Matalyza cv 4.

a) $f(x) = \sqrt{x}$ $\quad x = 382$

$x_0 = 400 = f(x) = 20$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{382} \simeq f(x_0) + f'(x_0) \cdot (x - x_0)$

$\sqrt{382} \approx 20 + \frac{1}{40} \cdot (382 - 400) = 20 - \frac{18}{40} = 20 - \frac{9}{20} = 19 + \frac{11}{20} =$

$= 19{,}55$

b) $\sqrt[5]{x} = x^{\frac{1}{5}}$ $\quad x = 36$

$f'(x) = \frac{1}{5} x^{-\frac{4}{5}}$ $\quad x_0 = 32 = 2$

$\sqrt[5]{36} \approx f(x_0) + f'(x_0)(36 - 32)$

$\sqrt[5]{36} \approx 2 + \frac{1}{5} \cdot \frac{1}{\sqrt[5]{32^4}} (4) =$

c) 2^x $\quad x = 1{,}9$

$f'(x) = 2^x \cdot \ln(2)$ $\quad x_0 = 2 \Rightarrow 4$

$2^{1,9} \approx f(x) + f'(x_0)(x - x_0)$

$2^{1,9} \approx 4 + 4\ln(2)(-0{,}1) = 4(1 + \ln(2)(-0{,}1)) = 4 - 0{,}4 = 3{,}6$

d) $\operatorname{arctg}(1{,}1) \Rightarrow f(x) = \operatorname{arctg}(x)$ $\quad x = 1{,}1$

$f'(x) = \frac{1}{1+x^2}$ $\quad x_0 = 1 \Rightarrow \frac{1}{2}$

$\operatorname{arctg}(1{,}1) \approx f(x_0) + f'(x_0)(x - x_0)$

$\operatorname{arctg}(1{,}1) \approx \frac{\pi}{4} + \frac{1}{2}(1{,}1 - 1) = \frac{\pi}{4} + 0{,}5 \cdot (0{,}1) = \frac{\pi}{4} + 0{,}05$

e) $f(x) = \arcsin(x)$ $\quad x = 0{,}2$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad x_0 = 0$

$\arcsin(0{,}2) \approx f(x_0) + f'(x_0)(x - x_0) = 0 + 1(0{,}2 - 0) = \underline{\underline{0{,}2}}$

PQ 3.

$e^2 = e^x \Rightarrow e = 3 ;\ x = 2$

$x_0 = 0 ;\ x_1 = 2$

$\frac{1}{(n+1)!} e^{\xi} x^{n+1} < 0{,}14$

$\frac{3^2 \cdot 2}{(n+1)!} < 0{,}14$

$\frac{1800}{(n+1)!} < 14$

$1800 < 14(n+1)!$

$128 \frac{1}{27} < (n+1)!$

$n = 0 \Rightarrow 1! = 1$

$n = 1 \Rightarrow 2! = 2$

$n = 2 \Rightarrow 3! = 6$

$n = 3 = 4! = 24$

$n = 4 = 5! = 120$

$n = 5 = 6! = 720$

$f(x) = e^x$

$f'(x) = e^x$

$f'' ; f''' ; f^{IV} ; f^{V}(x) = e^x$

$P_5 = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{IV}(0)x^4}{4!} + \frac{f^{V}(0)x^5}{5!}$

$P_5 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$

$P_5(2) = 1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120} = 5 + \frac{160 + 80 + 32}{120} = 5 + 2\frac{32}{120} =$

$7 + \frac{4}{15} = 7{,}2666 + 0{,}14 = 7{,}4066$

PQ 4.

$f(x) = \cos(61°) = \cos(x) ;\ n = 3 ;\ x_0 = 61°$

$f'(x) = -\sin(x)$

$f''(x) = -\cos(x)$

$f'''(x) = \sin(x)$

?

$T_3 = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3$

Vyšetrenie spojtosti f(x)

a) $f(x) = \ln(x) \cdot \log(1-x)$

$D(f) = x > 0 \wedge 1 - x > 0$

$x < 1 \Rightarrow D(f) = (0,1)$

b) $f(x) = x^{\frac{1}{x-1}}$ $\quad x-1 \neq 0$

$x \neq 1$

$\lim\limits_{x \to 1} x^{\frac{1}{x-1}} = 1^0 = 1$ Nieje spojitá

c) $x \cdot \operatorname{arctg}\left(\frac{1}{x}\right)$ $\quad -1 \leq \frac{1}{x} \leq 1$ $\quad \boxed{x \neq 0}$

$-x \leq 1 \wedge x \geq 1$

$x \geq -1 \quad D(f) = \langle 1, \infty)$

$\lim\limits_{x \to 0} x \cdot \operatorname{arctg}\left(\frac{1}{x}\right) = \lim\limits_{x \to 0} x \cdot \lim\limits_{x \to 0} \operatorname{arctg}\left(\frac{1}{x}\right) = 0 \int$

PR 2.

a) $f(x) = \ln(x) \quad x_0 = 1 \quad n = 4$

$$M_n = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 \ldots$$

$$M_4 = 0 + (x-1) - \frac{1}{2}(x-1)^2 + \frac{2}{6}(x-1)^3 - \frac{6}{24}(x-1)^4$$

$f(x_0) = 0$

$f'(x_0) = \frac{1}{x} \rightarrow 1$

$f''(x_0) = \frac{-1}{x^2} = -1$

$f'''(x_0) = \frac{2x}{x^4} = 2$

$f^{IV}(x_0) = \frac{2x^4 - 8x^4}{x^8} = -6$

b) $f(x) = \frac{1}{2^x} \; ; \; x_0 = 0 \; ; \; n = 3$

$f'(x) = 2^{-x} \cdot \ln(2) \cdot (-1) = -\ln(2) \cdot 2^{-x}$

$f''(x) = \ln^2(2) \cdot 2^{-x}$

$f'''(x) = -\ln^3(2) \cdot 2^{-x}$

$$M_3 = 1 - \ln(2) x + \frac{\ln^2(2) \cdot x^2}{2} - \frac{\ln^3(2) \cdot x^3}{6}$$

c) $f(x) = x^4 - 5x^3 + 2x - 3 \; ; \; x_0 = -1 \; ; n = 4$

$f'(x) = 4x^3 - 15x^2 + 2$

$f''(x) = 12x^2 - 15x$

$f'''(x) = 24x - 15$

$f^{IV}(x) = 24$

$$M_4 = (1 \cancel{+5} \cancel{-2} -3) + \frac{-17}{1}(x+1)^1 + \frac{27}{2}(x+1)^2 - \frac{39}{6}(x+1)^3 + \frac{24}{24}(x+1)^4$$

$$= 1 - 17(x+1) + \frac{27}{2}(x+1)^2 - \frac{39}{6}(x+1)^3 + (x+1)^4$$

d) $f(x) = e^{2x} \cdot \sin(x) \; ; \; x_0 = 0 \; ; \; n = 3$

$f'(x) = 2e^{2x} \cdot \sin(x) + e^{2x} \cdot \cos(x) = e^{2(x)} \cdot (2\sin(x) + \cos(x))$

$f''(x) = 2\, e^{2(x)} \cdot (2\sin(x) + \cos(x)) + e^{2(x)} \cdot (2\cos(x) - \sin(x))$

$= 4\sin(x)\, e^{2x} + 2\cos(x)\, e^{2x} + 2\cos e^{2x} - \cancel{\sin(x)\, e^{2x}}$

$3\sin(x)\, e^{2x} + 3\cos(x)\, e^{2x} = 3e^{2x}(\sin(x) + \cos(x))$

Priebeh funkcie

Vyšetruje sa:

1.) D(f)
2.) Párnosť
3.) spojitosť
4.) Nulové body
5.) Asymptoty
6.) Monotónnosť
7.) Extrémy
8.) Inflexný bod

a) $f(x) = \dfrac{2x^3}{x^2+1}$

1.) D(f)

OR: $x^2+1 \neq 0$

$x^2 \neq -1 \Rightarrow$ nikdy sa nestane

$D(f) = \mathbb{R}$

2.) Parita

Párnosť $\Rightarrow f(x) = f(-x)$

Nepárnosť $\Rightarrow f(x) = -f(-x)$

Párnosť $\dfrac{2x^3}{x^2+1} \neq \dfrac{-2x^3}{x^2+1}$

Nepárnosť $\dfrac{2x^3}{x^2+1} = -\left(\dfrac{-2x^3}{x^2+1}\right)$

$\dfrac{2x^3}{x^2+1} = \dfrac{2x^3}{x^2+1}$

Funkcia je Nepárna

3.) f(x) je spojitá na celom svojom D(f) lebo $D(f)=\mathbb{R}$

4.) Nieje periodická, nemá nulové body

5.) Asymptoty

ABS x

ASS $\Rightarrow y = kx + q$

$k_1 = \lim\limits_{x\to+\infty} \dfrac{\frac{2x^3}{x^2+1}}{x} = \lim\limits_{x\to+\infty} \dfrac{2x^3}{x^3+x} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim\limits_{x\to+\infty} \dfrac{2}{1+\underbrace{\frac{1}{x^2}}_{\to 0}} = \underline{\underline{2}}$

$q = \lim\limits_{x\to\infty} \dfrac{2x^3}{x^2+1} - 2x = \lim\limits_{x\to\infty} \dfrac{\cancel{2x^3} - \cancel{2x^3} - 2x}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{-2x}{x^2+1} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \dfrac{\overbrace{-\frac{1}{x}}^{\to 0}}{1+\underbrace{\frac{1}{x^2}}_{\to 0}} = \dfrac{0}{1} = 0$

$y = 2x$

6.) $f'(x) = \left(\frac{2x^3}{x^2+1}\right)' = \frac{6x^2 \cdot (x^2+1) - 2x^3 \cdot 2x}{(x^2+1)^2}$

$f'(x) = \frac{6x^4 + 6x^2 - 4x^4}{(x^2+1)^2} = \frac{2x^4+6x^2}{(x^2+1)^2} = \frac{2x^2(x^2+3)}{(x^2+1)^2}$

NB: $x = 0$

$x^2+3 = 0 \Rightarrow$ Nikdy

$x^2+1 = 0 \Rightarrow$ Nikdy

	$(-\infty, 0)$	$(0, \infty)$
$2x^2$	+	+
x^2+3	+	+
$(x^2+1)^2$	+	+
	$\nearrow$	$\nearrow$

$f(x)$ je na celom $\mathbb{R}_{\neq 0}$ rastúca na celom $D(f)$

7.) $f'(x) = \frac{2x^2(x^2+3)}{(x^2+1)^2}$

$f''(x) = \frac{(4x(x^2+3) + 2x^2 \cdot 2x) \cdot (x^2+1)^2 - (2x^2(x^2+3)) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$

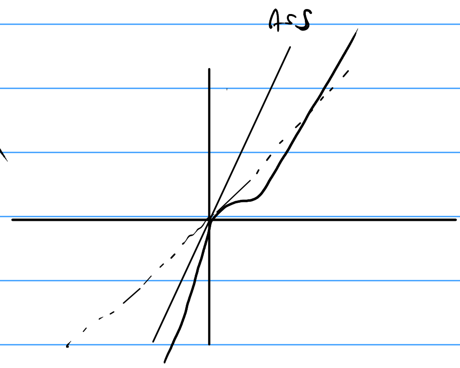
$f''(x) = \frac{(4x(x^2+3) + 4x^3)(x^2+1)^2 - 2x^2(x^2-3) \cdot 4x(x^2+1)}{(x^2+1)^4}$

$f''(x) = \frac{4x\cancel{(x^2+1)}(x^2(x^2+1) + (x^2+3)(x^2+1) - 2x^2(x^2-3))}{(x^2+1)^{\cancel{4}}}$

$f''(x) = \frac{4x(x^4+x^2+x^4+4x^2+3-4x^4+6x^2)}{(x^2+1)^3}$

$f''(x) = \frac{4x(11x^2 - 2x^4 + 3)}{(x^2+1)^3}$

$f''(0) = \frac{0}{1} = \underline{\underline{0}} \Rightarrow$ Inflexný bod



$(a-b)(a^2+ab+b^2) = a^3 - 3a^2b + 3ab^2 - b^3$

b) $f(x) > 16x(x-1)^3 = x^3 - 3x^2 + 3x - 1 = (x-1)(x^2-2x+1)$

1.) $OR = \mathbb{R}$

$D(f) = \mathbb{R}$

2.) Nieje Periodická

Parita:

$f(x) = f(-x) \Rightarrow$ párna $\quad x^3 - 3x^2 + 3x - 1 \neq (-x)^3 - 3(-x)^2 + 3(-x) - 1$

$f(x) = -f(-x) \Rightarrow$ Nepárna $\quad x^3 - 3x^2 + 3x - 1 \neq -x^3 - 3x^2 - 3x - 1$

$x^3 - 3x^2 + 3x - 1 \neq -(-x^3 - 3x^2 - 3x - 1)$ $\quad$ f(x) nieje párna

$x^3 - 3x^2 + 3x - 1 \neq x^3 + 3x^2 + 3x + 1$

f(x) Nieje Nepárna

f(x) nieje ani párna ani nepárna

3.) f(x) je spojitá na celom svojom D(f) lebo $D(f) = \mathbb{R}$

4.) NB: $(x-1)(x^2-2x+1)$

$x - 1 = 0 \quad (x-1)\underbrace{(x-1)^2}_{x=1}$

$x = 1$

5.) ABS $\Rightarrow$ Nemá

ASS $\Rightarrow y = kx + q$

$k = \lim\limits_{x \to \infty} \dfrac{x^3 - 3x^2 + 3x - 1}{x} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \infty \quad \nexists$

