PR 2.

a) $A = \begin{bmatrix} 2 & 4 & 3 \\ 3 & -1 & 6 \\ 5 & -2 & 4 \end{bmatrix}$

$M_{11} = (-1)\cdot 4 - 6\cdot(-2) = 8$ $\quad c_{11} = (-1)^{1+1} = +1$

$M_{12} = 3\cdot 4 - 5\cdot 6 = -18$ $\quad c_{12} = (-1)^{1+2} = -1$

$M_{13} = 3\cdot(-2) - (-1)\cdot 5 = -1$ $\quad c_{13} = (-1)^{1+3} = +1$

$M_{21} = 4\cdot 4 - 3\cdot(-2) = 22$ $\quad c_{21} = (-1)^{2+1} = -1$ $\quad M_{31} = 4\cdot 6 - 3\cdot(-1) = 27$ $\quad c_{31} = (-1)^{3+1} = +1$

$M_{22} = 2\cdot 4 - 5\cdot 3 = -7$ $\quad c_{22} = (-1)^{2+2} = +1$ $\quad M_{32} = 2\cdot 6 - 3\cdot 3 = 3$ $\quad c_{32} = (-1)^{3+2} = -1$

$M_{23} = 2\cdot(-2) - 5\cdot 4 = -24$ $\quad c_{23} = (-1)^{2+3} = -1$ $\quad M_{33} = 2\cdot(-1) - 4\cdot 3 = -14$ $\quad c_{33} = (-1)^{3+3} = +1$

$8 + 18 - 1 - 22 - 7 + 24 + 27 - 3 - 14 = 30$

b) $B = \begin{bmatrix} 0 & 2 & -3 & 1 \\ 1 & 4 & 2 & -1 \\ 3 & -2 & 4 & 0 \\ 4 & -1 & 1 & 0 \end{bmatrix}$

$c_{41} = (-1)^{4+1} = -1$ $\quad c_{12} = (-1)^{1+2} = -1$

$c_{42} = (-1)^{4+2} = +1$ $\quad c_{24} = (-1)^{2+4} = +1$

$c_{43} = (-1)^{4+3} = -1$ $\quad c_{31} = (-1)^{3+1} = +1$

$c_{44} = (-1)^{4+4} = +1$ $\quad c_{33} = (-1)^{3+3} = +1$

$M_{12} = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 3 & 4 & 0 & 3 & 4 \\ 4 & 1 & 0 & 4 & 1 \end{array}\right| = (0 + 0 - 3) - (-8 + 0 + 0) = \underline{\underline{5}}$

$M_{24} = \left|\begin{array}{ccc|cc} 0 & 2 & -3 & 0 & 2 \\ 3 & -2 & 4 & 3 & -2 \\ 4 & -1 & 1 & 4 & -1 \end{array}\right| = (0 + 8\cdot 4 + 9) - (24 + 0 + 0) = 41 - 24 = \underline{\underline{17}}$

$M_{31} = \left|\begin{array}{ccc|cc} 2 & -3 & 1 & 2 & -3 \\ 4 & 2 & -1 & 4 & 2 \\ -1 & 1 & 0 & -1 & 1 \end{array}\right| = (0 - 3 + 4) - (-2 - 2 + 0) = 1 + 4 = \underline{\underline{5}}$

$M_{34} = \left|\begin{array}{ccc|cc} 0 & 2 & -3 & 0 & 2 \\ 1 & 4 & 2 & 1 & 4 \\ 4 & -1 & 1 & 4 & -1 \end{array}\right| = (0 + 16 + 3) - (16\cdot(-3) + 0 + 2) = 19 + 46 = \underline{\underline{65}}$

$-4\cdot \begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & -1 \\ -2 & 4 & 0 \end{bmatrix} - 1 \begin{bmatrix} 0 & -3 & 1 \\ 1 & 2 & -1 \\ 3 & 4 & 0 \end{bmatrix} - 1 \begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & -1 \\ 3 & -2 & 0 \end{bmatrix} + 0\cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -4\cdot 22 - 7 + 20 = -75 \Rightarrow$ Determinant

$\begin{array}{ccc|cc} 2 & -3 & 1 & 2 & -3 \\ 4 & 2 & -1 & 4 & 2 \\ -2 & 4 & 0 & -2 & 4 \end{array}$

$(2\cdot 2\cdot 0 + (-3)\cdot(-1)\cdot(-2) + 1\cdot 4\cdot 4) - ((-2)\cdot 2\cdot 1 + 4\cdot(-1)\cdot 2 + 0\cdot x x)$

$(0 - 6 + 16) - (-4 - 8 + 0) = 10 + 12 = \underline{\underline{22}}$

$\begin{array}{ccc|cc} 0 & -3 & 1 & 0 & -3 \\ 1 & 2 & -1 & 1 & 2 \\ 3 & 4 & 0 & 3 & 4 \end{array}$

$(0\cdot 2\cdot 0 + (-3)\cdot(-1)\cdot 3 + 1\cdot 1\cdot 4) - (3\cdot 2\cdot 1 + 4\cdot(-1)\cdot 0 + 0\cdot 1\cdot(-3))$

