1.) a) √ d) X g) √ j) X m) X
b) X e) X h) √ k) √
c) √ f) √ i) X l) √

3. a) $\left[\begin{array}{c|c} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{array}\right]$ b) $\left[\begin{array}{ccccc|c} 6 & -1 & 3 & 0 & 0 & 4 \\ 0 & 5 & -1 & 0 & 0 & 1 \\ 0 & 2 & 0 & -3 & 1 & 0 \end{array}\right]$ d) $\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{array}\right]$

c) $\left[\begin{array}{ccccc|c} -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{array}\right]$ e) $\left[\begin{array}{ccc|c} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & -1 & -1 & 0 \end{array}\right]$

f) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

4. $\left.\begin{array}{l} 2x_1 - 4x_2 - x_3 = 1 \\ x_1 - 3x_2 + x_3 = 1 \\ 3x_1 - 5x_2 - 3x_3 = 1 \end{array}\right\}$ Transformácia $\Rightarrow$ $\left[\begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right]$ $(-1)R_3 + R_1$

$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array}\right]$ $-R_3 + R_2$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ -2 & 2 & 4 & 0 \\ 3 & -5 & -3 & 1 \end{array}\right]$ $(-2)R_1 + R_2$

$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & -5 & -3 & 1 \end{array}\right]$ $3R_1 + R_3$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 \end{array}\right]$ $\frac{1}{2}R_2 + R_1$

$\left[\begin{array}{ccc|c} -1 & 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 \end{array}\right]$

$-x_1 + \frac{5}{2}x_3 = \frac{1}{2} \quad / \cdot 2$

$-2x_2 + x_3 = 1 \quad / \cdot (-5)$

$-2x_1 + \cancel{5x_3} = 1$ $\qquad x_1 = -\frac{1}{2}$

$10x_2 - \cancel{5x_3} = -5$ $\qquad x_2 = -\frac{1}{2}$

$2 \cdot (-\frac{1}{2}) - 4x_2 + \frac{1}{2} = 1$

$-\frac{1}{2} - 3x_2 - \frac{1}{2} = 1$

$-\frac{3}{2} - 5x_2 + \frac{3}{2} = 1$

Rovnobežné p. Totožné p. Rôznobežné p.

5) a) nemá riešenie b) nekonečne veľa riešení c) práve jedno riešenie

a) $3x - 2y = 4 \quad / \cdot (-2)$ $\qquad \cancel{-6x} \cancel{+4y} = -8 \Rightarrow 0 = -8$

$6x - 4y = 9$ $\qquad \cancel{6x} \cancel{-4y} = 9 \qquad 0 = 9$ $\qquad$ NR

b) $2x - 4y = 1$ $\qquad 2x - 4t = 1 \Rightarrow x = \frac{1}{2}(1+4t)$

$4x - 8y = 2$ $\qquad$ Riešenie $\left(\frac{1}{2}(1+4t), b\right)$

Parametrické v $\Rightarrow y = t$

c) $x - 2y = 0$

$x - 4y = 8$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & -4 & 8 \end{bmatrix} -2R_1 + R_2 \begin{bmatrix} 1 & -2 & 0 \\ -1 & 0 & 8 \end{bmatrix} R_2 + R_1 \begin{bmatrix} 0 & -2 & 8 \\ -1 & 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 8 \\ 0 & -2 & 8 \end{bmatrix}$$

$+x_1 = -8$

$2x_2 = -8 \Rightarrow x_2 = -4$

6.) a) $2x - 5y = 3 \Rightarrow y = t$

$7x = 3 + 5t$

$x = \frac{3}{7} + \frac{5}{7}t$

b) $3x_1 - 5x_2 + 4x_3 = 7$

$P_1: x_1 = \frac{7}{3} + t$

$P_2: x_2 = \frac{4}{5} + t$

$P_3: x_3 = t$

?

