Lecture 1: Introduction to Non-parametric Modelling

MATH5824 Generalised Linear and Additive Models

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Reading

Course notes: Chapter 1, Sections 1.1–1.2

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Module Overview

Building on MATH3823: This module extends GLMs in two directions:

1. Allow the response to depend on a *smooth function* of the predictor, not just a linear function
2. Combine smooth functions with GLM structure $\Rightarrow$ **Generalised Additive Models**

New topics:

- Interpolating and smoothing splines
- Cross-validation for spline fitting
- Generalised additive models (GAMs)

Motivation: The Coal Seam Data

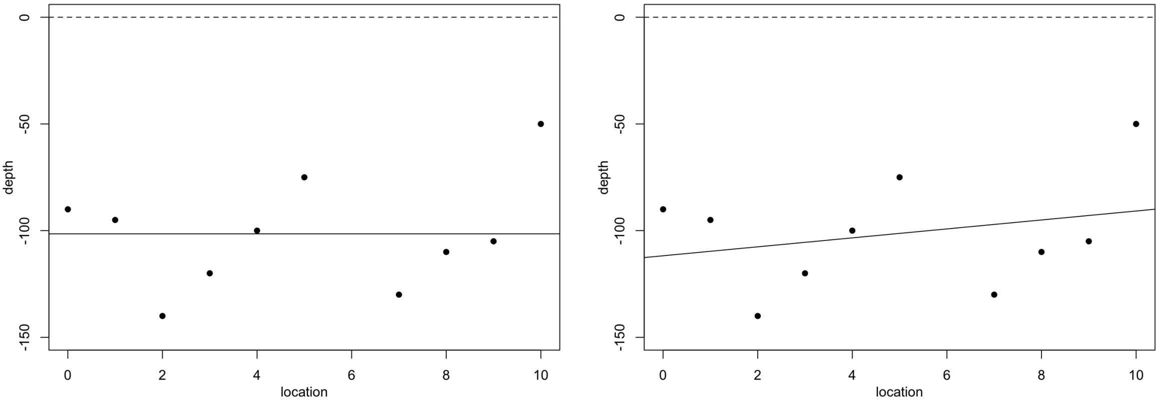
Data: Depths of a coal seam measured at 11 locations along a 10 km transect.

Location (km)	0	1	2	3	4	5	6	7	8	9	10
Depth	35	41	48	47	65	67	?	52	51	68	75

Question: Can we predict the missing depth at $x = 6$?

Different models give different predictions — which is best?

Polynomial Regression: Constant and Linear

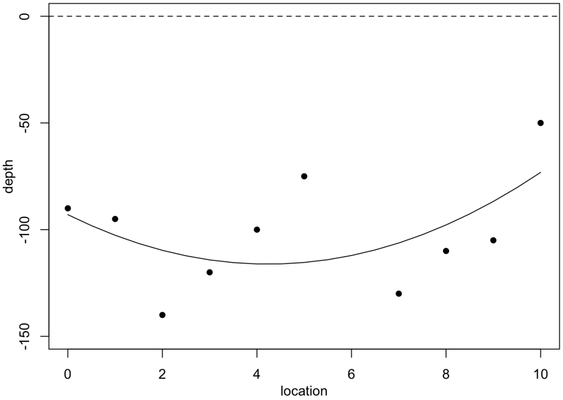


(a) Constant model

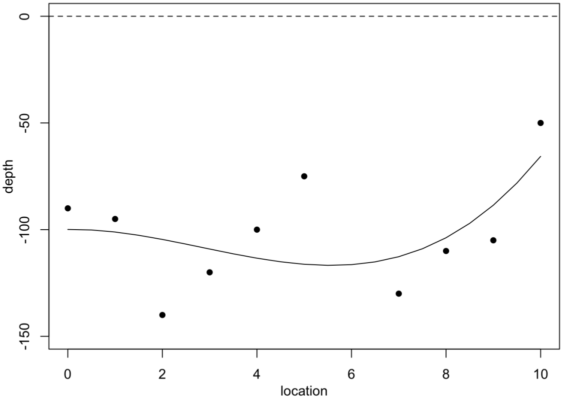
(b) Linear model

Neither captures the local fluctuations in the data.

Polynomial Regression: Quadratic and Cubic



(c) Quadratic model



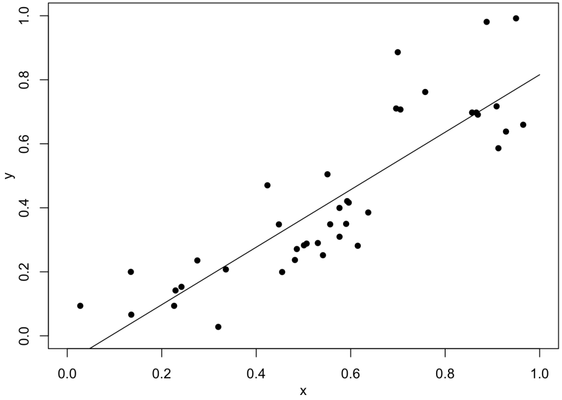
(d) Cubic model

RSS decreases with model complexity, but no polynomial fits convincingly.

