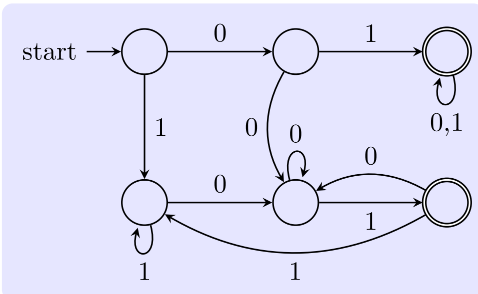
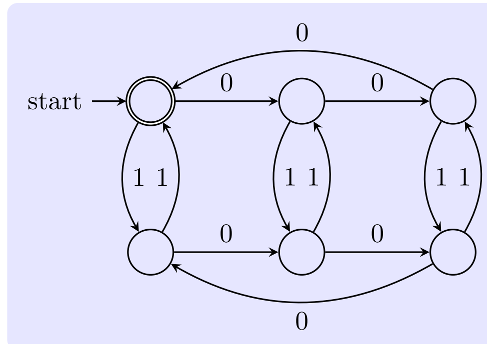
Homework 1 学号 姓名

1. Give DFA's accepting the languages over the alphabet $\{0, 1\}$ (notice : give diagram notation of DFA)

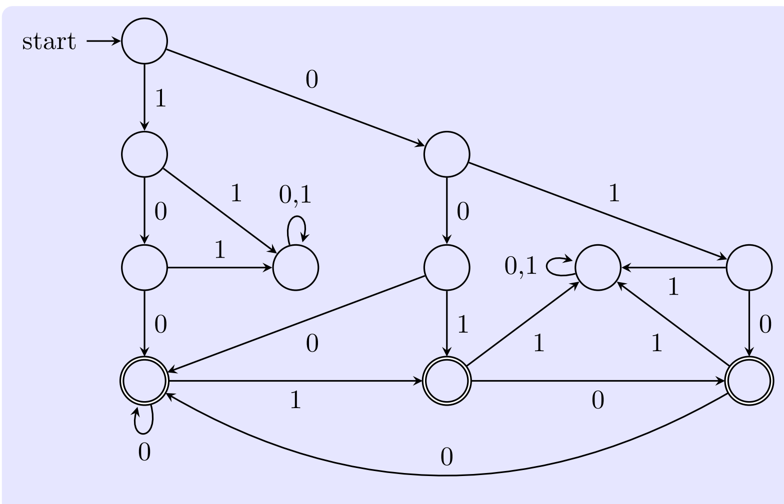
   a) The set of strings that either begin or end with 01.

   b) The set of strings such that the number of 0's is divisible by three, and the number of 1's is divisible by two.



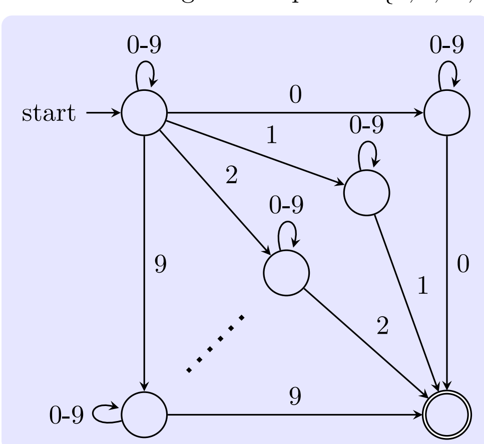


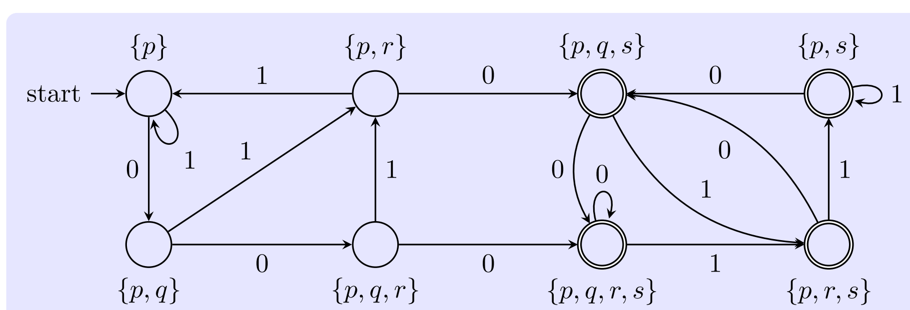
   c) The set of all strings such that each block of three consecutive symbols contains at least two 0's.



