Homework 3

1. Design context-free grammars for the following languages:

a) The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

b) The set of all strings with twice as many 0's as 1's.

c) $L(00^* 11^* 22^* 00^* 11^* 22^* 00^* 11^* 22^*)$

Hint : The language defined by the regular expression.

d) $L((0 + 1 + 2)^{81})$

2. Consider the CFG G defined by productions:

S → aS | Sb | a | b

Prove by induction on the string length that no string in $L(G)$ has ba as a substring.

1. Design context-free grammars for the following languages:

a) The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either different number of a's and b's or a different number of b's and c's or both.

解：a) 可以把这个DFA看成两个DFA L_1, L_2 的并，即 $L = L_1 \cup L_2$

$L_1 = \{a^i b^j c^k \mid i \neq j\}$　　$L_2 = \{a^i b^j c^k \mid j \neq k\}$.

对于 L_1，有：

$L_1 \to EC$

$E \to aEb \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to cC \mid c \mid \varepsilon$

其中 L_1 为 L_1 的初始符号

对于 L_2，有：

$L_2 \to AF$

$F \to bFc \mid B \mid C$

$B \to bB \mid b$

$C \to cC \mid c$

$A \to aA \mid a \mid \varepsilon$

其中 L_2 为 L_2 的初始符号

将以上两个语言取并集，我们有

$S \to L_1 \cup L_2$

$L_1 \to EX$

$L_2 \to YF$

$E \to aEb \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to cC \mid c$

$X \to C \mid \varepsilon$

$Y \to A \mid \varepsilon$

D生成a, b相等的串

$S \to ADC \mid DBC \mid ABE \mid AEC \mid A \mid B \mid C$

$D \to aDb \mid \varepsilon$

$E \to bEc \mid \varepsilon$

E生成b, c相等的串

$A \to aA \mid a$

$B \to bB \mid b$

$C \to cC \mid c$

b) The set of all strings with twice as many 0's as 1's.

c) $L(00^*11^*22^*00^*11^*22^*00^*11^*22^*)$

Hint : The language defined by the regular expression.

d) $L((0 + 1 + 2)^{81})$

b)解: $S \to SaSbSbS \mid SbSaSbS \mid SbSbSaS \mid \varepsilon$ abb

c)解: $S \to 0A1B2C\,0A1B2C\,0A1B2C$

$A \to 0A \mid \varepsilon$

$B \to 1B \mid \varepsilon$

$C \to 2C \mid \varepsilon$

d) $S \to (A|B|C)CA(B|C)\cdots\cdots(A|B|C)$ (81个) ✗

$A \to 0|1|2$

$B \to AAA$

$C \to BBB$

$D \to CCC$

$S \to DDD$

2. Consider the CFG G defined by productions:

$S \to aS \mid Sb \mid a \mid b$

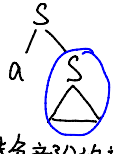
Prove by induction on the string length that no string in $L(G)$ has ba as a substring.

解:对于语法树的长度进行规那:

对 w 的长度 $|w|$ 归纳

①对于 $S \to a$, $S \to b$, 有 ba 显然不是 a 或 b 的子串

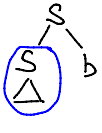
②对于如下所示的语法树我们有:



对蓝色部分的语法树,由于此语法树小于本语法树,由归纳假设,蓝色部分的产物必然不包括 ba 子串

于是,设蓝色部分的产物是 w,且 $ba \notin w$,有 ba 显然 $\notin aw$.

③对于如下所示的语法树,我们有:



同样,对于蓝色部分的语法树产物 w,有 $ba \notin w$,那么

同样显然 $ba \notin wb$.