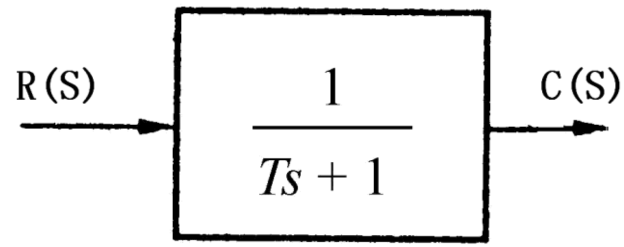
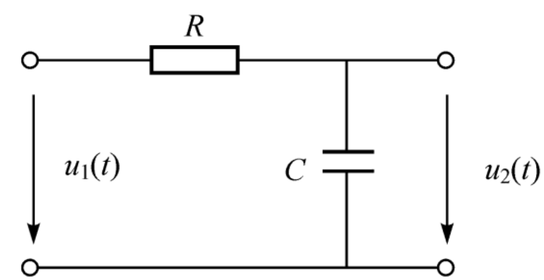
3.2 一阶系统时域分析

- 输入信号r(t)与输出信号c(t)的关系用一阶微分方程表示的称为一阶系统



$$T\frac{\mathrm{dc(t)}}{\mathrm{d}t}+c(t)=r(t)$$

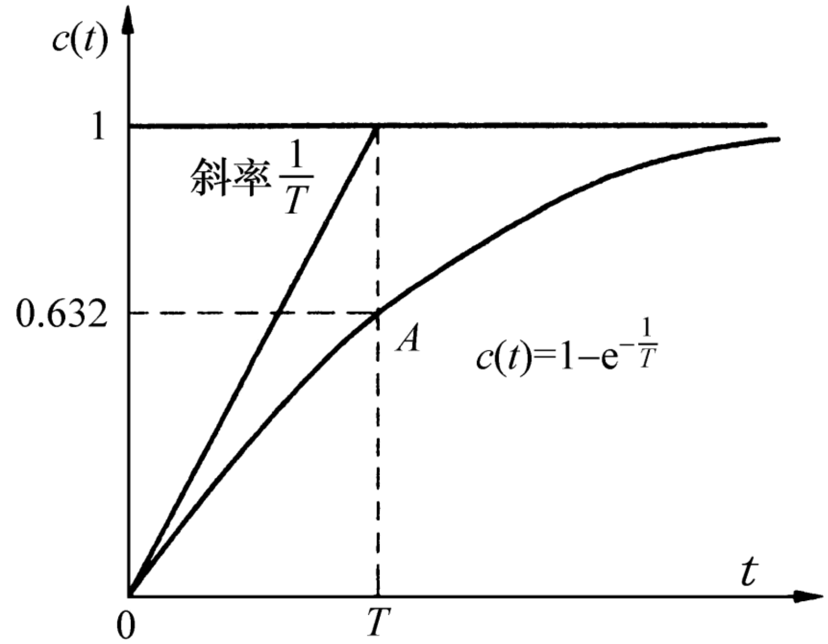
$$\Phi(\mathrm{s})=\frac{\mathrm{C(s)}}{\mathrm{R(s)}}=\frac{1}{Ts+1}$$



- 常见的温度控制系统和液压控制系统中的控制对象都是一阶系统。

3.2.1 一阶系统的单位阶跃响应

- 设r(t)=1(t),R(s)=1/s。于是有



$$C(s)=\Phi(s)R(s)=\frac{1}{Ts+1}\cdot\frac{1}{s}=\frac{1}{s}-\frac{T}{Ts+1}$$

$$c(t)=c_s(t)+c_t(t)=1-e^{-\frac{t}{T}} \qquad t\geq 0$$

- 单位阶跃响应的典型数值

$$c(0)=1-e^{0}=0,\quad c(T)=1-e^{-1}=0.632,\quad c(2T)=1-e^{-2}=0.865$$

$$c(3T)=1-e^{-3}=0.95,\quad c(4T)=1-e^{-4}=0.982,\quad c(\infty)=1$$

$$\dot{c}(0)=\frac{1}{T}e^{-\frac{t}{T}}\Big|_{t=0}=\frac{1}{T}$$

T为时间常数，1/T为初始斜率

3.2.2一阶系统的单位斜坡响应

- 令r(t)=t,则有 $R(s)=1/s^2$ 可求得输出信号的拉氏变换式

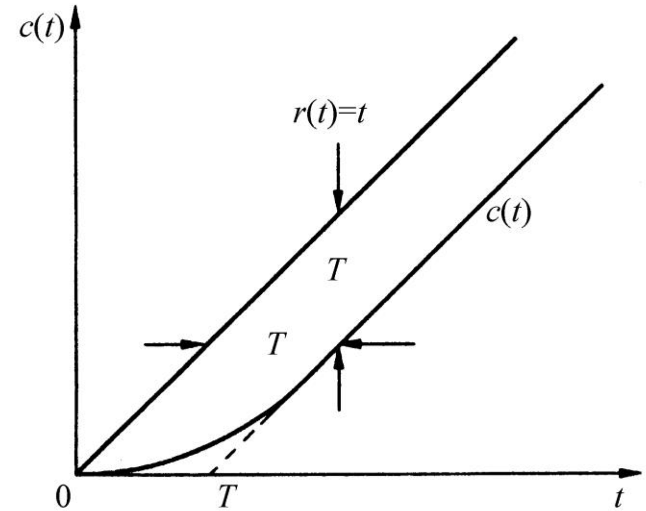
$$C(s)=\frac{1}{Ts+1}\cdot\frac{1}{s^2}=\frac{1}{s^2}-\frac{T}{s}+\frac{T^2}{Ts+1}$$

