Homework 4

1. Design a PDA(Diagram) to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

a) The set of all strings of 0's and 1's with an equal number of 0's and 1's.

b) The set of all strings of 0's and 1's with twice as many 0's as 1's.

c) $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$.

2. Let $S = \{0, 1\}$. Suppose *w* is a non-null string of even length so that *w* can be written as uxyv with *x*, *y* in S and |u| = |v| . Then we will say that *xy* is the middle of w. For example, in the string 00110011 we have 10 as its middle. Let $L \subseteq S^*$ be given by:

w is in *L* if and only if it is of non-null string of even length and its middle is 00 or 11.

Show that *L* is a context free language by constructing a (non-deterministic) push-down automaton that accepts *L*.

3. Convert the PDA $P = (\{p,q\},(O,1\},\{X,Z_0\},\delta,q,Z_0)$ to a CFG, if δ is given by:

(1) $\delta(q,1,Z_0) = \{(q,XZ_0)\}$.
(2) $\delta(q,1,X) = \{(q,XX)\}$.
(3) $\delta(q,0,X) = \{(p,X)\}$.
(4) $\delta(q,\varepsilon, Z_0) = \{(q, \varepsilon)\}$.
(5) $\delta(p,1,X) = \{(p,\varepsilon)\}$.
(6) $\delta(p,0,Z_0) = \{(q, Z_0)\}$.

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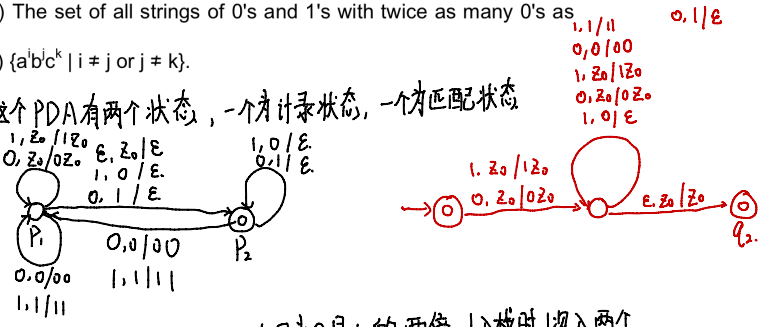
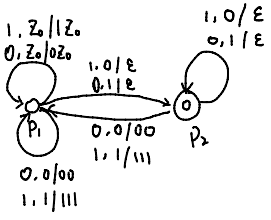
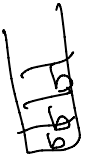
解：(a) 这个PDA有两个状态，一个为计录状态，一个为匹配状态



(b) 和上一个 PDA 差不多，只不过因为0是1的两倍，1入栈时1次入两个.



aaabbbccc



(c) 对于这样的语言，可以构造CFG如下：

$P = \{S = L_1 \cup L_2$

$L_1 \Rightarrow EX$ $\quad L_2 \to YF$

$E \Rightarrow aEb \mid A \mid B$ $\quad F \to bFc \mid B \mid C$

$X \to C \mid \varepsilon$ $\quad Y \to A \mid \varepsilon$

$A \to a \mid aA$

$B \to b \mid bB$

$C \to c \mid cC\}$

即 $G = (\{L_1, L_2, E, F, X, Y, A, B, C\}, \{a, b, c\}, S, P)$

由CFG生成的PDA，有

$P = (\{q\}, \{a, b, c\}, \{L_1, L_2, E, F, X, Y, A, B, C, a, b, c\}, \delta, q_0, S, \Phi) \Rightarrow$ 接空栈接收.

