Matalíza opak

Približná hodnota

a) $\sqrt{382}$ $x_0 = 400$ $x_1 = 382$ $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{382} \approx f(x_0) + f'(x_0) \cdot (x_1 - x_0)$

$\sqrt{382} \doteq 20 + \frac{1}{2 \cdot 20} \cdot (-18) = 20 - \frac{9}{20} = \frac{391}{20}$

b) $\sqrt[5]{36}$ $x_0 = 32$ $x_1 = 36$ $f(x) = x^{\frac{1}{5}}$

$f'(x) = \frac{1}{5\sqrt[5]{x^4}}$

$f(x_0) = 2$

$\sqrt[5]{36} \doteq f(x_0) + f'(x_0)(x_1 - x_0)$

$\sqrt[5]{36} \doteq 2 + \frac{1}{5\sqrt[5]{32^4}}(36-32) = 2 + \frac{4}{5\sqrt[5]{32^4}}$

c) $2^{1,9}$ $x_0 = 2$ $x_1 = 1,9$ $f(x) = 2^x$

$f'(x) = 2^x \cdot \ln(2)$

$2^{1,9} \doteq f(x_0) + f'(x_0)(x_1 - x_0)$

$2^{1,9} \doteq 4 + 4\ln(2)(-0,1) = 4 - \frac{4\ln(2)}{10}$

d) $\operatorname{arctg}(1,1)$ $x_0 = 1$ $x_1 = 1,1$

$f'(x) = \frac{1}{1+x^2}$

$\operatorname{arctg}(1,1) \doteq f(x_0) + f'(x_0)(x_1 - x_0)$

$\operatorname{arctg}(1,1) \doteq \frac{\pi}{4} + \frac{1}{20}$

e) $\arcsin(0,2)$ $x_0 = 0$ $x_1 = 0,2$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\arcsin(0,2) \doteq f(x_0) + f'(x_0)(x_1 - x_0)$

$\arcsin(0,2) \doteq 0 + 0,2 = 0,2$

Taylorov polinom

$$T_n = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2$$

a) $f(x) = \ln(x)$; $x_0 = 1$; $n=4$

$f(x_0) = 0$

$f'(x_0) = \frac{1}{x} = 1$

$f''(x_0) = (x^{-1})' \Rightarrow -x^{-2} = -\frac{1}{x^2} = -1$

$f'''(x_0) = \frac{1}{x^3} = 1$

$f^{IV}(x_0) = -\frac{1}{x^4} = -1$

$T_4 = 0 + \frac{1}{1}(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{24}(x-1)^4$

b) $f(x) = x^4 - 5x^3 + 2x - 3$; $x_0 = -1$; $n=4$

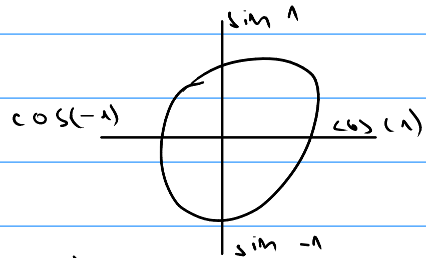
$f'(x) = 4x^3 - 15x^2 + 2 \Rightarrow -17$

$f''(x) = 12x^2 - 30x \Rightarrow 42$

$f'''(x) = 24x - 30 \Rightarrow -54$

$f^{IV}(x) = 24 \Rightarrow 24$

$T_4 = 1 - 17(x+1) + 21(x+1)^2 - 9(x+1)^3 + (x+1)^4$



c) $f(x) = e^{2x}\cdot\sin(x)$; $x_0 = 0$; $n=3$

$f'(x) = 2e^{2x}\cdot\sin(x) + e^{2x}\cdot\cos(x)$

$f''(x) = 4e^{2x}\cdot\sin(x) + 2e^{2x}\cdot\cos(x) + 2e^{2x}\cdot\sin(x) - e^{2x}\cdot\sin(x) =$

$= 3e^{2x}\cdot\sin(x) + 2e^{2x}(\cos(x) + \sin(x))$

$f'''(x) = 6e^{2x}\sin(x) + \underset{3}{3e^{2x}\cdot\cos(x)} + \underset{4}{4e^{2x}}(\cos(x) + \sin(x)) + 2e^{2x}(\cos(x) - \sin(x))$

$T_3 = 0 + x + x^2 + \frac{9x^3}{6}$

Maclurinov Rad

$e^2 = e^x$; $x_0 = 0$; $R < 0{,}14 \Rightarrow \frac{14}{100}$; $x_1 = 2$; $e \doteq 3$

$\frac{3 \cdot 2^{n+1}}{(n+1)!} < \frac{14}{100}$

$\frac{3 \cdot 2^{n+1} \cdot 100}{14} < (n+1)!$

$\frac{300 \cdot 2^{n+1}}{14} < (n+1)!$

$n=0 \Rightarrow 1$

$n=1 \Rightarrow 2$

$n=2 \Rightarrow 6$

$n=3 \Rightarrow 24$

$n=4 \Rightarrow 120$

$n=5 \Rightarrow 720$

$n=6 \Rightarrow 5040$

$n=7 \Rightarrow 40320$

$f', f'', f''', f^{IV} = e^x$ $\quad T_4 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} = 1 + 2 + 2 + \frac{8}{6} + \frac{16}{24} + \frac{32}{120}$

PR 4) $\quad n = 3$; $f(x) = \cos(x)$; $x_0 = 60°$; $x_1 = 61°$

$f'(x) = -\sin(x) = -\frac{\sqrt{3}}{2}$

$f''(x) = -\cos(x) = -\frac{1}{2}$

$f'''(x) = \sin(x) = \frac{\sqrt{3}}{2}$

	30°	45°	60°
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

$$T_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{\frac{1}{2}\left(x-\frac{\pi}{3}\right)^2}{2} + \frac{\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)^3}{6}$$

Limity

$$\lim_{x\to 1}\frac{x^3-2x^2-5x+6}{x^2+2x-3}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2-x-6)}{(x+3)\cancel{(x-1)}}=\lim_{x\to 1}\frac{x^2-x-6}{x+3}=-\frac{3}{2}$$

	1	-2	-5	6
1	1	-1	-6	0

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{\sin(2x)}+\lim_{x\to 0}\ln(1-x^2)=\lim_{x\to 0}\frac{\sqrt{x+4}-2}{\sin(2x)}\cdot\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}+0=$$

$$\lim_{x\to 0}\frac{x+\cancel{4}-\cancel{4}}{\sin(2x)(\sqrt{x+4}+2)}=\lim_{x\to 0}\frac{1}{2\cos(2x)(\sqrt{x+4}+2)+\sin(2x)\frac{1}{2\sqrt{x+4}}}$$

$$\frac{1}{2\cdot 4+0}=\frac{1}{8}$$

$$\lim_{x\to 0}\frac{tg(5x)}{tg(6x)}=\lim_{x\to 0}\frac{\frac{\sin(5x)}{\cos(5x)}}{\frac{\sin(6x)}{\cos(6x)}}=\lim_{x\to 0}\frac{\frac{d}{x}\left(\sin(5x)\cdot\cos(6x)\right)}{\frac{d}{x}\left(\cos(5x)\cdot\sin(6x)\right)}=$$

$$\lim_{x\to 0}\frac{5\cos(5x)\cdot\cos(6x)+0}{0+\cos(5x)\cdot 6\cos(6x)}=\frac{5}{6}$$

$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x=e^a$

$$\lim_{x\to\infty}\left(\frac{x+1+2-2}{x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x}\cdot\boxed{\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{-1}\Rightarrow 1}$$