2. Give nondeterministic finite automata to accept the following language. Try to take advantage of nondeterminism as much as possible.

   The set of strings over alphabet $\{0, 1, ..., 9\}$ such that the final digit has appeared before.





3. Convert the following NFA to a DFA by subset construction.(notice: give diagram notation of DFA, and label the states by subsets)

	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	$\emptyset$
$*s$	$\{s\}$	$\{s\}$

Homework 2 学号 姓名

1. Give regular expressions for the following languages.

   i) The set of all strings with an equal number of 0's and l's, such that no prefix has two more 0's than l's, nor two more l's than 0's.

   (01+10)*

   ii) $L = \{a^n b^m : n < 4, m \leq 3\}$.

   ((a+e)^3)((b+e)^3)

2. Prove $L = \{0^n | n$ is a perfect square$\}$ is not regular.

3. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

4. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   a) Give all the regular expressions $R_{ij}^{(0)}$, $R_{ij}^{(1)}$ and $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible. Note: Think of state q_i as if it were the state with integer number i.

   b) Give a regular expression for the language of the automaton.

3. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

证明：令 $L = L(M)$. 其中 $M = (Q, \Sigma, \delta, q_0, F)$.

构造 $M' = (Q, \Sigma, \delta, q_0, F')$

其中 $F' = \{q \mid q \in Q$ 且 $\delta(q, a) \in F$ 且 $a \in \Sigma\}$.

下证 $L(M')$ 为 L/a 语言.

$\forall w \in L(M')$ 有 $\hat{\delta}(q_0, w) \in F'$ 即 $\delta(\hat{\delta}(q_0, w), a) \in F$

$\therefore w \in L/a$

再证 $L(M')$ 是正则的.

$\forall w \in L/a$. 有 $wa \in L$. 有 $\hat{\delta}(q_0, wa) \in F$.

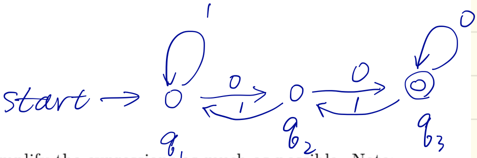
即 $\delta(\hat{\delta}(q_0, w), a) \in F$. 从而有 $\hat{\delta}(q_0, w) \in F'$ $w \in L(M')$

故 L 为正则的 $\therefore L(M')$ 是正则的. 从而 L/a 正则.

4. Here is a transition table for a DFA:

	0	1
$\to q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

DFA → RE



a) Give all the regular expressions $R_{ij}^{(0)}$, $R_{ij}^{(1)}$ and $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible. Note: Think of state q_i as if it were the state with integer number i.

b) Give a regular expression for the language of the automaton.

复习公式

$$R_{ij}^{k} = R_{ij}^{k-1} \cup R_{ik}^{k-1}(R_{kk}^{k-1})^* R_{kj}^{k-1}$$

$$R_{ij}^{0} = \begin{cases} \{a \mid \delta(q_i, a) = q_j\}. & i \neq j \\ \{a \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\}. & i = j \end{cases}$$

进而

	$k=0$	$k=1$	$k=2$	
R_{11}^k	$\varepsilon+1$	1^*	$1^*(\varepsilon+0(11^*0)^*11^*)$	$(1+01)^*$
R_{12}^k	0	1^*0	$1^*0(11^*0)^* \overset{?}{=} (1^*0)^* = (1+01)^*0$	
R_{13}^k	ϕ	ϕ	$1^*0(11^*0)^*0$	$= (1+01)^*00$
R_{21}^k	1	11^*	$(11^*0)^*11^*$	
R_{22}^k	ε	$\varepsilon+11^*0$	$(11^*0)^*$	
R_{23}^k	0	0	$(11^*0)^*0$	
R_{31}^k	ϕ	ϕ	$1(11^*+0)^*11^*$	
R_{32}^k	1	1	$1(11^*0)^*$	
R_{33}^k	$\varepsilon+0$	$\varepsilon+0$	$\varepsilon+0+1(11^*0)^*0$	$\varepsilon+0+(1+01)^*00+10$