$q = \lim\limits_{x \to \infty} f(x) + \infty x = \nexists$

f(x) nemá Asymptotu

6.) Monotónnosť

NB: 1

$f'(x) = (16x(x^3 - 3x^2 + 3x - 1))$

$f'(x) = 16(x^3 - 3x^2 + 3x - 1) + 16x(3x^2 - 6x + 3)$

$f'(x) = 16x^3 - 48x^2 + 48x - 16 + 48x^3 - 96x^2 + 48x$

$f'(x) = 64x^3 - 144x^2 + 96x - 16 / :16$

$f'(x) = 4x^3 - 9x^2 + 6x - 1$

	4	-9	6	-1
1	4	-5	1	0

	$(-\infty, \frac{1}{4})$	$(\frac{1}{4}; 1)$	$(1, \infty)$
$4x^3 - 9x^2 + 6x - 1$	−	+	+
	↙	↗	↗
	∪	∩	∩

$(x-1)(4x^2-5x+1)$

$x_1 = 1$

$x_2 = (4x^2 - 5x + 1)$

$D = 25 - 4 \cdot 4 \cdot 1 = \sqrt{9} = 3$

$$\frac{+5 \pm 3}{8} = \begin{cases} +1 \\ +\frac{1}{4} \end{cases}$$

$x = 1, \frac{1}{4}$

Inflexní bod: $f''(x) = 12x^2 - 18x + 6$

$2x^2 - 3x + 1$

$(2x-1)(x-1)$

$2x - 1 = 0$ $\quad \boxed{x = 1}$

$x = \frac{1}{2}$

NeNi v tabulke

Extréma:

$f''(x) = 12x^2 - 18x + 6 \Rightarrow x = \frac{1}{4}$

$f''(x) = \frac{12}{8} - \frac{18}{8} + \frac{48}{8} = \frac{42}{8} > 0 = \text{Min}$

$y = \frac{16}{4}\left(\frac{1}{4} - 1\right)^3 = 4 \cdot \left(-\frac{3}{4}\right)^3 = -4\left(\frac{27}{64}\right) = \frac{-108}{64} = \frac{-54}{32} = \frac{-27}{16}$

$16 \cdot 4 = 40 + 24 = 64$

$f(x)$ má v bode $x = \frac{1}{4}$ minimum s hodnotou $\frac{-27}{16}$

c) $f(x) = \frac{\ln(x)}{\sqrt{x}}$

1.) OR: $x \geq 0 \wedge x > 0$

$D(f) = x \in (0, \infty)$

2.) Parita, $f(x)$ nije Periodická

Párnosť:

$$\frac{\ln(x)}{\sqrt{x}} \neq \frac{\ln(-x)}{\sqrt{-x}}$$

Nepárnosť:

$$\frac{\ln(x)}{\sqrt{x}} \neq -\frac{\ln(-x)}{\sqrt{-x}}$$

$f(x)$ nije ani Párna ani Nepárna

3.) $f(x)$ je spojitá na celom svojom $D(f)$

4.) Asymptoty

ABS $\Rightarrow x = 0$

ASS $= kx + q$

$$k_1 = \lim_{x \to \infty} \frac{\ln(x)}{x \cdot \sqrt{x}} \cdot \frac{\frac{1}{x^{\frac{3}{2}}}}{\frac{1}{x^{\frac{3}{2}}}} = \frac{\frac{\ln(x)}{x^{\frac{3}{2}}}}{1} = \frac{0}{1} \Rightarrow 0$$

$$q = \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} - 0x = \lim_{x \to \infty} \frac{\frac{\ln(x)}{\sqrt{x}}}{1} = ?$$

ABS $= x = 0 \wedge$ ASS $\nexists$

4.) NB: $\frac{\ln(x)}{\sqrt{x}} \Rightarrow x = 1$

5.) Monotónnost

$f'(x) = \frac{\frac{\sqrt{x}}{x} - \frac{\ln(x)}{2\sqrt{x}}}{}$

$f'(x) = \frac{2\sqrt{x} - \ln(x)\sqrt{x}}{2x}$

	$(0, e^2)$	(e^2, ∞)
$2\sqrt{x} - \ln(x)\sqrt{x}$	$+$	$-$
$2x$	$\nearrow$	$\swarrow$
	$\cap$	$\cup$

$4 - \ln(2)\sqrt{2}$

NB: $2\sqrt{x} - \ln(x)\sqrt{x} = 0$

$\ln(x)\sqrt{x} = 2\sqrt{x} \quad / : \sqrt{x}$

$\ln(x) = 2$

$\log_e(x) = 2$

$\cancel{\log_e}(x) = \cancel{\log_e} e^2$

$x = e^2$

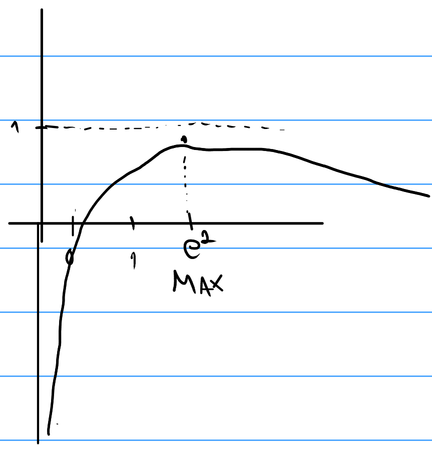
Inflexní bod $\wedge$ Extrémy

$f''(x) = \left(\frac{\cancel{\sqrt{x}}(2 - \ln(x))}{2\cancel{x}\,(2 \cdot \sqrt{x})}\right)'$

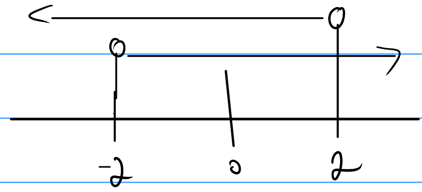
$f''(x) = \frac{-\frac{2\sqrt{x}}{x} - (2 - \ln(x)) \cdot \frac{2}{2\sqrt{x}}}{4x} = \frac{\frac{-2 - 2 + \ln(x)}{\sqrt{x}}}{4x} =$

$f''(x) = \frac{-4 + \ln(x)}{\sqrt{x} \cdot 4x} = \frac{-4 + \log_e e^2}{\sqrt{e^2} + 4 \cdot e^2} = \frac{-4 + 2}{e + e^2} < 0 \Rightarrow \text{Max}$

$f(e^2) = \frac{\ln(e^2)}{\sqrt{e^2}} = \frac{2}{e} \doteq \frac{2}{2{,}7} = 0{,}5\ldots$



d) $f(x) = \ln(4-x^2)$



1.) OR: $4-x^2=0$ $\quad 4-x^2>0$

$4 = x^2$ $\quad x < \pm 2$

$x = \pm 2$ $\quad D(f) = (-2, 2)$

2.) f(x) nie je periodická, Parita:

Párnosť:

$\ln(4-x^2) = \ln(4-(x)^2)$

$\ln(4-x^2) = \ln(4-x^2)$

f(x) je párna

3.) Spojitosť, f(x) je spojitá pretože nemá žiadne dieru v D(f)

4.) Asymptoty

ABS => $y = \pm 2$

ASS $\lim\limits_{x\to\infty} \dfrac{\ln(4-x^2)}{x} = \lim\limits_{x\to\infty} \dfrac{\frac{-2x}{4-x^2}}{\cancel{1}} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{\frac{-2}{x}\to 0}{\frac{4}{x^2}-1} = \dfrac{0}{-1} = \underline{\underline{0}}$

$q = \lim\limits_{x\to\infty}$

$ASS = \nexists$

5.) Monotónnosť

$f'(x) = \dfrac{1}{4-x^2} \cdot (-2x) = \dfrac{-2x}{4-x^2}$

$x = 0, \pm 2$

	$\cancel{(-\infty,-2)}$	$(-2,0)$	$(0,2)$	$\cancel{(2,\infty)}$
$-2x$	+	+	—	—
$4-x^2$	—	+	+	—
	↘	↗	↘	↗
	∪	∩	∪	∩

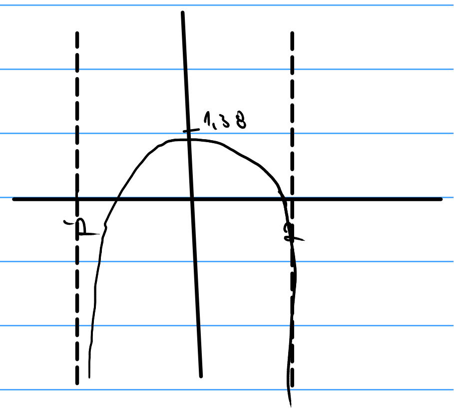
f(x) na intervale $(-\infty,-2)$, $(0,2)$ klesá

f(x) na intervale $(-2,0)$, $(2,\infty)$ rastie

6.) Inf, Min, Max

$f''(x) = \dfrac{-2(4-x^2) - (-2x)(-2x)}{(4-x^2)^2}$

$f''(x) = \dfrac{-8+2x^2-4x^2}{(4-x^2)^2} = -\dfrac{2x^2+8}{(4-x^2)^2}$



$f''(-2) = -\dfrac{2\cdot 4+8}{(4-4)^2} = \dfrac{16}{0}$ => Inflex

$f''(2) = 0$ => Inflexní bod

$f''(0) = -\dfrac{8}{16} = -\dfrac{1}{2} < 0$ => Max

$y = \ln(4)$

f(x) má v bode $x = \pm 2$ Inflexní bod a v bode $x=0$ má Max s hodnotou $\ln(4)$

e) $f(x) = x - 2\,\mathrm{arctg}(x)$

1.) OR:

$-1 \leq x \leq 1$

$D(f) = \mathbb{R}$

2.) Parita:

Párnosť:

$x - 2\,\mathrm{arctg}(x) \neq -x - 2\,\mathrm{arctg}(-x)$

$x - 2\,\mathrm{arctg}(x) = x + 2\,\mathrm{arctg}(-x) \Rightarrow x - 2\,\mathrm{arctg}(x)$

f(x) je nepárna

f(x) je periodická ale neviem ako ?

3.) spojitosť, f(x) je spojitá na celom svojom D(f)

4.) ABS $\Rightarrow x = \pm 1$

ASS $\Rightarrow y = k \cdot x + q$

$$k = \lim_{x\to\infty} \frac{x - 2\,\mathrm{arctg}(x)}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1 - \frac{2\,\mathrm{arctg}(x)}{x}}{1} = 1$$

$\frac{2\,\mathrm{arctg}(x)}{x} \to 0$

$$q = \lim_{x\to\infty} x - 2\,\mathrm{arctg}(x) \cancel{x} = \lim_{x\to\infty} -2\,\mathrm{arctg}(x) = -2x \cdot \left(\lim_{x\to\infty} \mathrm{arctg}(x)\right) =$$

$$= -2x \cdot \frac{\pi}{2} = -\pi$$

ASS $\Rightarrow y = x - \pi$ $\qquad \pi \doteq 3{,}14159$

5.) Monotónnosť

$f'(x) = 1 - \frac{2}{1+x^2}$

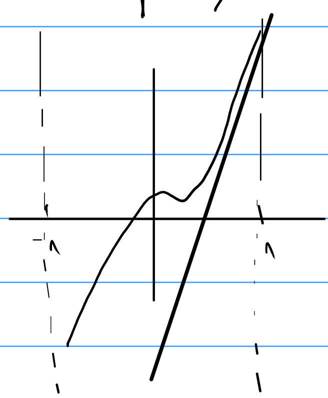
NB f(x) $\Rightarrow 1 - \frac{2}{1+x^2} = 0$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
$1-\frac{2}{1+x^2}$	+	−	+
	↗	↘	↗

$1 + x^2 = 2$

$x^2 = 1$

$x = \pm 1$



Perióda $= k \cdot \pi$

$x - 2\,\mathrm{arctg}(x) = 0$

$\frac{x}{2} = \mathrm{arctg}(x)$

f(x) je rastúca na intervale $(-\infty,-1) \wedge (1,\infty)$

f(x) je klesajúca na intervale $(-1,1)$

Extrémy a Inflexní body

$$f''(x) = \left(1 - \frac{2}{1+x^2}\right)' = \frac{2 \cdot 2x}{(1+x^2)^2} = \frac{4x}{(1+x^2)^2}$$

$$f''(-1) = \frac{-4}{4} = -1 < 0 \Rightarrow \text{Max}$$

$$f''(1) = \frac{4}{4} = 1 > 0 \Rightarrow \text{Min}$$

$$f(-1) = -1 - 2\operatorname{arctg}(-1) = -1 + 2\operatorname{arctg}(1) = \frac{\pi}{2} - 1 = 0{,}570$$

$$f(1) = 1 - 2\operatorname{arctg}(1) = 1 - \frac{\pi}{2} = -0{,}570$$

Max $\Rightarrow [-1, \frac{1}{2}]$

Min $\Rightarrow [1, -\frac{1}{2}]$

Inflex $\Rightarrow \nexists$

Integrálos ~ dívovzáhos

PR1.