$(0 + 9 + 4) - (6 + 0 + 0) = 13 - 6 = \underline{\underline{7}}$

$\begin{array}{ccc|cc} 0 & 2 & 1 & 0 & 2 \\ 1 & 4 & -1 & 1 & 4 \\ 3 & -2 & 0 & 3 & -2 \end{array}$

$(0 + 2\cdot(-1)\cdot 3 + 1\cdot 1\cdot(-2)) - (3\cdot 4\cdot 1 + (-2)\cdot(-1)\cdot 0 + 0\cdot 1\cdot 2)$

$(-6 - 2) - (12 + 0 + 0) = \underline{\underline{-20}}$

$$C = \begin{bmatrix} -3 & 3 & 0 & 5 \\ 2 & 1 & -1 & 4 \\ 6 & -3 & 4 & 0 \\ -1 & 5 & 1 & -2 \end{bmatrix}$$

$C_{12} = (-1)^{1+2} = -1 \qquad C_{42} = (-1)^{4+2} = +1$

$C_{24} = (-1)^{2+4} = +1 \qquad C_{33} = (-1)^{3+3} = +1$

$C_{31} = (-1)^{3+1} = +1 \qquad M_{11}, M_{41}, M_{23}, M_{32}, M_{44}$

$$M_{11} = \left|\begin{array}{ccc|cc} 1 & -1 & 4 & 1 & -1 \\ -3 & 4 & 0 & -3 & 4 \\ 5 & 1 & -2 & 5 & 1 \end{array}\right| = (-8+0-12) - (80+0-6) = -20-74 = \underline{\underline{-94}}$$

$$M_{44} = \left|\begin{array}{ccc|cc} -3 & 3 & 0 & -3 & 3 \\ 2 & 1 & -1 & 2 & 1 \\ 6 & -3 & 4 & 6 & -3 \end{array}\right| = (-12-18+0) - (0-3+24) = -30-21 = \underline{\underline{-51}}$$

$$M_{23} = \left|\begin{array}{ccc|cc} -3 & 3 & 5 & -3 & 3 \\ 6 & -3 & 0 & 6 & -3 \\ -1 & 5 & -2 & -1 & 5 \end{array}\right| = (-18+0+150) - (15+0-12\cdot 3) = 132+21 = 153$$

$$M_{32} = \left|\begin{array}{ccc|cc} -3 & 0 & 5 & -3 & 0 \\ 2 & -1 & 4 & 2 & -1 \\ -1 & 1 & -2 & -1 & 1 \end{array}\right| = (-6+0+10) - (5-12+0) = 4+7 = \underline{\underline{11}}$$

$$M_{41} = \left|\begin{array}{ccc|cc} 3 & 0 & 5 & 3 & 0 \\ 1 & -1 & 4 & 1 & -1 \\ -3 & 4 & 0 & -3 & 4 \end{array}\right| = (0+0+20) - (15+16\cdot 3+0) = 20-63 = \underline{\underline{-43}}$$

PR4.

a) $\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} = |A| = 3\cdot 4 - 5\cdot(-2) = 12+10 = \underline{\underline{22}} \Rightarrow \exists A^{-1}$

$A^{-1} = \frac{1}{|A|} = \left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right] R_2+R_1 \Rightarrow \left[\begin{array}{cc|cc} 1 & 9 & 1 & 1 \\ -2 & 4 & 0 & 1 \end{array}\right] 2R_1+R_2 = \left[\begin{array}{cc|cc} 1 & 9 & 1 & 1 \\ 0 & 22 & 2 & 3 \end{array}\right] -\frac{9}{22}R_2+R_1 = \left[\begin{array}{cc|cc} 1 & 0 & \frac{4}{22} & -\frac{5}{22} \\ 0 & 22 & 2 & 3 \end{array}\right] \frac{1}{22}R_2 =$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{4}{22} & -\frac{5}{22} \\ 0 & 1 & \frac{2}{22} & \frac{3}{22} \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} \frac{4}{22} & \frac{-5}{22} \\ \frac{2}{22} & \frac{3}{22} \end{bmatrix} =$$

b) $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix} = |B| = 4\cdot 2 - 1\cdot 8 = 8-8 \Rightarrow \nexists B^{-1}$

c) $\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix} = |C| = -5\cdot(-2) - 7\cdot(-7) = 10+49 = 59 \quad \exists C^{-1}$

$$C^{-1} = \frac{1}{59}\begin{bmatrix} -2 & 7 \\ 7 & -5 \end{bmatrix} = \begin{bmatrix} -\frac{2}{59} & -\frac{7}{59} \\ \frac{7}{59} & -\frac{5}{59} \end{bmatrix}$$

d) $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix} = |D| = \sqrt{2}\cdot\sqrt{3} - 4\sqrt{6} = \underline{\underline{-3\sqrt{6}}}$

$$D^{-1} = \frac{1}{-3\sqrt{6}}\begin{bmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{-3\sqrt{6}} & \frac{1}{3} \\ \frac{4}{3\sqrt{6}} & \frac{\sqrt{2}}{-3\sqrt{6}} \end{bmatrix} = \begin{matrix} \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\cdot(-\sqrt{3})\cdot\sqrt{6}} & \frac{1}{3} \\ \frac{4}{3\sqrt{6}} & \frac{\cancel{\sqrt{2}}}{-3\cancel{\sqrt{2}}\cdot\sqrt{3}} \end{matrix} = \begin{bmatrix} \frac{-1}{\sqrt{18}} & \frac{1}{3} \\ \frac{4}{3\sqrt{6}} & \frac{-1}{3\sqrt{3}} \end{bmatrix}$$