$$R_{13}^{3} = R_{13}^{2} + R_{13}^{2}(R_{33}^{2})^* R_{33}^{2}$$

$$= 1^*0(11^*0)^*0 + 1^*0(11^*0)^*0(\varepsilon+0+1(11^*0)^*0)^*(\varepsilon+\cdots$$

$$= 1^*0(11^*0)^*0(0+1(11^*0)^*0)^*$$

$$= (1+01)^*00(0+10+(1+01)^*00)^*$$

Homework 3 学号 姓名

1. Design context-free grammars for the following languages:

   a) L = $\{a^i b^j | i \neq j \text{ and } i \neq 2j\}$

   b) The set of all strings with twice as many 0's as 1's.

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

   a) The set of all strings of 0's and l's such that no prefix has more l's than 0's.

   b) $\{0^n 1^m | n < m < 2n\}$

3. Design a context-free grammar for the language consisting of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

1. Design context-free grammars for the following languages:

a) $L = \{a^i b^j \mid i \neq j \text{ and } i \neq 2j\}$

$i<j$ 或 $j<i<2j$ 或 $i>2j$

三种情况

① $A \to aAb \mid Ab \mid b$

② $B \to aBb \mid aaBb \mid aabbb$

③ $C \to aaCb \mid aC \mid a$

从而有 $S \to A \mid B \mid C$

b) The set of all strings with twice as many 0's as 1's.

类似于 $L = \{w \in \{0,1\}^* \mid$ 0和1数量相等$\}$

$S \to S0S1S \mid S1S0S \mid \varepsilon$ (每对0.1相对)

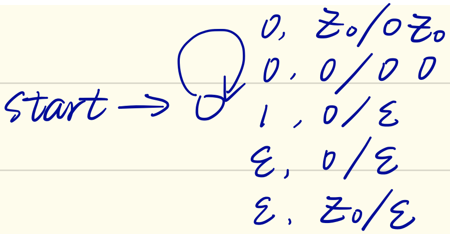
则每2个0对应1个1即有

$S \to S0S0S1S \mid S0S1S0S \mid S1S0S0S \mid \varepsilon$

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

a) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

① 直接画



空栈方式

② $S \to 0S1S \mid 0S \mid \varepsilon$ 先写CFG

构造PDA $P = (\{q\}, \{0,1\}, \{0,1,S\}, \delta, q, S, \phi)$

其中. $\delta(q,\varepsilon,S) = \{(q,0S1S),(q,0S),(q,\varepsilon)\}$.

$\delta(q,0,0) = \{(q,\varepsilon)\}$

$\delta(q,1,1) = \{(q,\varepsilon)\}$

b) $\{0^n1^m \mid n < m < 2n\}$

$S \to 0S1 \mid 0S11 \mid 00111$

构造PDA. $P = (\{q\}, \{0,1\}, \{0,1,S\}, \delta, q, S, \phi\}$.

$\delta(q,\varepsilon,S) = \{(q,0S1),(q,0S11),(q,00111)\}$.

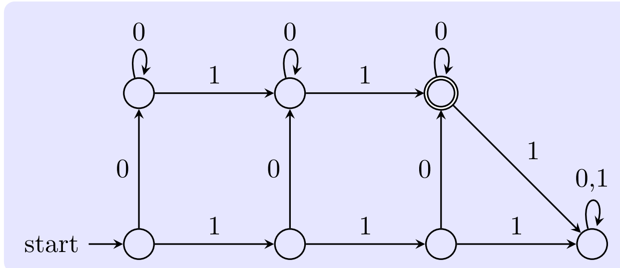
$\delta(q,0,0) = \{(q,\varepsilon)\}$.

$\delta(q,1,1) = \{(q,\varepsilon)\}$.