PR 1 d, e, m, s

a) $\int 3x^3+2x-4\ (dx) = \frac{3x^4}{4} + \frac{2x^2}{2} + c = \frac{3}{4}x^4 + x^2 - 4x$

b) $\int \frac{x^3}{3} - \frac{x}{5}\ (dx) = \frac{x^4}{12} - \frac{x^2}{10} + c = 0$

c) $\int \sqrt{x^3} - \frac{1}{\sqrt{x}}\ (dx) = \int x^{\frac{3}{2}} - x^{-\frac{1}{2}} = \frac{\sqrt{x^5}}{\frac{5}{2}} - \frac{\sqrt{x}}{\frac{1}{2}} = \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + c$

d) $\int \frac{\sqrt{x^4+2+x^{-4}}}{x^3}\ (dx) = \int \sqrt{x^4+2+x^{-4}} \cdot \int \frac{1}{x^3}$?

e) $\int \frac{x^3-1}{x-1}\ dx \Rightarrow \cancel{x^3}-1 : (x-1) = x^2+x+1$

$-(\cancel{x^3}-x^2)$

$\cancel{x^2}-1$

$-(\cancel{x^2}-x)$

$\cancel{x}-1$

$-(\cancel{x}-1)$

$\int x^2+x+1\ (dx) = \frac{x^3}{3} + \frac{x^2}{2} + x + c$

g) $\int e^x \cdot a^x\ (dx) = \int (ae)^x\ (dx) = \frac{a^x e^x}{\ln(ae)} = \frac{a^x e^x}{1+\ln(a)}$

h) $\int 5\cos(x) - 2x^5 + \frac{3}{1+x^2}\ (dx) = 5\sin(x) - \frac{2x^6}{6} + 3\,\mathrm{arctg}(x) = 5\sin(x) - \frac{x^6}{3} + 3\,\mathrm{arctg}(x)$

i) $\int 10^{-x} + \frac{x^2+3}{x^2+1}\ (dx) = \int 10^{-x} + \frac{x^2+1+2}{1+x^2} = \int 10^{-x} + 1 + \frac{2}{1+x^2} =$

$= \frac{\frac{\frac{1}{10^x}}{\ln(10)}}{1} = -\frac{1}{10^x \cdot \ln(10)} + x + 2\,\mathrm{arctg}(x) + c$

j) $\int 2\sin(x) - 3\cos(x)\ (dx) = -2\cos(x) - 3\sin(x) + c$

k) $\int \frac{1}{\sqrt{3-3x^2}} = \int \frac{1}{\sqrt{3(1-x^2)}} = \int \frac{1}{\sqrt{3}\cdot\sqrt{1-x^2}} = \frac{\sqrt{3}\arcsin(x)}{3}$

e) $\int \frac{3\cdot 2^x - 2\cdot 3^x}{2^x} (dx) = \int 3(dx) - \int 2\cdot\left(\frac{3}{2}\right)^x (dx) = 3x - \frac{2\cdot\left(\frac{3}{2}\right)^x}{\ln\left(\frac{3}{2}\right)}$

$3x - \frac{\frac{2^{1+x}\cdot 3^x}{2^x \cdot 2^x}}{\frac{\ln\left(\frac{3}{2}\right)}{1}} = 3x - \frac{2^{1-x}\cdot 3^x}{2^x \cdot \ln\left(\frac{3}{2}\right)} + c$

??

m) $\int \frac{1+\cos^2(x)}{1+\cos(2x)} dx = \int \frac{1+\cos^2(x)}{1+2\sin(x)\cdot\cos(x)} = \int \frac{2\cos^2(x)+\sin^2(x)}{\sin^2(x)+\cos^2(x)+2\sin(x)\cdot\cos(x)}$

n) $\int \frac{\cos(2x)}{\cos^2(x)\cdot\sin^2(x)} = \int \frac{\cos^2(x)-\sin^2(x)}{\cos^2(x)\cdot\sin^2(x)} = \int \frac{1}{\sin^2(x)}(dx) - \int \frac{1}{\cos^2(x)}(dx) =$

$= -\mathrm{cotg}(x) - \mathrm{tg}(x) + c \qquad \sin^2(x) \qquad = 1 - \cos^2(x)$

o) $\int \mathrm{tg}^2(x)\,(dx) = \int \frac{\sin^2(x)}{\cos^2(x)}(dx) = \int \frac{1}{\cos^2(x)}(dx) - \int 1 = \mathrm{tg}(x) - x + c$

p) $\int \mathrm{cotg}^2(x)\,(dx) = \int \frac{\cos^2(x)}{\sin^2(x)}(dx) = \int \frac{1}{\sin^2(x)}(dx) - x + c = -\mathrm{cotg}(x) - x + c$

r) $\int \frac{1}{\cos(2x)+\sin^2(x)} = \int \frac{1}{\cos^2(x) - \cancel{\sin^2(x)} + \cancel{\sin^2(x)}} = \mathrm{tg}(x) + c$

q) $\int \frac{1+2x^2}{x^2(1+x^2)}(dx) = \int \frac{1+x^2+x^2}{(1+x^2)\cdot x^2}(dx) = \int \frac{1}{x^2} + \frac{\cancel{x^2}}{(1+x^2)\cancel{x^2}} =$

$= \frac{x^{-1}}{-1} + \mathrm{arctg}(x) + c = -\frac{1}{x} + \mathrm{arctg}(x) + c$

s) $\int \frac{x^2+2x+1}{x+x^3}$?

t) $\int \frac{5x^2-7x+10}{(x-3)(x^2+2x+2)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+2x+2} = \frac{A(x^2+2x+2)+(Bx+C)(x-3)}{(x-3)(x^2+2x+2)}$

$5x^2-7x+10 = A(x^2+2x+2) + (Bx+C)(x-3)$

$5x^2-7x+10 = Ax^2 + Bx^2 + 2Ax - 3Bx + Cx + 2A - 3C$

$5x^2-7x+10 = x^2(A+B) + x(2A-3B+C) + 2A - 3C$

$5 = A+B \Rightarrow A = 5-B \qquad -17B = 9$

$-7 = 2A - 3B + C \qquad B = \frac{-9}{17} \qquad A = \frac{94}{17}$

$10 = 2A - 3C \Rightarrow C = \frac{2A-10}{3}$

$-7 = 10 - 5B + \frac{2A-10}{3} \quad / \cdot 3 \qquad C = \frac{188}{17} - 10 = \frac{6}{17}$? Ako prepisať

	2	12	11	5
-5	2	2	1	0

$$c)\int \frac{4x^2+x-13}{2x^3+12x^2+11x+5} = \int \frac{4x^2+x-13}{(x+5)(2x^2+2x+1)} = \frac{A}{x+5}+\frac{Bx+C}{2x^2+2x+1} = \frac{A(2x^2+2x+1)+(Bx+C)(x+5)}{(x+5)(2x^2+2x+1)}$$

$$4x^2+x-13 = 2Ax^2+2Ax+A+Bx^2+5Bx+Cx+5C$$

$$4x^2+x-13 = x^2(2A+B)+x(2A+5B+C)+A+5C$$

$$4 = 2A+B \Rightarrow 2-\frac{B}{2}=A$$

$$1 = 2A+5B+C$$

$$-13 = A+5C \Rightarrow C=\frac{-13-A}{5} = \frac{-13-(2-\frac{B}{2})}{5} = \frac{\frac{B}{2}-15}{5}$$

$$1 = 4-B+5B+\frac{\frac{B}{2}-15}{5} \quad / \cdot 5$$

$$5 = 20+20B+\frac{B}{2}-15$$

$$\cancel{5} = \cancel{5}+\frac{41}{2}B$$

$B=0$

$A=2$

$C=-3$

$$\int \frac{2}{x+5}+\frac{0x-3}{2x^2+2x+1} = \underbrace{2\int\frac{1}{x+5}dx}_{\mathrm{I.}}+\underbrace{\int\frac{-3}{2x^2+2x+1}}_{\mathrm{II.}} =$$

I. $2\ln(|x+5|)$

$\int \frac{1}{x^2+a^2} = \frac{1}{a}\cdot \operatorname{arctg}(\frac{x}{a})$

$$\mathrm{II.}\int\frac{-3}{2(x^2+x+\frac{1}{2})} = \int\frac{-3}{2(x^2+x+\frac{1}{4}+\frac{1}{4})} = \int\frac{-3}{2((x+\frac{1}{2})^2+\frac{1}{4})} \Rightarrow \left|\begin{matrix} t=x+\frac{1}{2} \\ dt=dx\end{matrix}\right| =$$

$$-\frac{3}{2}\int\frac{1}{t^2+\frac{1}{4}} = -\frac{3}{2}\int\frac{1}{t^2+(\frac{1}{2})^2} = -\frac{3}{2}\cdot\frac{1}{2}\cdot\operatorname{arctg}(x) = -\frac{3}{4}\operatorname{arctg}(x)$$

$$2\ln(|x+5|)-\frac{3}{4}\operatorname{arctg}(x)+C$$

$\int u'\cdot v = uv-\int u\cdot v'$

$$d)\int\frac{\sqrt{x^4+2+\frac{1}{x^4}}}{x^3}dt = \left|\begin{matrix} u=\sqrt{x^4+2+x^{-4}} & u'=\frac{4x^3-4x^{-5}}{2\sqrt{x^4+2+x^{-4}}} \\ v'=x^{-3} & v=\frac{x^{-2}}{-2}\end{matrix}\right|$$

$$\frac{\sqrt{x^4+2+x^{-4}}}{-2x^2}-\int\frac{4x^3-4x^{-5}}{2\sqrt{x^4+2+x^{-4}}\cdot(-2x^2)} = \frac{\sqrt{x^4+2+x^{-4}}}{-2x^2}-\int\frac{\cancel{4}(x^3-x^{-5})}{\cancel{-4}x^2\sqrt{\text{—}\,''\,\text{—}}}$$

$$= \frac{\sqrt{x^4+2+x^{-4}}}{-2x^2}--\int\frac{(x^3-x^{-5})}{x^2\sqrt{\text{—}\,''\,\text{—}}} = \frac{\sqrt{x^4+2+x^{-4}}}{-2x^2}+\int\frac{(x^3-x^{-5})}{x^2\sqrt{\text{—}\,''\,\text{—}}}$$

$$\left|\begin{matrix} u=x^2\sqrt{x^4+2+x^{-4}} & u'=2x(\sqrt{\text{—}\,''\,\text{—}})+\frac{4x^3-4x^{-5}}{2\sqrt{x^4+2+x^{-4}}} \\ v'=x^3-x^{-5} & v=\frac{x^4}{4}-\frac{x^{-6}}{-6}\end{matrix}\right.$$

e) $\int \frac{x(x^{\frac{1}{3}} - x^{\frac{4}{3}})}{x^{\frac{1}{4}}} = \int \frac{x^{\frac{4}{3}} - x^{\frac{7}{3}}}{x^{\frac{1}{4}}} =$

PR 2.

a) $\int \frac{1}{3+4x^2}\,dx = \frac{1}{4}\int \frac{1}{\frac{3}{4}+x^2} = \frac{1}{4}\cdot\frac{2}{\sqrt{3}}\cdot \operatorname{arctg}\left(\frac{\sqrt{3}}{2}\right) =$

$\frac{2\operatorname{arctg}\left(\frac{\sqrt{3}}{2}x\right)}{4\cdot\sqrt{3}}$ ✓

b) $\int \frac{x}{3+4x^2}\,dx = \left|\begin{matrix} t = 3+4x^2 \\ dt = 8x\,dx \\ \frac{dt}{8} = x\,dx \end{matrix}\right| = \int \frac{1}{8t}\,dt = \frac{1}{8}\int \frac{1}{t}\,dt = \frac{1}{8}\ln|t| =$

$= \frac{1}{8}\ln(3+4x^2)$ ✓

c) $\int \frac{x}{(x^2+5)^4}\,dx = \left|\begin{matrix} t = x^2+5 \\ dt = 2x\cdot dx \\ \frac{dt}{2} = x\,dx \end{matrix}\right| = \int \frac{1}{t^4}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{1}{t^4}\,dt =$

$= \frac{1}{2}\cdot\int t^{-4}\,dt = \frac{1}{2}\cdot\frac{t^{-3}}{-3} = \frac{1}{2}\cdot\frac{1}{-3(x^2+5)^3} = \frac{1}{-6(x^2+5)^3} + C$

d) $\int e^x \operatorname{tg}(e^x)\,dx = \left|\begin{matrix} t = e^x \\ dt = e^x\,dx \end{matrix}\right| = \int \operatorname{tg}(t)\,dt = \int \frac{\sin(t)}{\cos(t)}\,dt =$

$= \left|\begin{matrix} s = \cos(t) \\ ds = -\sin(t)\,dt \end{matrix}\right| = -1\int \frac{1}{s}\,ds = -\ln(|s(t)|) = -\ln(|\cos(e^x)|)$ ✓

e) $\int \frac{3x+2}{x^2+4x+5}\,dx = \frac{\frac{3}{2}(2x+4)-4}{x^2+4x+5}\,dx = \int \frac{\frac{3}{2}(2x+4)}{x^2+4x+5}\,dx + \int \frac{-4}{x^2+4x+5}\,dx =$

$= \int \frac{\frac{3}{2}(2x+4)}{x^2+4x+5}\,dx + \int \frac{-4}{(x+2)^2+1}$

I. II.

I. $= \left|\begin{matrix} t = x^2+4x+5 \\ dt = (2x+4)\,dx \end{matrix}\right| = \frac{3}{2}\int \frac{1}{t}\,dt = \frac{3}{2}\ln(|x^2+4x+5|)$

II. $= \left|\begin{matrix} t = x+2 \\ dt = dx \end{matrix}\right| = -4\int \frac{1}{t^2+1} \Rightarrow \int \frac{1}{x^2+a^2} = \frac{1}{a}\cdot \operatorname{arctg}(x) = -4\cdot\frac{1}{1}\operatorname{arctg}(1)$

