Homework 2

Give regular expressions for the following languages 1-4.

1. $L = \{\, a^n b^m : n < 4,\ m \leq 3 \,\}$.

2. The set of all strings of 0's and 1's not containing 101 as a substring. $(\varepsilon + 0 + 1)\,(000 + 00 + 1)^*\,(\varepsilon + 0 + 1)$

3. The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's.

4. $L = \{\, w \mid w$ represents the variable of C programming language $\}$.

5. Here is a transition table for a DFA:

	0	1
®q_1	q_2	q_1
q_2	q_3	q_1
*q_3	q_3	q_2

a) Give all the regular expressions $R_{ij}^{(0)}$. Note: Think of state q_i as if it were the state with integer number i.

b) Give all the regular expressions $R_{ij}^{(1)}$. Try to simplify the expressions as much as possible.

c) Give all the regular expressions $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible.

d) Give a regular expression for the language of the automaton.

e) Construct the transition diagram for the DFA and give a regular expression for its language by eliminating state q_2.

*6. The set of strings of 0's and 1's whose number of 0's is divisible by five and whose number of 1's is even. (It is not necessary to finish this one. But you should think about it at least. We will discuss it on following lesson.)

$R_{11}^{(0)} = \varepsilon + 1$

$R_{12}^{(0)} = 0$

$R_{13}^{(0)} = \phi$

$R_{21}^{(0)} = 1$

$R_{22}^{(0)} = \varepsilon$

$R_{23}^{(0)} = 0$

$R_{31}^{(0)} = \phi$

$R_{32}^{(0)} = 1$

$R_{33}^{(0)} = (0+\varepsilon)$

$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} = (\varepsilon+1) + (\varepsilon+1)(\varepsilon+1)^*(\varepsilon+1) = (\varepsilon+1)^* = 1^*$

$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} = 0 + (\varepsilon+1)(\varepsilon+1)^* 0 = 0 + (\varepsilon+1)^* 0 = (\varepsilon+1)^* 0 = 1^*0$

$R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} = \phi + (\varepsilon+1)(\varepsilon+1)^* \varepsilon = \varepsilon + (\varepsilon+1)^* \phi = \phi$

$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} = 1 + 1(\varepsilon+1)^*(\varepsilon+1) = 1 + 1(\varepsilon+1)^* = 1(\varepsilon+1)^* = 11^*$

$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} = \varepsilon + 1(\varepsilon+1)^* 0 = \varepsilon + 1^*0$

$R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} = 0 + 1(\varepsilon+1)^* \phi = 0$

$R_{31}^{(1)} = R_{31}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} = \varepsilon + \varepsilon(\varepsilon+1)^*(\varepsilon+1) = (\varepsilon+1)^* = 1^*$

$R_{32}^{(1)} = R_{32}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} = 1 + \varepsilon(\varepsilon+1)^* 0 = 1 + 1^*0$

$R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} = 0 + \varepsilon(\varepsilon+1)^* \phi = 0$

$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{21} = 1^* + (1^*0)(\varepsilon+11^*0)^*(11^*) = 1^* + (1^*0)(11^*0)^*(11^*) = 1^*(011^*)^*$

$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{22} = 1^*0 + 1^*0)(\varepsilon+11^*0)^* = 1^*0(11^*0)^*$

$R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{23} = \phi + (1^*0)(\varepsilon+11^*0)^* 0 = 1^*0(11^*0)^*0$

$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{21} = 11^* + (\varepsilon+11^*0)^* 11^* = (\varepsilon+11^*0)^* 11^*$

$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{22} = (\varepsilon+11^*0)^* = (11^*0)^*$

$R_{23}^{(2)} = R_{23}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{23} = 0 + (\varepsilon+11^*0)^* 0 = (11^*0)^*0$

$R_{31}^{(2)} = R_{31}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{21} = 1^* + (1+1^*0)(\varepsilon+11^*0)^* 11^* =$

$R_{32}^{(2)} = R_{32}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{22} = (1+1^*0) + (1+1^*0)(\varepsilon+11^*0)^* = (1+1^*0)(11^*0)^*$

$R_{33}^{(3)} = R_{33}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{23} = 0 + (1+1^*0)(11^*0)^* 0 =$