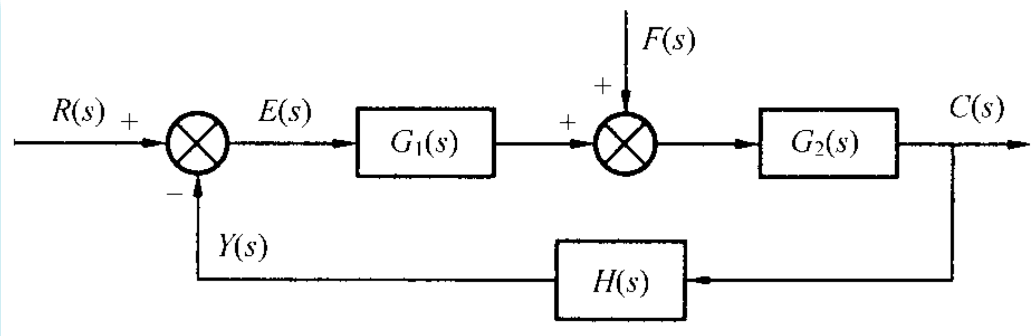
# 2.3.3 闭环系统的传递函数

- 典型控制系统框图→



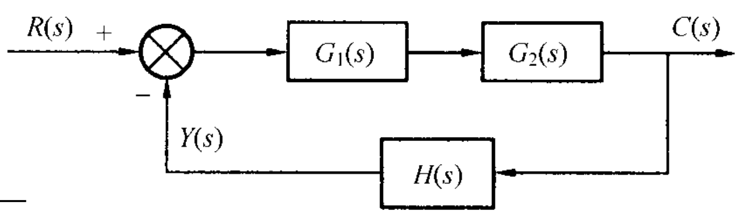

- 前向通路的传递函数为

$$G(s) = G_1(s)G_2(s)$$

- 1）系统的开环传递函数

$$G(s)H(s) = G_1(s)G_2(s)H(s)$$

- 2）输出对参考输入的闭环传递函数



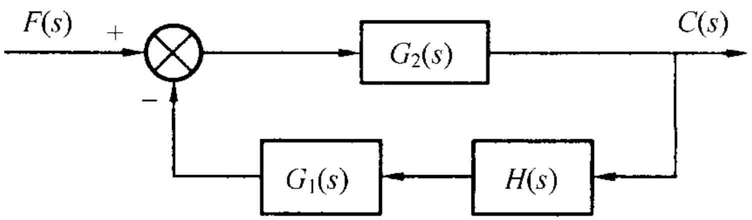

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$C(s) = \Phi(s)R(s) = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}R(s) = \frac{G(s)}{1+G(s)H(s)}R(s)$$

当 $H(s)=1$时

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)} = \frac{G(s)}{1+G(s)}$$

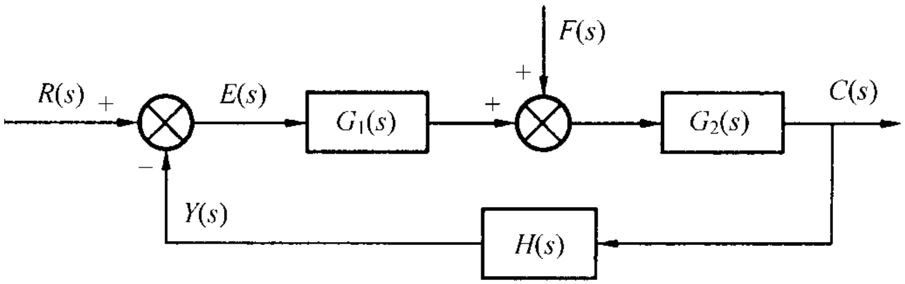
## 3）输出对于扰动输入的 闭环传递函数




$$\Phi_F(s)=\frac{C(s)}{F(s)}=\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}=\frac{G_2(s)}{1+G(s)H(s)}$$

$$C(s)=\Phi_F(s)F(s)=\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}F(s)=\frac{G_2(s)}{1+G(s)H(s)}F(s)$$