$= -4\operatorname{arctg}(t) = -4\operatorname{arctg}(x+2)$

$\frac{3}{2}\ln(|x^2+4x+5|) - 4\operatorname{arctg}(x+2)$

f) $\int \frac{2x+1}{x^2+2x+5}\,dx = \overset{\mathrm{I.}}{\int \frac{2x+2}{x^2+2x+5}\,dx} + \overset{\mathrm{II.}}{\int \frac{-1}{x^2+2x+5}\,dx}$

I. $\Rightarrow \left| \begin{matrix} t = x^2+2x+5 \\ dt = (2x+2)\,dx \end{matrix} \right| = \int \frac{1}{t}\,dt = \ln(|t|) = \ln(|x^2+2x+5|)$

II. $\Rightarrow \int \frac{-1}{(x+1)^2+4} = -1 \int \frac{1}{(x+1)^2+2^2} = \left| \begin{matrix} t = x+1 \\ dt = dx \end{matrix} \right| = -1 \int \frac{1}{t^2+2^2} = -\frac{1}{2}\operatorname{arctg}\left(\frac{t}{4}\right)$

$= -\frac{1}{2}\operatorname{arctg}(x+1)$

Výsledok: $\ln(|x^2+2x+5|) - \frac{1}{2}\operatorname{arctg}\frac{(x+1)}{2}$

g) $\int \frac{5x-1}{x^2+2x+3}\,dx = \overset{\mathrm{I.}}{\int \frac{4x+4}{x^2+2x+3}} + \overset{\mathrm{II.}}{\int \frac{x-5+1-1}{x^2+2x+3}} =$

I. $= \left| \begin{matrix} t = x^2+2x+3 \\ dt = (2x+2)\,dx \end{matrix} \right| = \int \frac{2}{t}\,dt = 2\int \frac{1}{t}\,dt = 2\ln(|x^2+2x+3|)$

II. $\Rightarrow \underbrace{\int \frac{x+1}{x^2+2x+3}\,dx}_{\mathrm{II.}} - \underbrace{\int \frac{6}{x^2+2x+3}}_{\mathrm{III.}} = \left| \begin{matrix} t = x^2+2x+3 \\ dt = 2(x+1)\,dx \\ \frac{dt}{2} = (x+1)\,dx \end{matrix} \right|$

II. $\Rightarrow \int \frac{1}{t} \cdot \frac{dt}{2} = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln(|x^2+2x+3|)$

III. $\Rightarrow -6\int \frac{1}{(x+1)^2+\sqrt{2}^2}\,dx = \left| \begin{matrix} t = x+1 \\ dt = dx \end{matrix} \right| = -6\int \frac{1}{t^2+\sqrt{2}^2}\,dt = \frac{-6}{\sqrt{2}}\operatorname{arct}\left(\frac{x+1}{\sqrt{2}}\right)$

Výsledok: $\frac{5}{2}\ln(|x^2+2x+3|) - \frac{6}{\sqrt{2}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + c$

h) $\int \frac{2^x}{(2^x+3)^7}\,dx = \left| \begin{matrix} t = 2^x+3 \\ dt = 2^x \cdot \ln(2)\,dx \\ \frac{dt}{\ln(2)} = 2^x\,dx \end{matrix} \right| = \int \frac{1}{t^7}\frac{dt}{\ln(2)} = \frac{1}{\ln(2)}\int t^{-7} = \frac{1}{\ln(2)} \cdot \frac{t^{-6}}{-6} =$

$= \frac{1}{\ln(2) - 6 \cdot (2^x+3)^6}$

i) $\int \frac{1}{\sqrt{3-4x^2}}\,dx = \int \frac{1}{\sqrt{4\left(\frac{3}{4}-x^2\right)}} = \int \frac{1}{\sqrt{4}\cdot\sqrt{\frac{3}{4}-x^2}} = \frac{1}{2}\int \frac{1}{\sqrt{\frac{3}{4}-x^2}} =$

$= \frac{1}{2}\cdot\arcsin\left(\frac{x}{\frac{\sqrt{3}}{2}}\right) = \frac{\arcsin\left(\frac{2x}{\sqrt{3}}\right)}{2}$

j) $\int \frac{x}{\sqrt{3-4x^2}}\,dx = \left| \begin{matrix} t = \sqrt{3-4x^2} \\ dt = \frac{-8x}{2\sqrt{3-4x^2}}\,dx \end{matrix} \right| = \int$

$\frac{dt\,(2\sqrt{3-4x^2})}{-8} = x\,dx$

PR 3. Per Partes

a) $\int x \cdot \sin(x)\,(dx) = \left| \begin{matrix} u = x & u' = 1 \\ v' = \sin(x) & v = -\cos(x) \end{matrix} \right|$

$$\int x \cdot \sin(x) = u \cdot v - \int u' \cdot v$$

$$\int x \cdot \sin(x) = -\cos(x) \cdot x - \int -\cos(x) = \sin x - \cos(x) \cdot x$$

b) $\int x \cdot e^{2x} = \left| \begin{matrix} u = x & u' = 1 \\ v = e^{2x} & v' = \frac{e^{2x}}{2} \end{matrix} \right|$

$$\int x\, e^{2x} = u \cdot v - \int u' \cdot v = \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} = \frac{x e^{2x}}{2} - \frac{1}{2} \cdot \int e^{2x} =$$

$$\frac{x \cdot e^{2x}}{2} - \frac{1}{2} \cdot \frac{e^{2x}}{2} = \frac{x \cdot e^{2x}}{2} - \frac{e^{2x}}{4}$$

c) $\int (x^3 - x + 1)\, e^{2x}\,(dx) = \left| \begin{matrix} u = x^3 - x + 1 & u' = 3x^2 - 1 \\ v' = e^{2x} & v = \frac{e^{2x}}{2} \end{matrix} \right|$

$$\int (x^3 - x + 1) \cdot e^{2x} = \frac{(x^3 - x + 1)\, e^{2x}}{2} - \int (3x^2 - 1) \cdot \frac{e^{2x}}{2} =$$

$$= \frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \int (3x^2 - 1) \cdot e^{2x} = \left| \begin{matrix} u_1 = 3x^2 - 1 & u'_1 = 6x \\ v'_1 = e^{2x} & v_1 = \frac{e^{2x}}{2} \end{matrix} \right|$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \cdot \left(\frac{(3x^2 - 1) \cdot e^{2x}}{2} - \int \frac{\cancel{6}x\, e^{2x}}{\cancel{2}} \right) =$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \cdot \left(\frac{(3x^2 - 1) \cdot e^{2x}}{2} - \int x e^{2x} \right) = \left| \begin{matrix} u_2 = x & u'_2 = 1 \\ v'_2 = e^{2x} & v_2 = \frac{e^{2x}}{2} \end{matrix} \right|$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \cdot \left(\frac{(3x^2 - 1) \cdot e^{2x}}{2} + \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} \right) =$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \cdot \left(\frac{(3x^2 - 1) \cdot e^{2x}}{2} + \frac{x e^{2x}}{2} - \frac{1}{2} \cdot \int e^{2x} \right) =$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{1}{2} \cdot \left(\frac{(3x^2 - 1) \cdot e^{2x}}{2} + \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} \right)$$

$$\frac{(x^3 - x + 1) \cdot e^{2x}}{2} - \frac{(3x^2 - 1) \cdot e^{2x}}{4} - \frac{x e^{2x}}{4} + \frac{e^{2x}}{8}$$

$$\frac{4e^{2x}x^3 - 4e^{2x}x + 4e^{2x} - 6e^{2x}x^2 - 2e^{2x} + 2xe^{2x} + e^{2x}}{8} =$$

$$4e^{2x}x^3 - 2e^{2x}x + 3e^{2x} - 6e^{2x}x^2$$

Cviko 7-8

PR 1.

$\frac{3x^4}{4} + \cancel{\frac{2x^2}{2}} - 4x$ ✓

✗ a) $\int (3x^3 + 2x - 4)\,dx = \frac{3x^4}{4} + \frac{2x^3}{3} - 4x + c$

✓ b) $\int \left(\frac{x^3}{3} - \frac{x}{5}\right)dx = \int\left(\frac{5x^3 - 3x}{15}\right)dx = \frac{1}{15}\int 5x^3 - 3x = \frac{1}{15}\left(\frac{5x^4}{4} - \frac{3x^2}{2}\right) + c$

✗ c) $\int x^{\frac{3}{2}} - x^{-\frac{1}{2}} = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2x^{\frac{5}{2}}}{5} - 2x^{\frac{1}{2}} + c$

✗ d) $\int \frac{\sqrt{x^4 + 2 + x^{-4}}}{x^3}\,dx = \int \frac{\cancel{x^2} \cdot \sqrt{2} \cdot x^{-2}}{\cancel{x^3}} = \int \frac{\sqrt{2}}{x^3} = \sqrt{2}\int \frac{1}{x^3} = \sqrt{2} \cdot \ln|x^3| + c$

✓ e) $\int \frac{x \cdot (x^{\frac{1}{3}} - x^{\frac{4}{3}})}{x^{\frac{1}{4}}}\,dx = \int \left(x^{\frac{4}{3}} - x^{\frac{7}{3}}\right) \cdot x^{-\frac{1}{4}} = \int x^{\frac{13}{12}} - x^{\frac{25}{12}} = \frac{x^{\frac{25}{12}}}{\frac{25}{12}} - \frac{x^{\frac{37}{12}}}{\frac{37}{12}} =$

$= \frac{12x^{\frac{25}{12}}}{25} - \frac{12x^{\frac{37}{12}}}{37} + c$

✓ f) $\int \frac{x^3 - 1}{x - 1}\,dx \Rightarrow (x^3 - 1) : (x - 1) = x^2 + x + 1$

$-(x^3 - x^2)$

$x^2 - 1$

$-(x^2 - x)$

$x - 1$

$-(x - 1)$

0

$\Rightarrow \int \frac{(x^2 + x + 1)\cancel{(x - 1)}}{\cancel{(x - 1)}}\,dx = \frac{x^3}{3} + \frac{x^2}{2} + x + c$

2 g) $\int e^x a^x\,dx = \int ea^x = \frac{ea^x}{\ln(a)} + c$

✓ h) $\int 5\cos(x) - 2x^5 + \frac{3}{1 + x^2}\,dx = 5\sin(x) - \frac{2x^6}{6} + 3\operatorname{arctg}(x) + c$

✓ i) $\int 10^{-x} + \frac{x^2 + 3}{x^2 + 1}\,dx = -\frac{10^{-x}}{\ln(10)} + x + 2\operatorname{arctg}(x)$

↳ keď je pravda dodobná na $-x$ niečo tak $(-)$ sa dáva pred integrál

✓ j) $\int 2\sin(x) - 3\cos(x)\, dx = -2\cos(x) - 3\sin(x) + C$

✓ k) $\int \frac{1}{\sqrt{3-3x^2}}\, dx = \int \frac{1}{\sqrt{3(1-x^2)}}\, dx = \int \frac{1}{\sqrt{3}\cdot\sqrt{1-x^2}} = \frac{\arcsin(x)}{\sqrt{3}} + C$

✗ l) $\int \frac{3\cdot 2^x - 2\cdot 3^x}{2^x}\, dx = \int \frac{\cancel{2^x}\left(3 - 2\cdot\left(\frac{3}{2}\right)^x\right)}{\cancel{2^x}} = 3x - 2\cdot \frac{\left(\frac{3}{2}\right)^x}{\ln\left(\frac{3}{2}\right)} + C = 3x - \frac{-2\cdot 3^x}{2^x\cdot\ln\left(\frac{3}{2}\right)} + C$

✓ m) $\int \frac{1+\cos^2(x)}{1+\cos(2x)} = \int \frac{1+\cos^2(x)}{2\cos^2(x)} = \frac{1}{2}\int \frac{1+\cos^2(x)}{\cos^2(x)} = \frac{1}{2}\left(\operatorname{tg}(x) + x\right) + C$

✓ n) $\int \frac{\cos(2x)}{\cos^2(x)\cdot\sin^2(x)} = \int \frac{\cos^2(x) - \sin^2(x)}{\cos^2(x)\cdot\sin^2(x)} = \int \frac{1}{\sin^2(x)} - \frac{1}{\cos^2(x)} = -\cot(x) - \operatorname{tg}(x) + C$

✓ o) $\int \operatorname{tg}^2(x)\, dx = \int \frac{\sin^2(x)}{\cos^2(x)}\, dx = \int \frac{1-\cos^2(x)}{\cos^2(x)}\, dx = \int \frac{1}{\cos^2(x)} - 1 = \operatorname{tg}(x) - x$

✓ p) $\int \cot g^2(x)\, dx = \int \frac{\cos^2(x)}{\sin^2(x)}\, dx = \int \frac{1-\sin^2(x)}{\sin^2(x)}\, dx = \int \frac{1}{\sin^2(x)} - 1 =$
$-\cot(x) - x + C$

✓ r) $\int \frac{1+2x^2}{x^2(1+x^2)}\, dx = \int x^{-2} + \frac{1}{1+x^2} = \frac{x^{-1}}{-1} + \operatorname{arctg}(x) + C = -\frac{1}{x} + \operatorname{arctg}(x) + C$

✓ u) $\int \frac{1}{\cos(2x) + \sin^2(x)}\, dx = \int \frac{1}{\cos^2(x)} = \operatorname{tg}(x) + C$

✗ s) $\int \frac{(1+x)^2}{x(1+x^2)}\, dx = \int \frac{1+x^2+2x}{x(1+x^2)} = \int \left(\frac{\cancel{1+x^2}}{x(\cancel{1+x^2})} + \frac{2\cancel{x}}{\cancel{x}(1+x^2)}\right) dx = \int x^{-1} + 2\int \frac{1}{1+x^2} =$
$= 1 + 2\operatorname{arctg}(x) + C$ $\quad \frac{1}{x} = \ln(|x|) + C$

PR 2.