c) $-8x_1 + 2x_2 - 5x_3 + 6x_4 = 1$

$P_1: x = \frac{1}{8} + t$

$P_2: x = -\frac{1}{2}t$

$P_3: x = \frac{t}{5}$

$P_4: x = -\frac{t}{6}$

e) $x + 10y = 3 \Rightarrow y = t$

$x = 3 + 10t$

7.)

a) $2x - 3y = 1 \Rightarrow y = t$

$6x - 9y = 3$

$2x = 1 + 3t$

$x = \frac{1}{2} + \frac{3}{2}t$

c) $6x_1 + 2x_2 = -8$

$3x_1 + x_2 = -4 \Rightarrow x_2 = t$

$3x_1 = -4 - t$

$x_1 = -\frac{4}{3} - \frac{t}{3}$

b) $x_1 + 3x_2 - x_3 = -4$

$3x_1 + 9x_2 - 3x_3 = -12$

$-x_1 - 3x_2 + x_3 = 4$

$$\begin{bmatrix} 1 & 3 & -1 & -4 \\ 3 & 9 & -3 & -12 \\ -1 & -3 & 1 & 4 \end{bmatrix} 3R_1 + R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & -4 \\ 0 & 0 & 0 & 0 \\ -1 & -3 & 1 & 4 \end{bmatrix}$$

NR

d)

$$6x - 3y + 6z = -12$$
$$-4x + 2y - 4z = 8$$

8.)

a) $\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$ $2R_2 + R_1$ $\begin{bmatrix} 1 & -7 & 8 & 8 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -6 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix} \xRightarrow{R_3+R_1} \begin{bmatrix} 1 & 3 & -8 & 3 \\ 2 & -6 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 4 & -6 & 8 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix} \xRightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$

d) $\begin{bmatrix} 7 & -4 & -2 & 2 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix} \xRightarrow{R_3+R_1} \begin{bmatrix} 1 & -1 & -3 & 6 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$

9. Určte, či nasledovné matice sú v riadkovom echelonovom tvare, alebo v redukovanom riadkovom echelonovom tvare, alebo v oboch

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
Red. riadkovy echelovy tvar
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

Riadkovy echelovy tvar
$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

To be in row echelon form, a matrix must have the following properties:

1. If a row does not consist entirely of zeros, then the first nonzero number in the row is a 1.
2. All zero rows are at the bottom of the matrix.
3. In any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row.

★ Row echelon form is not unique

For reduced row echelon form:

4. Each column containing a leading 1 has zeros in all its other entries.

★ Reduced row echelon form is unique

10.)

a) $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 10 & 13 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \xrightarrow{(-10)R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -37 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] -2R_3+R_2$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{array}\right]$ $x_1 = -37$, $x_2 = -8$, $x_3 = 5$

b) $\left[\begin{array}{cccc|c} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} -8R_3+R_1 \\ -4R_3+R_2 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -13 & -10 \\ 0 & 1 & 0 & -13 & -5 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$

$x_1 - 13x_4 = -10$

$x_2 - 13x_4 = -5$

$x_3 + x_4 = 2 \Rightarrow x_4 = 2 - x_3$

$x_1 - 13x_4 = -5(x_3+x_4)$

$2x_2 - 13x_4 = -5(x_3+x_4)$

$x_1 - \cancel{13x_4} = -5x_3 - \cancel{5x_4}$

$2x_2 - \cancel{13x_4} = -5x_3 - \cancel{5x_4}$

$x_1 - 8x_4 = -5x_3 \quad / \cdot(-1)$

$2x_2 - 8x_4 = -5x_3$

$-x_1 + \cancel{8x_4} = \cancel{5x_3}$

$2x_2 - \cancel{8x_4} = -\cancel{5x_3}$

$-x_1 + 2x_2 = 0$

$x_1 = 2x_2$

$x_1 - 13(2-x_3) = -10$

$x_2 - 13(2-x_3) = -5$

$x_1 + 13x_3 - 26 = -10$

$x_2 + 13x_3 - 26 = -5$

$x_1 + 13x_3 = 16$

$x_2 + 13x_3 = 21$

?

