^3)}$

r) $y = (\ln(x))^x$ $\quad y' = (\ln(x))^x\cdot\ln(\ln(x))\cdot\frac{1}{x}$?

s) $y = (3x)^{\sin(x)}$ $\quad y' = (3x)^{\sin(x)}\cdot\ln(\sin(x))\cdot\cos(x)$?

PR 2.

a) $y = x^{-1}$ $\quad y' = -1\cdot x^{-2}$

$f'(a) = -1\cdot\frac{1}{x^2}$ $\quad a = 2 = -\frac{1}{4}$

b) $a = 4$ $\quad f(x) = (x+4)^{\frac{1}{3}}$ $\quad f'(x) = \frac{1}{3}\cdot(x+4)^{-\frac{2}{3}} = \frac{1}{3\cdot\sqrt[3]{(x+4)^2}}$

$f'(a) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\cdot 4} = \frac{1}{12}$

c) $a = 0$ $\quad f(x) = x\cdot\sin(x)$ $\quad f'(x) = \sin(x) + x\cdot\cos x$

$f_2(x) = x^2$ $\quad f'(a) = \underbrace{\sin(0)}_{0} + 0\cdot\ldots = 0$

$f_2'(x) = 2x$

$f_2'(a) = 2\cdot 0 = 0$

d) $a = 2$ $\quad f(x) = |3x-6|$

$f'(x) = \left(\sqrt{(3x-6)^2}\right)' = \frac{1}{2\sqrt{(3x-6)^2}} \cdot 2(3x-6)\cdot 3 = \frac{6(3x-6)}{2\sqrt{(3x-6)^2}} =$

$\frac{9x-18}{|3x-6|}$ $\quad f'(2) = 0 \leftarrow$ ✗ $\quad f'(x) \nexists$

e) $a = 2$ $\quad f(x) = (4x+1)^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2\sqrt{4x+1}} \cdot 4 = \frac{2}{\sqrt{4x+1}}$

$f'(a) = \frac{2}{3}$

f) $a = 3$ $\quad f(x) = |x-3|$ $\quad f'(a) = \nexists$

DR 3.

a) $t: y - y_0 = f'(x_0)(x-x_0)$ $\quad f(x) = e^{-x} \cdot \cos(2x)$ $\quad A = [0;?] \Rightarrow A[0,1]$

$n: y - y_0 = -\frac{1}{f'(x_0)}(x-x_0)$ $\quad f(0) = e^0 \cdot \cos(2\cdot 0) = 1\cdot 1 = 1 \Rightarrow x_0 = 0, y_0 = 1$

$t: y - 1 = -1(x-0)$ $\quad f'(x) = e^{-x}\cdot \ln(e)\cdot(-1)\cdot\cos(2x) + e^{-x}\cdot(-\sin(2x))\cdot 2$

$n: y - 1 = -\frac{1}{-1}(x-0)$ $\quad f'(x) = e^{-x}\cdot(-\cos(2x) - 2\sin(2x))$

$t: y = 1 - x$ $\quad f'(x) = -\frac{\cos(2x) + 2\sin(2x)}{e^x}$

$n: y = x+1$ $\quad f'(0) = -\frac{1+2\cdot 0}{1} = -1$

b) $f(x) = \frac{3x-4}{2x-3}$ $\quad A[2, y_0]$

$f'(x) = \frac{3\cdot(2x-3) - 2\cdot(3x-4)}{(2x-3)^2} =$

$y_0 = f(2)$

$y_0 = \frac{6-4}{4-3} = 2 \Rightarrow A[2,2]$ $\quad = \frac{\cancel{6x} - 9 \cancel{-6x} + 8}{(2x-3)^2} = \frac{-1}{(2x-3)^2}$

$t: y - y_0 = f'(x_0)(x-x_0)$ $\quad f'(x_0) = -\frac{1}{(2\cdot 2-3)^2} = -\frac{1}{1} = -1$