## 4）系统总输出

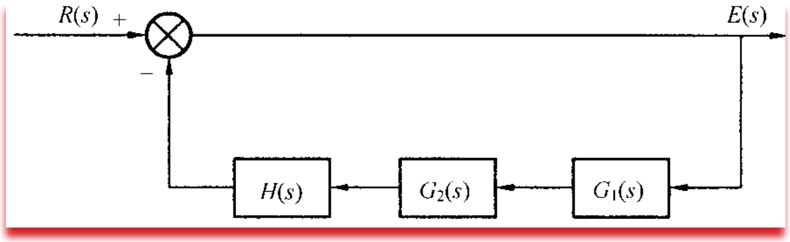



$$C(s)=\Phi(s)R(s)+\Phi_F(s)F(s)$$

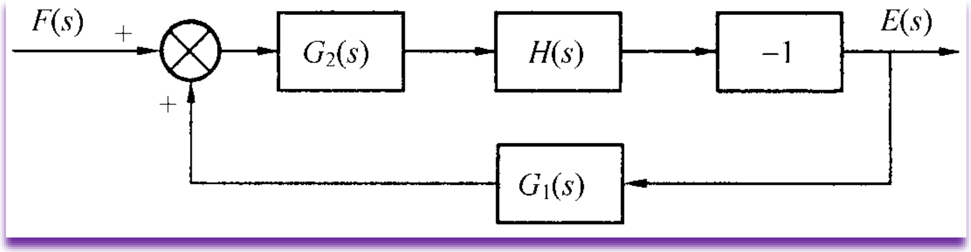
$$=\frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}R(s)+\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}F(s)$$

## 5）偏差信号对参考输入的闭环传递函数

$$\Phi_E(s)=\frac{E(s)}{R(s)}=\frac{1}{1+G_1(s)G_2(s)H(s)}=\frac{1}{1+G(s)H(s)}$$




## 6）偏差信号对扰动输入的闭环传递函数



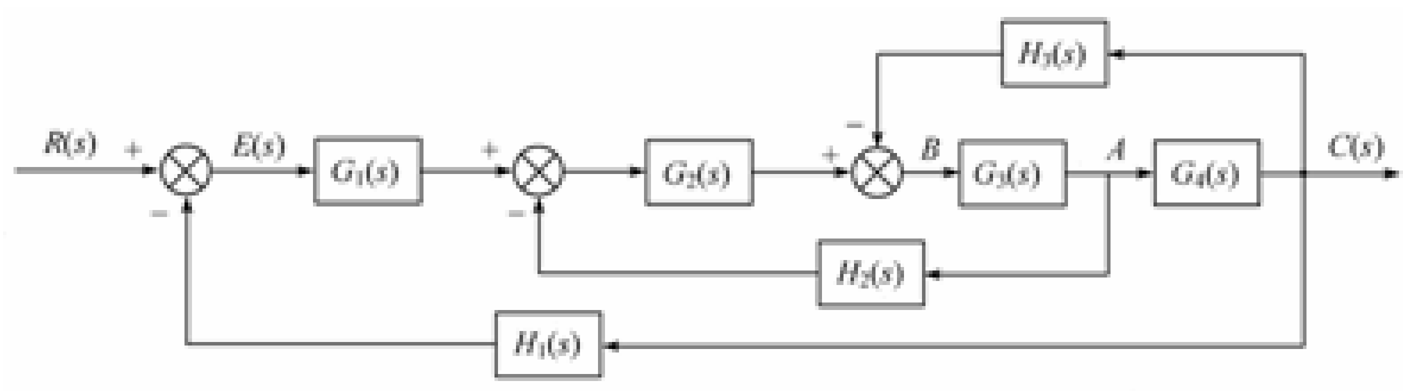

$$\Phi_{EF}(s)=\frac{E(s)}{F(s)}=\frac{-G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}=\frac{-G_2(s)H(s)}{1+G(s)H(s)}$$