$$c(t)=c_s(t)+c_t(t)=(t-T)+Te^{-\frac{t}{T}}\qquad t\geq 0$$

$c_s(t)=(t-T)$是稳态分量，$c_t(t)=Te^{-\frac{t}{T}}$是瞬态分量。

- 系统的误差信号 e (t)为

$$e(t)=r(t)-c(t)=T(1-e^{-\frac{t}{T}})$$



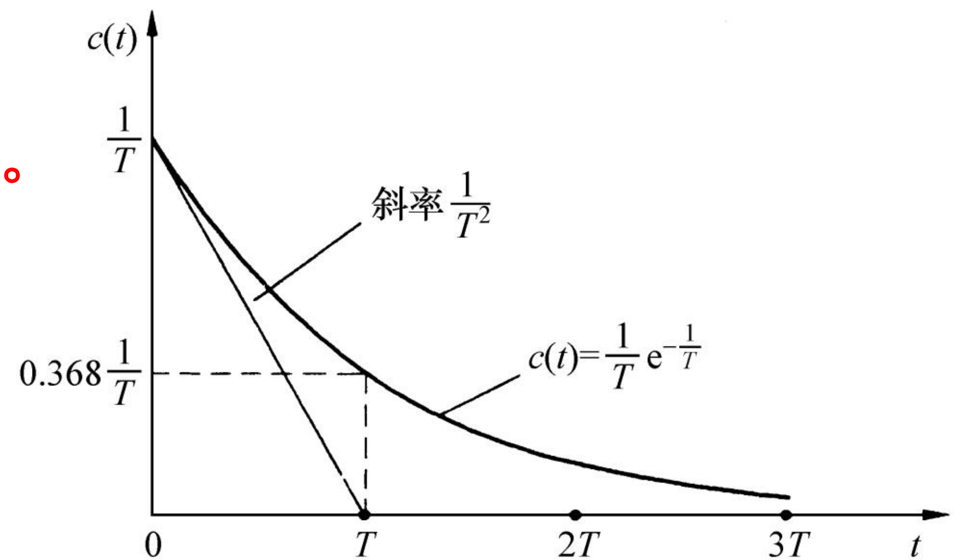
3.2.3 单位冲激响应

$$r(t)=\delta(t) \quad \Rightarrow \quad R(s)=1$$

$$\Rightarrow \qquad C(s)=\frac{1}{Ts+1}$$

$$g(t)=c(t)=L^{-1}\left(\frac{1}{Ts+1}\right)=\frac{1}{T}\mathrm{e}^{-\frac{t}{T}} \qquad (t\geq 0)$$

单位冲激响应中只有瞬态响应。



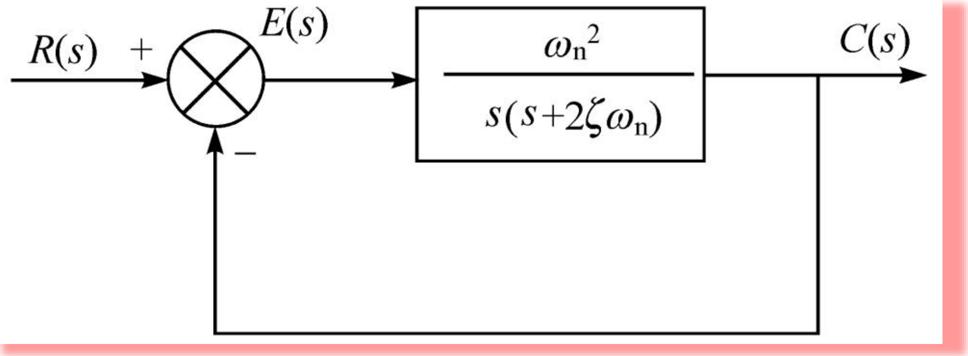
3.3 二阶系统的时域分析

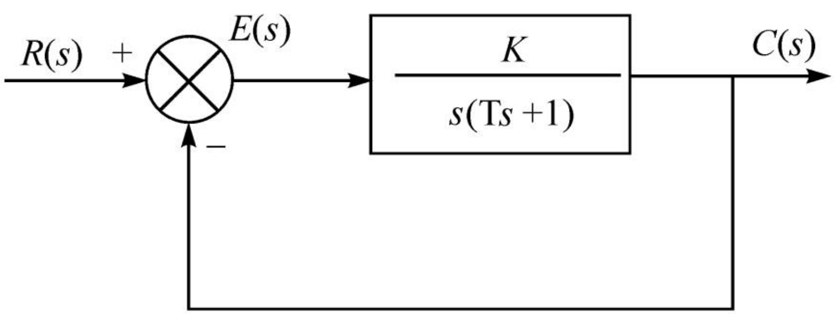
3.3.1 二阶系统的典型形式

- 典型形式

$$\ddot{c}(t)+2\zeta\omega_n\dot{c}(t)+\omega_n^2c(t)=\omega_n^2r(t)$$

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$





- 特征方程及特征根(极点)

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

$$s_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

$$\omega_n=\sqrt{\frac{K}{T}},\ \zeta=\frac{1}{2\sqrt{KT}}$$

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

$$s_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

(a).欠阻尼（0<ζ<1）

$$s_{1,2}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$

(b).临界阻尼（ζ=1）

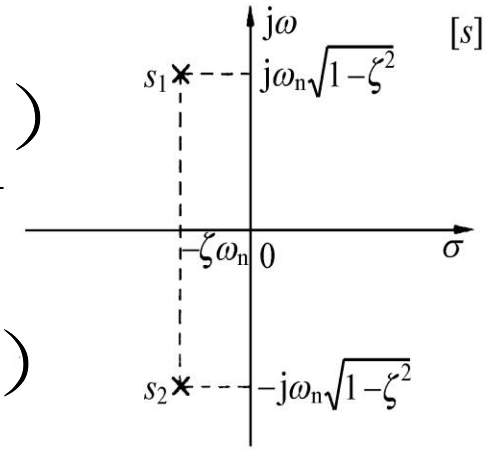
$$s_{1,2}=-\omega_n$$

(c).过阻尼（ζ>1）

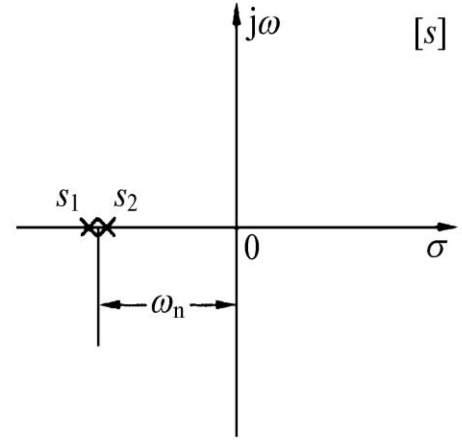
$$s_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