$n: y - y_0 = \frac{-1}{f'(x_0)}(x-x_0)$

$t: y - 2 = -1(x-2) \Rightarrow y = 2 - x + 2$

$n: y - 2 = \frac{-1}{-1}(x-2) \Rightarrow y = 2 + x - 2$

$t: \underline{\underline{y = 4 - x}}$

$n: \underline{\underline{y = x}}$

c) $f(x) = e^{1-x^2}$ $p: y = 1$ $A[x_0, 1] \Rightarrow A[(-1, +1), 1]$

$1 = e^{1-x^2}$

$\cancel{e}^0 = \cancel{e}^{1-x^2}$

$x = \pm 1$

$f'(x) = e^{1-x^2} \cdot (-2x) = -2x \cdot e^{1-x^2}$

$f'(-1) = 2 \cdot e^0 = 2$

$f'(+1) = -2 \cdot e^{1-1} = -2$

$t: y = y_0 + f'(x_{0,1})(x - (x_{0,1}))$

$n: y = y_0 - \frac{1}{f'(x_{0,1})}(x - (x_{0,1}))$

$t_1: y = 1 + 2 \cdot (x - (-1)) \Rightarrow$

$n_1: y = 1 - \frac{1}{2}(x - (-1)) \Rightarrow$

$y = 1 + 2x + 2$ $t_1: y = 3 + 2x$

$y = 1 - \frac{1}{2}x - \frac{1}{2}$ $n_1: y = \frac{1}{2} - \frac{1}{2}x$

$t_2: y = 1 - 2(x - 1)$

$n_2: y = 1 - \frac{1}{-2}(x - 1)$

$t_2: y = 1 - 2x + 2$

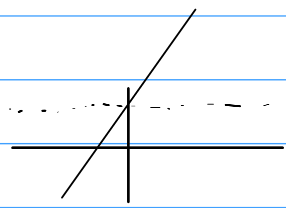
$n_2: y = 1 + \frac{1}{2}x - \frac{1}{2}$

$t_2: y = 3 - 2x$

$n_2: y = \frac{1}{2}x + \frac{1}{2}$

d) $f(x) = x^2 - 2x + 3$ $t \parallel p: 3x - y + 5 = 0 \Rightarrow y = 3x + 5$

$k_p = 3$



$f'(x) = 2x - 2$

$2x_0 - 2 = 3$

$2x_0 = 5$

$x_0 = \frac{5}{2}$

$A[\frac{5}{2}, \frac{17}{4}]$

$(\frac{5}{2})^2 - 2\frac{5}{2} + 3 = \frac{25}{4} - 5 + 3$

$\frac{25}{4} - 2 = \frac{17}{4}$

$t: y = 3x - \frac{13}{4}$

$n: y = -\frac{1}{3}x + \frac{51 + 10}{12} = -\frac{1}{3}x + \frac{61}{12}$

$t: y = y_0 + f'(x_0)(x - x_0)$

$n: y = y_0 - \frac{1}{f'(x_0)}(x - x_0)$

$t: y = \frac{17}{4} + 3(x - \frac{5}{2})$

$n: y = \frac{17}{4} - \frac{1}{3}(x - \frac{5}{2})$

$t: y = \frac{17}{4} + 3x - \frac{30}{4}$

$n: y = \frac{17}{4} - \frac{1}{3}x + \frac{5}{6}$

e) $f(x) = \ln(x)$ $t \perp p: x + 2y - 2 = 0$

$f'(x) = \frac{1}{x}$

$2y = 2 - x$

$\frac{1}{x_0} = \frac{1}{2}$

$y = 1 - \frac{1}{2}x$

$\frac{2}{x_0} = 1$

$k_t = \frac{1}{2}$

$2 = x_0$

$A[+2, \ln(2)]$

?

$t: y = y_0 + f'(x_0)(x - x_0)$

$n: y = y_0 - \frac{1}{f'(x_0)}(x - x_0)$

$y - \ln(2) = \frac{1}{2}(x - 2)$

$y - \ln(2) = -\frac{1}{\frac{1}{2}}(x - 2)$

$y = \ln(2) + \frac{1}{2}x$

5 + 5 + 1 + 5 = 16

Skušobný test

1 - 2017 17:10 štart

1) $f(x) = \sqrt{2 - \ln(3-x)}$

$2 - \ln(3-x) \geq 0 \wedge 3 - x > 0$

$2 \geq \ln(3-x)$ $\quad x < 3$

$\log_e e^2 \geq \ln(3-x)$ $\quad D(f) = \langle 3 - e^2, 3)$ ✓

$e^2 \geq 3 - x$

$x \geq 3 - e^2$

$x \geq 3 - 2{,}71^2$

2.) $\lim\limits_{x \to 0} \left(\frac{\sin(3x)}{\sqrt{x+1} - 1} + \ln(1+x) \right)$