$\frac{1}{1}\Rightarrow 1$

$$\lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x}\cdot 1\Rightarrow \text{subst } t=x-2$$
$$x=t+2$$

$$\lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t+4}=\lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t}\cdot 1^4={e^3}^2=e^6 \quad \checkmark$$

$$\lim_{x\to 0}\left(\frac{3-2x}{2+5x}\right)^{\frac{\sqrt{x+1}-1}{x}}$$

$$\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=\frac{\cancel{x}+\cancel{1}-\cancel{1}}{\cancel{x}(\sqrt{x+1}+1)}=\frac{1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0}\left(\frac{3-2x}{2+5x}\right)^{\frac{1}{\sqrt{x+1}+1}}=\frac{3}{2}^{\frac{1}{2}}=\sqrt{\frac{3}{2}}$$

$$\lim_{x\to\infty}\left(-4x^5+5x^3-7x+10\right)\quad \underline{\underline{(-\infty)}}$$

$$\lim_{x\to-\infty}\left(-4x^5+5x^3-7x+10\right)\quad \underline{\underline{(+\infty)}}$$

$$\lim_{x\to\infty}\frac{x^2+3x-5}{2x^3-4x+1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}=\frac{\frac{1}{x}+\frac{3}{x^2}-\frac{5}{x^3}}{2-\frac{4}{x^2}+\frac{1}{x^3}}=\frac{0}{2}=0$$

$$\lim_{x\to\infty}\frac{4x^3-2x^2+7}{7x^3-3x^2-6x+4}\Rightarrow\frac{4}{7}$$

$$\lim_{x\to\infty}\frac{x^5-3x^2+2x-1}{2x^3-x^2+x-1}=\frac{\infty}{2}\Rightarrow+\infty$$

$$\lim_{x\to0}\left(2^{\cot(x)}-1\right)\Rightarrow\cot(0)\ \cancel{\exists}$$

$$\lim_{x\to\frac{\pi}{4}}\left(2^{\cot(x)}-1\right)=2^1-1=2-1=1$$

$$\lim_{x\to\frac{\pi}{2}}\left(2^{\cot(x)}-1\right)=2^0-1=1-1\Rightarrow0$$

$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{\ln(x)}}\Rightarrow\lim_{x\to\infty}\quad ?$$

$$\lim_{x\to\infty}x\left[\ln(x+1)-\ln(x)\right]=\lim_{x\to\infty}x\left[\ln\left(\frac{x+1}{x}\right)\right]=\lim_{x\to\infty}\ln\left(\frac{x+1}{x}\right)^x=$$
$$\ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right)=\ln(e)=1$$

$$\lim_{x\to\infty}\left(\frac{2x+1+2-2}{2x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x-1}\Rightarrow t=2x-2,\ \frac{t+2}{2}=x$$

$$\underbrace{\lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{2t+4}}_{\mathrm{I.}}\cdot\underbrace{\lim_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{-1}}_{\mathrm{II.}}$$

$$\mathrm{II.}\ \frac{1}{\lim_{x\to\infty}\left(\frac{2+\frac{1}{x}\to0}{2-\frac{2}{x}\to0}\right)}=\frac{1}{1}=1^{-1}=1$$

$$\mathrm{I.}\ e^2\cdot1^4=e^2$$

Pokus 2. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{2x-1} = \lim\limits_{x\to\infty}\left(\frac{2x+1+2-2}{2x-2}\right)^{2x-1} = \lim\limits_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x-1}$

$$= \underbrace{\lim_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x}}_{\mathrm{I.}} \cdot \underbrace{\lim_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{-1}}_{\mathrm{II}}$$

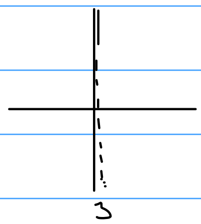
$$\mathrm{I.}\ \lim_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x} \Rightarrow \begin{matrix} t=2x-2 \\ \frac{t+2}{2}=x \end{matrix} \Rightarrow \lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{t+2} = \underbrace{\lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{t}}_{\mathrm{III.}} \cdot \underbrace{\lim_{t\to\infty}\left(1+\frac{3}{t}\right)^{2}}_{\mathrm{IV}}$$

$\mathrm{III.}\ e^3$ $\quad \mathrm{IV.}\ (1+0)^2 = 1$ $\quad \mathrm{I.} \Rightarrow e^3\cdot 1 = \underline{e^3}$ $\quad \mathrm{II.} \Rightarrow 1$

$$\lim_{x\to\infty}\frac{|x|}{\sqrt{x^2-1}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{|1|}{\sqrt{1-\frac{1}{x^2}}} = \frac{1}{1} = \underline{1}$$

$$\lim_{x\to\infty}\left(\frac{x+5}{x-3}\right) = \lim_{x\to\infty}\left(\frac{x+5+3-3}{x-3}\right) = \lim_{x\to\infty}\left(1+\frac{8}{x-3}\right) = 1+0 = 1$$

$\lim\limits_{x\to3^+}\left(1+\frac{8}{x-3}\right) \Rightarrow$



$3^+ \Rightarrow \frac{8}{0{,}000\ldots1} \Rightarrow \infty$

$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{\ln(x)}} = e^{\lim\limits_{x\to\infty}\left(\ln(x^{-1})\cdot\ln(x)^{-1}\right)}$$

$$\lim_{x\to\infty}\frac{\frac{\pi}{2}-\operatorname{arctg}(x)}{\ln\left(\sqrt{\frac{x-1}{x+1}}\right)} = \frac{\lim\limits_{x\to\infty}\left(\frac{\pi}{2}-\operatorname{arctg}(x)\right)'}{\lim\limits_{x\to\infty}\left(\ln\left(\sqrt{\frac{x-1}{x+1}}\right)\right)'}$$

$$= \frac{\lim\limits_{x\to\infty}\left(-\frac{1}{\sqrt{1-x^2}}\right)}{\lim\limits_{x\to\infty}\left(\frac{1}{\sqrt{\frac{x-1}{x+1}}}\cdot\frac{1}{2\sqrt{\frac{x-1}{x+1}}}\cdot\frac{x+1-x+1}{(x+1)^2}\right)} = \frac{\lim\limits_{x\to\infty} -\frac{1}{\sqrt{1-x^2}}}{\lim\limits_{x\to\infty}\frac{1}{\frac{2x-2}{x+1}\cdot\frac{2}{(x+1)^2}}}$$

$$= \frac{\lim\limits_{x\to\infty} -\frac{1}{\sqrt{1-x^2}}}{\frac{1}{\frac{4x-4}{(x+1)^3}}}$$

$$\lim_{x\to 0^+} \frac{\ln(\sin 3x)}{\ln(\sin 5x)} = \lim_{x\to 0^+} \left(\frac{\frac{3\cos 3x}{\sin 3x}}{\frac{5\cos 5x}{\sin 5x}}\right) = \lim_{x\to 0^+} \frac{3\cos(3x)\cdot\sin(5x)}{5\cos(3x)\cdot\sin(3x)}$$

$$\frac{3\cdot 0{,}05}{5\cdot 0{,}03} = \frac{0{,}15}{0{,}15} = \underline{\underline{1}}$$

$$\lim_{x\to 2}\left(\frac{x^2-4}{x^2}\cdot tg\left(\frac{\pi x}{4}\right)\right) = \underbrace{\lim_{x\to 2}\left(\frac{x^2-4}{x^2}\right)}_{\mathrm{I.}}\cdot\underbrace{\lim_{x\to 2} tg\left(\frac{\pi x}{4}\right)}_{\mathrm{II.}}$$

$$\mathrm{I.}\ \lim_{x\to 2}\left(\frac{2x}{2x}\right) = 1$$

$$\mathrm{II.}\ \lim_{x\to 2}\frac{\sin\left(\frac{\pi x}{4}\right)}{\cos\left(\frac{\pi x}{4}\right)}\ \text{lh} \Rightarrow$$

$$\sin\left(\frac{\pi x}{4}\right)' = \cos\left(\frac{\pi x}{4}\right)\cdot\frac{\pi}{4}$$

$$\cos\left(\frac{\pi x}{4}\right)' = -\sin\left(\frac{\pi x}{4}\right)\cdot\frac{\pi}{4}$$

$$\lim_{x\to 0}\frac{tg(x)-x}{x-\sin(x)}\ \text{"Lh"} \Rightarrow \lim_{x\to 0}\frac{\frac{1}{\cos^2(x)}-1}{1-\cos(x)}\ \text{"Lh"} \Rightarrow \lim_{x\to 0}\frac{-2\cos^{-3}(x)\cdot\cancel{\sin(x)}}{\cancel{\sin(x)}}$$

$$-2\cdot 1^{-3} = -2\cdot(-1) = \underline{\underline{+2}}$$

$$\lim_{x\to 0^+}(e^x-1)\cot g(x)\ \text{"Lh"} \Rightarrow \underbrace{\lim_{x\to 0^+}\frac{e^x\cdot\cos(x)}{\sin(x)}}_{\mathrm{I.}} + \underbrace{\lim_{x\to 0^+} -\frac{e^x-1}{\sin^2(x)}}_{\mathrm{II.}}$$

$$\mathrm{I.}\ \text{"Lh"}\lim_{x\to 0^+}\frac{e^x\cdot\cos(x)-e^x\cdot\sin(x)}{\cos(x)} = \frac{1-0}{1} = 1$$

$$\mathrm{II.}\ \text{"Lh"} \Rightarrow -\lim_{x\to 0^+}\frac{e^x}{2\sin(x)\cdot\cos(x)} \Rightarrow \text{"Lh"} - \lim_{x\to 0^+}\frac{e^x}{2\cos^2(x)-2\sin^2(x)} = -\frac{1}{2}$$

$$1-\frac{1}{2} = \underline{\underline{\frac{1}{2}}}$$

Priebeh funkcie

a) $f(x) = \frac{2x^3}{x^2+1}$ OR: $x^2+1 \neq 0 \Rightarrow$ nikdy nenastane

1. definičný obor

$D(f) = \mathbb{R}$

2. párnosť

$f(x) = f(-x) \Rightarrow$ párna $\frac{2(-x)^3}{(-x)^2+1} = -\frac{2x^3}{x^2+1}$