✗ a) $\int \frac{1}{3+4x^2}\, dx = \frac{1}{3}\int \frac{1}{1+\frac{4}{3}x^2}\, dx = \left| \begin{matrix} t = \frac{4}{3}x \\ dt = \frac{4}{3}dx \end{matrix} \right| = \frac{1}{3}\int \frac{1}{1+t^2}\, dt = \frac{\operatorname{arctg}\left(\frac{4}{3}x\right)}{3} + C$

✓ b) $\int \frac{x}{3+4x^2}\, dx = \left| \begin{matrix} t = 3+4x^2 \\ dt = 8x\, dx \\ \frac{dt}{8} = x\, dx \end{matrix} \right| = \frac{1}{8}\int \frac{1}{t} = \frac{\ln(|3+4x^2|)}{8} + C$

✓ c) $\int \frac{x}{(x^2+5)^4}\, dx = \left| \begin{matrix} t = x^2+5 \\ dt = 2x\, dx \\ x\, dx = \frac{dt}{2} \end{matrix} \right| = \frac{1}{2}\int \frac{1}{t^4}\, dt = \frac{1}{2}\cdot\frac{t^{-3}}{-3} = \frac{1}{2}\cdot\frac{-1}{3\cdot(x^2+5)^3} + C$

✓ d) $\int e^x \cdot \operatorname{tg}(e^x)\, dx = \left| \begin{matrix} t = e^x \\ dt = e^x\, dx \end{matrix} \right| = -1\cdot\int \frac{-\sin(t)}{\cos(t)}\, dt = -\ln(|\cos(e^x)|) + C$

✗ e) $\int \frac{\overset{x-2}{3x+2+2-2}}{x^2+4x+5}dx = \left| \begin{array}{l} t=x^2+4x+5 \\ dt=(2x+4)dx \end{array}\right| = \underbrace{\int \frac{1}{t}dt}_{I.} + \underbrace{\int \frac{x-2}{t}dt}_{II.}$

I. ⇒ $\ln(|x^2+4x+5|)+C$

II. ⇒ $\int \frac{\cancel{x-2}}{\cancel{(x-2)}(x+6+\frac{17}{x-2})} = == == \left|\begin{array}{l} s=x+6+\frac{17}{x-2}\\ ds = 1+\frac{-17}{(x-2)^2}dx\end{array}\right| = \int \frac{1}{s}ds =$

$(x^2+4x+5):(x-2) = x+6+\frac{17}{x-2}$

$-(x^2-2x)$

$6x+5$

$-(6x-12)$

17

-17

0

$= \ln(|x+6+\frac{17}{x-2}|)$

Výsledok: $\ln(|x^2+4x+5|)+\ln(|x+6+\frac{17}{x-2}|)+C$

f) $\int \frac{2x+1}{x^2+2x+5}dx = \left|\begin{array}{l} t=x^2+2x+5\\ dt=(2x+2)dx\\ \frac{dt}{2}=(x+1)dx\end{array}\right| = \underbrace{\int \frac{1}{(x+1)^2+2^2}dt}_{I.} - \underbrace{\int \frac{-1}{t}}_{II.}$

I. $\frac{1}{2}\operatorname{arctg}\frac{(x+1)}{2}+c$

$\boxed{\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\operatorname{arctg}\left(\frac{x}{a}\right)}$!!!

II. $\ln(|x^2+2x+5|)+c$

... ✓ Výsledok: $\frac{\operatorname{arctg}\left(\frac{x+1}{2}\right)}{2} + \ln(|x^2+2x+5|)+c$

✗ g) $\int \frac{5x-1}{x^2+2x+3} = \left|\begin{array}{l} t=x^2+2x+3\\ dt=(2x+2)dx\end{array}\right| = \int \frac{\frac{5}{2}(2x+2)-6}{x^2+2x+3}dx = \int \frac{\frac{5}{2}-6}{t}dt$

$= -\frac{7}{2}\int \frac{1}{t}dt = -\frac{7\cdot\ln(|x^2+2x+3|)}{2}$

... ✓ h) $\int \frac{2^x}{(2^x+3)^7}dx = \left|\begin{array}{l} t=2^x+3\\ dt=2^x\cdot\ln(2)\,dx\\ \frac{dt}{\ln(2)}=2^x dx\end{array}\right| = \int t^{-7}dt = \frac{-1}{6t^6} = -\frac{1}{6\cdot(2^x+3)^6}$

✓ i) $\int \frac{1}{\sqrt{3-4x^2}}dx = \int \frac{1}{\sqrt{4\cdot(\frac{3}{4}-x^2)}} = \frac{1}{\sqrt{4}}\cdot\int \frac{1}{\sqrt{\frac{3}{4}-x^2}} = \frac{\arcsin\left(\frac{2x}{\sqrt{3}}\right)}{2}$

... ✓ j) $\int \frac{x}{\sqrt{3-4x^2}}dx = \left|\begin{array}{l} t=3-4x^2\\ dt=-8x\,dx\\ -\frac{dt}{8}=x\,dx\end{array}\right| = -\frac{1}{8}\int t^{-\frac{1}{2}}dt = -\frac{1}{8}\cdot\sqrt{3-4x^2}\cdot 2 = -\frac{\sqrt{3-4x^2}}{4}$

✗ k) $\int \frac{e^x(e^x-1)}{e^{2x}+1}dx = \int \frac{e^{2x}-e^x+1-1}{e^{2x}+1} = \int \frac{e^{2x}+1}{e^{2x}+1}dx + \int \frac{-1-e^x}{e^{2x}+1}dx = x - \int \frac{e^x+1}{e^{2x}+1} = \left|\begin{array}{l} t=e^x\\ dt=e^x dx\end{array}\right|$

$= x - \int \frac{1}{t^2+1} = x - \operatorname{arctg}(e^x)$

X l) $\int \frac{\sin(x)\cos(x)}{\sin^2(x)+\sin(x)+3}dx = \left| \begin{matrix} t=\sin(x) \\ dt = \cos(x)\,dx \end{matrix} \right| = \int \frac{t}{t^2+t+3}dt = \int \frac{1}{(t+1+\frac{3}{t})} = \left| \begin{matrix} s=t+1+\frac{3}{t} \\ ds = 1+\frac{3}{t^2} \end{matrix} \right|$

$\int \frac{1}{s} = \ln\left(\left|\sin(x)+1+\frac{3}{\sin(x)}\right|\right)$

X m) $\int x\cdot(x+2)^{\frac{1}{3}}dx = \int x^{\frac{4}{3}} + 2^{\frac{1}{3}}\cdot x\,dx = \frac{3x^{\frac{7}{3}}}{7} + \frac{2^{\frac{1}{3}}x^2}{2}$

X h) $\int \frac{\sqrt{x}}{x^{\frac{4}{3}}+x^{\frac{3}{2}}}dx = \int \frac{\cancel{x^{\frac{1}{2}}}}{\cancel{x^{\frac{1}{2}}}(x^{\frac{5}{6}}+x)}dx = \int x^{-\frac{5}{6}} + x^{-1}dx = 6x^{\frac{1}{6}} + \partial$

2. o)

a) $\int \frac{1}{3+4x^2}dx = \frac{1}{4}\int \frac{1}{\frac{3}{4}+x^2} = \frac{\operatorname{arctg}\left(\frac{2x}{\sqrt{3}}\right)}{4}$

e) $\int \frac{3x+2}{x^2+4x+5}dx = \left| \begin{matrix} t = x^2 \cancel{+4x} +5 \\ dt = (2x+4)\,dx \end{matrix} \right| = \int \frac{\frac{3}{2}(2x+4)-4}{x^2+4x+5}dx = \underbrace{\frac{3}{2}\int \frac{2x+4}{x^2+4x+5}dx}_{\mathrm{I.}} \underbrace{-4\int \frac{1}{x^2+4x+5}}_{\mathrm{II.}}$

I. $\frac{3}{2}\int \frac{1}{t}dt = \frac{3}{2}\ln(|x^2+4x+5|)$

II. $-4\int \frac{1}{x^2+4x+5}dx = -4\int \frac{1}{(x+2)^2+1}dx = \left| \begin{matrix} t=x+2 \\ dt = dx \end{matrix} \right| = -4\int \frac{1}{t^2+1} = -4\operatorname{arctg}(x+2)$

Výsledok $\frac{3\ln(|x^2+4x+5|)}{2} - 4\operatorname{arctg}(x+2)$

g) $\int \frac{5x-1}{x^2+2x+3}dx = \left| \begin{matrix} t = x^2+2x+3 \\ dt = (2x+2)\,dx \end{matrix} \right| = \int \frac{\frac{5}{2}(2x+2)-6}{x^2+2x+3} = \underbrace{\frac{5}{2}\int \frac{2x+2}{x^2+2x+3}dx}_{\mathrm{I.}} \underbrace{-6\int \frac{1}{x^2+2x+3}}_{\mathrm{II.}}$

I. $\frac{5}{2}\int \frac{1}{t}dt = \frac{5\ln(|x^2+2x+3|)}{2}$

II. $-6\int \frac{1}{(x+1)^2+2}dx = \left| \begin{matrix} s=x+1 \\ ds = dx \end{matrix} \right| = -6\int \frac{1}{s^2+\sqrt{2}^2} \Rightarrow \frac{1}{a}\operatorname{arctg}\left(\frac{t}{a}\right)$

$= \frac{-6}{\sqrt{2}}\operatorname{arctg}\left(\frac{s}{\sqrt{2}}\right) = \frac{-6\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$

Výsledok: $\frac{5\ln(|x^2+2x+3|)}{2} - \frac{6\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$

k) $\int \frac{e^x(e^x-1)}{e^{2x}+1}dx = \int \frac{e^{2x}-e^x}{e^{2x}+1} = \underbrace{\int \frac{e^{2x}}{e^{2x}+1}dx}_{\mathrm{I.}} - \underbrace{\int \frac{e^x}{e^{2x}+1}dx}_{\mathrm{II.}}$

I. $\int \frac{e^{2x}}{e^{2x}+1} = \left| \begin{matrix} t=e^{2x}+1 \\ dt = e^{2x}\cdot 2\,dx \\ \frac{dt}{2} = e^{2x}\,dx \end{matrix} \right| = \frac{1}{2}\int \frac{1}{t}dt = \frac{\ln(|e^{2x}+1|)}{2}$

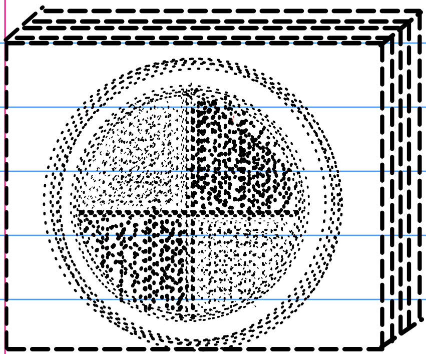
$$\text{II.}\ -\int \frac{e^x}{e^{2x}+1}dx = \begin{vmatrix} t=e^x \\ dt=e^x dx \end{vmatrix} = -\int \frac{1}{t^2+1}dt = -\operatorname{arctg}(e^x)$$

$$e)\ \int \frac{\sin(x)\cos(x)}{\sin^2(x)+\sin(x)+3}dx = \begin{vmatrix} t=\sin(x) \\ dt=\cos(x)\,dx \end{vmatrix} = \int \frac{t}{t^2+t+3}dt = \begin{vmatrix} s=t^2+t+3 \\ ds=(2t+1)dt \end{vmatrix} =$$

$$=\int \frac{(2t+1)}{2(t^2+t+3)}dt - \int \frac{1}{2(t^2+t+3)}dt = \frac{1}{2}\int \frac{1}{s}ds - \int \frac{1}{2(t^2+t+3)}dt =$$

$$= \frac{\ln(|\sin^2(x)+\sin(x)+3|)}{2} - \int \frac{1}{2(t^2+t+3)}dt = \text{—||—} - \frac{1}{2}\int \frac{1}{(t+\frac{1}{2})^2+\frac{11}{4}} = \begin{vmatrix} z=t+\frac{1}{2} \\ dz=dt \end{vmatrix}$$

$$= \text{—||—} - \frac{1}{2}\int \frac{1}{z^2+\frac{\sqrt{11}}{2}} = \frac{\ln(|\sin^2(x)+\sin(x)+3|)}{2} - \frac{\operatorname{arctg}\left(\frac{2x}{\sqrt{11}}\right)}{\sqrt{11}}$$



BMW -> BIG MIGET WINDOW

PR 3.