(d).无阻尼（ζ=0）

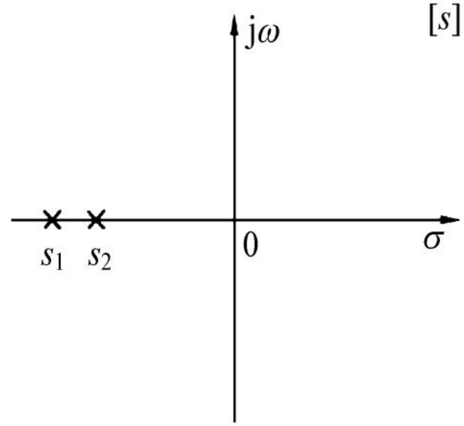
$$s_{1,2}=\pm j\omega_n$$



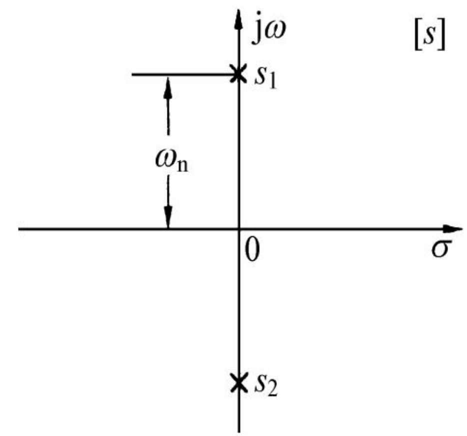
(a) $0<\zeta<1$



(b) $\zeta=1$

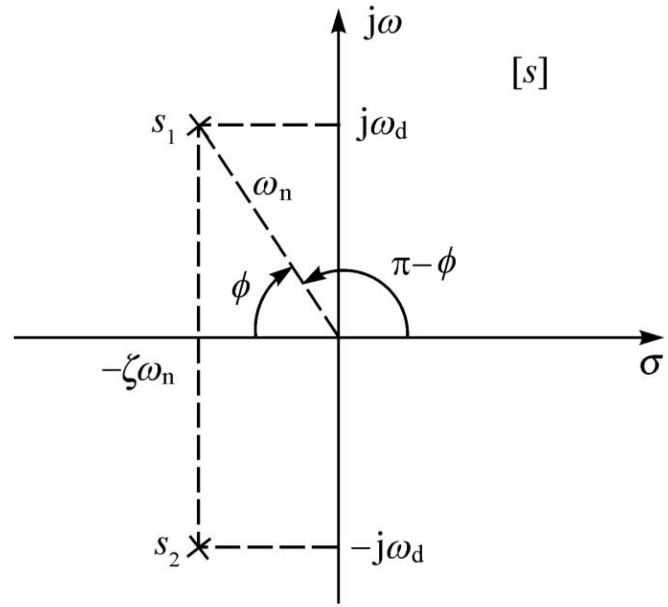


(c) $\zeta>1$



(d) $\zeta=0$

3.3.2 二阶系统的单位阶跃响应



- 令r(t)=1(t),则有R(s)=1/s

$$C(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}\qquad c(t)=L^{-1}[C(s)]$$

- 1.欠阻尼状态（0<ζ<1）

$$C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)}$$

$$=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^{\ 2}}-\frac{\zeta\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^{\ 2}}\qquad \omega_d=\omega_n\sqrt{1-\zeta^2}$$

$$c(t)=1-\mathrm{e}^{-\zeta\omega_n t}\cos\omega_d t-\frac{\zeta\omega_n}{\omega_d}\cdot\mathrm{e}^{-\zeta\omega_n t}\sin\omega_d t$$

$$=1-\mathrm{e}^{-\zeta\omega_n t}(\cos\omega_d t+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t)\qquad (t\ge 0)$$

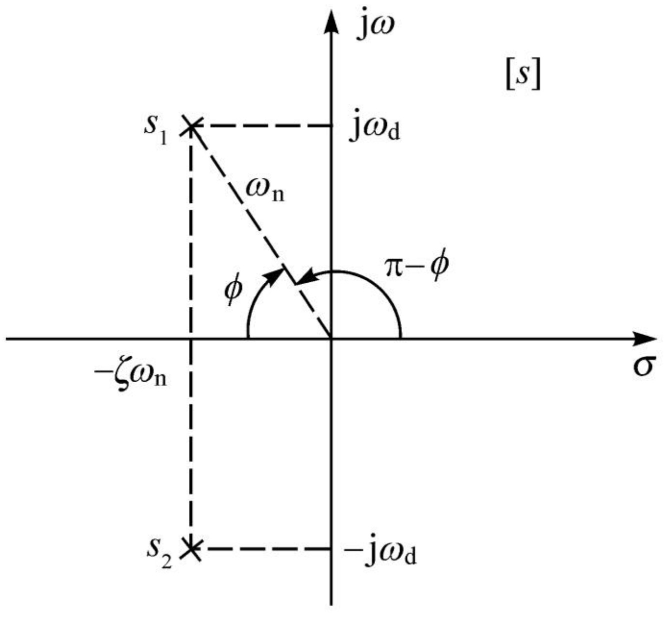
$$c(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\sqrt{1-\zeta^2}\cos\omega_d t+\zeta\sin\omega_d t)$$

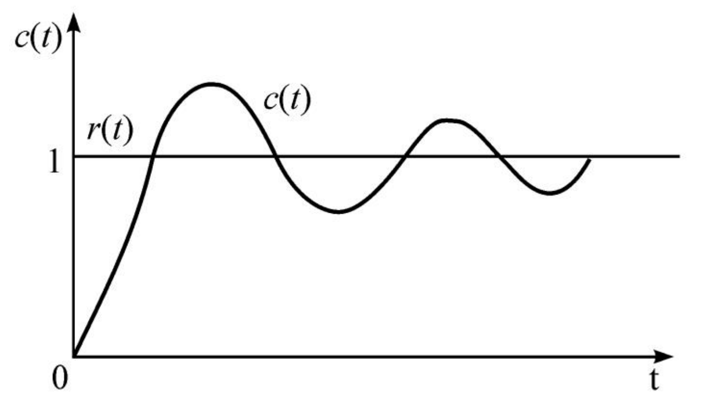
$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi) \qquad (t\geq 0)$$

$$\phi=\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}=\arccos\zeta$$

$$c_s(t)=1, \qquad c_t(t)=-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)$$

$$T_d=\frac{2\pi}{\omega_d}=\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