## 7）系统总偏差

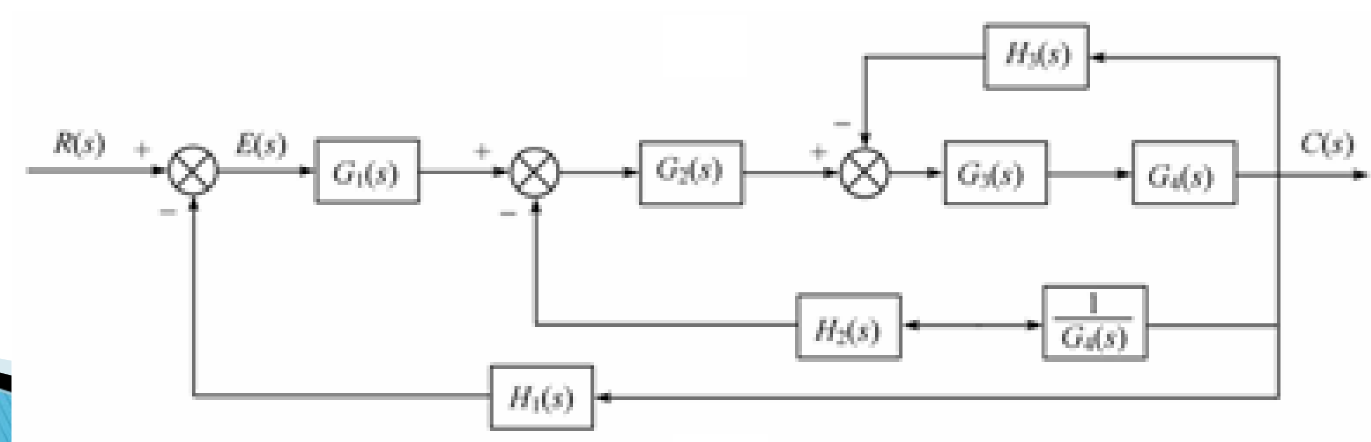
$$E(s)=\Phi_E(s)R(s)+\Phi_{EF}(s)F(s)$$

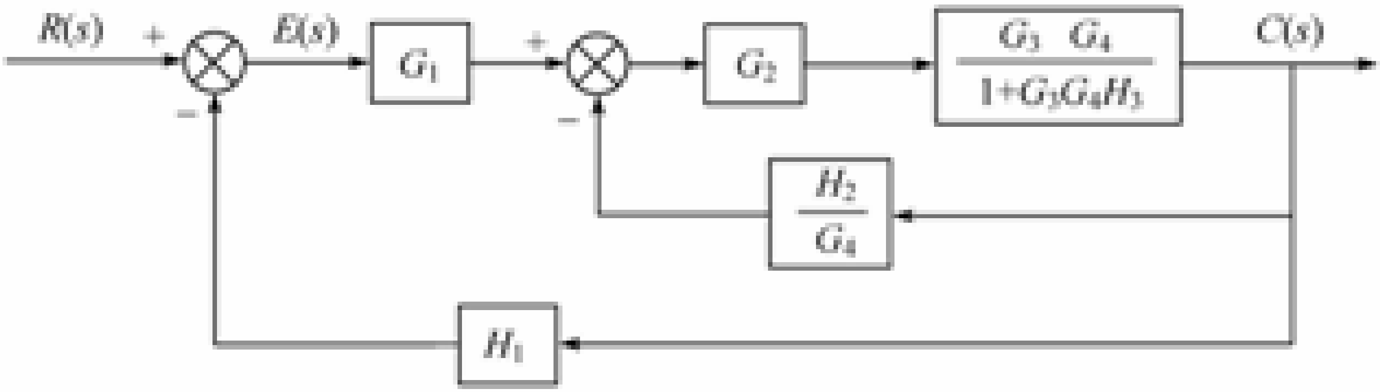
## 2.3.4框图的化简

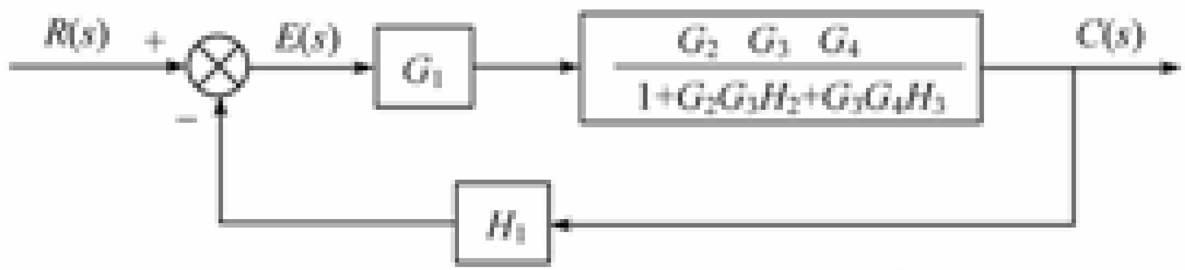
- 将框图变换成串联、并联环节和反馈回路，再用等效环节代替。
- 化简框图的关键是**解交叉**结构，办法是移动分支点和相加点。
- 例2-3-2 求闭环传递函数 $C(s)/R(s)$ 和 $E(s)/R(s)$。




- **解：**

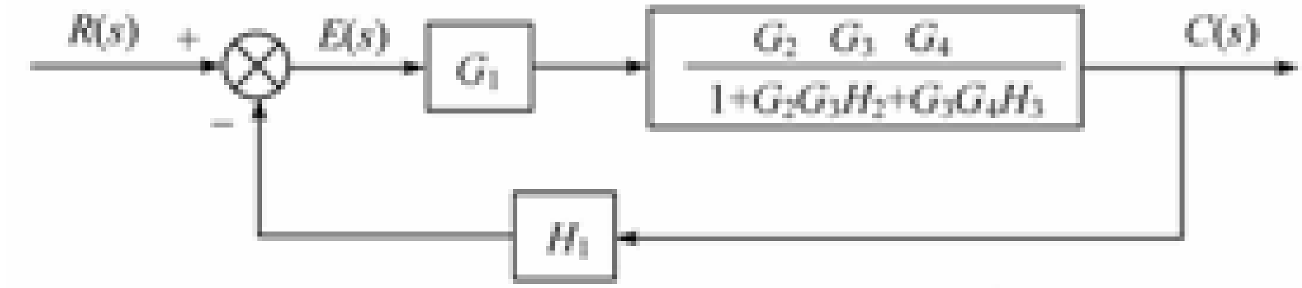





$$\frac{C(s)}{R(s)}=\frac{\dfrac{G_1G_2G_3G_4}{1+G_2G_3H_2+G_3G_4H_3}}{1+\dfrac{G_1G_2G_3G_4H_1}{1+G_2G_3H_2+G_3G_4H_3}}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_2+G_3G_4H_3+G_1G_2G_3G_4H_1}$$


误差传递函数




$$\frac{E(s)}{R(s)}=\frac{1}{1+\dfrac{G_1G_2G_3G_4H_1}{1+G_2G_3H_2+G_3G_4H_3}}=\frac{1+G_2G_3H_2+G_3G_4H_3}{1+G_2G_3H_2+G_3G_4H_3+G_1G_2G_3G_4H_1}$$

$$\frac{E(s)}{R(s)}=\frac{R(s)-H_1(s)C(s)}{R(s)}=1-H_1(s)\frac{C(s)}{R(s)}$$

# 2.3.5梅森增益公式

- 梅森增益公式的一般形式 $\Phi(s)=\dfrac{\sum\limits_{k=1}^{n}P_k\Delta_k}{\Delta}$
- 式中，$\Phi(s)$就是系统的输出信号和输入信号之间的传递函数，Δ 称为特征式，$\Delta=1-\Sigma L_i+\Sigma L_i L_j-\Sigma L_i L_j L_k+\cdots$
- 式中，$\Sigma L_i$ ——所有各回路的回路传递函数之和；