d) $\left[\begin{array}{cccc} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{3R_2+R_1} \left[\begin{array}{cccc} 1 & 0 & 19 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$x_1 + 19x_3 = 1$

$x_2 + 4x_3 = 0$

NR

11)

a) $x_1+x_2+2x_3=8$
$-x_1-2x_2+3x_3=1$
$3x_1-7x_2+4x_3=10$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} R_2+R_1 \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{bmatrix} -3R_1+R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] -\frac{1}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 5 & 1 & 7 \end{array}\right] -2R_3+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -9 & 0 & -6 \\ 0 & -1 & 5 & 9 \\ 0 & 5 & 1 & 7 \end{array}\right] -5R_3+R_2$$

$$\left[\begin{array}{ccc|c} 1 & -9 & 0 & -6 \\ 0 & -26 & 0 & -26 \\ 0 & 5 & 1 & 7 \end{array}\right] \begin{matrix} 9R_2+R_1 \\ -\frac{1}{26}R_2 \end{matrix} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 5 & 1 & 7 \end{array}\right] -5R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{matrix} x_1=3 \\ x_2=1 \\ x_3=2 \end{matrix}$$

$x_1+x_2+2x_3=8$ 2
$-x_1-2x_2+3x_3=1$
$3x_1-7x_2+4x_3=10$

$3+1+2\cdot 2=8$ ✓
$-3-2+6=1$ ✓
$3\cdot 3-7+8=10$ ✓

b) $2x_1+2x_2+2x_3=0$
$-2x_1+5x_2+2x_3=1$
$8x_1+x_2+4x_3=-1$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \frac{1}{2}R_1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix} \begin{matrix} 2R_1+R_2 \Rightarrow \\ -8R_1+R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix} \xrightarrow{} R_3+R_2 \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \end{bmatrix} -\frac{1}{4}R_2+R_1$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{3}{4} & 0 & -\frac{1}{4} \\ 0 & 7 & 4 & 1 \end{array}\right]$$

$x_1-\frac{3}{4}x_2=-\frac{1}{4} \Rightarrow x_1=\frac{3}{4}x_2-\frac{1}{4}$

$x_2-4x_3=1 \Rightarrow 4x_3=x_2-1 \Rightarrow x_3=\frac{1}{4}(x_2-1)$

$(x_1,x_2,x_3)=\left(\frac{1}{4}(3x_2-1), x_2, \frac{1}{4}(x_2-1)\right)$

c)

$x - y + 2z - w = -1$
$2x + y - 2z - 2w = -2$
$-x + 2y - 4z - 2w = 1$
$3x - 3w = -3$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & -2 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \\ R_1+R_3 \\ -3R_1+R_4 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ R_3 \Leftrightarrow R_2 \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -3R_2+R_3 \\ -3R_2+R_3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{bmatrix} \begin{array}{l} \frac{1}{9}R_4+R_1 \\ \frac{1}{3}R_4+R_2 \\ \\ \\ \end{array} \Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{bmatrix} \begin{array}{l} R_2+R_1 \\ \\ \\ \end{array}$$

$$\begin{array}{ccccc} x & y & z & w & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{bmatrix}$$

$x = -1$ | $x - y + 2z - w = -1$ | $-1 - 0 - 0 = -1$ ✓
$y - 2z = 0$ | $2x + y - 2z - 2w = -2$ | $-2 + 0 - 0 = -2$ ✓
$9w = 0$ | $-x + 2y - 4z - 2w = 1$ | $-(-1) + 2\cdot 0 - 2\cdot 0 = 1$ ✓
$w = 0$ | $3x - 3w = -3$ | $-3 - 3\cdot 0 = -3$ ✓

$x = -1$

$y - 2z = 0 \quad 2z = y \quad z = \frac{1}{2}y$

$(x, y, z, w) = (-1, y, \frac{1}{2}y, 0)$

d)