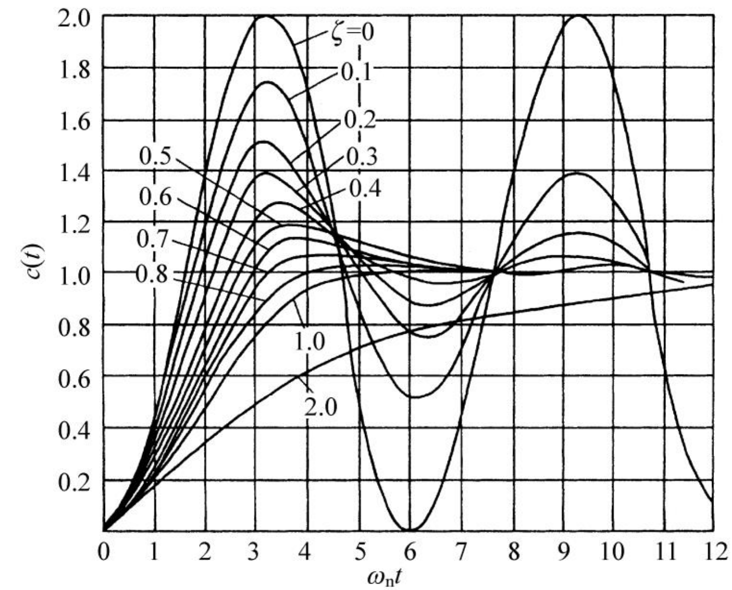




- 2.无阻尼状态（ζ=0）

$$c(t) = 1 - \cos\omega_n t \qquad t \geq 0$$

- 3.临界阻尼（ζ=1）



$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

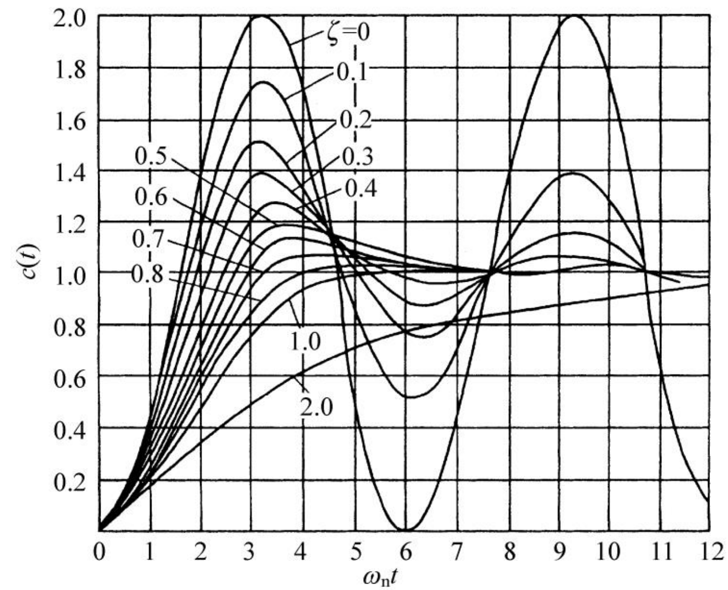
$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

$$c(t) = 1 - (\omega_n t + 1)\mathrm{e}^{-\omega_n t} \qquad (\mathrm{t} \geq 0)$$

- 4.过阻尼（ζ>1）

$$s_1 = -(\zeta + \sqrt{\zeta^2 - 1})\omega_n$$

$$s_2 = -(\zeta - \sqrt{\zeta^2 - 1})\omega_n$$



$$C(s) = \frac{s_1 s_2}{(s - s_1)(s - s_2)} \cdot \frac{1}{s}$$

$$= \frac{1}{s} + \frac{A_1}{(s - s_1)} + \frac{A_2}{(s - s_2)}$$

式中 $A_1 = \dfrac{1}{2\sqrt{\zeta^2 - 1}(\zeta + \sqrt{\zeta^2 - 1})}$，$A_2 = -\dfrac{1}{2\sqrt{\zeta^2 - 1}(\zeta - \sqrt{\zeta^2 - 1})}$

$$c(t) = 1 + A_1 e^{s_1 t} + A_2 e^{s_2 t} = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{e^{s_1 t}}{-s_1} - \frac{e^{s_2 t}}{-s_2}\right)$$

3.3.3 二阶欠阻尼系统的动态性能指标

- 1.上升时间t_r的计算

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

当$t=t_r$时，$c(t_r)=1$

$$c(t_r)=1-\mathrm{e}^{-\zeta\omega_n t_r}\left(\cos\omega_d t_r+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right)=1$$

即 $\mathrm{e}^{-\zeta\omega_n t_r}\left(\cos\omega_d t_r+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right)=0$ 因为 $\mathrm{e}^{-\zeta\omega_n t_r}\neq 0$,

所以 $\cos\omega_d t_r+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r=0$ 或 $\tan\omega_d \mathrm{t_r}=\frac{\omega_\mathrm{n}\sqrt{1-\zeta^2}}{-\zeta\omega_n}=\frac{\omega_\mathrm{d}}{-\zeta\omega_n}$

$\tan\omega_d \mathrm{t_r}=\tan(\pi-\phi)$ $t_r=\frac{\pi-\phi}{\omega_d}=\frac{\pi-\phi}{\omega_n\sqrt{1-\zeta^2}}$