$$a)\ \int x\cdot\sin(x)\,dx = \begin{vmatrix} u=x & u'=1 \\ v'=\sin(x) & v=-\cos(x) \end{vmatrix}$$

$$u\cdot v - \int u'\cdot v = -x\cdot\cos(x) - \int(-\cos(x)) = -x\cdot\cos(x) + \int\cos(x)$$

$$= -x\cdot\cos(x) + \sin(x) + c$$

$$b)\ \int x e^{2x}dx = \begin{vmatrix} u=x & u'=1 \\ v'=e^{2x} & v=\frac{e^{2x}}{2} \end{vmatrix} = \frac{xe^{2x}}{2} - \int\frac{e^{2x}}{2} = \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x} = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4}$$

$$c)\ \int (x^3-x+1)e^{2x}dx = \begin{vmatrix} u=x^3-x+1 & u'=3x^2-1 \\ v'=e^{2x} & v=\frac{e^{2x}}{2} \end{vmatrix} = \underbrace{\frac{(x^3-x+1)e^{2x}}{2}}_{\text{I.}} - \frac{1}{2}\int(3x^2-1)e^{2x}$$

$$= \begin{vmatrix} u_1=3x^2-1 & u_1'=6x \\ v_1'=e^{2x} & v_1'=\frac{e^{2x}}{2} \end{vmatrix} = \text{I.} - \frac{1}{2}\left(\underbrace{\frac{(3x^2-1)e^{2x}}{2}}_{\text{II.}} - 3\int xe^{2x}\right) = \begin{vmatrix} u_2=x & u_2'=1 \\ v_2'=e^{2x} & v_2=\frac{e^{2x}}{2} \end{vmatrix}$$

$$= \text{I.} - \frac{1}{2}\left(\text{II.} - 3\cdot\left(\frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}\right)\right) = \text{I.} - \frac{1}{2}\left(\text{II.} - 3\left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right)\right) =$$

$$\frac{(x^3-x+1)e^{2x}}{2} - \frac{(3x^2-1)e^{2x}}{4} + \frac{3xe^{2x}}{4} - \frac{3e^{2x}}{8}$$

X d) $\int \ln(x)\,dx = \begin{vmatrix} u = 1 & u' = 0 \\ v' = \ln(x) & v = \frac{1}{x} \end{vmatrix} = \frac{1}{x} - \int 0 = \underline{\underline{\frac{1}{x}}}$

yb X e) $\int x \cdot \log_{10}(2x)\,dx = \begin{vmatrix} u = \log(2x) & u' = \frac{1}{2x \cdot \ln(10)} \cdot \cancel{2} \\ v' = x & v = 1 \end{vmatrix} = \log(2x) - \int \frac{1}{x \ln(10)}$

$= \log(2x) - \frac{1}{\ln(10)} \int x = \log(2x) - \frac{x^2}{2\ln(10)}$

X … f) $\int e^x \sin(x)\,dx = \begin{vmatrix} u = \sin(x) & u' = \cos(x) \\ v' = e^x & v = e^x \end{vmatrix} = \sin(x)\,e^x - \int \cos(x)\,e^x = \begin{vmatrix} u_1' = -\sin(x) \\ u_1 = \cos(x) \\ v_1' = e^x & v = e^x \end{vmatrix}$

$= \sin(x)\,e^x - \cos(x)\,e^x - \int -\sin(x)\,e^x = \sin(x)\,e^x - \cos(x)\,e^x + \boxed{\int \sin(x)\,e^x}$

2 ?

Výsledok: $\sin(x)\,e^x - \cos(x)\,e^x$

✓ g) $\int \frac{x}{\cos^2(x)}\,dx = \begin{vmatrix} u = x & u' = 1 \\ v' = \cos^{-2}(x) & v = tg(x) \end{vmatrix} = x \cdot tg(x) - \int \frac{\sin(x)}{\cos(x)} = x \cdot tg(x) + \int \frac{-\sin(x)}{\cos(x)} =$

$x \cdot tg(x) + \ln(|\cos(x)|) + c$

$\mathrm{Cotg}^{-1}(x)$ i) $\int \mathrm{arccotg}(x)\,dx = \int \frac{\cos(x)}{\sin(x)} = \ln(|\sin(x)|) + c$ X

$\int \frac{f'(x)}{f(x)} = \ln(|f(x)|) + c$

X j) $\int x \cdot \ln(x^2 + 3x - 10)\,dx = \begin{vmatrix} u = \ln(x^2+3x-10) & u' = \frac{2x+3}{x^2+3x-10} \\ v' = x & v = \frac{x^2}{2} \end{vmatrix}$

$= \frac{\ln(x^2+3x-10)\,x^2}{2} - \frac{1}{2}\int \frac{(2x+3)\,x^2}{x^2+3x-10}\,dx = \begin{vmatrix} t = x^2+3x-10 \\ dt = (2x+3)\,dx \end{vmatrix} =$

$\int 1 \cdot \ln(x) = \begin{vmatrix} u = \ln(x) & u' = \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} = x \cdot \ln(x) - \int 1 = x\ln(x) - x$

$= I_0 - \frac{1}{2}\int \frac{x^2}{t}\,dt = \begin{vmatrix} u = x^2 & u' = 2x \\ v' = t^{-1} & v = 0 \end{vmatrix} = I_0 - \frac{1}{2}\left(0 - \int 0\right) = \frac{\ln(x^2+3x-10)\,x^2}{2}$

✓ d) $\int \ln(x)\,dx = \begin{vmatrix} u = \ln(x) & u' = \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} = x\ln(x) - \int \frac{x}{x} = x\ln(x) - x$

X d) $\int \ln(x^2-4x+6)\,dx = \left| \begin{matrix} u=\ln(x^2-4x+6) & u'=\frac{(2x-4)}{x^2-4x+6} \\ v'=1 & v=x \end{matrix} \right| =$

$= x\cdot\ln(x^2-4x+6) - \int \frac{x\,(2x-4)}{x^2-4x+6} = \left| \begin{matrix} t=x^2-4x+6 \\ dt=(2x-4)\,dx \end{matrix} \right| = t. - \int \frac{x}{t} = \left| \begin{matrix} u=x & u'=1 \\ v'=t^{-1} & v=0 \end{matrix} \right|$

$x\ln(x^2-4x+6)$

✓ e) $\int x\log(2x) = \left| \begin{matrix} u=\log(2x) & u'=\frac{2}{2x\cdot\ln(10} \\ v'=x & v=\frac{x^2}{2} \end{matrix} \right| =$

$= \frac{\log(2x)\,x^2}{2} - \int \frac{x^{\cancel{2}}}{2\cancel{x}\ln(10)} = \frac{\log(2x)\,x^2}{2} - \frac{1}{2\ln(10)}\int x = \frac{\log(2x)\,x^2}{2} - \frac{x^2}{4\ln(10)}$

f) $\int e^x\sin(x)\,dx = \left| \begin{matrix} u=\sin(x) & u'=\cos(x) \\ v'=e^x & v=e^x \end{matrix} \right| = e^x\sin(x) - \int \cos(x)\,e^x = \left| \begin{matrix} u_2=\cos(x) & u_2'=-\sin(x) \\ v'=e^x & v=e^x \end{matrix} \right|$

$e^x\sin(x) - \left(\cos(x)e^x + \int \sin(x)e^x\right) =$

$\int e^x\sin(x) = e^x\sin(x) - \cos(x)\,e^x - \int \sin(x)e^x \;/+$

$2\int e^x\sin(x) = e^x\sin(x) - \cos(x)e^x \;/:2$

$\int e^x\sin(x) = \frac{e^x\sin(x)}{2} - \frac{\cos(x)e^x}{2} + C$

i) $\int \operatorname{arccotg}(x)\,dx = \left| \begin{matrix} u=\operatorname{arccotg}(x) & u'=-\frac{1}{1+x^2} \\ v'=1 & v=x \end{matrix} \right| =$

$x\cdot u - \int -\frac{x}{1+x^2} = x\cdot u + \int \frac{x}{1+x^2} = \left| \begin{matrix} t=1+x^2 \\ dt=2x\,dx \\ \frac{dt}{2}=x\,dx \end{matrix} \right| = x\,u + \frac{1}{2}\int \frac{1}{t} =$

✓ $x\cdot\operatorname{arccotg}(x) + \frac{\ln(|1+x^2|)}{2} + C$

X j) $\int x\cdot\ln(x^2+3x-10) = \left| \begin{matrix} u=\ln(x^2+3x-10) & u'=\frac{(2x+3)}{x^2+3x-10} \\ v'=x & v=\frac{x^2}{2} \end{matrix} \right| =$

$= \frac{\ln(x^2+3x-10)\,x^2}{2} - \frac{1}{2}\int \frac{(2x+3)\,x^2}{x^2+3x-10} = A\,B\,C$ $\boxed{\frac{2x^3+3x^2}{(x+5)(x-2)}}$

g, h, l

PR 2. g, k, l, m, n, o

✓ a) $\int \frac{1}{3+4x^2} = \frac{1}{4}\int \frac{1}{\frac{3}{4}+x^2} = \frac{\frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x}{\sqrt{3}}\right)}{4}$

✓ e) $\int \frac{3x+2}{x^2+4x+5} = \left| \begin{matrix} t = x^2+4x+5 \\ dt = 2x+4 \end{matrix} \right| = \int \frac{\frac{3}{2}(2x+4)-4}{x^2+4x+5} =$

$\frac{3}{2}\int \frac{1}{t}\,dt - 4\int \frac{1}{x^2+4x+5} = \frac{3\ln(|x^2+4x+5|)}{2} - 4\int \frac{1}{(x+2)^2+1} = \left| \begin{matrix} t_1 = x+2 \\ dt = dx \end{matrix} \right| =$

I. $-4\int \frac{1}{t^2+1^2}\,dt = \frac{3\ln(|x^2+4x+5|)}{2} - 4\cdot\operatorname{arctg}(x+2)$

✗ g) $\int \frac{5x-1}{x^2+2x+5} = \left| \begin{matrix} t = x^2+2x+5 \\ dt = (2x+2)\,dx \end{matrix} \right| = \int \frac{\frac{5}{2}(2x+2)-6}{x^2+2x+5}\,dx = \frac{5}{2}\int \frac{1}{t}\,dt - 6\int \frac{1}{x^2+2x+5} =$

$= \frac{5\ln(|x^2+2x+5|)}{2} - 6\int \frac{1}{(x+1)^2+2^2} = \frac{5\ln(|x^2+2x+5|)}{2} - \boxed{3\operatorname{arctg}\left(\frac{x+1}{2}\right)}$ Nesedí

✗ k) $\int \frac{e^x(e^x-1)}{e^{2x}+1}\,dx = \int \frac{e^{2x}-e^x}{e^{2x}+1}\,dx = \left| \begin{matrix} t = e^x \\ dt = e^x\,dx \end{matrix} \right| = -\int \frac{1}{t^2+1} + \int \frac{e^{2x}}{e^{2x}+1} =$

$-\operatorname{arctg}(e^x) + \int \frac{e^{2x}+1-1}{e^{2x}+1} = \text{I.} + \int 1 - \frac{1}{t^2+1} = x - 2\operatorname{arctg}(e^x)$

✗ m) $\int \sqrt{2}\cdot x\,dx + \int x^{\frac{4}{3}} = \frac{\sqrt{2}\,x^2}{2} + \frac{3x^{\frac{7}{3}}}{7}$

o) $\int \frac{x}{1+\sqrt{x-1}}\cdot\frac{1-\sqrt{x-1}}{1-\sqrt{x-1}} = \int \frac{x\left(1-\sqrt{x-1}\right)}{2-x}\,dx$

Integrály z racionálnych funkcií

$5x^3+9x^2-22x-8 : (x^3-4x) = 5 + \cdots$
$-(5x^3-20x)$

✓ a) $\int \frac{5x^3+9x^2-22x-8}{x^3-4x} = \int 5 + \frac{9x^2-2x-8}{x^3-4x} = \int 5 + \frac{A(x-2)(x+2)+Bx(x+2)+Cx(x-2)}{x(x-2)(x+2)}$

$9x^2-2x-8 = Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$

$9x^2-2x-8 = x^2(A+B+C)-4A+x(2B-2C)$

$9 = A+B+C \Rightarrow 7 = B+C$ $\qquad A=2$

$-8 = -4A \qquad 7-C = B$ $\qquad B=3$

$\qquad C=4$

$A = 2$

$-2 = 2B-2C \Rightarrow -2 = 14-4C$

$-16 = -4C \qquad C=4$

$$\int 5 + \frac{2}{x} + \frac{3}{x-2} + \frac{4}{x+2} = 5x + 2\ln(|x|) + \underbrace{\int \frac{3}{x-2}}_{\mathrm{I.}} + \underbrace{\int \frac{4}{x+2}}_{\mathrm{II.}}$$

I. $3\int \frac{1}{x-2} = \left| \begin{array}{l} t = x-2 \\ dt = dx \end{array} \right| = 3\int \frac{1}{t}\,dt = 3\ln(|x-2|)$

II. $4\ln(|x+2|)$

Výsledok $5x + 2\ln(|x|) + 3\ln(|x-2|) + 4\ln(|x+2|) + C$

✓ b) $\int \frac{-2x+19}{x^2+x-6}\,dx = \int \frac{A(x-2)+B(x+3)}{(x+3)(x-2)} \Rightarrow -2x+19 = Ax - 2A + Bx + 3B$

$-2x + 19 = x(A+B) - 2A + 3B$

$-2 = A+B \Rightarrow A = -2-B$

$19 = 3B - 2A$

$19 = 4 + 5B$

$15 = 5B$

$B = 3$

$A = -2 - 3 = -5$

$-\int \frac{5}{(x-2)}\,dx + \int \frac{3}{x+3}\,dx =$

$= -5\ln(|x+3|) + 3\ln(|x-2|)$

X c) $\int \frac{x^2+1}{x^4+x^3}\,dx = \frac{Ax^2(x+1) + Bx(x+1) + C(x+1) + Dx^3}{x^3(x+1)}$