$f(x) = -f(-x) \Rightarrow$ nepárna

$f(x) \neq f(-x)$; $f(x) = -f(-x)$; $f(x)$ je nepárna

3. spojitosť ; $\Rightarrow f(x)$ je spojitá na celom svojom $D(f)$ lebo je $D(f)$ je $\mathbb{R}$

4. NB: 0

5. Asymptota

ABS x

ASS $\Rightarrow kx + q$

$k = \lim_{x\to\infty} \frac{2x^3}{(x^2+1)x} = \lim_{x\to\infty} \frac{2x^3}{x^3+x} = 2$

$q = \lim_{x\to\infty} \frac{2x^3}{x^2+1} \overset{-kx}{-2x} = \lim_{x\to\infty} \frac{\cancel{2x^3} - \cancel{2x^3} - 2x}{x^2+1} = 0$

ASS: $y = 2x$

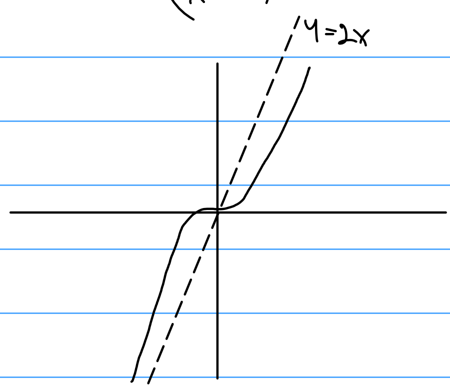
6. monotónnosť

$f'(x) = \left(\frac{2x^3}{x^2+1}\right)' = \frac{6x^2(x^2+1) - 2x^3 \cdot 2x}{(x^2+1)^2} = \frac{\cancel{6x^4} + 6x^2 - \cancel{4x^4}}{(x^2+1)^2} =$

$= \frac{2x^4 + 6x^2}{(x^2+1)^2}$ NB: 0

7. extrémy nemá

8. Inflexný bod



$f''(x) = \frac{(8x^3 + 12x)(x^4+2x^2+1) - (4x^3+4x) \cdot (2x^4+6x^2)}{(x^2+1)^4} =$

$f''(x) = \frac{\cancel{8x^7} + 16x^5 + 8x^3 + 12x^5 + \cancel{24x^3} + 12x - (\cancel{8x^7} + 24x^5 + 8x^5 + \cancel{24x^3})}{(x^2+1)^4}$

$$f''(x) = \frac{28x^5 - 32x^5 + 8x^3 + 12x}{(x^2+1)^4} = \frac{-4x^5 + 8x^3 + 12x}{(x^2+1)^4}$$

$f(x) = 16x(x-1)^3 \Rightarrow 16x(x^3 - 3x^2 + 3x - 1) = 16(x^4 - 3x^3 + 3x^2 - 1)$

1.) OR : $\mathbb{R} \Rightarrow D(f) = \mathbb{R}$ ✓

2.) Párnosť

$f(-x) = 16(x^4 + 3x^3 + 3x^2 - 1)$

$-f(-x) = -16(x^4 + 3x^3 + 3x^2 - 1) = 16(-x^4 - 3x^3 - 3x^2 + 1)$

f(x) nieje ani párna ani nepárna ✓

3.) Spojitosť $\Rightarrow$ f(x) je spojitá na celom svojom D(f) ?

4.) NB: 0, 1

5.) Asymptoty

ABS x

ASS: $kx + q$

$k \Rightarrow \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{16\cancel{x}(x-1)^3}{\cancel{x}} = \lim_{x\to\infty} ((x-1)^3) \Rightarrow \infty \Rightarrow \nexists$

$q \Rightarrow \lim_{x\to\infty} 16x(x-1)^3 - \infty x \Rightarrow \infty \nexists$

f(x) nemá asymptoty ✓

5.) Monotónnosť ✓

$f'(x) = (16x(x-1)^3)' = 16(x^3 - 3x^2 + 3x - 1) + 16x \cdot 3(x^2 - 2x + 1)$

$f'(x) = 16x^3 - 48x^2 + 48x - 16 + 48x^3 - 96x^2 + 48x = 64x^3 - 144x^2 + 96x - 16$

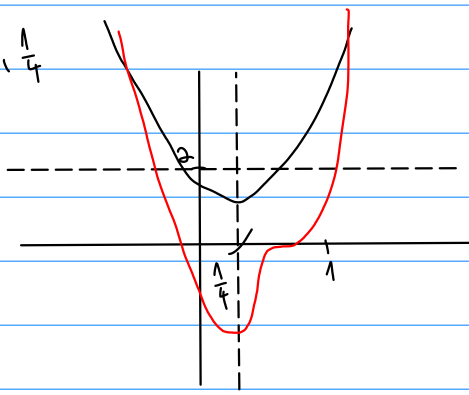
$4x^3 - 9x^2 + 6x - 1 = 0$

	4	-9	6	-1
1	4	-5	1	0
1	4	-1	0	

$(x-1)(4x^2 - 5x + 1) = 0$

$(x-1)^2(4x-1) = 0 \Rightarrow 1, \frac{1}{4}$

NB: $0, 1, \frac{1}{4}$



	$(-\infty, 0)$	$(0, \frac{1}{4})$	$(\frac{1}{4}, 1)$	$(1, \infty)$
$(x-1)$	−	−	−	+
$(x-1)$	−	−	−	+
$(4x-1)$	−	−	+	+
	−	−	+	+

$\frac{1}{4} \Rightarrow$ Konkávna ∪ $\Rightarrow$ min

$f(x) \Rightarrow (-\infty; \frac{1}{4}) \Rightarrow$ klesá ; $(\frac{1}{4}, \infty) \Rightarrow$ rastie

7. extrémy   beriem len to čoje z Provej $\frac{el}{x}$

$f''(x) = (4x^3 - 9x^2 + 6x - 1)' = 12x^2 - 18x + 6$

$\cancel{0}, 1, \frac{1}{4}$

$f''(0) = 6 > 0 \Rightarrow \min$ treba zahrnúť ?? ✗

$f''(1) = 0 \Rightarrow$ Inflexný bod

$f''(\frac{1}{4}) = \frac{12}{8} - \frac{36}{8} + \frac{48}{8} > 0$ min ✓

Inflex $16x(x-1)^3 = 16 \cdot 1(0)^3 = 0$

c) $f(x) = \frac{\ln(x)}{\sqrt{x}}$

1. Definičný obor

$OR: \ln(x) \Rightarrow x \neq 0$

$\sqrt{x} \Rightarrow x >= 0$

$D(f) = (0, \infty)$ ✓

2. Párnosť

$f(-x) = \frac{\ln(-x)}{\sqrt{-x}}$ f(x) nije ani párna ✓ ani nepárna ✓

$-f(-x) = -\frac{\ln(-x)}{\sqrt{-x}}$

3. Nulové body: 1 ✓

4. Spojitosť $\Rightarrow$ f(x) je spojitá lebo nemá dieru v D(f) ✗ !

5. Asymptota

ABS: $\cancel{y} = 0$

ASS: $kx + q$

$k: \lim_{x \to \infty} \frac{\ln(x)}{x^{\frac{3}{2}}}$ "LH" $\Rightarrow \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{3}{2}\sqrt{x}} = 0$

$q: \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} \Rightarrow 0$

ASS $\nexists$ ✓

6. Monotónnosť

$f'(x) = \left(\frac{\ln(x)}{\sqrt{x}}\right)' = \frac{\frac{\sqrt{x}}{x} - \frac{\ln(x)}{2\sqrt{x}}}{x} = \frac{\frac{1}{\sqrt{x}} - \frac{\ln(x)}{2\sqrt{x}}}{x} = \frac{2 - \ln(x)}{2x^{\frac{3}{2}}}$ ✓