$\lim\limits_{x \to 0} \left(\frac{\sin(3x)}{\sqrt{x+1} - 1} \right) + \lim\limits_{x \to 0} \left(\ln(1+x) \right) = \lim\limits_{x \to 0} \left(\frac{\sin(3x)}{\sqrt{x+1} - 1} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right) + \ln(1) =$

$= \lim\limits_{x \to 0} \frac{\sin(3x)(\sqrt{x+1} + 1)}{x + 1 - 1} = \lim\limits_{x \to 0} \frac{+3\cos(3x)(\sqrt{x+1} + 1) + \sin(3x)(\sqrt{x+1} + 1)'}{1} =$

$\frac{-3 \cdot 2 + 0 \cdot (\sqrt{x+1} + 1)'}{1} = +6$

3.)

a) $f(x) = 2e^{\tan^2(x)}$

$f'(x) = 2e^{\tan^2(x)} \cdot \ln(e) \cdot 2\tan(x) \cdot \frac{1}{\cos^2 x}$

$f'(x) = \frac{2e^{\tan^2(x)} \cdot 2\tan(x)}{\frac{\cos^2 x}{1}} = \frac{\frac{2\sin(x) \cdot 2e^{\tan^2(x)}}{\cos x}}{\frac{\cos^2 x}{1}} = \frac{4\sin(x) \cdot e^{\tan^2(x)}}{\cos(x)^3}$

b) $h(x) = (2x)^{\cos(x)}$

$h'(x) = (2x)^{\cos(x)} \cdot \ln(2x) \cdot (-\sin(x)) =$

PR 4. $f(x) = -\left(\frac{x^2}{4}\right) + \frac{4}{9}$ $\quad t \perp p: y = 3x + 1$

$f'(x) = -\left(\frac{2x \cdot 4 - 0 \cdot x^2}{16}\right) + 0$ $\quad k_p = 3$

$f'(x) = -\frac{8x}{16} = -\frac{1}{2}x$

$f(x_0) = -\left(\frac{(-6)^2}{4}\right) + \frac{4}{9} = -9 + \frac{4}{9} = \frac{4}{9} - \frac{81}{9} = -\frac{77}{9}$

$3 = -\frac{1}{2}x_0 \quad | \cdot (-2)$ $\quad A\left[-6, -\frac{77}{9}\right]$

$x_0 = -6$

$t: y = y_0 + f'(x)(x - x_0)$ $\quad t: y = \frac{-77}{9} + 3x + 18$ $\quad 90 + 72 = 162 - 77$

$n: y = y_0 - \frac{1}{f'(x)}(x - x_0)$ $\quad n: y = \frac{-77}{9} - \frac{1}{3}x - \frac{18}{9}$

$t: y = -\frac{77}{9} + 3(x+6)$ $\quad t: y = \frac{85}{9} + 3x$

$n: y = -\frac{77}{9} - \frac{1}{3}(x+6)$ $\quad n: y = -\frac{95}{9} - \frac{1}{3}x$

$$\lim_{x \to -\infty} \left(\left(\frac{2x+3}{2x-1}\right)^{2x+1} + \left(\overset{\overset{0}{\downarrow}}{3^{2x}}\right) \right)$$

$$\left(\frac{2x+3+1-1}{2x-1}\right)^{2x+1} \Rightarrow \left(1 + \frac{4}{2x-1}\right)^{2x+1}$$

$$t = 2x-1$$

$$x = \frac{1}{2}(t+1)$$

$$\lim_{t \to -\infty} \left(1+\frac{4}{t}\right)^{t+2}$$

$$\left(1+\frac{4}{t}\right)^{t} \cdot \overbrace{\left(1+\frac{4}{t}\right)^{2}}^{1}$$

$$e^4$$