- 2.峰值时间t_p的计算

$$\left.\frac{\mathrm{d}c(t)}{\mathrm{d}t}\right|_{t=t_0}=0$$

$$\frac{\zeta\omega_n \mathrm{e}^{-\zeta\omega_\mathrm{n}t_p}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_p+\phi)-\frac{\omega_d \mathrm{e}^{-\zeta\omega_\mathrm{n}t_p}}{\sqrt{1-\zeta^2}}\cos(\omega_d t_p+\phi)=0$$

$$\sin(\omega_d t_p+\phi)=\frac{\sqrt{1-\zeta^2}}{\zeta}\cos(\omega_d t_p+\phi)$$

$$\tan(\omega_d t_p+\phi)=\tan\phi \qquad \omega_d t_p=0,\pi,2\pi,3\pi,\cdots$$

$$t_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=\frac{1}{2}T_d$$

- 3.最大超调（量）σ_p 的计算

$$\sigma_p=\frac{c(t_p)-c(\infty)}{c(\infty)}=-\mathrm{e}^{\zeta\omega_n t_p}\left(\cos\omega_d t_p+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_p\right)\times 100\%$$

$$=-\mathrm{e}^{\zeta\omega_n t_p}\left(\cos\pi+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi\right)\times 100\%$$

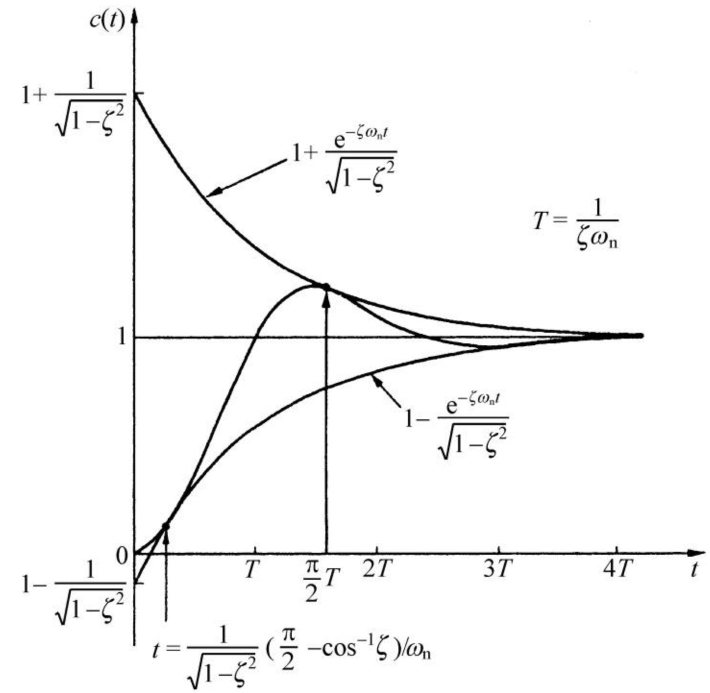
即 $$\sigma_p=\mathrm{e}^{-\zeta\pi/\sqrt{1-\zeta^2}}\times 100\%=\mathrm{e}^{-\pi\cot\phi}$$

- 4.过渡过程时间 t_s 的计算

$c(t)$位于响应曲线包络线 $1\pm\dfrac{\mathrm{e}^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}$ 内，

$$\frac{\mathrm{e}^{-\zeta\omega_n t_s}}{\sqrt{1-\zeta^2}}=\Delta;\quad t_s=\frac{1}{\zeta\omega_n}\left(\ln\frac{1}{\Delta}+\ln\frac{1}{1-\zeta^2}\right)$$

$$\Delta=5\%\qquad t_s\approx\frac{3}{\zeta\omega_n};\Delta=2\%\qquad t_s\approx\frac{4}{\zeta\omega_n};$$



- 5.振荡次数N的计算

$$N=\frac{t_s}{T_d}=\frac{t_s}{2t_p}$$

当$\Delta=2\%$时，$t_s=\dfrac{4}{\zeta\omega_n}$则有 $\mathrm{N}=\dfrac{2\sqrt{1-\zeta^2}}{\pi\zeta}$

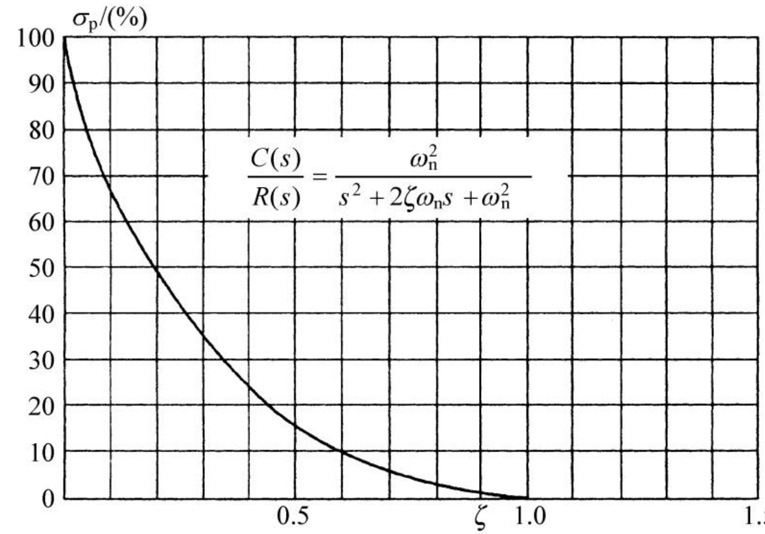
当$\Delta=5\%$时，$t_s=\dfrac{3}{\zeta\omega_n}$则有 $\mathrm{N}=\dfrac{1.5\sqrt{1-\zeta^2}}{\pi\zeta}$

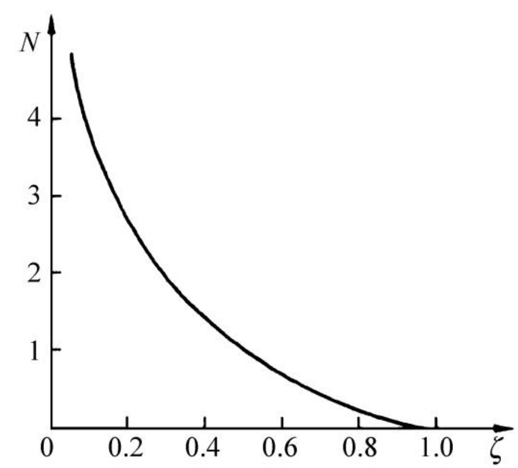
若已知σ_p，$\sigma_p=\mathrm{e}^{-\pi\zeta/\sqrt{1-\zeta^2}}$