$x^2 + 1 = Ax^3 + Ax^2 + Bx^2 + Bx + Cx + C + Dx^3$

$0x^3 + 1x^2 + 0x + 1 = x^3(A+D) + x^2(A+B) + x(B+C) + C$

$0 = A + D$

$1 = A + B \Rightarrow$

$0 = B + C$

$C = 1 \quad B = -1 \quad A = 2 \quad D = -2$

$\int \frac{2}{x} - x^{-2} + x^{-3} - \frac{2}{x+1} = 2\ln(|x|) + \frac{1}{x} - \frac{1}{2x^2} - 2\ln(|x+1|) + C$

d) $$\int \frac{5x^2-7x+10}{x^3-x^2-4x-6}\,dx = \int \frac{5x^2-7x+10}{(x-3)(x^2+2x+2)} = \int \frac{A(x^2+2x+2)+(Cx+D)(x-3)}{(x-3)(x^2+2x+2)}$$

$$5x^2-7x+10 = Ax^2+2Ax+2A+Cx^2-3Cx+Dx-3D$$

$$5x^2-7x+10 = x^2(A+C)+x\cdot(2A+D-3C)+2A-3D$$

$5 = A+C \Rightarrow C = 5-A$

$7 = 2A+D-3C$

$10 = 2A-3D$

$D = \frac{2A-10}{3}$

$-7 = 2A-15+3A+\frac{2A-10}{3}$

$8 = 5A+\frac{2A-10}{3} \quad /\cdot 3$

$24 = 17A-10$

$34 = 17A \Rightarrow A = 2$

$C = 3$

$D = -2$

$$\int \frac{2}{x-3}+\frac{3x-2}{x^2+2x+2}\,dx = 2\ln(|x-3|)+$$

$$\int \frac{3x-2}{x^2+2x+2} = \left| \begin{array}{l} t = x^2+2x+2 \\ \frac{dt}{2} = (x+1)\,dx \end{array} \right| = \int \frac{3(x+1)-5}{x^2+2x+2}\,dx = \int \frac{3(x+1)}{x^2+2x+2}\,dx - 5\int \frac{1}{(x+1)^2+1}$$

$$\frac{3}{2}\int \frac{1}{t}\,dt - 5\,\mathrm{arctg}(x+1) = 2\ln(|x-3|)+\frac{3}{2}\ln(|x^2+2x+2|)-5\,\mathrm{arctg}(x+1)$$

	2	12	11	5
-5	2	2	1	0

e) $$\int \frac{4x^2+x-13}{2x^3+12x^2+11x+5} = \frac{4x^2+x-13}{(x+5)(2x^2+2x+1)} = \frac{A(2x^2+2x+1)+(Bx+C)(x+5)}{—\,||\,—}$$

$$4x^2+x-13 = 2Ax^2+2Ax+A+Bx^2+5Bx+Cx+5C$$

$4 = 2A+B \Rightarrow B = 4-2A$

$1 = 2A+5B+C$

$-13 = A+5C \Rightarrow C = \frac{-13-A}{5}$

$1 = 2A+20-10A\;\frac{-13-A}{5} \quad /\cdot 5$

$5 = -41A+100-13 \qquad A = 2 \quad B = 0 \quad C = -3$

$41A = 82$

$$\int \frac{2}{x+5}-\frac{3}{2x^2+2x+1}\,dx = 2\ln(|x+5|)-3\int \frac{1}{2x^2+2x+1}$$

$$= -3\int \frac{1}{2x^2+2x+1} = -3\int \frac{1}{2(x^2+x+\frac{1}{2})} = -\frac{3}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{1}{4}} = -\frac{3}{2}\cdot 2\,\mathrm{arctg}(2x+1)$$

$$= 2\ln(|x+5|)-3\,\mathrm{arctg}(2x+1)$$

	1	2	0	1
-1	1	-1	1	0

f) $\int \frac{1}{x^3+1}\,dx = \int \frac{1}{(x+1)(x^2-x+1)} = \frac{A(x^2-x+1)+(Bx+c)(x+1)}{— 1 —}$

$1 = x^2(A+B) + x(B+C-A) + A + C$

$0 = A+B \Rightarrow B = -A$

$0 = B+C-A \qquad 0 = -A+1-A-A \Rightarrow A = \frac{1}{3} \qquad C = \frac{2}{3}$

$1 = A+C \Rightarrow 1-A = C \qquad B = -\frac{1}{3}$

$\underbrace{\int 3(x+1)dx}_{I} + \underbrace{\int \frac{\frac{2}{3}-\frac{1}{3}x}{x^2-x+1}dx}_{II.}$

I. $3\int x+1 = 3\left(\frac{x^2}{2}+x\right)$

II. $\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx = \frac{2\ln(|x^2-x+1|)}{3} - \int \frac{x}{x^2-x+1}dx = \left| \begin{array}{l} t = x^2-x+1 \\ dt = (2x-1)dx \\ \frac{dt}{2}+\frac{1}{2} = x\,dx \end{array} \right| =$

$= -\int \frac{1}{t}\frac{dt}{2}+\frac{1}{2} \Rightarrow \quad -\frac{\ln(|x^2-x+1|)}{2} - \frac{x}{4}$

Výsledok: $\frac{3x^2}{2}+3x - \frac{\ln(|x^2-x+1|)}{2} - \frac{x}{4}$

g) $\int \frac{x^5+x^4-7x^3+8x-3}{x^3+x^2-6x}$

$(\cancel{x^5}+\cancel{x^4}-7x^3+8x-3 : (x^3+x^2-6x) = x^2-1+\frac{x^2+2x-3}{x^3+x^2-6x}$

$-(\cancel{x^5}+\cancel{x^4}-6x^3)$

$\quad -\cancel{x^3}+8x-3$

$\quad -(-\cancel{x^3}-x^2+6x)$

$\quad\quad x^2+2x-3$

	1	1	-6	0
-3	1	-2	0	0

$\int x^2-1+\frac{x^2+2x-3}{(x+3)(x^2-2x)}dx = \frac{x^3}{3}+x+\frac{x^2+2x-3}{x(x-2)(x+3)} = \frac{A(x-2)(x+3)+Bx(x+3)+Cx(x-2)}{— 1 —}$

$x^2+2x-3 = A\overset{x^2+x-6}{(x-2)(x+3)} + Bx^2+3Bx+Cx^2-2Cx$

$x^2+2x-3 = x^2(A+B+C)+x(A+3B-2C)-6A$

$1 = A+B+C \Rightarrow \frac{1}{2}-B = C \qquad 2 = \frac{1}{2}+3B+2B-1$

$2 = A+3B-2C \Rightarrow \qquad 2 = -\frac{1}{2}+5B$

$-3 = -6A \Rightarrow A = \frac{1}{2} \qquad C = 0 \qquad \frac{5}{2} = 5B$

$B = \frac{1}{2}$

$$\int 2x + 2x(x-2) + 0 = \frac{\cancel{2}x^2}{\cancel{2}} + \int 2x(x-2)\,dx = \left| \begin{matrix} u = 2x & u' = 2 \\ v' = x-2 & v = \frac{x^2}{2} - 2x \end{matrix} \right|$$

$$= x^2 + x^3 - 4x^2 - \int x^2 - 2x = \cancel{x^2} + x^3 - 4x^2 - \cancel{\frac{x^3}{3}} - \cancel{x^2} + \cancel{\frac{x^3}{3}} + x = x^3 - 4x^2 + x$$

h) $$\int \frac{6x-13}{4x^2+4x+17}\,dx = \underbrace{-\frac{13}{4}\int \frac{1}{(x+\frac{1}{2})^2 + (\frac{4}{2})^2}}_{\text{I.}} + \underbrace{6\int \frac{x}{4x^2+4x+17}\,dx}_{\text{II.}}$$

$$\text{I.} = -\frac{\frac{13}{2}\,\mathrm{arctg}\left(\frac{x}{2}\right)}{4}$$

$$\text{II.} \quad 6\int \frac{x}{4x^2+4x+17} = \left| \begin{matrix} t = 4x^2+4x+17 \\ dt = 8x+4 \\ \frac{dt-4}{8} = x\,dx \end{matrix} \right| = \frac{6}{8}\int \frac{1}{t}\,\frac{dt-4}{}$$

Výsledek: $$\frac{6\cdot(-3)\ln(|4x^2+4x+17|)}{8} - \frac{\frac{13}{2}\,\mathrm{arctg}\left(\frac{x}{2}\right)}{4}$$

i) $$\int \frac{\cos(x)}{\sin^2(x)+5\sin(x)-6}\,dx = \int \frac{\cos(x)}{(\sin(x)+6)(\sin(x)-1)}\,dx -$$

ODMíetam toto Počítat

Testovací test 10:105

$$\int \frac{x^2+x+12}{x^3+7x^2+11x+5}\,dx = \int \frac{x^2+x+12}{\underbrace{(x+1)(x+5)(x+1)}_{(x+1)^2(x+5)}} = \int \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+5)}$$

$$A(x+1)(x+5) = Ax^2 + 6Ax + 5A$$

$$B(x+5) = Bx + 5B$$

$$C(x+1)^2 = Cx^2 + 2Cx + C$$

$$x^2 + x + 12 = x^2(A+C) + x(6A+5B+2C) + 5A + 5B + C$$

$1 = A + C \Rightarrow A = 1 - C$ $\qquad$ $1 = 6 - 6C + 2C + B \Rightarrow B = 4C - 5$

$1 = 6A + B + 2C$ $\qquad$ $12 = 5 - 5C + C + 5B$

$12 = 5A + 5B + C$ $\qquad$ $7 = -4C + 20C - 25$ $\qquad$ $A = -1$

$32 = 16C$ $\qquad$ $B = 3$

$C = 2$

$$\int -\frac{1}{x+1} + \frac{3}{(x+1)^2} + \frac{2}{x+5}\,dx = -\ln(|x+1|) + 3\underbrace{\int \frac{1}{(x+1)^2}\,dx}_{I.} + 2\ln(|x+5|)$$

$$I.\quad 3\int t^{-2}\,dt = 3t^{-1} = \frac{3}{x+1}$$

Výsledok: $2\ln(|x+5|) - \ln(|x+1|) - \frac{3}{x+1}$

b) $\int x\cdot\ln(x^2)\,dx = \left| \begin{array}{ll} u = \ln(x^2) & u' = \frac{2\cancel{x}}{x^{\cancel{2}}} \\ v' = x & v = \frac{x^2}{2} \end{array} \right|$

$$\frac{\ln(x^2)\,x^2}{2} - \int \frac{\cancel{2}\,x^{\cancel{2}}}{\cancel{2}\,\cancel{x}} = \frac{\ln(x^2)x^2 - x^2}{2}$$

PR 2.

$$\int \frac{1}{x^2}\sqrt{\frac{2x+1}{x+1}}\,dx = \left| t = \sqrt{\frac{2x+1}{x+1}} \right.$$

$$dt = \left(\sqrt{1+\frac{x}{x+1}}\right)' = \left(\sqrt{\frac{x}{x+1}}\right)' = \frac{1}{2\sqrt{\frac{x}{x+1}}}\cdot\frac{(x+1)-x}{(x+1)^2}$$

$$= \frac{1}{2\sqrt{\frac{x}{x+1}}\,(x+1)^2}$$

$$\int \cos(\ln(x))\,dx = \left| \begin{array}{l} t = \ln(x) \\ dt = \frac{1}{x}\,dx \\ x\,dt = dx \end{array} \right| = \int \cos(t)\times dt = \int \cos(t)\cdot e^t\,dt = \left| \begin{array}{l} u = \cos(t) \\ u' = -\sin(t) \\ v' = e^t \\ v = e^t \end{array} \right|$$

$\ln(x) = t$

$\cancel{\log_e(x)} = \cancel{\log_e e^t}$

$x = e^t$

$$\cos(t)e^t - \int -\sin(t)e^t = \cos(t)e^t + \int \sin(t)e^t = \left| \begin{array}{ll} u = \sin(t) & u' = \cos(t) \\ v' = e^t & v = e^t \end{array} \right|$$

$$\cos(t)e^t + \sin(t)e^t - \int \cos(t)e^t = 0$$

$$\int \cos(t)e^t = \frac{\cos(t)e^t + \sin(t)e^t}{2} = \frac{\cos(\ln(x))\,e^{\ln(x)} + \sin(\ln(x))\,e^{\ln(x)}}{2}$$

$$\int x^{-2}\sqrt{\frac{2x+1}{x+1}} = \left| t = \sqrt{\frac{2x+1}{x+1}} \right.$$

$$t^2 = \frac{2x+1}{x+1}$$

$$\frac{t^2x + t^2 - 1}{2} = x$$

$$dt = \frac{1}{2t}'$$

$$i)\int \frac{\cos(x)\,dx}{\sin^2(x)+5\sin(x)-6} = \int \frac{\frac{1-t^2}{\cancel{1+t^2}}}{\frac{t^2+10t-6-6t^2}{\cancel{1+t^2}}} = \int \frac{1-t^2}{-5t^2+10t-6}\,dt =$$

$$\int \frac{1 \boxed{-t^2} -4t^2 \boxed{+4t^2} \boxed{+10t} -10t +6 \boxed{-6}}{-\text{ll}-} = \int 1\,dt + \int \frac{4t^2-10t+7}{-5t^2+10t-6}$$

$$(4t^2-10t+7):(-5t^2+10t-6) = -\frac{4}{5} + \frac{2t+\frac{11}{5}}{-\text{ll}-}$$

$$-\left(\cancel{4t^2} -8t+\frac{24}{5}\right)$$

$$2t+\frac{11}{5}$$

$$t+\int -\frac{4}{5}+\frac{2t+\frac{11}{5}}{-5t^2+10t-6}\,dt = \frac{1}{5}t - \frac{1}{5}\int \frac{-10t+11}{-5t^2+10t-6}\,dt = \left| \begin{matrix} s=-5t^2+10t-6 \\ ds=(-10t+10)\,dt \end{matrix} \right| =$$