PR 5.

a) $A = \begin{bmatrix} x-2 & 1 \\ -5 & x+4 \end{bmatrix}$ $|A| = 0$ Sk $\begin{bmatrix} 0 & 1 \\ -5 & 6 \end{bmatrix}$ $0+5 \neq 0$ X

$(x-2)(x+4)+5=0$

$x^2+4x-2x-8+5=0$

$x^2+2x-3=0$

$(x+3)(x-2)=0 \quad x_{1,2}=-3,2$

$\begin{bmatrix} -5 & 1 \\ -5 & 1 \end{bmatrix} = -5-(-5)=0$ ✓

$\underline{\underline{x=-3}}$

b) $B = \left[\begin{array}{ccc|cc} x-4 & 0 & 0 & x-4 & 0 \\ 0 & x & 2 & 0 & x \\ 0 & 3 & x-1 & 0 & 3 \end{array}\right]$ $Det(B) = ((x-4)(x)(x-1)+0+0)-(0+6(x-4)+0)=0$

$= (x-4)(x)(x-1)-6x+24=0 \qquad x^3-5x^2-2x+28=0 \Rightarrow NR$

$(x^2-5x+4)(x)-6x+24=0$

$x^3-5x^2+4x-6x+28=0$

c) $C = \left[\begin{array}{ccc|cc} x-4 & 4 & 0 & x-4 & 4 \\ -1 & x & 0 & -1 & x \\ 0 & 0 & x-5 & 0 & 0 \end{array}\right]$ $Det(C) = ((x-4)(x)(x-5)+0+0)-(0+0-4(x-5))=0$

$x^3-9x^2+20x-(-4x+20)=0$

$x^3-9x^2+24x-20=0$

	1	-9	+24	-20
2	1	-7	10	0

$(x-2)(x^2-7x+10)=0 \qquad (x-2)^2(x-5) \Rightarrow x_{1,2}=2,5$

$(\; 2)(x-2)(\; x-) \; x-5 \; =$

$Sk: x=2 = \left[\begin{array}{ccc|cc} -2 & 4 & 0 & -2 & 4 \\ -1 & 2 & 0 & -1 & 2 \\ 0 & 0 & -3 & 0 & 0 \end{array}\right] = (12+0+0)-(0+0+12) = 12-12 = \underline{\underline{0}}$ ✓

$Sk: x=5 = \left[\begin{array}{ccc|cc} 1 & 4 & 0 & 1 & 4 \\ -1 & 5 & 0 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] = (0+0+0)-(0+0+0) = \underline{\underline{0}}$ ✓

PR 6.

$$A = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 1 & -2 \\ -1 & 3 & 6 \end{bmatrix} \begin{matrix} 3 & -2 \\ 5 & 1 \\ -1 & 3 \end{matrix} \quad |A| = (18 - 4 + 60) - (-4 - 18 - 60) = 74 + 82 = 156$$

a) $|A| = 3 \cdot c_{11} \cdot M_{11} + 2 c_{12} \cdot M_{12} + 4 \cdot C_{13} \cdot M_{13}$

$|A| = 3 \cdot 12 + 2 \cdot 28 + 4 \cdot 16 = 36 + 56 + 64 = \underline{156}$ ✓

b) $|A| = -5(-24) + 22 + 2 \cdot (7) = 120 + 22 + 14 = \underline{156}$ ✓

c) $|A| = -1(0) - 3(-26) + 6 \cdot 13 = 0 + 78 + 78 = \underline{156}$ ✓

d) $|A| = 3 \cdot (12) - 5 \cdot (-24) - 1 \cdot (0) = 36 + 120 = \underline{156}$ ✓

e) $|A| = 2 \cdot (28) + 22 - 3 \cdot (-26) = 56 + 22 + 78 = \underline{156}$ ✓

f) $|A| = 4 \cdot (16) + 2(7) + 6 \cdot (13) = 64 + 14 + 78 = \underline{156}$ ✓

$C_{21} = -1$, $C_{31} = +1$, $C_{12} = -1$, $C_{22} = +1$, $C_{23} = -1$, $C_{13} = +1$, $C_{23} = -1$, $C_{33} = +1$

$C_{11} = (-1)^{1+1} = +1$

$C_{12} = (-1)^{1+2} = -1$

$C_{13} = (-1)^{1+3} = +1$

$C_{21} = (-1)^{2+1} = -1$

$C_{22} = (-1)^{2+2} = +1$

$C_{23} = (-1)^{2+3} = -1$

$C_{31} = (-1)^{3+1} = +1$

$C_{32} = (-1)^{3+2} = -1$

$C_{33} = (-1)^{3+3} = +1$

b) $$B = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix} = |B| = (0 - 5 + 42) - (0 + 35 + 6) = 37 - 41 = -4$$