$0a - 2b + 3c = 1$
$3a + 6b - 3c = -2$
$6a + 6b + 3c = 5$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix} \begin{array}{l} \\ \\ \hookrightarrow -2R_1+R_3 \end{array}$$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix} \circlearrowleft \quad \hookrightarrow -2R_1+R_3$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right] -3R_2+R_3 \Rightarrow \left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{array}\right] 3R_2+R_1$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 6 & 1 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{array}\right] \begin{matrix} \frac{1}{3}R_1 \\ -\frac{1}{2}R_2 \end{matrix} \quad \left[\begin{array}{ccc|c} 1 & 0 & 2 & \frac{1}{3} \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 6 \end{array}\right] \text{NR.}$$

13.)

a) $2x - 3y = 5$
$-x + y = -3$

$$\left[\begin{array}{cc|c} 2 & -3 & 5 \\ -1 & 1 & -3 \end{array}\right] -R_2 \circlearrowleft \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & -3 & 5 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & -1 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 1 & 1 \end{array}\right] R_2+R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right] \begin{matrix} x=4 \\ y=1 \end{matrix}$$

$2\cdot(4) - 3\cdot(1) = 5$ ✓
$-(4) + (1) = -3$ ✓

---

b) $2x - 2y = 1$
$3x = 1$

$$\left[\begin{array}{cc|c} 2 & -2 & 1 \\ 3 & 0 & 1 \end{array}\right] \circlearrowleft \left[\begin{array}{cc|c} 3 & 0 & 1 \\ 2 & -2 & 1 \end{array}\right] \frac{1}{3}R_1 \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 2 & -2 & 1 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & -2 & \frac{1}{3} \end{array}\right] \cdot\left(-\frac{1}{2}\right)R_2 \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{6} \end{array}\right]$$

$x = \frac{1}{3}$ $\quad 2\left(\frac{1}{3}\right) - 2\left(-\frac{1}{6}\right) =$
$y = -\frac{1}{6}$ $\quad = \frac{2}{3} + \frac{2}{6} = 1$ ✓
$3\left(\frac{1}{3}\right) = 1$
$1 = 1$ ✓

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c) $2x - z = 4$
$x + 4y + z = 2$
$4x + y - z = 1$

$$\begin{bmatrix} 2 & 0 & -1 & 4 \\ 1 & 4 & 1 & 2 \\ 4 & 1 & -1 & 1 \end{bmatrix} \circlearrowleft \Rightarrow$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 2 & 0 & -1 & 4 \\ 4 & 1 & -1 & 1 \end{bmatrix} \circlearrowleft -2R_2+R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 2 & 0 & -1 & 4 \\ 0 & 1 & 1 & -7 \end{array}\right] -2R_1+R_2$$

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & -8 & 1 & 0 \\ 0 & 1 & 1 & -7 \end{bmatrix} \circlearrowleft \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 1 & -7 \\ 0 & -8 & 1 & 0 \end{bmatrix} 8R_2+R_3 \quad \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & 1 & -7 \\ 0 & 0 & 9 & -56 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 4 & 1 & 2 \\ 0 & 1 & 1 & -7 \\ 0 & 0 & 9 & -56 \end{array}\right] -4R_2+R_1 \quad \left[\begin{array}{cccc} 1 & 0 & -3 & 30 \\ 0 & 1 & 1 & -7 \\ 0 & 0 & 9 & -56 \end{array}\right] \begin{array}{l} \frac{1}{3}R_3+R_1 \\ -\frac{1}{9}R_3+R_2 \\ \frac{1}{9}R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{56}{3}+\frac{90}{3} \\ 0 & 1 & 0 & \frac{56}{9}-\frac{63}{9} \\ 0 & 0 & 1 & -\frac{56}{9} \end{array}\right] \quad \begin{array}{l} x = \frac{34}{3} \\ y = -\frac{7}{3} \\ z = -\frac{56}{3} \end{array}$$