NB $f'(x) \Rightarrow e^2$

$2 - \ln(x) = 0$

$\cancel{\log_e}(x) = \cancel{\log_e} e^2$

$x = e^2$

	$(0, e^2)$	(e^2, ∞)
$2 - \ln(x)$	+	−
$2x^{\frac{3}{2}}$	+	+
	+	−

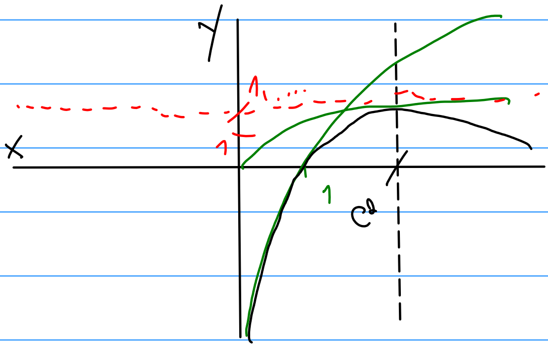
7\. Extrémy

$$f''(x) = \left(\frac{\ln(x)}{2x}\right)' = \frac{2-2\ln(x)}{4x^2} = \frac{2(1-\ln(x))}{4x^2} = \frac{1-\ln(x)}{2x^2}$$

$f(e^2) = \frac{-1}{2e^4} < 0 \Rightarrow$ ~~min~~ max

$f(x)$ je v bode e^2 konkávna

max $\frac{\ln(e^2)}{\cancel{2}e^{\cancel{2}1}} = \frac{2}{e}$



d) $f(x) = \ln(4-x^2)$

1.) Definičný obor

$4-x^2 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$D(f) = (-2, 2)$

2.) Párnosť

$f(-x) = \ln(4-(-x)^2) = \ln(4-x^2)$

$f(x)$ je párna

3.) NB: $\pm\sqrt{3}$

? 4.) Spojitosť $f(x)$ je spojitá lebo nemá dieru v $D(f)$

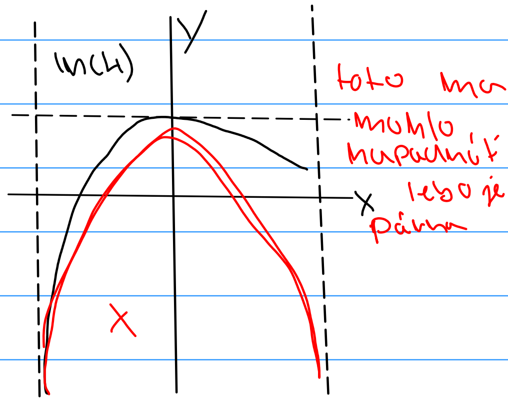
5.) Asymptoty

ABS: $x = \pm 2$

ASS: $kx+q$

$k \Rightarrow \lim_{x\to\infty} \frac{\ln(4-x^2)}{x}$ "LH" $= \lim_{x\to\infty} \frac{\frac{-2x}{4-x^2}}{1} \Rightarrow 0$

$q = \lim_{x\to\infty} \ln(4-x^2) - 0x = \nexists$



6\. Monotónnosť

$f'(x) = (\ln(4-x^2))' = \frac{-2x}{4-x^2}$

NB: 0

	$(-2,0)$	$(0,2)$
$-2x$	+	−
$4-x^2$	+	+
	+	−

$\cap \Rightarrow$ konkávna

8\. Extrémy

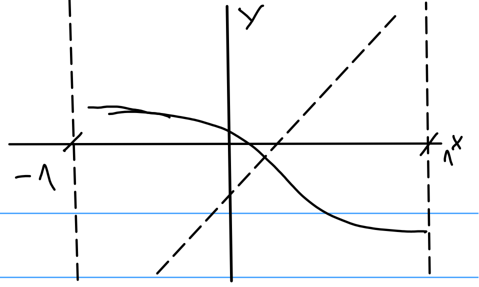
$$f''(x) = \frac{-2(4-x^2)-(-2x)(-2x)}{(4-x^2)^2}$$

$$f''(x) = \frac{-8+2x^2-4x^2}{(4-x^2)^2} =$$

$$f''(x) = -\frac{2x^2+8}{(4-x^2)}$$

$f''(0) = -\frac{8}{4} < 0 \Rightarrow \max \Rightarrow f(x)$ je v bode $-\frac{8}{4}$ maximum s hodnotou $\ln(4)$

a je konkávna $[0, \ln(4)]$



e) $f(x) = x - 2\,\mathrm{arctg}(x)$

1. Definičný obor

$OR: x \in \mathbb{R} \wedge -1 \leq x \leq 1$

$D_f = x \in \langle -1; 1 \rangle$

2.) Párnosť

$f(-x) = -x + 2\,\mathrm{arctg}(x)$

$-f(-x) = x - 2\,\mathrm{arctg}(x)$

$f(x)$ je nepárna

3.) Spojitosť

$f(x)$ je spojitá lebo nemá dieru v $D(f)$

4.) Asymptoty

NB: $\frac{x}{2} = \mathrm{arctg}(x)$

ABS $x = +1^{+}, -1^{-}$

ASS: $kx + q$

$$\lim_{x \to \infty} \left(\frac{x - 2\,\mathrm{arctg}(x)}{x} \right) \text{"Lh"} \Rightarrow \lim_{x \to \infty} \frac{1 - \frac{2}{1+x^2}}{\cancel{1}} = \lim_{x \to \infty} \frac{x^2 - 1 + 1 - 1}{x^2 + 1} =$$

$$\lim_{x \to \infty} 1 - \frac{2}{x^2 + 1} \Rightarrow 1$$

$$q \Rightarrow \lim_{x \to \infty} \cancel{x} - 2\,\mathrm{arctg}(x) \; \cancel{-x} \Rightarrow \frac{-2}{1 + x^2} \Rightarrow -2$$

ASS: $y = x - 2$

6.) Monotónnosť

$f'(x) = 1 - \frac{2}{1+x^2} = \frac{x^2 - 1}{x^2 + 1}$ NB: ± 1

	$\langle -1, 0 \rangle$	$\langle 0, 1 \rangle$
$x^2 - 1$	$-$	$-$
$x^2 + 1$	$+$	$+$
	$-$ ↘	$-$ ↘

7.) Extrémy

$$f''(x) = \frac{2x(x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2}$$

$$f''(x) = \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2}$$

$$f''(x) = \frac{4x}{(x^2+1)^2}$$

$f''(-1) = \frac{-4}{+} < 0 \Rightarrow \max$

$f''(1) = \frac{+}{+} > 0 \Rightarrow \min$

Určité Integrály

$$\int \cos(2x)\,dx = \left| \begin{matrix} t = 2x \\ dt = 2\,dx \\ dx = \frac{dt}{2} \end{matrix} \right| = \frac{1}{2}\int \cos(t)\,dt = \frac{\sin(t)}{2} = \frac{\sin(2x)}{2}$$

1.) $$\int_0^{\pi} (2x^2+3)\cdot\cos(2x)\,dx = \left| \begin{matrix} u = 2x^2+3 & u' = 4x \\ v' = \cos(2x) & v = \frac{\sin(2x)}{2} \end{matrix} \right|$$

$$\left[\frac{(2x^2+3)\cdot\sin(2x)}{2}\right]_0^{\pi} - \underbrace{\int_0^{\pi} 2x\,\sin(2x)\,dx}_{I}$$

I. $$\int_0^{\pi} 2x\cdot\sin(2x)\,dx = \left| \begin{matrix} u = 2x & u' = 2 \\ v' = \sin(2x) & v = -\frac{\cos(2x)}{2} \end{matrix} \right| = -x\cdot\cos(2x) - \int_0^{\pi} -\cos(2x)\,dx$$

$$= \frac{(2x^2+3)\cdot\sin(2x)}{2} - \left(-x\cdot\cos(2x) - \int -\cos(2x)\,dx\right)$$

$$= \int_0^{\pi} -\cos(2x)\,dx = \left| \begin{matrix} t = 2x \\ \frac{dt}{2} = dx \end{matrix} \right| = -\frac{\sin(2x)}{2}$$

$$= \left[\frac{(2x^2+3)\sin(2x)}{2} + x\cdot\cos(2x) - \frac{\sin(2x)}{2}\right]_0^{\pi}$$

$$\frac{(2\pi^2+3)(\sin(0))}{2} + \pi - 0 - (0 + 0 - 0) = 0 + \pi - 0 \ldots = \pi$$

$$\int_{x_1=\ln(2)}^{x_2=\ln(5)} \frac{(e^x-2)\,e^x}{e^{2x}+2e^x+7}\,dx = \left| \begin{matrix} t = e^x \\ dt = e^x\,dx \\ x_1 = e^{\ln(2)} = 2 \\ x_2 = e^{\ln(5)} = 5 \end{matrix} \right| = \int_2^5 \frac{(t-2)}{t^2+2t+7}\,dt$$

$$= \int_2^5 \frac{t-2+1-1}{(t+1)^2+6}\,dt = \left| \begin{matrix} u = t+1 \\ du = dt \\ x_1 = 3 \\ x_2 = 6 \end{matrix} \right| = \underbrace{\int_3^6 \frac{u}{u^2+6}\,du}_{I.} - \underbrace{\int_3^6 \frac{3}{u^2+6}\,du}_{II.}$$

Zlý postup riešenia I.