a) $-1 \cdot (35) + 11 + 2 \cdot 21 = -35 + 11 + 42 = 53 - 35 = 18$ ✗

$$D = \begin{bmatrix} -5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix} = |D| = (-10 + 0 - 4) - (6 + 20 + 0) = -14 - 26 = \underline{-40}$$

a) $-5(6) + 0 - 1 \cdot (10) = -30 - 10 = \underline{-40}$ ✓ $C_{21} = -1$ $C_{31} = +1$

b) $1 \cdot 4 + 2 \cdot (-8) + 1(-20) = -4 - 16 - 20 = -40$ ✓ $C_{22} = +1$ $C_{32} = -1$

c) $-3 \cdot (2) - 4 \cdot 6 + 1 \cdot (-10) = -6 - 24 - 10 = \underline{-40}$ ✓ $C_{23} = -1$ $C_{33} = +1$

d) $-5 \cdot (6) - 1(4) - 3 \cdot (2) = -30 - 4 - 6 = -40$ ✓ $C_{11} = +1$ $C_{12} = -1$

e) $0 \cdot (☺) + 2 \cdot (-8) - 4 \cdot (6) = -24 - 16 = \underline{-40}$ ✓ $C_{21} = -1$ $C_{22} = +1$

f) $-1(10) + 1 \cdot (-20) + 1 \cdot (-10) = \underline{-40}$ ✓ $C_{31} = +1$ $C_{32} = -1$

$C_{13} = +1$ $C_{33} = +1$

Na imej matrici kak je v zadani konvrhalza ?!?! $C_{23} = -1$

Pokračovanie PR.6

$$A=\left[\begin{array}{ccc|cc} 3 & -2 & 4 & 3 & -2 \\ 5 & 1 & -2 & 5 & 1 \\ -1 & 3 & 6 & -1 & 3 \end{array}\right] \quad |A| = (18-4+60)-(-4-18-60) = 74+82 = \underline{\underline{156}} \checkmark \;??$$

$$\begin{bmatrix} \boxed{3 \quad -2 \quad 4} \\ 5 \quad 1 \quad -2 \\ -1 \quad 3 \quad 6 \end{bmatrix} \quad 3\cdot\begin{vmatrix} 1 & -2 \\ 3 & 6 \end{vmatrix} + 2\begin{vmatrix} 5 & -2 \\ -1 & 6 \end{vmatrix} + 4\cdot\begin{vmatrix} 5 & 1 \\ -1 & 3 \end{vmatrix} =$$

$$= 3\cdot(6-(-6)) + 2\cdot(30-2) + 4\cdot(15-(-1)) =$$

$$= 3\cdot 12 + 2\cdot 28 + 4\cdot 16 = \underline{\underline{156}} \checkmark \text{ konečne}$$

$C_{11} = +1$

$C_{12} = -1$

$C_{13} = +1$

PR 2.

a) $A \begin{bmatrix} -3 & \boxed{0} & 7 \\ 2 & \boxed{5} & 1 \\ -1 & \boxed{0} & 5 \end{bmatrix} \quad |A| = 0\cdot(\ddot\smile) + 5\cdot(-15-(-7)) + 0\cdot(\ddot\smile) = 5\cdot(-8) = \underline{\underline{-40}} \checkmark$

b) $\begin{bmatrix} 3 & 3 & 1 \\ \boxed{1 \quad 0 \quad -4} \\ 1 & -3 & 5 \end{bmatrix} = |A| = -1\cdot(18) + 0\cdot(\ddot\smile) + 4\cdot(-12) = -18-48 = -66$

c) $\begin{bmatrix} k+1 & k-1 & \boxed{7} \\ 2 & k-3 & \boxed{4} \\ 5 & k+1 & \boxed{k} \end{bmatrix} = |A| = 7\cdot(2k+2-(5k-15)) - 4\cdot((k+1)^2 - 5(k-1)) + k\cdot((k+1)(k-3)-2k+2)$

$|A| = 7\cdot(17-3k) - 4\cdot(k^2+2k+1-5k+5) + k\cdot(k^2-2k-3-2k+2)$

$|A| = 7\cdot(17-3k) - 4\cdot(k^2-3k+6) + k\cdot(k^2-4k-1)$

$|A| = 119 - 21k - 4k^2 + 12k - 24 + k^3 - 4k^2 - k$

$|A| = \underline{\underline{k^3 - 8k^2 - 10k + 95}} \checkmark$

d) $|A| = \begin{bmatrix} 5 & 2 & 1 & 0 \\ -1 & 3 & 5 & 2 \\ 4 & 1 & 0 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix} = 0\cdot(\ddot\smile) + 2\cdot\begin{vmatrix} 5 & 2 & 1 \\ 4 & 1 & 0 \\ 0 & 2 & 3 \end{vmatrix} - 2\cdot\begin{vmatrix} 5 & 2 & 1 \\ -1 & 3 & 5 \\ 0 & 2 & 3 \end{vmatrix} + 0\cdot(\ddot\smile) =$