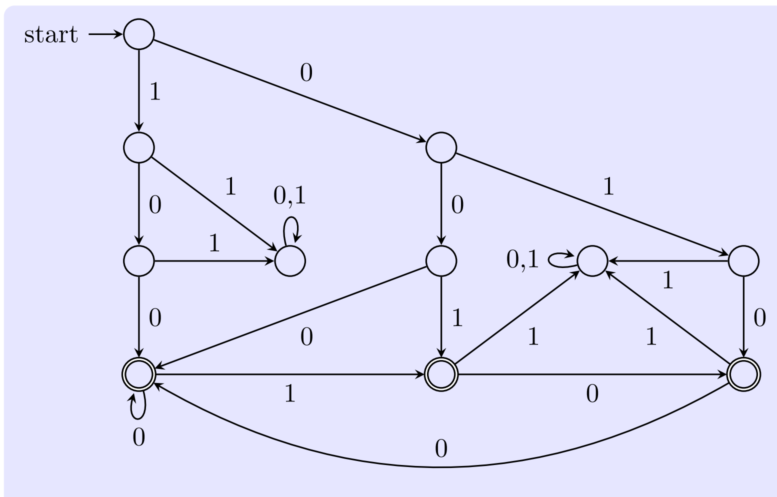
Homework 1 　　　　班号　　　　学号　　　　姓名

Give DFA's accepting the languages over the alphabet $\{0, 1\}$.

1. the set of all strings with at least one 0 and exactly two 1's.(所有以 01 开始或结尾的串.)

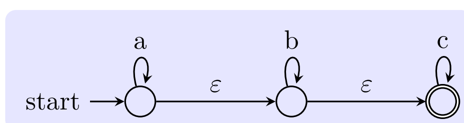


2. The set of all strings such that each block of three consecutive symbols contains at least two 0's.(任何 3 个连续的字符都至少有两个 0.)

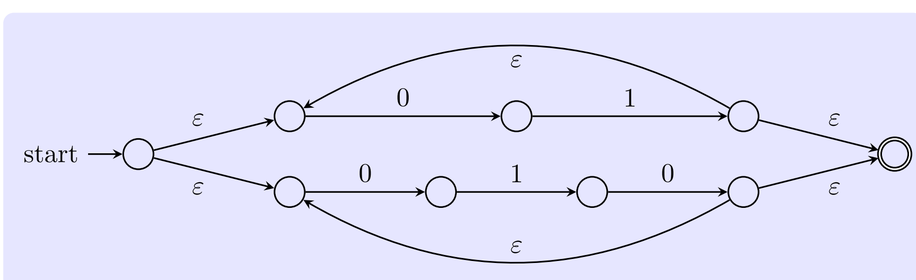


Design ε-NFA's for the following languages. Try to use ε-transitions to simplify your design.

3. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.



4. The set of strings that consist of either 01 repeated one or more times or 010 repeated one or more times.



Design regular expression:

5. The set of all strings of 0's and l's not containing 101 as a substring.

$0^*(1+000^*)^*0^*$ or $(0+\varepsilon)(1+000^*)^*(0+\varepsilon)$

Homework 2　　　　　班号　　　　　学号　　　　　姓名

1. Prove that language $L = \{0^n \mid n \text{ is a power of } 2\}$ is not regular.

   证明思路：泵引理，反证法。取 $s = 0^{2^N}$, 则 $2^N < |xy^2z| < 2^N + N < 2^N + 2^N = 2^{(N+1)}$

2. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

   证明：令 $L = L(M)$, 其中 $M = (Q, \Sigma, \delta, q_0, F)$

   构造 $M' = (Q, \Sigma, \delta, q_0, F')$, 其中 $F' = \{q | \delta(q, a) \in F\}, q \in Q, a \in \Sigma$

   证明 $L(M') = L/a$, $\because \forall w \in L(M')$ 即 $\hat{\delta}(q_0, w) \in F'$ 即 $\delta(\hat{\delta}(q_0, w), a) \in F$ $\therefore w \in L/a$