即 $$\ln\sigma_p=-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

N与σ_p的关系为

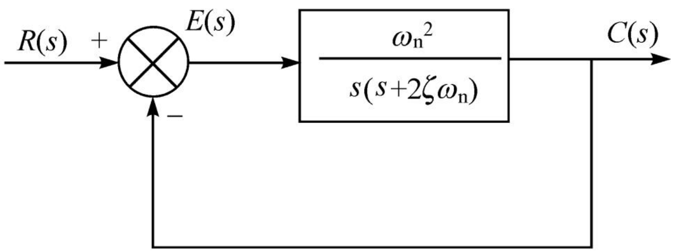
$$N=\frac{-2}{\ln\sigma_p}\quad(\Delta=2\%);\qquad N=\frac{-1.5}{\ln\sigma_p}\quad(\Delta=5\%)$$





3.3.4 二阶系统的计算举例

- 例 3-3-1



二阶系统如图所示，其中$\zeta=0.6$，$\omega_n=5\text{rad/s}$。

$r(t)=1(t)$,求t_r,t_p,t_s,σ_p和N。

解：$\sqrt{1-\zeta^2}=\sqrt{1-0.6^2}=0.8,\quad \omega_d=\omega_n\sqrt{1-\zeta^2}=5\times0.8=4$，$\zeta\omega_n=0.6\times5=3$

$$\phi=\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}=\arctan\frac{0.6}{0.8}=0.93\text{rad},\quad t_r=\frac{\pi-\phi}{\omega_d}=\frac{\pi-0.93}{4}=0.55\text{s}$$

$$t_p=\frac{\pi}{\omega_d}=\frac{3.14}{4}=0.785s,\quad \sigma_p=e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\times100\%=e^{-\frac{3.14\times0.6}{0.8}}\times100\%=9.5\%$$

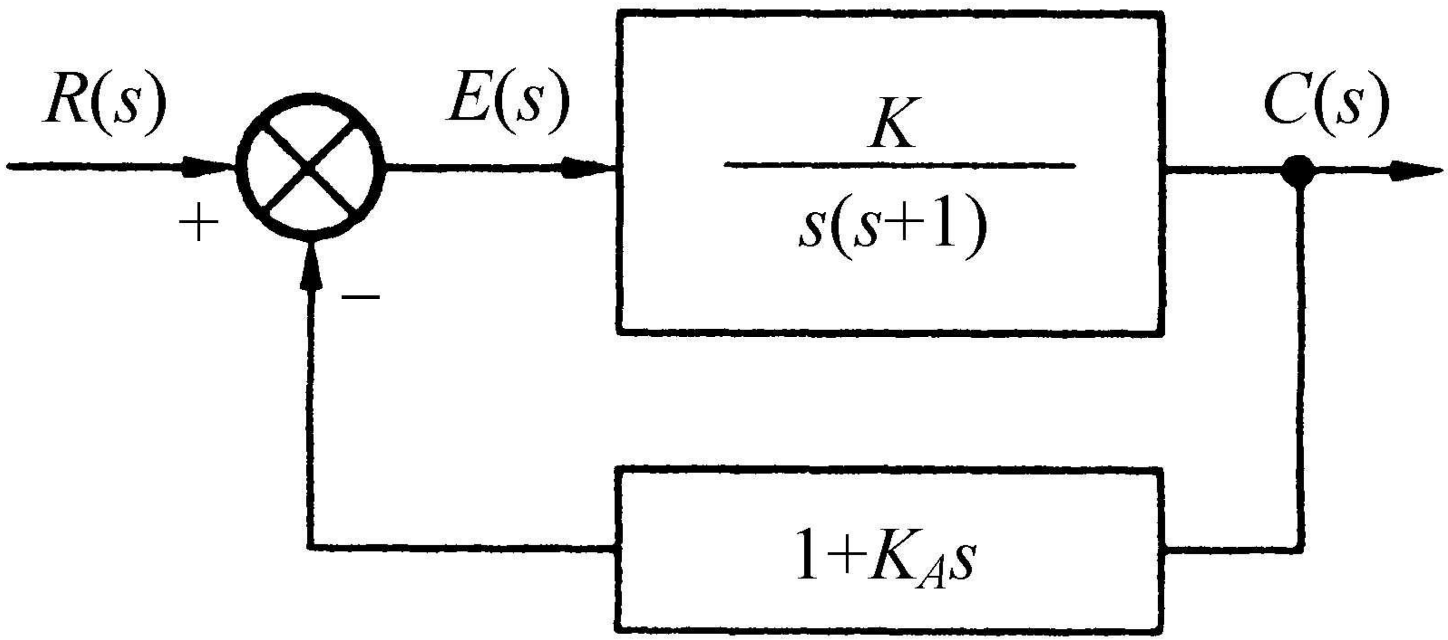
$$t_s\approx\frac{3}{\zeta\omega_n}=1s\qquad(\Delta=5\%);\qquad t_s\approx\frac{4}{\zeta\omega_n}=1.33s\qquad(\Delta=2\%)$$

$$N=\frac{t_s}{2t_p}=\frac{1.33}{2\times0.785}=0.8\quad(\Delta=2\%);\quad N=\frac{t_s}{2t_p}=\frac{1}{2\times0.785}=0.6\quad(\Delta=5\%)$$

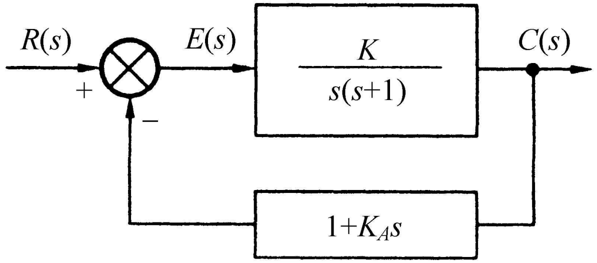
- 例3-3-2 要求系统性能指标为

$$\sigma_p = 20\%, \qquad t_p = 1s$$

确定K值和K_A值,并计算t_r, t_s及N值。



- 例3-3-2 要求系统性能指标为 $\sigma_p = 20\%, t_p = 1s$。确定$K$值和$K_A$值,并计算$t_r, t_s$及$N$值。



解 $\sigma_p = \mathrm{e}^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$, $\quad \dfrac{\pi\zeta}{\sqrt{1-\zeta^2}} = \ln\dfrac{1}{\sigma_p} = \ln\dfrac{1}{0.2} = 1.61$

$$\Rightarrow \quad \zeta = 0.456 \qquad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad \Rightarrow \quad \omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 3.53\,\mathrm{rad/s}$$