$= 2\cdot((15+0+8)-(0+0+24)) - 2\cdot((45+0-2)-(0+50-6))$

$= -2 - 2\cdot(43-44) = -2 - 2\cdot(-1) = \underline{\underline{0}}$

e) $|A| = \begin{bmatrix} 0 & 5 & 4 & 0 \\ 4 & 1 & -2 & 7 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{bmatrix} \quad -1\cdot\begin{vmatrix} 5 & 4 & 0 \\ 1 & -2 & 7 \\ 2 & 1 & 5 \end{vmatrix} + 0\cdot(\ddot\smile) + 3\cdot\begin{vmatrix} 0 & 5 & 0 \\ 4 & 1 & 7 \\ 0 & 2 & 5 \end{vmatrix} + 0\cdot(\ddot\smile)$

$= -1\cdot((-50+56+0)-(0+35+20)) + 3\cdot((0+0+0)-(0+0+100)) =$

$= -1\cdot(-49) - 300 = \underline{\underline{-251}} \checkmark$

f) $|A| = \begin{vmatrix} 2 & 1 & 9 & 7 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \end{vmatrix} = 0\cdot(☺) + 0\cdot(☺) + 0\cdot(☺) + 6\cdot\begin{vmatrix} 2 & 1 & 9 \\ 0 & -1 & 3 \\ 0 & 0 & 5 \end{vmatrix} = 6\cdot\big((-10+0+0)-(0+0+0)\big)$

$= \underline{\underline{-60}}$

g) $\begin{vmatrix} 0 & 4 & 1 & 3 & -2 \\ 2 & 2 & 3 & -1 & 0 \\ 3 & 1 & 2 & -5 & 1 \\ 1 & 0 & -4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \end{vmatrix}$ $|A| = 0\cdot X4 - 3\cdot\begin{vmatrix} 0 & 1 & 3 & -2 \\ 2 & 3 & -1 & 0 \\ 3 & 2 & -5 & 1 \\ 1 & -4 & 0 & 0 \end{vmatrix} + 2\cdot\begin{vmatrix} 0 & 4 & 1 & 3 \\ 2 & 2 & 3 & -1 \\ 3 & 1 & 2 & -5 \\ 1 & 0 & -4 & 0 \end{vmatrix}$

$= -3\cdot\big((0-1+0+0)-(4+0+0+0)\big) + 2\big((0-60+0-24)-(9+0+160+0)\big)$

$= 15 + 2\cdot(-84-169) = \underline{\underline{-491}}$ x nesedí!

DR 8.

a) $\begin{vmatrix} x & 2 \\ 5 & x+3 \end{vmatrix} = 0$

$x\cdot(x+3) - 2\cdot 5 = 0$

$x^2+3x-10=0$

$(x+5)(x-2)=0$ $x_{1,2} = 2, -5$

Sk: $x \in 2$ $\begin{vmatrix} 2 & 2 \\ 5 & 5 \end{vmatrix} = 10-10 = 0$ ✓

$x \in (-5)$ $\begin{vmatrix} -5 & 2 \\ 5 & -2 \end{vmatrix} = 10-10 = 0$ ✓

b) $\begin{vmatrix} 15 & x-4 \\ x+7 & -2 \end{vmatrix} = 0$

$-30 - (x^2+3x-28) = 0$

$-x^2-3x+28-30=0$

$-x^2-3x-2=0$

$(x+2)(-x-1)=0$

$x_{1,2} = -2, -1$

Sk: $x \in (-1)$ $\begin{vmatrix} 15 & -5 \\ 6 & -2 \end{vmatrix} = -30-(-30) = \underline{0}$ ✓

$x \in (-2)$ $\begin{vmatrix} 15 & -6 \\ 5 & -2 \end{vmatrix} = -30-(-30) = \underline{0}$ ✓

c) $\begin{vmatrix} (x-3) & 5 & -19 \\ 0 & (x-1) & 6 \\ 0 & 0 & (x-2) \end{vmatrix} = (x-3)(x-1)(x-2) = 0$

$\underline{\underline{x_{1,2,3} = 1, 2, 3}}$

Sk $x \in 1$ $\begin{bmatrix} -2 & 5 & -19 \\ 0 & 0 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \underline{0}$ ✓

Funguje to protože v diagonále vždy bude 0 tou 0 vynásobí a celý dt vyjde vždy 0

PR 11.

a) $\begin{bmatrix}1&-3&0\\0&1&0\\0&0&1\end{bmatrix} 3R_2+R_1 \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\text{Det}(A)=\underline{\underline{1}}$

b) $\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix} -2R_1+R_2 \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\det(A)=\underline{\underline{1}}$

e) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&-4\end{bmatrix} \cdot(-\frac{1}{4})R_3 \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\det(A)=\underline{\underline{1}}$

c) $\begin{bmatrix}0&0&0\\0&1&0\\0&1&0\end{bmatrix} -R_2+R_3 \Rightarrow \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}$ $\det(A)=0$

f) $\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix} \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\det(A)=\underline{\underline{1}}$

d) $\begin{bmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{bmatrix} \cdot\frac{1}{\frac{1}{2}} \Rightarrow \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\det(A)=\underline{\underline{1}}$

PR 12.