$$= -\frac{1}{5}\int \frac{(-10t+10)-21}{-5t^2+10t-6}\,dt = -\frac{1}{5}\int \frac{1}{s}\,ds - 21\int \frac{1}{(-\sqrt{5}t-\sqrt{5})^2-1}\,dt = \left| \begin{matrix} s_2=-\sqrt{5}t-\sqrt{5} \\ ds_2=-\frac{1}{2\sqrt{5}}\,dt \end{matrix} \right|$$

$$\frac{1}{5}t - \frac{\ln(|-5t^2+10t-6|)}{5} + \frac{21}{5}\operatorname{arctg}(-\sqrt{5}t-\sqrt{5})$$

$$\frac{\tan\left(\frac{x}{2}\right) - \ln(|-5tg^2\left(\frac{x}{2}\right)-10tg\left(\frac{x}{2}\right)-6|)+21\operatorname{arctg}\left(-\sqrt{5}\,tg\left(\frac{x}{2}\right)-\sqrt{5}\right)}{5}$$

$$\int \frac{\cos(x)}{\sin^2(x)+5\sin(x)-6}$$

$$\tan\left(\frac{x}{2}\right) \Rightarrow \cos(x) = \frac{1-t^2}{t^2+1}$$

$$\sin(x) = \frac{2t}{t^2+1}$$

$$=\int \frac{\frac{(1-t^2)}{t^2+1}}{\left(\frac{2t}{t^2+1}\right)^2+\frac{10t}{t^2+1}-6} = \frac{\frac{1-t^2}{\cancel{t^2+1}}\,1}{\frac{4t^2+10t(t^2+1)-6(t^2+1)^{\cancel{2}}}{(t^2+1)^{\cancel{2}}}}$$

$$\int \frac{(1-t^2)(1+t^2)}{4t^2+10t^3+10t-6t^4-12t^2-6}\,dt = \frac{1-t^4}{-6t^4+10t^3-8t^2+10t-6}$$

	-6	10	-8	10	-6
1	-6	4	-4	6	0

$$\left(1-\cancel{t^4}\right) : \left(-6t^4+10t^3-8t^2+10t-6\right) = \frac{1}{6} + \frac{2}{-6t^4+10t^3-8t^2+10t-6} + \frac{8t^2-10t^3-10t}{-6t^4+10t^3-8t^2+10t-6}$$

$$-\left(-\cancel{t^4}+\frac{10t^3-8t^2+10t}{6}-1\right)$$

$$2+\frac{8t^2-10t^3-10t}{6}$$

$$\int \frac{1}{6}+\frac{2}{-6t^4+10t^3-8t^2+10t-6}+\frac{8t^2-10t^3-10t}{-6t^4+10t^3-8t^2+10t-6}\,dt$$

$$=\frac{t}{6}+2\ln\left(\left|-6t^4+10t^3-8t^2+10t-6\right|\right)+\int\frac{8t^2-10t^3-10t}{\dots}$$

$$8t^2-10t^3-10t=A(-6t^3+4t^2-4t+6)+(Bt^2+Ct+D)(t-1)$$

$$8t^2-10t^3-10t=-6At^3+4At^2-4At+6A+Bt^3-Bt^2+Ct^2-Ct+Dt-D$$

$$-10t^3+8t^2-10t=t^3(B-6A)+t^2(4A-B+C)+t(D-4A-C)+6A-D$$

$$-10=B-6A \Rightarrow B=6A-10$$

$$8=4A-B+C$$

$$-10=D-4A-C \Rightarrow C=2A+10$$

$$0=6A-D \Rightarrow D=6A$$

$$8=4A-6A+10+2A$$

$$\int\frac{1}{2\sin(x)-\cos(x)+5}\,dx=\int\frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+\frac{5+5t^2}{1+t^2}}\,dt$$

$$\sin(x) \Rightarrow \frac{2t}{1+t^2}$$

$$\cos \Rightarrow \frac{1-t^2}{1+t^2} \qquad t=\tan\left(\frac{x}{2}\right)$$

$$\int\frac{\cancel{1+t^2}}{6t^2+4t+4}\cdot\frac{2}{\cancel{1+t^2}}\,dt=\int\frac{1}{3t^2+2t+2}=\frac{1}{3}\int\frac{1}{t^2+\frac{2}{3}t+\frac{2}{3}}$$

$$=\frac{1}{3}\int\frac{1}{\left(t+\frac{1}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}\,dt=\int\frac{1}{s^2+\frac{\sqrt{5}}{3}} \qquad \frac{S}{A} \qquad \frac{3t+1}{\sqrt{5}}$$

$$=\frac{\cancel{3}}{\cancel{3}\sqrt{5}}\cdot\operatorname{arctg}\left(\frac{3t}{\sqrt{5}}\right)=\frac{\operatorname{arctg}\left(\frac{3\operatorname{tg}\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

$$\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx \Rightarrow t=\sqrt{\frac{1-x}{1+x}}$$

$$t^2+xt^2=1-x$$
$$-t^2-xt^2+1=x$$

$$dt=\frac{1}{2\sqrt{\frac{1-x}{1+x}}}\cdot\frac{-2}{(1+x)^2}\,dx$$

$$t^2=\frac{1-x}{1+x}$$

$$\int \frac{1-\sqrt[6]{x+1}}{x+1+\sqrt[3]{(x+1)^4}}\,dx = \left|\begin{array}{l} t=\sqrt[6]{x+1} \Rightarrow (x+1)=t^6 \\ t^8=\sqrt[3]{(x+1)^4} \\ dt=\frac{1}{6\sqrt{x+1}}\,dx \end{array}\right.$$

$$t^{\frac{1}{2}\cdot\frac{1}{3}}$$

$$\int \frac{1-t}{t^6+t^8}\cdot\frac{dt}{6\,t^3}=\frac{1}{6}\int\frac{1-t}{t^2+t^{100}}$$

$$\int \frac{1}{x^2}\sqrt{\frac{2x+1}{x+1}} = \left|\; t=\sqrt{\frac{2x+1}{x+1}}\right.$$

$$t^2=\frac{2x+1}{x+1}$$
$$t^2x+t^2=2x+1$$
$$t^2-1=2x-t^2x$$
$$t^2-1=x(2-t^2)$$
$$x=\frac{t^2-1}{2-t^2}$$

$$dx=\frac{2t(2-t^2)-(-2)t(t^2-1)}{(2-t^2)^2}=\frac{4t-\cancel{2t^3}+\cancel{2t^3}-2t}{(2-t^2)^2}\,dt$$

$$\int \frac{\cancel{(2-t^2)^2}}{(t^2-1)^2}\cdot t\cdot\frac{2t}{\cancel{(2-t^2)^2}}\,dt=\int\frac{2t^2}{(t^2-1)^2}\,dt=2\int\frac{t^2}{(t^2-1)^2}\,dt=$$

$$=2\int\frac{(At+B)}{t^2-1}+\frac{(Ct+D)}{(t^2-1)^2}=2\int\frac{(At+B)(t^2-1)+Ct+D}{-\,\|\,-}\,dt=$$

$$t^2=At^3-At+Bt^2-B+Ct+D$$
$$0t^3+t^2+0t+0=At^3+Bt^2+t(C-A)+D-B$$

$$\int\frac{1}{x^2-a^2}\,dx=\frac{\ln\left(\left|\frac{x-a}{x+a}\right|\right)}{2a}$$

$A=0$

$B=1$

$C=0$

$D=-1$

$$2\left(\int\frac{1}{t^2-1}\,dt-\int\frac{1}{(t^2-1)^2}\,dt\right)=2\left(\frac{\ln\left(\left|\frac{t-1}{t+1}\right|\right)}{2}-\underbrace{\int\frac{1}{(t^2-1)^2}\,dt}_{\mathrm{I.}}\right.$$

$$\text{I.}\quad \int \frac{1}{(t^2-1)^2}\,dt = \left| \begin{array}{l} s = t^2-1 \\ ds = 2t\,dt \\ \frac{t\,ds}{2} = dt \end{array} \right| = \int \frac{t}{s^2}\,\frac{ds}{2} = \frac{1}{2}\int t\cdot s^{-2}\,ds = \left| \begin{array}{ll} u=t & u'=1 \\ v'=s^{-2} & v=\frac{s^{-1}}{-1} \end{array} \right|$$

$$\frac{1}{2}\left(t\cdot\left(-\frac{1}{s}\right) - \int -s^{-1}\right) = \frac{1}{2}\left(-\frac{t}{s} - 1\right) = \frac{1}{2}\left(\frac{t}{t^2-1} - 1\right)$$

$$\ln\left(\left|\frac{t-1}{t+1}\right|\right) + \left(\frac{t}{t^2-1} - 1\right)$$

$$\ln\left(\left|\frac{\sqrt{\frac{2x+1}{x+1}} - 1}{\sqrt{\frac{2x+1}{x+1}} + 1}\right|\right) + \left(\frac{\sqrt{\frac{2x+1}{x+1}}}{\frac{2x+1}{x+1} - 1} - 1\right)$$

$$\int \frac{1 - \sqrt[6]{x+1}}{x+1 + \sqrt[3]{(x+1)^4}} \quad \left| \begin{array}{ll} t = \sqrt[6]{x+1} & 6t^5\,dt = dx \\ t^6 = x+1 & dt = \frac{1}{6\sqrt[6]{(x+1)^5}}\,dx \\ t^8 = \sqrt[3]{(x+1)^4} & \end{array} \right|$$

$$= \int \frac{(1-t)}{t^6+t^8}\cdot 6t^5\,dt = 6\cdot\int \frac{(1-t)}{t^{\cancel{6}}(1+t^2)}\,\cancel{t^5}\,dt = 6\int \frac{1-t}{t+t^3}\,dt$$

$$6\left(\int \frac{1}{t+t^3}\,dt - \int \frac{1}{1+t^2}\right)$$

$$-6\,\mathrm{arctg}\left(\sqrt[6]{x+1}\right) + 6\underbrace{\int \frac{1}{t+t^3}\,dt}_{\text{I.}}$$

$$\text{I.}\quad \int \frac{1}{t(1+t^2)}\,dt$$

$$1 = A(1+t^2) + (Bt+C)t$$

$$1 = A + At^2 + Bt^2 + Ct$$

$$1 = A$$

$$0 = A + B \Rightarrow B = -1$$

$$0 = C$$

$$\int \frac{1}{t} - \frac{t}{1+t^2}\,dt = 6\ln(t) - \int \frac{t}{1+t^2}\,dt = \left| \begin{array}{l} s = 1+t^2 \\ ds = 2t\,dt \\ \frac{ds}{2} = t\,dt \end{array} \right| =$$

$$= 6\cdot -\frac{1}{2}\int \frac{1}{s}\,ds = 6\ln(t) - 3\ln\left(\left|1 + \sqrt[6]{(x+1)^2}\right|\right)$$

Výsledok: $6\ln(t) - 3\ln\left(\left|1 + \sqrt[6]{(x+1)^2}\right|\right) - 6\,\mathrm{arctg}\left(\sqrt[6]{x+1}\right)$

$$6\ln\left(\sqrt[6]{x+1}\right) - 3\ln\left(\left|1 + \sqrt[6]{(x+1)^2}\right|\right) - 6\,\mathrm{arctg}\left(\sqrt[6]{x+1}\right)$$

$$\int \frac{1}{x+\sqrt{2x-1}}\,dx = \left| \begin{array}{l} t=\sqrt{2x-1} \\ dt = \frac{\cancel{2}}{\cancel{2}\sqrt{2x-1}}\,dx \\ t^2 = 2x-1 \\ \frac{t^2+1}{2} = x \\ \cancel{2}t\,dt = \cancel{2}\,dx \\ t\,dt = dx \end{array} \right| = \int \frac{t}{\frac{t^2+1}{2}+t}\,dt = \int \frac{2t}{t^2+2t+1}\,dt$$

$$2\int \frac{t}{(t+1)^2}\,dt = \left| \begin{array}{l} s=t+1 \\ ds = dt \end{array} \right| = 2\int \frac{s-1}{s^2} = 2\left(\int \frac{1}{s}\,ds - \int s^{-2}\right) =$$

výsledok: $2\ln(|\sqrt{2x-1}+1|) + \frac{2}{\sqrt{2x-1}+1}$

$$\int \frac{\sqrt{x-1}}{x+2\sqrt{x-1}}\,dx = \left| \begin{array}{l} t=\sqrt{x-1} \\ dt = \frac{1}{2\sqrt{x-1}} \\ t^2 = x-1 \\ t^2+1 = x \end{array} \right| = \int \frac{2t^2}{t^2+1+2t}\,dt = \int \frac{2t^2}{t^2+2t+1}\,dt$$

$$= (2t^2) : (t^2+2t+1) = 2 + \frac{2-4t}{t^2+2t+1}$$

$$-(2t^2+4t+2)$$

$$-4t+2$$

$$2\sqrt{x-1} + -2\cdot\int \frac{-1+2t}{t^2+2t+1} \quad \left| \begin{array}{l} s=t^2+2t+1 \\ ds = 2t+2\,dt \end{array} \right|$$

$$\int \frac{2t+2-3}{t^2+2t+1}\,dt = \int \frac{1}{s} - 3\int (t+1)^{-2} = -2\ln(|x+2\sqrt{x-1}|) + \frac{-6}{\sqrt{x-1}+1}$$

???