I. $$\int_3^6 \frac{u}{u^2+6}\,du = \left| \begin{matrix} f = u^2+6 \\ df = 2u \\ \frac{df}{2} = u\,du \end{matrix} \right| = \frac{1}{2}\int f^{-1}\,df \Rightarrow 0$$

II. $$3\left(\frac{1}{\sqrt{6}}\operatorname{arctg}\left(\frac{u}{\sqrt{6}}\right)\right) = \left[\frac{3\operatorname{arctg}\left(\frac{e^x+1}{\sqrt{6}}\right)}{\sqrt{6}}\right]_{\ln(2)}^{\ln(5)}$$

$$\int_2^5 \frac{t-2}{t^2+2t+7}\,dt = \left| \begin{matrix} w = t^2+2t+7 \\ dw = 2t+2\,dt \end{matrix} \right| = \int_2^5 \frac{\frac{1}{2}(2t+2)-3}{t^2+2t+7} = \underbrace{\int \frac{\frac{1}{2}(2t+2)}{t^2+2t+7}\,dt}_{\mathrm{I.}} - \underbrace{\int \frac{3}{t^2+2t+7}}_{\mathrm{II.}}$$

$$\mathrm{I.}\quad \frac{1}{2}\int w^{-1} = \frac{\ln(e^{2x}+2e^x+7)}{2}$$

$$\mathrm{II.}\quad -3\int \frac{dt}{(t+1)^2+6} = \left| \begin{matrix} w = t+1 \\ dw = dt \end{matrix} \right| = -3\int \frac{1}{w^2+6} = \frac{-3\,\mathrm{arctg}\left(\frac{e^x+1}{\sqrt{6}}\right)}{\sqrt{6}}$$

$$\left[\frac{\ln(e^{2x}+2e^x+7)}{2} - \frac{3\,\mathrm{arctg}\left(\frac{e^x+1}{\sqrt{6}}\right)}{\sqrt{6}} \right]_{\ln(2)}^{\ln(5)}$$

nevícm je to zle píči bot

$$\frac{\ln(e^{10}+2e^5+7)}{2} - \frac{3\,\mathrm{arctg}\left(\frac{e^5+1}{\sqrt{6}}\right)}{\sqrt{6}} - \left(\frac{\ln(e^4+2e^2+7)}{2} - \frac{3\,\mathrm{arctg}\left(\frac{e^2+1}{\sqrt{6}}\right)}{\sqrt{6}} \right)$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\cos^2(x)-5\cos(x)+6}\,dx = \left| \begin{matrix} t = \cos(x) \\ dt = -\sin(x)\,dx \end{matrix} \right| = -\int_0^{\frac{\pi}{2}} \frac{1}{(t^2-5t+6)}\,dt =$$

$$= -\int \frac{1}{\left(t-\frac{5}{2}\right)^2-\frac{1}{4}}\,dt = \left| \begin{matrix} s = t-\frac{5}{2} \\ ds = dt \end{matrix} \right| = -\int \frac{1}{s^2-\left(\frac{1}{2}\right)^2}\,ds$$

$$= \ln\left|\frac{s-\frac{1}{2}}{s+\frac{1}{2}}\right| = \left[\ln\left|\frac{\cos(x)-3}{\cos(x)-2}\right| \right]_0^{\frac{\pi}{2}} \Rightarrow \ln\left|\frac{3}{2}\right| - \ln|2|$$

$$\int_1^{64} \frac{2\sqrt{x}}{x\left(x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)} = 2\int_1^{64} \frac{1}{x^2+x}\,dx = \frac{A}{x} + \frac{B}{x+1}$$

$$0x+0+1 = x(A+B)+Ax \qquad 2\left(\int_1^{64} \frac{1}{x+1} - \frac{1}{x} \right)$$

$A = 1$

$A+B = 0$

$B = -A = -1$

$$\left[2\left(\ln(x+1) - \ln(x)\right) \right]_1^{64}$$

$$\left[2\ln\left(\frac{x+1}{x}\right) \right]_1^{64} = 2\ln\left(\frac{65}{64}\right) - 2\ln(2) \quad \checkmark$$

$$\int_0^{\frac{\pi}{3}} \frac{(\cos^2(x)+1)\sin(x)}{\cos^4(x)+\cos^3(x)}\,dx = \left| \begin{array}{l} t=\cos(x) \\ dt = -\sin(x)\,dx \end{array} \right| = -\int_0^{\frac{\pi}{3}} \frac{t^2+1}{t^3(t+1)}\,dt =$$

$$\frac{A}{t} + \frac{B}{t^2} + \frac{C}{t^3} + \frac{D}{t+1}$$

$$t^2+1 = At^2(t+1) + Bt(t+1) + Ct + C + Dt^3$$

$$t^2+1 = At^3 + At^2 + Bt^2 + Bt + Ct + C + Dt^3$$

$$t^2+1 = t^3(A+D) + t^2(A+B) + t(B+C) + C$$

$C = 1$

$B = -1$

$A = 2$

$D = -2$

$$-\int_0^{\frac{\pi}{3}} \frac{2}{t} - \frac{1}{t^2} + \frac{1}{t^3} - \frac{2}{t+1}\,dt$$

$$= -\left(2\ln(\cos(x)) + \frac{1}{\cos(x)} - \frac{1}{2\cos^2(x)} - 2\ln(\cos(x)+1)\right)$$

$$= \left[\frac{1}{2\cos^2(x)} + 2\ln(\cos(x)+1) - \frac{1}{\cos(x)} - 2\ln(\cos(x))\right]_0^{\frac{\pi}{3}}$$

$$\left[\frac{1-2\cos(x)}{2\cos^2(x)} + 2\ln(\cos(x)+1) - 2\ln(\cos(x))\right]_0^{\frac{\pi}{3}}$$

$$0 + 2\ln\left(\frac{3}{2}\right) - 2\ln(0{,}5) - \left(-\frac{1}{2} + 2\ln(2) - 0\right) \checkmark$$

22. $\int_0^{\frac{\pi}{4}} \frac{tg(x)}{tg^2(x)+tg(x)+1}\,dx = \int_0^{\frac{\pi}{4}} \frac{\frac{tg(x)}{\cos^2(x)}}{\frac{tg^2(x)+tg(x)+1}{\cos^2(x)}}\,dx = \left| \begin{array}{l} t = tg(x) \\ dt = \frac{1}{\cos^2(x)}\,dx \end{array}\right| =$

$\int_0^{\frac{\pi}{4}} \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = \int_0^{\frac{\pi}{4}} \frac{2\sin(x)\cdot\cos(x)}{\sin^4(x)+\cos^4(x)}\,dx = \left| \begin{array}{l} t = \sin^2(x) \\ dt = 2\sin(x)\cdot\cos(x)\,dx \end{array}\right|$

$\int_0^{\frac{\pi}{4}} \frac{1}{2t^2-2t+1}\,dt = \int_0^{\frac{\pi}{4}} = \frac{1}{2(t^2-t+\frac{1}{2})} = \frac{1}{2}\int_0^{\frac{\pi}{4}}$

$\cos^4(x) \Rightarrow \sin^4(x)$

A) tu sa musí použit Pojebaní secans

$\cos^4(x) = \cos^2(x)\cdot\cos^2(x)$

$(1-\sin^2(x))^2 = 1-2\sin^2(x)+\sin^4(x) + \sin^4(x)$

$\int_0^{\frac{\pi}{4}} \frac{\tan(x)}{\tan^2(x)+\tan(x)+1} = \int_0^{\frac{\pi}{4}} \frac{\sec^2(x)\tan(x)}{\tan^2(x)\sec^2(x)+\tan(x)\sec^2(x)+\sec^2(x)}\,dx = \left| \begin{array}{l} t = \tan(x) \\ dt = \sec^2(x)\,dx \\ x_1 = 0 \\ x_2 = 1 \end{array}\right.$

$\sec^2(x) = \tan^2(x)+1$

$\int_0^1 \frac{t}{\tan^2(x)\cdot(\tan^2(x)+1)+\tan(x)(\tan^2(x)+1)+\tan^2(x)+1}\,dt$

$\int_0^1 \frac{t}{t^4+t^2+t^3+t+t^2+1}\,dt = \int_0^1 \frac{t}{t^4+t^3+2t^2+t+1}$