Nevychádza to bohužiaľ

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d) $-3x + y + z = 2$
$-4z = 0$
$-4x + 2y - 3z = 1$

$$\left[\begin{array}{cccc} -3 & 1 & 1 & 2 \\ 0 & 0 & -4 & 0 \\ -4 & 2 & -3 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc} -3 & 1 & 1 & 2 \\ -4 & 2 & -3 & 1 \\ 0 & 0 & -4 & 0 \end{array}\right] -R_2+R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ -4 & 2 & -3 & 1 \\ 0 & 0 & -4 & 0 \end{array}\right] 4R_1+R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & -2 & 13 & 5 \\ 0 & 0 & -4 & 0 \end{array}\right] R_3+R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & -2 & 13 & 5 \\ 0 & 0 & -4 & 0 \end{array}\right] 13\tfrac{1}{4}(R_3)+R_2 \Rightarrow \left[\begin{array}{cccc} 1 & -1 & 0 & 1 \\ 0 & -2 & 0 & 5 \\ 0 & 0 & -4 & 0 \end{array}\right] \left(-\tfrac{1}{2}\right)R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & 0 \end{array}\right] R_2+R_1 \Rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & -\frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x = -\frac{3}{2}$ SkP:
$y = -\frac{5}{2}$
$z = 0$

$-3x + y + z = 2$
$-4z = 0$
$-4x + 2y - 3z = 1$
$\Rightarrow$
$-3\left(-\frac{3}{2}\right) - \frac{5}{2} + 0 = \frac{9}{2} - \frac{5}{2} = \frac{4}{2} = 2$ ✓
$-4 \cdot 0 = 0$ ✓
$-4\left(-\frac{3}{2}\right) + 2\left(-\frac{5}{2}\right) - 3 \cdot 0 = 6 - 5 = 1$ ✓

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e) $2x_1 - x_3 = 4$
$x_1 + 4x_2 + x_3 = 2$

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 4 \\ 1 & 4 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 2 & 0 & -1 & 4 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & -8 & -3 & 0 \end{array}\right] \tfrac{1}{2}R_2+R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & -8 & -3 & 0 \end{array}\right] \cdot\left(-\tfrac{1}{8}\right) \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & \frac{3}{8} & 0 \end{array}\right]$$

$x_1 - \frac{1}{2}x_3 = 2 \Rightarrow x_1 = 2 + \frac{1}{2}x_3$

$x_2 + \frac{3}{8}x_3 = 0 \Rightarrow x_2 = -\frac{3}{8}x_3$

$$(x_1, x_2, x_3) = \left(2 + \tfrac{1}{2}x_3,\ -\tfrac{3}{8}x_3,\ x_3\right)$$

f) $4x_1 + 1x_2 - 4x_3 = 1$
$4x_1 - 4x_2 + 2x_3 = -2$

$$\left[\begin{array}{ccc|c} 4 & 1 & -4 & 1 \\ 4 & -4 & 2 & -2 \end{array}\right] -R_1+R_2 \Rightarrow$$

$$\left[\begin{array}{ccc|c} 4 & 1 & -4 & 1 \\ 0 & -5 & 6 & -3 \end{array}\right] \tfrac{1}{5}R_2+R_1 \Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & -\frac{14}{5} & \frac{2}{5} \\ 0 & -5 & 6 & -3 \end{array}\right] \begin{array}{l} \cdot(\frac{1}{4}) \\ \cdot(-\frac{1}{5}) \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{14}{20} & \frac{2}{20} \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \end{array}\right] \quad \begin{array}{l} x_1 - \frac{14}{20}x_3 = \frac{1}{10} \Rightarrow x_1 = \frac{1}{10} + \frac{7}{10}x_3 \\ x_2 - \frac{6}{5}x_3 = \frac{3}{5} \qquad x_2 = \frac{3}{5} + \frac{6}{5}x_3 \end{array}$$

$$(x_1, x_2, x_3) = \left(\frac{1}{10} + \frac{7}{10}x_3, \frac{3}{5} + \frac{6}{5}x_3, x_3\right)$$