   又 $\because \forall w \in L/a$ 有 $wa \in L$ 即 $\hat{\delta}(q_0, wa) \in F$ 即 $\delta(\hat{\delta}(q_0, w), a) \in F$ 即 $\hat{\delta}(q_0, w) \in F'$ $\therefore w \in L(M')$

Design context-free grammars for the following languages:

3. The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

   $S \to A_1C|A_2C|AB_1|AB_2$
   $A_1 \to aA_1b|aA_1|a$
   $A_2 \to aA_2b|A_2b|b$
   $C \to Cc|\varepsilon$
   $B_1 \to bB_1c|bB_1|b$
   $B_2 \to bB_2c|B_2c|c$
   $A \to Aa|\varepsilon$

   (注意：$Cc|\varepsilon$ 若为 $Cc|c$ 则不能产生 a, c 同时为 0 个，或 b, c)

4. The set of all strings over $\{0, 1\}$ with twice as many 0's as 1's.

   $S \to S0S0S1S|S0S1S0S|S1S0S0S|\varepsilon$

5. The set of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

   如果串长为奇数，显然不是 ww 形式 (对应下面文法中的 A 或 B)。而对于长度为偶数 ($2n$) 的串，至少存在一对儿距离为 n(串长度的一半) 的两字符不相同。为了能够产生两个不相同字符的距离刚好是整个长度的一半，使用两个变元 A 和 B 分别产生基数长的串，然后合并即可。对 A 或 B，在为产生串时，如果增加了两字符间的字符数，那么也要增加两字符外的字符数。如 $aaaabbbb = \underline{aa\check{a}ab}\ \underline{b\check{b}b}$ 或 $aabaaa = \underline{aa\check{b}aa}\ \underline{\check{a}}$.

   $S \to A \mid B \mid AB \mid BA$
   $A \to XAX \mid a$
   $B \to XBX \mid b$
   $X \to a \mid b$

6. Give regular expressions for the following languages.

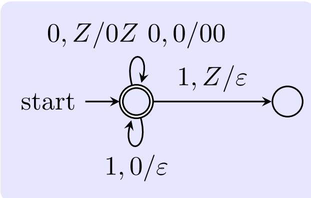
   i) The set of all strings with an equal number of 0's and l's, such that no prefix has two more 0's than l's, nor two more l's than 0's.

   ii) $L = \{a^n b^m : n < 4, m \leq 3\}$.

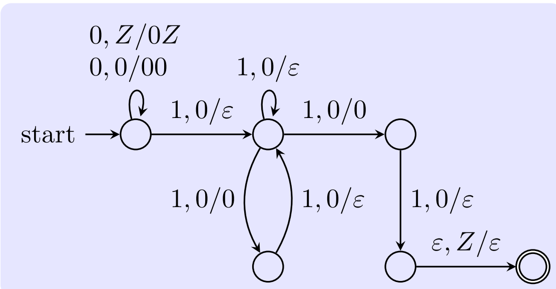
Homework 3 班号 学号 姓名

Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

1. The set of all strings of 0's and l's such that no prefix has more l's than 0's.



2. $L = \{0^n 1^m \mid 0 < n < m < 2n\}$



3. Use the CFL pumping lemma to show following language not to be context-free: $\{a^i b^j c^k | i < j < k\}$.

   反证法。假设 L=$\{a^i b^j c^k | i < j < k\}$ 是 CFL，由 CFL 泵引理，存在正整数 N，使长度超过 N 的串符合 CFL 泵引理。取 $s = a^N b^{N+1} c^{N+2}$ 则 $s = uvwxy$ 中，因为 $|vwx| \leq N$ vwx 可能几种分布:

   i) 都在 a 或 b 中，取 $i = 2$ 则 $s' = uv^i wx^i y$ 中 a 或 b 可能不小于 c

   ii) 在 c 中，取 $i = 0$，$\cdots$

   iii) 在 ab 之间，取 $i = 2$，$\cdots$

   iv) 在 bc 之间，取 $i = 0$，$\cdots$

   无论何种情况，都与假设矛盾。得证

4. Design a Turing machine for the language $L = \{w \mid w \in \{0, 1\}^*, w = w^R\}$.

   略