$\frac{A}{(t^2+1)} + \frac{B}{(t^2+t+1)}$ $\quad t = t^2(A+B)+Bt+A+B$

$A+B=0 \quad B=1 \quad A=-1$

$\int_0^1 \underbrace{\frac{1}{(t+1)^2}}_{I.} - \underbrace{\frac{1}{t^2+1}}_{II.}$

I. $\int_0^1 \frac{1}{(t+1)^2}\,dt = \left| \begin{array}{l} s = t+1 \\ ds = dt \\ x_1 = 1 \\ x_2 = 2 \end{array}\right| = \int_1^2 s^{-2} = \left[-\frac{1}{tg(x)+1}\right]_1^2$

$$\mathrm{II.}\quad -\int_0^1 \frac{1}{t^2+1}\,dt = \Big[-\operatorname{arctg}(tg(x)+1)\Big]_0^1$$

$$\Big[-\operatorname{arctg}(tg(x)+1)\Big]_0^1 + \Big[-\frac{1}{tg(x)+1}\Big]_1^2$$

$$-\operatorname{arctg}(tg(1)+1) + \operatorname{arctg}(tg(0)+1) - \frac{1}{tg(2)+1} + \frac{1}{tg(1)+1} =$$

$$=$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin(2x)}{\sin^4(x)+\cos^4(x)}\,dx = \int_0^{\frac{\pi}{4}} \frac{2\sin(x)\cdot\cos(x)}{\sin^4(x)+\cos^4(x)}\,dx = \left| \begin{array}{l} t=\sin^2(x) \\ dt = 2\sin(x)\cdot\cos(x) \\ x_1 = \frac{1}{2} \quad x_2 = 0 \end{array} \right|$$

$$\int_{\frac{1}{2}}^{0} \frac{dt}{t^2+\cos^4(x)} = \int_{\frac{1}{2}}^{0} \frac{dt}{2t^2-2t+1} = \frac{1}{2}\int_{\frac{1}{2}}^{0} \frac{dt}{t^2-t+\frac{1}{2}} = \frac{1}{2}\int_{\frac{1}{2}}^{0} \frac{dt}{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}} =$$

$$= \left| \begin{array}{l} s = t-\frac{1}{2} \\ ds = dt \\ x_1 = 0 \\ x_2 = -\frac{1}{2} \end{array} \right| = \frac{1}{2}\int_0^{-\frac{1}{2}} \frac{1\,ds}{s^2+\left(\frac{1}{2}\right)^2} \Rightarrow \left[\operatorname{arctg}\left(\frac{\sin^2(x)-\frac{1}{2}}{\frac{1}{2}}\right)\right]_0^{-\frac{1}{2}}$$

$$\operatorname{arctg}\left(\frac{\sin^2(-0{,}5)-\frac{1}{2}}{\frac{1}{2}}\right) - \operatorname{arctg}(-1)$$

$$\int_1^e \ln^2 x\,dx = \left| \begin{array}{ll} u=\ln^2(x) & u' = \frac{2\ln(x)}{x} \\ v'=1 & v = x \end{array} \right| = \Big[x\ln^2(x)\Big]_1^e - \int_1^e 2\ln(x) = \left| \begin{array}{ll} u=\ln(x) & u'=\frac{1}{x} \\ v'=1 & v=x \end{array} \right|$$

$$\Big[x\ln^2(x)\Big]_1^e - 2\left(x\ln(x) - \int_1^e 1\right) = \Big[x\ln^2(x) - 2x\ln(x) + 2x\Big]_1^e$$

$$= e - 2e + 2e - (0 - 0 + 2) = e - 2$$

$$\int_{\ln 3}^{\ln 5} \frac{e^{2x}+2e^{x}-3}{e^{2x}+e^{x}-6}\,dx = \left| \begin{array}{ll} t=e^{x} & x_1=3 \\ dt=e^{x}\,dx & x_2=5 \\ \frac{dt}{t}=dx & \end{array} \right|$$

$$\int_{3}^{5} \frac{t^2+2t-3}{t^3+t^2-6t}\,dt = \int_{3}^{5} \frac{t^2+2t-3}{t(t+3)(t-2)}$$

$$\frac{A}{t}+\frac{B}{t+3}+\frac{C}{t-2} \qquad \frac{\frac{1}{2}}{\frac{x}{1}}$$

$$t^2+2t-3 = At^2+At-6A+Bt^2-2Bt+Ct^2+3Ct$$

$$t^2+2t-3 = t^2(A+B+C)+t(A-2B+3C)-6A$$

$-3=-6A$ $\qquad A-2B+3C=2$ $\qquad 3C-2B=\frac{3}{2}$

$A=\frac{1}{2}$ $\qquad \underline{A+B+C=1}$ $\qquad \underline{B+C=\frac{1}{2} \Rightarrow B=\frac{1}{2}-C}$

$3C-1+2C=\frac{3}{2}$ $\qquad 10C=5$ $\qquad B=0$

$5C=\frac{5}{2}$ $\qquad C=\frac{1}{2}$

$$\frac{1}{2}\int_{3}^{5}\frac{1}{t}\,dt+\frac{1}{2}\int_{3}^{5}\frac{1}{(t-2)}\,dt = \left| \begin{array}{ll} s_1=t & s_2=t-2 \\ ds_1=dt & ds_2=dt \\ s_1x_1=5 & s_2x_1=1 \\ s_1x_2=3 & s_2x_2=3 \end{array} \right|$$

$$\left[\frac{1}{2}\ln(t)\right]_3^5+\left[\frac{1}{2}\ln(s)\right]_1^3$$

$$\frac{1}{2}\ln\left(\frac{5}{3}\right)+\frac{1}{2}\ln(3)$$

11.) $$\int_{0}^{\ln 2} \frac{2e^{x}+3}{e^{2x}+2e^{x}+2}\,dx = \left| \begin{array}{ll} t=e^{x} & x_1=1 \\ dt=e^{x}\,dx & x_2=2 \\ \frac{dt}{t}=dx & \end{array} \right|$$

$$\int_{1}^{2} \frac{2t+3}{t(t^2+2t+2)}\,dt$$

$$\frac{A}{t}+\frac{(Bt+C)}{(t^2+2t+2)}$$

$$0t^2+2t+3 = A(t^2+2t+2)+(Bt+C)t$$

$$0t^2+2t+3 = At^2+2At+2A+Bt^2+Ct$$

$$2t+3 = t^2(A+B)+t(2A+C)+2A$$

$2 = 2A + C \qquad 2A = 3$

$A + B = 0 \qquad A = \frac{3}{2} \quad B = -\frac{3}{2}$

$A = -B \qquad C = 2 - 3 = -1$

$$\int_1^2 \frac{3}{2t} + \frac{-\frac{3}{2}t - 1}{t^2+2t+2} = \underbrace{\frac{3}{2}\int_1^2 \frac{1}{t}\, dt}_{\text{I.}} - \underbrace{\int_1^2 \frac{\left(\frac{3t+2}{2}\right)}{t^2+2t+2}}_{\text{II.}}$$

I. $\left[\frac{3}{2}\ln(t)\right]_1^2$

II. $-\frac{1}{2}\int_1^2 \frac{3t+2}{t^2+2t+2}\, dt = \left| \begin{array}{ll} s = t^2+2t+2 & x_1 = 5 \\ ds = (2t+2)dt & x_2 = 10 \end{array} \right|$

$$-\frac{1}{2}\int_1^2 \frac{\frac{3}{2}(2t+2) - 1}{t^2+2t+2}\, dt = -\frac{1}{2}\left(\underbrace{\int_1^2 \frac{\frac{3}{2}(2t+2)}{t^2+2t+2}\, dt}_{\text{III.}} - \underbrace{\int_1^2 \frac{1}{t^2+2t+2}\, dt}_{\text{IV.}} \right)$$

III. $-\frac{3}{4}\int_5^{10} \frac{1}{s} = \left[\frac{3}{4}\ln(s)\right]_5^{10}$

IV. $+\frac{1}{2}\int_1^2 \frac{1}{(t+1)^2+1}\, dt = \left| \begin{array}{ll} s = t+1 & x_1 = 2 \\ ds = dt & x_2 = 3 \end{array} \right| = +\frac{1}{2}\int_2^3 \frac{1}{s^2+1}\, ds = \left[+\frac{\operatorname{arctg}(s)}{2}\right]_2^3$

$$\left[\frac{3}{2}\ln(t)\right]_1^2 - \frac{3}{4}\left[\ln(s)\right]_5^{10} + \left[\frac{\operatorname{arctg}(s)}{2}\right]_2^3$$

$$\frac{6\ln(2)}{4} - \frac{3\ln(2)}{4} + \frac{\operatorname{arctg}(3) - \operatorname{arctg}(2)}{2}$$

$$\frac{3}{4}\ln(2) + \frac{\operatorname{arctg}(3) - \operatorname{arctg}(2)}{2}$$