其中 δ 定义如下

$\delta(q, \varepsilon, L_1) = (q, EX)$

$\delta(q, \varepsilon, L_2) = (q, YF)$

$\delta(q, \varepsilon, E) = \{(q, aEb), (q, A), (q, B)\}$

$\delta(q, \varepsilon, F) = \{(q, bFc), (q, B), (q, C)\}$

$\delta(q, \varepsilon, X) = \{(q, C), (q, \varepsilon)\}$

$\delta(q, \varepsilon, Y) = \{(q, A), (q, \varepsilon)\}$

$\delta(q, \varepsilon, A) = \{(q, a), (q, aA)\}$

$\delta(q, \varepsilon, A) = \{(q, \beta) \mid A \to \beta$ 为G的一个产生式$\}$

$\delta(q, a, a) = (q, \varepsilon)$

$\delta(q, b, b) = (q, \varepsilon)$

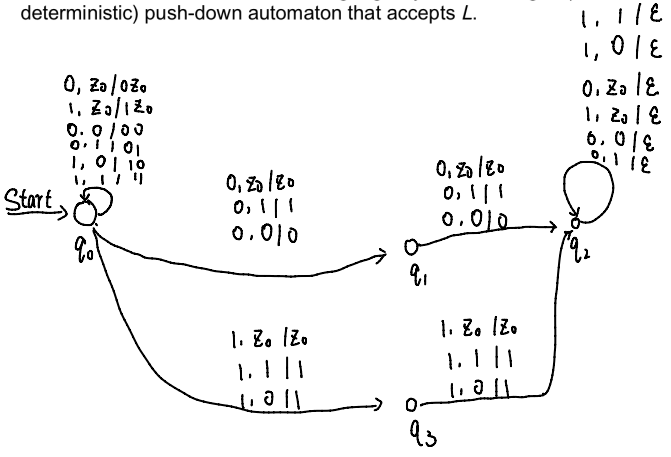
$\delta(q, c, c) = (q, \varepsilon)$

2. Let Σ = {0, 1}. Suppose *w* is a non-null string of even length so that *w* can be written as uxyv with *x*, *y* in Σ and |u| = |v| . Then we will say that *xy* is the middle of w. For example, in the string 00110011 we have 10 as its middle. Let *L* $\subseteq \Sigma^*$ be given by:

w is in *L* if and only if it is of non-null string of even length and its middle is 00 or 11.

Show that *L* is a context free language by constructing a (non-deterministic) push-down automaton that accepts *L*.

解:



构造一个以空栈方式接收的PDA

W = u00v |u|=|v|
u11v

3. Convert the PDA $P = (\{p,q\},(0,1),\{X,Z_0\},\delta,q,Z_0)$ to a CFG, if δ is given by:

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解：对于 PDA $P=(\{p,q\},(0,1),\{X,Z_0\},\delta,q,Z_0)$，有

$Q=\{p,q\}$，$\Sigma=(0,1)$，$\Gamma=\{X,Z_0\}$，那么

它生成的 CFG，有 $G=(V,\Sigma,S,P)$，有

S为开始符号，于是

$V=\{S, pXp, pXq, qXq, qXp, pZ_0p, pZ_0q, qZ_0q, qZ_0p\}$

a) $S\to[qZ_0p]$ $S\to[qZ_0q]$

b) ① $\delta(q,1,Z_0)=(q,XZ_0)$

所有的状态序列为pq或qp

$\therefore [qZ_0q]=|[qXp][pZ_0q]\,|\,|[qXq][qZ_0q]$

$[qZ_0p]=|[qXq][qZ_0p]\,|\,|[qXp][pZ_0p]$

② $\delta(q,1,X)=\{(q,XX)\}$

$[qXq]=|[qXp][qXq]$

$[qXp]=|[qXq][qXp]$

③ $\delta(q,0,X)=\{(p,X)\}$

所有状态序列为p或q.

$[qXq]=0[pXq]$

$[qXp]=0[pXp]$

④ $\delta(q,\varepsilon,Z_0)=\{(q,\varepsilon)\}$

$[qZ_0$ ← 这里写什么呢？

⑤ $\delta(p,1,X)=\{(p,\varepsilon)\}$

$[$

⑥ $\delta(p,0,Z_0)=\{(q,Z_0)\}$

$[pZ_0p]\to 0[qZ_0p]$

$[pZ_0q]\to 0[qZ_0q]$