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g) $2x_1 + 4x_2 + 2x_3 + 2x_4 = -2$
$4x_1 - 2x_2 - 3x_3 - 2x_4 = 2$
$x_1 + 3x_2 + 3x_3 - 3x_4 = -4$

$$\left[\begin{array}{cccc|c} 2 & 4 & 2 & 2 & -2 \\ 4 & -2 & -3 & -2 & 2 \\ 1 & 3 & 3 & -3 & -4 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{cccc|c} 2 & 4 & 2 & 2 & -2 \\ 0 & -10 & -7 & -6 & 6 \\ 1 & 3 & 3 & -3 & -4 \end{array}\right] (\tfrac{1}{2}) \Rightarrow \left[\begin{array}{ccccc} 1 & 2 & 1 & 1 & -1 \\ 0 & -10 & -7 & -6 & 6 \\ 1 & 3 & 3 & -3 & -4 \end{array}\right] -R_1+R_3$$

$$\left[\begin{array}{ccccc} 1 & 2 & 1 & 1 & -1 \\ 0 & -10 & -7 & -6 & 6 \\ 0 & 1 & 2 & -4 & -3 \end{array}\right] \left[\begin{array}{ccccc} 1 & 2 & 1 & 1 & -1 \\ 0 & 1 & 2 & -4 & -3 \\ 0 & -10 & -7 & -6 & 6 \end{array}\right] 10R_2+R_3$$

$$\left[\begin{array}{ccccc} 1 & 2 & 1 & 1 & -1 \\ 0 & 1 & 2 & -4 & -3 \\ 0 & 0 & 13 & -46 & -24 \end{array}\right] -2R_2+R_1 \quad \left[\begin{array}{ccccc} 1 & 0 & -3 & 9 & 5 \\ 0 & 1 & 2 & -4 & -3 \\ 0 & 0 & 13 & -46 & -24 \end{array}\right] \begin{array}{l} \frac{3}{13}R_3+R_1 \\ -\frac{2}{13}R_3+R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{21}{13} & -\frac{27}{13} \\ 0 & 1 & 0 & \frac{100}{13} & \frac{54}{13} \\ 0 & 0 & 1 & -\frac{46}{13} & -\frac{24}{13} \end{array}\right] \quad \begin{array}{l} x_1 - \frac{21}{13}x_4 = -\frac{27}{13} \Rightarrow x_1 = \frac{21}{13}x_4 - \frac{27}{13} \\ x_2 + \frac{100}{13}x_4 = \frac{54}{13} \Rightarrow x_2 = \frac{54}{13} - \frac{100}{13}x_4 \\ x_3 - \frac{46}{13}x_4 = -\frac{24}{13} \Rightarrow x_3 = \frac{46}{13}x_4 - \frac{24}{13} \end{array}$$

$$(x_1, x_2, x_3, x_4) = \left(\frac{21}{13}x_4 - \frac{27}{13}; \frac{54}{13} - \frac{100}{13}x_4; \frac{46}{13}x_4 - \frac{24}{13}; x_4\right)$$

h)

$$\begin{aligned} 3x_1 - 3x_3 + 4x_4 &= -3 \\ -4x_1 + 2x_2 - 2x_3 - 4x_4 &= 4 \\ 4x_2 - 3x_3 + 2x_4 &= -3 \end{aligned} \qquad \begin{bmatrix} 3 & 0 & -3 & 4 & -3 \\ -4 & 2 & -2 & -4 & 4 \\ 0 & 4 & -3 & 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & -2 & -4 & 4 \\ 3 & 0 & -3 & 4 & -3 \\ 0 & 4 & -3 & 2 & -3 \end{bmatrix} \begin{matrix} R_2+R_1 \\ \Rightarrow \\ \end{matrix} \begin{bmatrix} -1 & 2 & -5 & 0 & 1 \\ 3 & 0 & -3 & 4 & -3 \\ 0 & 4 & -3 & 2 & -3 \end{bmatrix} 3R_1+R_2$$