a) $\begin{bmatrix}10&4&21\\0&-4&3\\-5&-1&-12\end{bmatrix} 2R_3+R_1 = \begin{bmatrix}0&2&-3\\0&-4&3\\-5&-1&-12\end{bmatrix} = -\begin{bmatrix}-5&-1&-12\\0&-4&3\\0&2&-3\end{bmatrix} 2R_3+R_2 =$

$\begin{bmatrix}-5&-1&-12\\0&0&-3\\0&2&-3\end{bmatrix} \Rightarrow \begin{bmatrix}-5&-1&-12\\0&2&-3\\0&0&-3\end{bmatrix}$ $\det(A)=\underline{\underline{30}}$

b) $\begin{bmatrix}18&-9&-14\\6&-3&-5\\-3&1&2\end{bmatrix} 2R_3+R_2 \Rightarrow \begin{bmatrix}-3&1&2\\0&-1&-1\\18&-9&-14\end{bmatrix} 6R_1+R_3 \Rightarrow \begin{bmatrix}-3&1&2\\0&-1&-1\\0&-3&-2\end{bmatrix} -3R_2+R_3 \Rightarrow$

$\Rightarrow \begin{bmatrix}-3&1&2\\0&-1&-1\\0&0&1\end{bmatrix} \cdot(-1) \begin{bmatrix}-3&1&2\\0&1&1\\0&0&1\end{bmatrix} = \det(A) = \underline{\underline{-3}}$

c) $\begin{bmatrix}1&-1&5&1\\-2&1&-7&1\\-3&2&-12&-2\\2&-1&9&1\end{bmatrix} \Rightarrow$ R_4+R_2, 2.) $3R_1+R_3$, 1.) $2R_1+R_4$ $\begin{bmatrix}1&-1&5&1\\0&0&2&2\\0&-1&3&1\\0&1&-1&-1\end{bmatrix}$ 2.) $R_2 \Leftrightarrow R_3$, 1.) R_3+R_4 $\Rightarrow \begin{bmatrix}1&-1&5&1\\0&-1&3&1\\0&0&2&2\\0&0&2&0\end{bmatrix} -R_3+R_4 \Rightarrow$

$\Rightarrow \begin{bmatrix}1&-1&5&1\\0&-1&3&1\\0&0&2&2\\0&0&0&2\end{bmatrix} = \det(A) = -4$

Dotaz ku 12. temu cvičeniu. Niekedy vychádza dt s opačným znamienkom. Opýtať sa na ·(-1) kedy používať?

d) $\begin{bmatrix}-8 & 4 & -3 & 2\\ 2 & 1 & -1 & -1\\ -3 & -5 & 4 & 0\\ 2 & -4 & 3 & -1\end{bmatrix} \begin{matrix}-R_4+R_2\\ R_4+R_3\end{matrix} \Rightarrow \begin{bmatrix}-8 & 4 & -3 & 2\\ 0 & 5 & -4 & 0\\ -1 & -9 & 7 & -1\\ 2 & -4 & 3 & -1\end{bmatrix} \begin{matrix}-8R_3+R_1\\ \Rightarrow\\ 2R_3+R_4\end{matrix} \begin{bmatrix}0 & 76 & -59 & 10\\ 0 & 5 & -4 & 0\\ -1 & -9 & 7 & -1\\ 0 & -22 & 17 & -3\end{bmatrix}$

$\begin{bmatrix}-1 & -9 & 7 & -1\\ 0 & 5 & -4 & 0\\ 0 & 76 & -59 & 10\\ 0 & -22 & 17 & -3\end{bmatrix} \begin{matrix}-15R_2+R_3\\ 4R_2+R_4\end{matrix} \Rightarrow \begin{bmatrix}-1 & -9 & 7 & -1\\ 0 & 5 & -4 & 0\\ 0 & 1 & 1 & 10\\ 0 & -2 & 1 & -3\end{bmatrix} \begin{matrix}1.)\ -5R_3+R_2\\ 3.)\ R_3 \Leftrightarrow R_2 \Rightarrow\\ 2.)\ 2R_3+R_4\end{matrix} \begin{bmatrix}-1 & -9 & 7 & -1\\ 0 & 1 & 1 & 10\\ 0 & 0 & -9 & -50\\ 0 & 0 & 3 & 17\end{bmatrix}$

$\begin{bmatrix}-1 & -9 & 7 & -1\\ 0 & 1 & 1 & 10\\ 0 & 0 & 0 & 1\\ 0 & 0 & 3 & 17\end{bmatrix} \Rightarrow \begin{bmatrix}-1 & -9 & 7 & -1\\ 0 & 1 & 1 & 10\\ 0 & 0 & 3 & 17\\ 0 & 0 & 0 & 1\end{bmatrix}$ $\det(A) = \underline{\underline{-3}}$

PR 13.