$\sqrt{x} : (\sqrt{x} - 1) = 1 + \frac{1}{\sqrt{x} - 1}$
$-\sqrt{x} + 1$

12.) $$\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1}\,dx = \underbrace{\int_4^9 1\,dx}_{\mathrm{I.}} + \underbrace{\int_4^9 \frac{1}{\sqrt{x}-1}\,dx}_{\mathrm{II.}}$$

I. $[x]_4^9 \Rightarrow 9-4=5$

II. $= \left| \begin{array}{l} t = \sqrt{x}-1 \\ dt = \frac{1}{2\sqrt{x}}\,dx \\ dt\ 2(t+1) = dx \end{array} \right| = 2\int_1^2 \frac{t+1}{t} = 2\left(\int_1^2 1 + \frac{1}{t}\right)$

$x_1 = 1$
$x_2 = 2$

$\left[2x + 2\ln(t))\right]_1^2 \Rightarrow 5 + 4 + 2\ln(2) - 2 = 7 + 2\ln(2)$

13.) $$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin^2(x)+\cos^2(x)}{\cos(x)-2\sin(x)+3} = \underbrace{\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin^2(x)}{\cos(x)-2\sin(x)+3}\,dx}_{\mathrm{I.}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos^2(x)}{\cos(x)-2\sin(x)+3}$$

I. $= \left| \begin{array}{l} t = \cos(x) \\ dt = -\sin(x)\,dx \end{array} \right|$

14) $$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1 + tg^2(x)}{(1+tg(x))^2} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+tg^2(x)}{1+tg^2(x)+2tg(x)}\,dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin^2 + \cos^2(x) + \frac{\sin^2(x)}{\cos^2(x)}}{\sin^2(x)+\cos^2(x) + \frac{2\sin(x)}{\cos(x)} + \frac{\sin^2(x)}{\cos^2(x)}}\,dx = \frac{\frac{\sin^2(x)\cdot\cos^2(x)+\cos^4(x)+\sin^2(x)}{\cancel{\cos^2(x)}}}{\frac{\sin^2(x)\cdot\cos^2(x)+\cos^4(x)+\sin(2x)+\sin^2(x)}{\cancel{\cos^2(x)}}}$$

nevedie to nikom znova by sme skončili pri zlomku

$$\int_0^{\frac{\pi}{2}} \sin(5x)\cdot\cos(x)\,dx = \left| \begin{matrix} u = \sin(5x) & u' = 5\cos(5x) \\ v' = \cos(x) & v = \sin(x) \end{matrix} \right|$$

$$\sin(x)\cdot\sin(5x) - 5\int_0^{\frac{\pi}{2}} \cos(5x)\cdot\sin(x)\,dx = \left| \begin{matrix} u_2 = \cos(5x) & u_2' = -5\sin(5x) \\ v' = \sin(x) & v = -\cos(x) \end{matrix} \right|$$

$$\sin(x)\cdot\sin(5x) \;-5\left(-\cos(x)\cdot\cos(5x) \;-5\int_0^{\frac{\pi}{2}} \sin(5x)\cdot\cos(x)\right)$$

$$\Big[\sin(x)\cdot\sin(5x) + 5\cos(x)\cdot\cos(5x)\Big]_0^{\frac{\pi}{2}}$$

$$1\cdot\sin\left(\frac{5}{2}\pi\right)+0 \quad -\left(0+5\right) \quad = \sin\left(\frac{5}{2}\pi\right)-5$$

$$\int_0^{\frac{\pi}{3}} \frac{\left(\cos^2(x)+1\right)\sin(x)}{\cos^4(x)+\cos^3(x)} = \left| \begin{matrix} t = \cos(x) \\ dt = -\sin(x) \\ x_1 = 1 \\ x_2 = \frac{1}{2} \end{matrix} \right| = -\int_1^{\frac{1}{2}} \frac{t^2+1}{t^3(t+1)}$$

$$\frac{A}{t} + \frac{B}{t^2} + \frac{C}{t^3} + \frac{D}{t+1}$$

$$t^2+1 = At^2(t+1) + Bt(t+1) + Ct + C + Dt^3$$

$$t^2+1 = At^3 + At^2 + Bt^2 + Bt + Ct + C + Dt^3$$

$$t^2+1 = t^3(A+D) + t^2(A+B) + t(B+C) + C$$

$C=1 \qquad A+B=1 \qquad A=2 \qquad D=-2$

$A=-D \qquad B+C=0 \Rightarrow B=-1 \qquad C=1$

$$-\left(\underbrace{\int_1^{\frac{1}{2}} \frac{2}{t} - t^{-2} + t^{-3}\,dt}_{\mathrm{I.}} + \int_1^{\frac{1}{2}} \frac{-2}{t+1}\,dt\right)$$

$$\mathrm{I.} \quad -\left(2\ln(t) + \frac{1}{t} - \frac{1}{2t^2}\right) = \left[\frac{1}{2t^2} - \frac{1}{t} - 2\ln(t)\right]_1^{\frac{1}{2}}$$

$$\mathrm{II.} \quad = \left| \begin{matrix} s = t+1 & x_1 = 2 \\ ds = dt & x_2 = \frac{3}{2} \end{matrix} \right| = -2\int_2^{\frac{3}{2}} \frac{1}{s}\,ds = \Big[+2\ln(s)\Big]_2^{\frac{3}{2}}$$

$$-2\ln(0{,}5) + \frac{1}{2} + 2\ln\left(\frac{3}{2}\right) - 2\ln(2) \quad \checkmark$$

1.) $\sum_{n=1}^{\infty} \frac{n^n}{2^3 \cdot 2^n} = \frac{1}{8} \sum_{n=1}^{\infty} \left(\frac{n}{2}\right)^n \Rightarrow \lim_{n\to\infty} \left(\frac{n}{2}\right) = \infty > 1$ Diverguje

2.) $\sum_{n=1}^{\infty} \frac{1}{2^n + 1} \Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \Rightarrow \lim_{n\to\infty} \frac{2^n + 1}{2 \cdot (2^n + \frac{1}{2})} = \frac{1}{2} \lim_{n\to\infty} \frac{2^n + 1}{2^n + \frac{1}{2}} \cdot \frac{\frac{1}{2^n}}{\frac{1}{2^n}} = \frac{1}{2} \cdot \lim (1)$

$\frac{1}{2} < 1 \Rightarrow$ konverguje

3.) $\sum_{n=1}^{\infty} \frac{3}{n-1} \Rightarrow 3 \int_1^{\infty} \frac{1}{x-1} dx = \left| \begin{matrix} t = \cancel{x} - 1 & x_2 = \infty \\ dt = d\cancel{x} & \\ x_1 = 0 & \end{matrix} \right| = 3 \int_0^{\infty} \frac{1}{t} dt = \left[3 \ln(t) \right]_0^{\infty}$

4.) $\sum_{n=3}^{\infty} \frac{3}{n (\ln(n))^{\frac{3}{2}}} \Rightarrow \lim_{a\to\infty} \int_3^a \frac{3}{x \cdot \sqrt{\ln^3(x)}} dx = \left| \begin{matrix} t = \ln(x) & x_1 = \ln(3) \\ dt = \frac{1}{x} dx & x_2 = \infty \end{matrix} \right|$

$= \lim_{a\to\infty} 3 \int_{\ln(3)}^{a} t^{-\frac{3}{2}} = \frac{3 t^{-\frac{1}{2}}}{-\frac{1}{2}} = -\frac{\frac{3}{1}}{\frac{\sqrt{t}}{2}} = \lim_{a\to\infty} \left[\frac{-6}{\sqrt{t}} \right]_{\ln(3)}^{a}$

$F(x) \Big|_a^b = F(b) - F(a) = \lim_{a\to\infty} \left(-\frac{6}{\sqrt{a}} - \left(-\frac{6}{\sqrt{\ln(3)}} \right) \right)$

$= -\frac{6}{\sqrt{\infty}} + \frac{6}{\sqrt{\ln(3)}} = 0 + \frac{6}{\sqrt{\ln(3)}} \Rightarrow$ keďže limita je Konečná
nevlastný integrál **konverguje**

$\sum_{n=1}^{\infty} \frac{(n+1)^n}{(n+1)!} \Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\frac{(n+2)^{n+1}}{(n+2)!}}{\frac{(n+1)^n}{(n+1)!}}$