$$\begin{bmatrix} -1 & 2 & -5 & 0 & 1 \\ 0 & 6 & -18 & 4 & 0 \\ 0 & 4 & -3 & 2 & -3 \end{bmatrix} \begin{matrix} \cdot(-1) \\ \cdot(\frac{1}{2}) \Rightarrow \\ \end{matrix} \begin{bmatrix} 1 & -2 & 5 & 0 & -1 \\ 0 & 3 & -9 & 2 & 0 \\ 0 & 4 & -3 & 2 & -3 \end{bmatrix} \begin{matrix} \\ \\ -R_2+R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 5 & 0 & -1 \\ 0 & 3 & -9 & 2 & 0 \\ 0 & 1 & 6 & 0 & -3 \end{bmatrix} \begin{matrix} 2R_3+R_1 \\ \Rightarrow \\ \end{matrix} \begin{bmatrix} 1 & 0 & 17 & 0 & -4 \\ 0 & 1 & 6 & 0 & -3 \\ 0 & 3 & -9 & 2 & 0 \end{bmatrix} \begin{matrix} \\ \Rightarrow \\ -3R_2+R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 17 & 0 & -4 \\ 0 & 1 & 6 & 0 & -3 \\ 0 & 0 & -27 & 2 & 9 \end{bmatrix} \begin{matrix} (\frac{17}{27})R_3+R_1 \\ (\frac{6}{27})R_3+R_2 \\ \end{matrix} \quad \begin{bmatrix} 1 & 0 & 0 & \frac{34}{27} & \frac{45}{27} \\ 0 & 1 & 0 & \frac{4}{9} & -1 \\ 0 & 0 & 1 & -\frac{2}{27} & \frac{1}{3} \end{bmatrix}$$

$x_1 + \frac{34}{27}x_4 = \frac{5}{3} \Rightarrow x_1 = \frac{5}{3} - \frac{34}{27}x_4$ $\qquad (17\cdot 9)-(4\cdot 27)$

$x_2 + \frac{4}{9}x_4 = -1 \Rightarrow x_2 = -\frac{4}{9}x_4 - 1$ $\qquad (6\cdot 9)-(3\cdot 27)$

$x_3 - \frac{2}{27}x_4 = \frac{1}{3} \Rightarrow x_3 = \frac{1}{3} - \frac{2}{27}x_4$

$(x_1, x_2, x_3, x_4) = \left(\frac{5}{3} - \frac{34}{27}x_4 ; -\frac{4}{9}x_4 - 1 ; \frac{1}{3} - \frac{2}{27}x_4 , x_4\right)$

a) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2/3 \end{bmatrix}$ k) $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -1/3 & 4 \\ 0 & 1 & 3 & 4/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ g) $\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ L) $\begin{bmatrix} 1 & 5 & 5 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ h) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ M) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 1 & -1 & 2 & 2 \end{bmatrix}$ i) $\begin{bmatrix} 1 & 3 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$ N) $\begin{bmatrix} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1 & 7 \\ 0 & 0 & 1 & 2 & -1 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 & 2/3 & 0 & -1 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4/5 \end{bmatrix}$ j) $\begin{bmatrix} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1 & 7 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ o) $\begin{bmatrix} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1 & 7 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $x_1 = -1$, $x_2 = \frac{1}{2}$, $x_3 = 0$

$(x_1, x_2, x_3) = (-1, \frac{1}{2}, 0)$

b) $\begin{bmatrix} 1 & 0 & -\frac{1}{3} & 4 \\ 0 & 1 & 3 & \frac{4}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_1 - \frac{1}{3}x_3 = 4$, $x_2 + 3x_3 = \frac{4}{3}$