Singulárna ⇒ neexistuje inverzná

a) $\begin{bmatrix}5 & 6\\ -3 & -4\end{bmatrix} = -20 - (6\cdot(-3)) = -20 + 18 = \underline{\underline{-2}} \Rightarrow$ Reg. M $A^{-1} = -\frac{1}{2}\cdot\begin{bmatrix}-4 & -6\\ 3 & 5\end{bmatrix} = \begin{bmatrix}2 & 3\\ \frac{-3}{2} & \frac{-5}{2}\end{bmatrix}$

b) $\begin{bmatrix}12 & 7 & -27\\ 4 & -1 & 2\\ 3 & 2 & -8\end{bmatrix} = dt(A) = (12\cdot(-8) + 14\cdot 3 - 27\cdot 8) - (3\cdot 27 - 4\cdot 12 - 8\cdot 28) =$
$= -270 + 191 = \underline{\underline{-79}}$ Reg. M

$M_{11} = 4 \qquad M_{21} = -56 + 54 = -2 \qquad M_{31} = 14 - 27 = -13$
$M_{12} = -32 - 6 = -38 \qquad M_{22} = 96 + 81 = 177 \qquad M_{32} = -24 + 4\cdot 27 = 84$
$M_{13} = 11 \qquad M_{23} = -24 - 21 = -45 \qquad M_{33} = 12 - 28 = -16$

$B^{-1} = \frac{1}{-79}\cdot\begin{bmatrix}4 & -38 & 11\\ -2 & 177 & -45\\ -13 & 84 & -16\end{bmatrix} = \begin{bmatrix}\frac{-4}{79} & \frac{38}{79} & \frac{-11}{79}\\ \frac{2}{79} & \frac{-177}{79} & \frac{45}{79}\\ \frac{13}{79} & \frac{-84}{79} & \frac{16}{79}\end{bmatrix} \Rightarrow B^{T} =$

$= \begin{bmatrix}\frac{-4}{79} & \frac{-2}{79} & \frac{13}{79}\\ \frac{-38}{79} & \frac{-177}{79} & \frac{+84}{79}\\ \frac{-11}{79} & \frac{-45}{79} & \frac{16}{79}\end{bmatrix}$

c) $\begin{bmatrix}\cos(0) & -\sin(0)\\ \sin(0) & \cos(0)\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \Rightarrow \det(A) = \underline{\underline{1}}$ Reg. M. $\quad C^{-1} = \begin{bmatrix}1 & -0\\ -0 & 1\end{bmatrix}$

d) $\begin{bmatrix}31 & -20 & 106\\ -11 & 7 & -37\\ -9 & 6 & -32\end{bmatrix} = \det(A) = \left((31\cdot 7\cdot(-32) - 20\cdot 37\cdot 9 - 106\cdot 11\cdot 6\right) -$

$- \left(-9\cdot 7\cdot 106 - 6\cdot 37\cdot 31 - 32\cdot 11\cdot 20\right)$

$= -20\,600 + 20600 = \underline{\underline{0}}$ Singulárna M.

PR 14.

a) $|A| = \begin{bmatrix}2 & 5 & 5\\ -1 & -1 & 0\\ 2 & 4 & 3\end{bmatrix}$ $\quad \det(A) = (-6 + 0 - 20) - (-10 + 0 - 15) = -26 + 25 = -1$

$\underline{\underline{\text{Reg M.}}}$

$M_{11} = -3 \qquad M_{21} = +5 \qquad M_{31} = 5$

$M_{12} = +3 \qquad M_{22} = -4 \qquad M_{32} = -5$

$M_{13} = -2 \qquad M_{23} = +2 \qquad M_{33} = 3$

$A^{-1} = \frac{1}{-1}\cdot A^T = -1\cdot \begin{bmatrix}-3 & 5 & 5\\ 3 & -4 & -5\\ -2 & 2 & 3\end{bmatrix} = \begin{bmatrix}3 & -5 & -5\\ -3 & 4 & 5\\ 2 & -2 & -3\end{bmatrix} \underline{\checkmark}$

b) $B = \begin{bmatrix}2 & 0 & 3\\ 0 & 3 & 2\\ -2 & 0 & -4\end{bmatrix}$ $\quad \det(B) = (-24 + 0 + 0) - (-18 + 0 + 0) = -24 + 18 = -6$

$\underline{\underline{\text{Reg Mat.}}} \Rightarrow \exists B^{-1}$

$C_{11} = -12\cdot 1 = -12 \qquad C_{21} = 0 \qquad C_{31} = -9$

$C_{12} = 4\cdot(-1) = -4 \qquad C_{22} = -2\cdot 1 = -2 \qquad C_{32} = 4\cdot(-1) = -4$

$C_{13} = 6\cdot 1 = 6 \qquad C_{23} = 0 \qquad C_{33} = 6$

$B^{-1} = \frac{1}{-6}\begin{bmatrix}-12 & 0 & -9\\ -4 & -2 & -4\\ 6 & 0 & 6\end{bmatrix} = \begin{bmatrix}2 & 0 & \frac{3}{2}\\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3}\\ -1 & 0 & -1\end{bmatrix} \underline{\checkmark}$

c) $C = \begin{bmatrix}2 & -3 & 5\\ 0 & 1 & -3\\ 0 & 0 & 2\end{bmatrix}$ $\quad \det(C) = \underline{\underline{4}} = \underline{\underline{\text{Reg. Mat.}}} \Rightarrow \exists C^{-1}$