$\lim_{n\to\infty} \frac{\cancel{(n+1)!} (n+2)^n \cancel{(n+2)}}{(n+1)^n \cancel{(n+2)} \cancel{(n+1)!}} = \lim_{n\to\infty} \left(\frac{n+2}{n+1} \right)^n \Rightarrow \lim_{n\to\infty} \left(1 + \frac{1}{n+1} \right)^{n+1-1}$

$\Rightarrow \underbrace{\lim_{n\to\infty} \left(1 + \frac{1}{n+1} \right)^{n+1}}_{e} \cdot \underbrace{\lim_{n\to\infty} \left(1 + \frac{1}{n+1} \right)^{-1}}_{1} \Rightarrow e \cdot 1 > 1 \Rightarrow$ diverguje

f) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n^2+1}\right)^n \Rightarrow \lim_{n\to\infty} \cancel{\sqrt[n]{\left(\frac{n+1}{2n^2+1}\right)^{n}}} = \lim_{n\to\infty}\left(\frac{n+1}{2n^2+1}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right) =$

$= \lim_{n\to\infty}\left(\frac{\frac{1}{n}+\frac{1}{n^2}}{2+\frac{1}{n^2}}\right) \Rightarrow \frac{0}{2} = 0 < 1$ konverguje

g) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n} \Rightarrow \lim_{a\to\infty} \int_1^a \frac{\ln(x)}{x}\,dx = \left| \begin{matrix} t=\ln(x) & x_1 = 0 \\ dt = \frac{1}{x}dx & x_2 = a \end{matrix} \right|$

$= \lim_{a\to\infty} \int_0^a t\,dt = \lim_{a\to\infty}\left[\frac{t^2}{2}\right]_0^a$

$F(x)\Big|_a^b = F(b) - F(a) = \lim_{n\to\infty}\left(\frac{a^2}{2} - 0\right) = \frac{\infty}{2} \Rightarrow \infty$

nevlastní hodnota $\Rightarrow$ diverguje

h) $\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n \Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)\cancel{\left(\frac{1}{2}\right)^n}\cdot\left(\frac{1}{2}\right)}{n\cancel{\left(\frac{1}{2}\right)^n}} =$

$\lim_{n\to\infty}\left(\frac{\frac{n}{2}+\frac{1}{2}}{n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right) = \frac{\frac{1}{2}+\frac{1}{\infty}}{1} = \frac{1}{2}$

$L = \frac{1}{2} < 1 \Rightarrow$ konverguje

i) $\sum_{n=1}^{\infty} \frac{n^2}{3^n} \Rightarrow \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^2}{3^n\cdot 3}}{\frac{n^2}{3^n}} = \lim_{n\to\infty} \frac{\cancel{3^n}(n+1)^2}{3\cdot\cancel{3^n}\cdot n^2}$

$= \frac{1}{3}\lim_{n\to\infty} \frac{n^2+2n+1}{n^2} = \frac{1}{3} < 1 \Rightarrow$ konverguje

j) $\sum_{n=1}^{\infty} \frac{n!}{3^n} \Rightarrow \lim_{n\to\infty} \frac{\frac{(n+1)!}{3^n\cdot 3}}{\frac{n!}{3^n}} = \lim_{n\to\infty} \frac{\cancel{3^n}(n+1)\cancel{n!}}{3\cdot\cancel{3^n}\cdot\cancel{n!}} =$

$\frac{1}{3}\lim_{n\to\infty}(n+1) \Rightarrow \frac{\infty}{3} > 1$ Diverguje

Ideálne nepoužívať nutnú podmienku

j) $\sum_{n=1}^{\infty} \frac{n!}{3^n} \Rightarrow \lim_{n\to\infty} \frac{\infty}{\infty} = \infty > 1$ Diverguje

k) $\sum_{n=1}^{\infty} \frac{3}{n^2+1} \Rightarrow \lim_{n\to\infty} \left(\frac{3}{n^2+1}\right) = \frac{3}{\infty} = 0 < 1$ konverguje

l) $\sum_{n=1}^{\infty} \frac{6n}{(n^2+4)^{\frac{3}{2}}} \Rightarrow \lim_{n\to\infty} \left(\frac{6n}{\sqrt{(n^2+4)^3}}\right) = \frac{\infty}{\infty} > 1$ Diverguje

d) $\sum_{n=1}^{\infty} \frac{3}{n^2+1} = 3\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{(n+1)^2+1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+2} = 1$

$= 3 \cdot 1 > 1$ Diverguje

Pr: D'alembertovho vyšla lim 1 tak sa to nedá použiť

e) $\sum_{n=1}^{\infty} \frac{(n+1)^n}{(n+1)!} \Rightarrow \lim_{n\to\infty} \frac{(n+2)^n \cdot \cancel{(n+2)} \cdot \cancel{(n+1)!}}{\cancel{(n+2)}\cancel{(n+1)!}(n+1)^n} = \lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^n = \infty$

Diverguje

e) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n^2+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{2n^2+1}\right) \Rightarrow 0$ konverguje lebo limita existuje

g) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n} = \lim_{a\to\infty} \int_1^a \frac{\ln(x)}{x} dx = \left| \begin{matrix} t = \ln(x) & x_1 = 0 \\ dt = \frac{1}{x} & x_2 = a \end{matrix} \right| = \lim_{a\to\infty} \int_0^a (t)$

$\left[\frac{t^2}{2}\right]_0^a \Rightarrow F(x)\Big|_a^b F(b) - F(a) \Rightarrow \lim_{a\to\infty} \left(\frac{a^2}{2} - 0\right) \Rightarrow \infty$ diverguje

h) $\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n \Rightarrow \lim_{n\to\infty} \frac{(n+1)\cdot \cancel{\left(\frac{1}{2}\right)^n} \cdot \frac{1}{2}}{n\cancel{\left(\frac{1}{2}\right)^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}$ konverguje

NWM toto nesedí

a) $y = x^3$; $y = 4x$

$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$\Downarrow$ 0 $\quad \Downarrow$ ± 2

$$\int_0^2 x^3 + 4x \, dx = \left[\frac{x^4}{4} + 2x^2 \right]_0^2$$

c) $y = e^{-x}$; $y = 1 + \cos(x)$; $x \in \langle 0, \frac{\pi}{2} \rangle$

$$\int_0^{\frac{\pi}{2}} e^{-x} - 1 - \cos(x) \, dx = \left[te^{-x} - x - \sin(x) \right]_0^{\frac{\pi}{2}}$$

$te^{-\frac{\pi}{2}} - \frac{\pi}{2} \cdot 1 - (1)$ nesedí zase podľa cvičenia

d) $y = |\cos(x)|$; $y = \sin(x) + 1$; $x \in \langle 0, \pi \rangle$

$$\int_0^{\pi} \cos(x) - \sin(x) - 1 \, dx = \left[\sin(x) + \cos(x) - x \right]_0^{\pi}$$

$-1 - \pi - (1) = -\pi - 2$

g) $\frac{1}{2}y^2 - 3 = y + 1$

$y^2 - 6 - 2y - 2 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$\Downarrow$ 4 $\quad \Downarrow$ -2

$$\int_{-2}^{4} x^2 - 2x - 8 \, dx = \left[\frac{x^3}{3} - x^2 - 8x \right]_{-2}^{4} =$$

$$= \frac{64}{3} - 16 - 32 - \left(-\frac{8}{3} - 4 + 16\right)$$

nejaký príklad)

$y = x^2 + 1$; $y = 3 - x$

$x^2 + 1 - 3 + x = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1)$

$\Downarrow$ -2 $\quad \Downarrow$ 1

$$\int_{-2}^{1} 3 - x - x^2 - 1 \, dx = \int_{-2}^{1} -x^2 - x + 2 \, dx$$

$y = x^2 - 2$, $y = 2$

$(x^2 - 4) = 0$

± 2

$$\int_{-2}^{2} 2 - x^2 + 2 \, dx = \int_{-2}^{2} 4 - x^2 \, dx = \left[4x - \frac{x^3}{3} \right]_{-2}^{2}$$

$y = 2x^2 + 10$; $y = 4x + 16$; $x = -2$, $x = 5$

$2x^2 + 10 - 4x - 16 = 0$

$2x^2 - 4x - 6 = 0$

$(2x + 2)(x - 3)$

$\Downarrow$ $\Downarrow$

-1 3

$$\int_{-1}^{3} 4x + 16 - 2x^2 - 10 \, dx = \left[\frac{-2x^3}{3} + 2x^2 + 6x \right]_{-1}^{3}$$

$$\cancel{-18 + 18} + 18 - \left(\frac{2}{3} - 4 \right) = 18 + \frac{10}{3} = \frac{64}{3}$$