  $\Sigma L_i L_j$ ——两两互不接触的回路，其回路传递函数乘积之和；

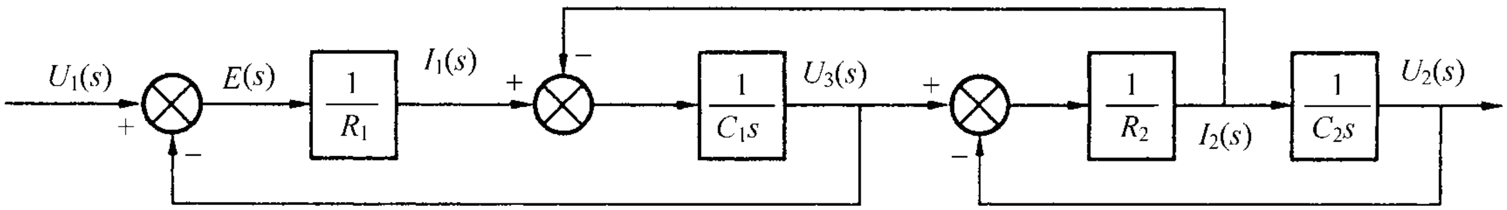
  $\Sigma L_i L_j L_k$——所有的三个互不接触的回路，其回路传递函数乘积之和；

  n ——系统前向通路个数；

  $P_k$——从输入端到输出端的第k条前向通路上各传递函数之积

  $\Delta_k$——在 Δ 中，将与第k条前向通路相接触的回路所在项除去后所余下的部分，称余子式。

例2-3-3 求传递函数$\Phi(s)=U_2(s)/U_1(s)$及$\Phi_E(S)=E(s)/U_1(s)$




(c)

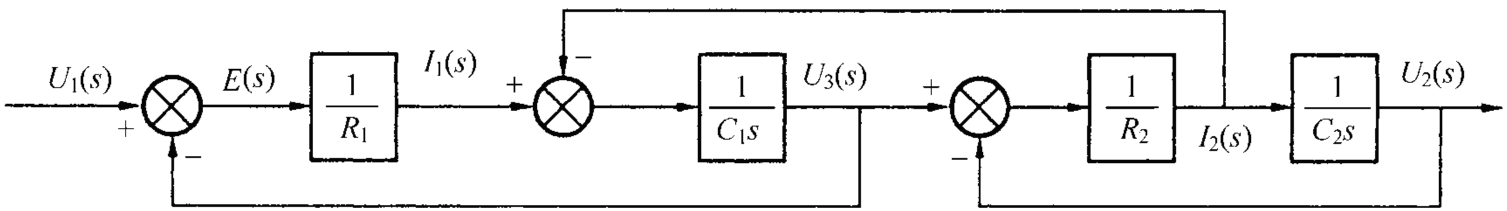
解

$$\sum_{i=1}^{3} L_i = L_1 + L_2 + L_3 = -\frac{1}{R_1 C_1 s} - \frac{1}{R_2 C_1 s} - \frac{1}{R_2 C_2 s}$$

$$\Sigma L_i L_j = L_1 L_3 = \frac{1}{R_1 R_2 C_1 C_2 s^2}$$

$$\Delta = 1 + \frac{1}{R_1 C_1 s} + \frac{1}{R_2 C_1 s} + \frac{1}{R_2 C_2 s} + \frac{1}{R_1 R_2 C_1 C_2 s^2} \qquad P_1 = \frac{1}{R_1 R_2 C_1 C_2 s^2} \qquad \Delta_1 = 1$$

$$\Phi(s) = \frac{U_2(s)}{U_1(s)} = \frac{\frac{1}{R_1 R_2 C_1 C_2 s^2}}{1 + \frac{1}{R_1 C_1 s} + \frac{1}{R_2 C_1 s} + \frac{1}{R_2 C_2 s} + \frac{1}{R_1 R_2 C_1 C_2 s^2}} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2)s + 1}$$



(c)

$$\Phi_E(s) = \frac{E(s)}{U_1(s)} \qquad P_1 = 1 \qquad \Delta_1 = 1 + \frac{1}{R_2 C_1 s} + \frac{1}{R_2 C_2 s}$$

$$\Phi_E(s) = \frac{E(s)}{U_1(s)} = \frac{1 + \dfrac{1}{R_2 C_1 s} + \dfrac{1}{R_2 C_2 s}}{1 + \dfrac{1}{R_1 C_1 s} + \dfrac{1}{R_2 C_1 s} + \dfrac{1}{R_2 C_2 s} + \dfrac{1}{R_1 R_2 C_1 C_2 s^2}}$$

$$= \frac{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2) s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}$$