$C_{11} = 2 \qquad C_{21} = -6\cdot(-1) \qquad C_{31} = 4$

$C_{12} = 0 \qquad C_{22} = 4 \qquad C_{32} = 6$

$C_{13} = 0 \qquad C_{23} = 0 \qquad C_{33} = 2$

$$C^{-1} = \frac{1}{4} \cdot \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix} = C^{-1} \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

d) $D = \begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix}$ $\det(D) = (24 - 24 - 16) - (24 - 16 - 24) = -16 + 16 = 0$

$\underline{\underline{\text{Sing Mat.}}}$ $\nexists D^{-1}$

e) $E = \begin{bmatrix} 1 & 0 & -1 \\ 9 & -1 & 4 \\ 8 & 9 & -1 \end{bmatrix}$ $\det(E) = (1 + 0 - 81) - (8 + 36 + 0) = -80 - 44 = -124$

Reg Mat. $\exists E^{-1}$

$C_{11} = \underline{\underline{-35}}$ $C_{21} = \underline{\underline{-9}}$ $C_{31} = \underline{\underline{-1}}$

$C_{12} = (-9 - 32) \cdot (-1) = \underline{\underline{41}}$ $C_{22} = -1 + 8 = \underline{\underline{7}}$ $C_{32} = (4 + 9) \cdot (-1) = \underline{\underline{-13}}$

$C_{13} = 81 + 8 = \underline{\underline{89}}$ $C_{23} = 9 \cdot (-1) = \underline{\underline{-9}}$ $C_{33} = \underline{\underline{-1}}$

$$E^{-1} = \frac{1}{-124} \begin{bmatrix} -35 & -9 & -1 \\ 41 & 7 & -13 \\ 89 & -9 & -1 \end{bmatrix} = \begin{bmatrix} \frac{35}{124} & \frac{9}{124} & \frac{1}{124} \\ \frac{-41}{124} & \frac{-7}{124} & \frac{13}{124} \\ \frac{-89}{124} & \frac{9}{124} & \frac{1}{124} \end{bmatrix}$$

f) $F = \begin{bmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{bmatrix}$ $\det(F) = (0 + 0 + 0) - (0 + 0 + 0) = \underline{\underline{0}}$ $\underline{\underline{\text{Sing. Mat.}}}$ $\nexists F^{-1}$

g) $G = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix} =$ $\det(G) = \underline{\underline{12}}$ $\underline{\underline{\text{Reg Mat.}}}$

$C_{11} = 6$ $C_{21} = 0$ $C_{31} = 0$

$C_{12} = -48$ $C_{22} = 12$ $C_{32} = 0$

$C_{13} = 24 + 5 = 29$ $C_{23} = -6$ $C_{33} = 2$

$$G^{-1} = \frac{1}{12} \cdot \begin{bmatrix} 6 & 0 & 0 \\ -48 & 12 & 0 \\ 29 & -6 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -4 & 1 & 0 \\ \frac{29}{12} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

h) $H = \begin{bmatrix} \sqrt{2} & -\sqrt{7} & 0 \\ 3\sqrt{2} & -3\sqrt{7} & 0 \\ 5 & -9 & 0 \end{bmatrix}$ $\det(H) = \underline{\underline{0}}$ $\nexists H^{-1}$

i) $I = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -1 & 0 & -4 \end{bmatrix}$ $\det(I) = (-24 + 0 + 0) - (-9 + 0 + 0) = -24 + 9 = -15$

$C_{11} = -12$ $C_{21} = 0$ $C_{31} = -9$

$C_{12} = -2$ $C_{22} = -5$ $C_{32} = -4$

$C_{13} = 3$ $C_{23} = 0$ $C_{33} = 6$

$$I^{-1} = \frac{1}{-15} \begin{bmatrix} -12 & 0 & -9 \\ -2 & -5 & -4 \\ 3 & 0 & 6 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & 0 & \frac{3}{5} \\ \frac{2}{15} & \frac{1}{3} & \frac{4}{15} \\ -\frac{1}{5} & 0 & -\frac{6}{15} \end{bmatrix}$$

PR16.

a) $\left[\begin{array}{cc|c} 7 & -2 & 3 \\ 3 & 1 & 5 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{cc|c} 1 & -4 & -7 \\ 3 & 1 & 5 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{cc|c} 1 & -4 & -7 \\ 0 & 13 & 26 \end{array}\right] \cdot \frac{1}{13} \left[\begin{array}{cc|c} 1 & -4 & -7 \\ 0 & 1 & 2 \end{array}\right] 4R_2+R_1$

$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow x_1 = 1, x_2 = 2$ $\quad 7x - 2y = 3 \Rightarrow 7 - 4 = 3 \checkmark$

$3x + y = 5 \quad 3 + 2 = 5 \checkmark$

b) $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 2 & 2 & 4 \end{array}\right] \cdot \frac{1}{2} \Rightarrow \left[\begin{array}{ccc} 2 & 3 & 4 \\ 1 & 1 & 2 \end{array}\right] -2R_2+R_1 \Rightarrow \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 1 & 2 \end{array}\right] \begin{array}{l} 2.)\ R_1 \Leftrightarrow R_2 \\ 1.)\ -R_1+R_2 \end{array} \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right]$

$x_1 = 2, x_2 = 0 \checkmark$