$(x_1, x_2, x_3) = (4 + \frac{1}{3}x_3, \frac{4}{3} - 3x_3, x_3)$

c) NR. d) NR.

e) $x_4 = \frac{4}{5}$

$x_1 + \frac{2}{3}x_3 = -1 \Rightarrow x_1 = -1 - \frac{2}{3}x_3$

$x_2 - 3x_3 = 1 \qquad x_2 = 3x_3$

$(x_1, x_2, x_3, x_4) = (-1 - \frac{2}{3}x_3, 3x_3, x_3, \frac{4}{5})$

f) $(x_1, x_2, x_3) = (2, 0, -\frac{2}{3})$

g) $\left.\begin{matrix} x_1 - 2x_2 = -3 \\ x_3 = 2 \end{matrix}\right\}$ $(x_1, x_2, x_3) = (2x_2 - 3, x_2, 2)$

j) $x_1 - 3x_4 = 1$, $x_2 - x_4 = 7$, $x_3 = 0$

$(x_1, x_2, x_3, x_4) = (1 + 3x_4, 7 + x_4, 0, x_4)$

h) NR. i) NR.

K) NR. L) NR.

M) NR.

N) $x_1 - 3x_4 = 1$, $x_2 - x_4 = 7$, $x_3 + 2x_4 = -1$

$(x_1, x_2, x_3, x_4) = (1 + 3x_4, 7 + x_4, -2x_4 - 1, x_4)$

o) NR.

15.)

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \quad x_1 = 2 \quad x_2 = 3$$

b) NR

c) $$\begin{bmatrix} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 5 & 2 \end{bmatrix} \quad \begin{aligned} x_1 + x_4 &= -1 \\ x_2 &= -1 \\ x_3 + 5x_4 &= 2 \end{aligned}$$

$(x_1, x_2, x_3, x_4) = (-1 - x_4, -1, 2 - 5x_4, x_4)$

d)

$$\begin{bmatrix} 4 & -3 & -4 & -2 \\ -4 & 2 & 1 & -4 \\ -1 & -3 & 1 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 2 & 1 & -4 \\ 4 & -3 & -4 & -2 \\ -1 & -3 & 1 & -4 \end{bmatrix} R_1 + R_2$$

$$\begin{bmatrix} -4 & 2 & 1 & -4 \\ 0 & -1 & -3 & -6 \\ -1 & -3 & 1 & -4 \end{bmatrix} \begin{matrix} -3R_3 + R_1 \\ \cdot(-1) \\ \\ \end{matrix} \Rightarrow \begin{bmatrix} -1 & 11 & -2 & 8 \\ 0 & 1 & 3 & 6 \\ -1 & -3 & 1 & -4 \end{bmatrix} \begin{matrix} \cdot(-1) \\ \\ -R_1 + R_3 \end{matrix} \Rightarrow$$

$$\begin{bmatrix} 1 & -11 & 2 & -8 \\ 0 & 1 & 3 & 6 \\ 0 & -14 & 3 & -12 \end{bmatrix} \begin{matrix} 11R_2 + R_3 \\ \\ 14R_2 + R_3 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 0 & 35 & 58 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 55 & 72 \end{bmatrix} \frac{1}{55}R_3$$

$$\begin{bmatrix} 1 & 0 & 35 & 58 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & \frac{72}{55} \end{bmatrix} \begin{matrix} -35R_3 + R_1 \\ -3R_3 + R_2 \\ \\ \end{matrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{670}{55} \\ 0 & 1 & 0 & \frac{-102}{55} \\ 0 & 0 & 1 & \frac{72}{55} \end{bmatrix} \Rightarrow$$

$x_1 = \frac{670}{55}$ $\quad (x_1, x_2, x_3) = \left(\frac{132}{5}, \frac{-102}{55}, \frac{72}{55}\right)$

$x_2 = \frac{-102}{55}$

$x_3 = \frac{72}{55}$