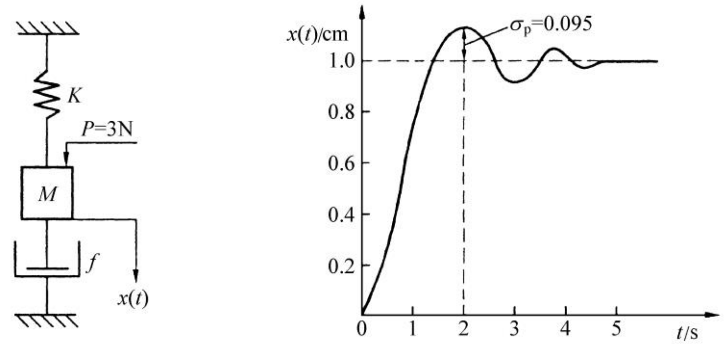
$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + (1+KK_A)s + K} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = K,\ \ 2\zeta\omega_n = (1+KK_A) \quad K = \omega_n^2 = 3.53^2 = 12.5 \qquad K_A = \frac{2\zeta\omega_n - 1}{K} = 0.178$$

$$t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\zeta^2}},\ \ \phi = \arccos\zeta = \arctan\frac{\sqrt{1-\zeta^2}}{\zeta} = 1.1rad \qquad \Rightarrow \ \ t_r = 0.65\,\mathrm{s}$$

$$\Delta = 5\% \quad t_s = \frac{3}{\zeta\omega_n} = 1.86\,\mathrm{s} \quad N = \frac{t_s}{2t_p} = 0.93; \ \ \Delta = 2\% \quad t_s = \frac{4}{\zeta\omega_n} = 2.48\,\mathrm{s} \ \ N = \frac{t_s}{2t_p} = 1.2$$

- 例3-3-3 根据过渡过程曲线确定质量M、黏性摩差系数*f* 和弹簧刚度K的值。



(a)机械平移系统 (b)阶跃响应曲线

解 $\sigma_p = 0.095,\ t_p = 2\text{s},\quad x(\infty) = \underset{t\to\infty}{\text{lin}}\, x(t) = 0.01\text{m}$

$$M\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + f\frac{\mathrm{d}x}{\mathrm{d}t} + Kx = P$$

$$\frac{X(s)}{P(s)} = \frac{1}{Ms^2 + fs + K} = \frac{1}{K}\cdot\frac{K/M}{s^2 + (f/M)s + K/M}$$ ，$\omega_n^2 = K/M, 2\zeta\omega_n = f/M$。

当$p(t) = 3\cdot 1(t)$时, $X(s) = \dfrac{1}{Ms^2 + fs + K}\cdot\dfrac{3}{s}$

$$x(\infty) = \underset{t\to\infty}{\text{lin}}\, x(t) = \underset{s\to 0}{\text{lin}}\, s\cdot X(s) = \underset{s\to 0}{\text{lin}}\, s\cdot\frac{1}{Ms^2 + fs + K}\cdot\frac{3}{s} = \frac{3}{K} = 0.01 \qquad K = 300N/\text{m}$$

$$\sigma_\text{p} = 0.095 \Rightarrow \zeta = 0.6, \qquad t_\text{p} = 2 \Rightarrow \omega_n = 1.96\,\text{rad/s}$$

$$\omega_n^2 = K/M \Rightarrow 300/M = 1.96^2 \Rightarrow M = 78\,\text{kg}$$

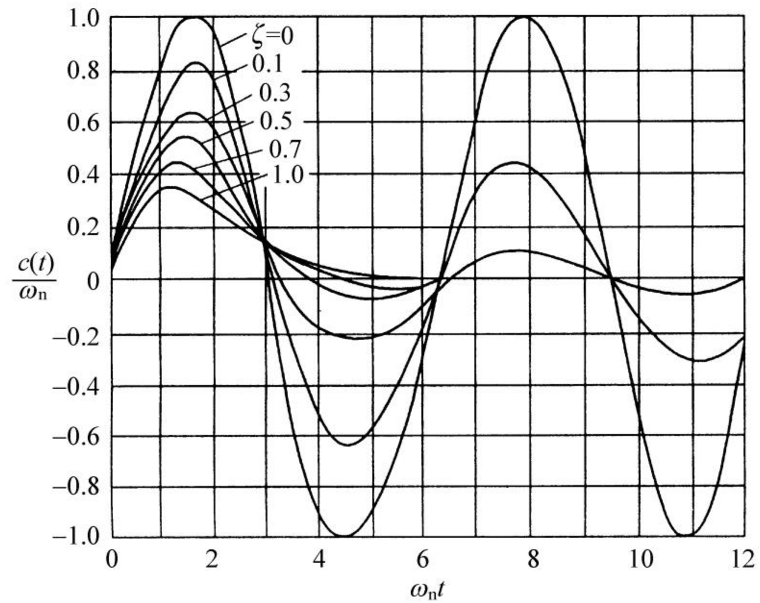
$$2\zeta\omega_n = f/M \Rightarrow 2\times 0.6\times 1.96 = f/78 \Rightarrow f = 180\text{Ns/m}$$

3.3.5 二阶系统的单位冲激响应

$$r(t)=\delta(t),\quad R(s)=1 \quad \Rightarrow \quad C(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$0<\zeta<1$

$$g(t)=c(t)=\frac{\omega_n}{\sqrt{1-\zeta^2}}\mathrm{e}^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t$$



$\zeta=0$

$$g(t)=c(t)=\omega_n\sin\omega_n t$$

$\zeta=1$

$$g(t)=c(t)=\omega_n^2 t\,\mathrm{e}^{-\omega_n t}$$

$\zeta>1$

$$g(t)=c(t)=\frac{\omega_n}{2\sqrt{\zeta^2-1}}[\mathrm{e}^{-(\zeta-\sqrt{\zeta^2-1})\omega_n t}-\mathrm{e}^{-(\zeta+\sqrt{\zeta^2-1})\omega_n t}]$$

3.3.6 二阶系统的单位斜坡响应

$$r(t)=t,\quad R(s)=\frac{1}{s^2}\quad \Rightarrow\ C(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s^2}$$

1. $0<\zeta<1$

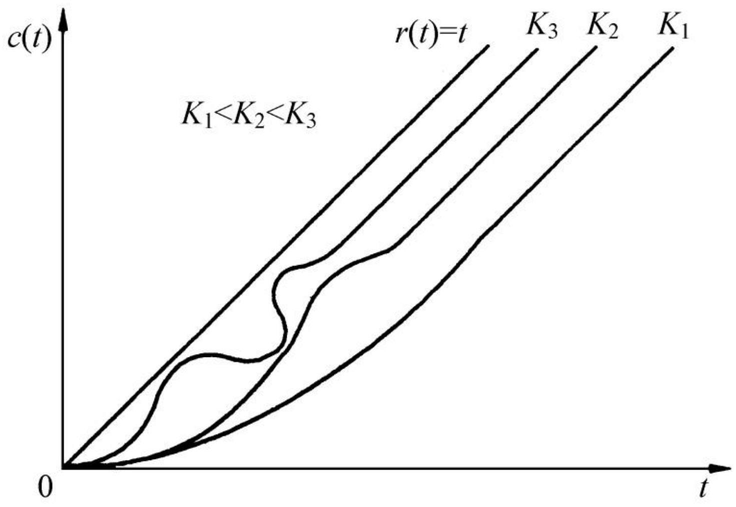
$$\mathrm{C(s)}=\frac{1}{\mathrm{s}^2}-\frac{\frac{2\zeta}{\omega_n}}{s}+\frac{\frac{2\zeta}{\omega_n}(s+\zeta\omega_n)+(2\zeta^2-1)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$c(t)=t-\frac{2\zeta}{\omega_n}+\mathrm{e}^{-\zeta\omega_n t}\left(\frac{2\zeta}{\omega_n}\cos\omega_d t+\frac{2\zeta^2-1}{\omega_n\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

$$=t-\frac{2\zeta}{\omega_n}+\frac{\mathrm{e}^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_d t+\arctan\frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}\right)$$

$$\arctan\frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}=2\arctan\frac{\sqrt{1-\zeta^2}}{\zeta}=2\phi$$

$$r(t)=t,\quad R(s)=\frac{1}{s^2}\qquad\Rightarrow\quad C(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s^2}$$



2. $\zeta=1$ $\quad C(\mathrm{s})=\dfrac{1}{\mathrm{s}^2}-\dfrac{\frac{2}{\omega_n}}{s}+\dfrac{1}{(s+\omega_n)^2}+\dfrac{\frac{2}{\omega_n}}{s+\omega_n}$

$$c(t)=t-\frac{2}{\omega_n}+\frac{2}{\omega_n}\left(1+\frac{\omega_n}{2}t\right)\mathrm{e}^{-\zeta\omega_n t}$$

3 $\zeta>1$

$$c(t)=t-\frac{2\zeta}{\omega_n}-\frac{2\zeta^2-1-2\zeta\sqrt{\zeta^2-1}}{2\omega_n\sqrt{\zeta^2-1}}\mathrm{e}^{-(\zeta+\sqrt{\zeta^2-1})\omega_n t}+\frac{2\zeta^2-1+2\zeta\sqrt{\zeta^2-1}}{2\omega_n\sqrt{\zeta^2-1}}\mathrm{e}^{-(\zeta-\sqrt{\zeta^2-1})\omega_n t}$$

$$c_s(t)=t-\frac{2\zeta}{\omega_n},\qquad c_t(\infty)=\lim_{t\to\infty}c_t(t)=0$$

$$e(t)=r(t)-c(t)=r(t)-c_s(t)-c_t(t)=\frac{2\zeta}{\omega_n}-c_t(t)\qquad e(\infty)=\lim_{t\to\infty}e(t)=\frac{2\zeta}{\omega_n}$$

$$K=\omega_n/(2\zeta)$$

3.3.7 初始条件不为零时二阶系统的时间响应

$$\ddot{c}(t)+2\zeta\omega_n\dot{c}(t)+\omega_n^2c(t)=\omega_n^2r(t)$$

$$s^2C(s)-sc(0)-\dot{c}(0)+2\zeta\omega_n[sC(s)-c(0)]+\omega_n^2C(s)=\omega_n^2R(s)$$

$$C(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}R(s)+\frac{c(0)[s+2\zeta\omega_n]+\dot{c}(0)}{s^2+2\zeta\omega_n s+\omega_n^2}\qquad c(t)=c_1(t)+c_2(t)$$

$0<\zeta<1,$

$$c_2(t)=L^{-1}\left[\frac{c(0)[s+2\zeta\omega_n]+\dot{c}(0)}{s^2+2\zeta\omega_n s+\omega_n^2}\right]=\sqrt{[c(0)]^2+\left[\frac{c(0)\zeta\omega_n+\dot{c}(0)}{\omega_n\sqrt{1-\zeta^2}}\right]^2}\,\mathrm{e}^{-\zeta\omega_n t}\sin(\omega_d t+\theta)$$

$$\theta=\arctan\frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n+\frac{\dot{c}(0)}{c(0)}}\quad;\qquad \zeta=0\quad c_2(t)=\sqrt{[c(0)]^2+\left[\frac{\dot{c}(0)}{\omega_n}\right]^2}\sin\left[\omega_n t+\arctan\frac{\omega_n}{\frac{\dot{c}(0)}{c(0)}}\right]$$

3.4 高阶系统的时间响应概述

- 高于二阶的系统称高阶系统
- 数字仿真是分析高阶系统时间响应最有效的方法（定量）
- 高阶系统时间响应可分为稳态分量和瞬态分量（定性）
- 1) 瞬态分量的各个运动模态衰减的快慢取决于对应的极点和虚轴的距离
- 2) 各模态所对应的系数和初相角取决于零、极点的分布
- 3) 系统的零点和极点共同决定了系统响应曲线的形状
- **4) 对系统响应起主要作用的极点称为主导极点**
- 5) 非零初始条件时的响应由零初始条件时的响应和零输入响应组成——重复上节