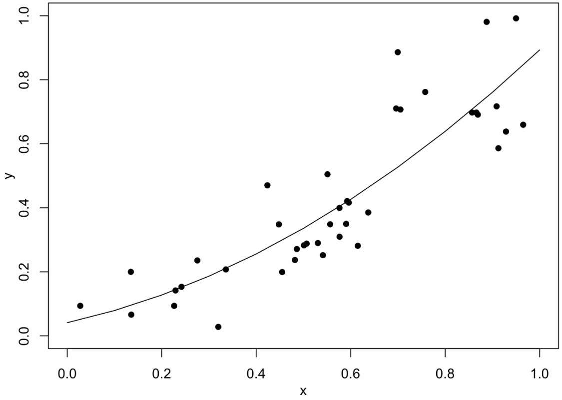
Model	Parameters	RSS

Simulated Change-Point Data: Polynomial Fits

Data: Simulated from a piecewise model with a discontinuity at $x = 0.67$.



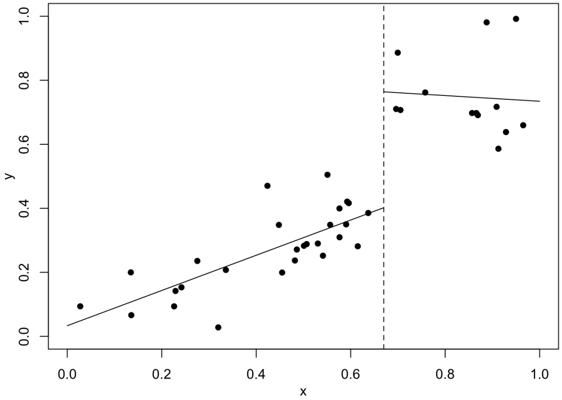
(a) Linear fit



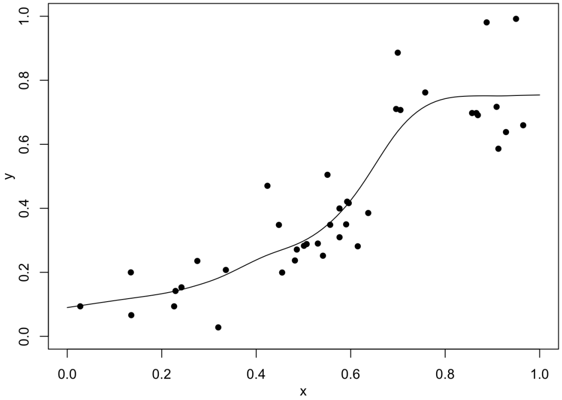
(b) Quadratic fit

Simple polynomial models fail to capture the change-point.

Change-Point Data: Better Approaches



(c) Piecewise linear (known change-point)

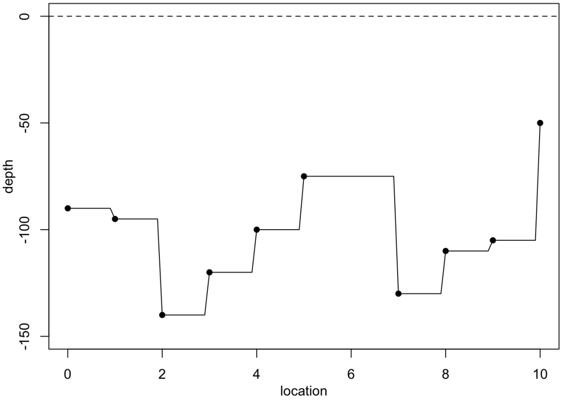


(d) Cubic smoothing spline

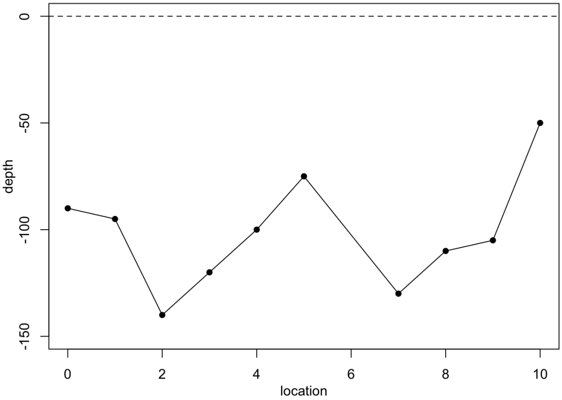
The cubic smoothing spline reveals the pattern *without* assuming the change-point location is known.

$\Rightarrow$ We need methods that adapt locally to the data.

Spline Approaches: Constant and Linear



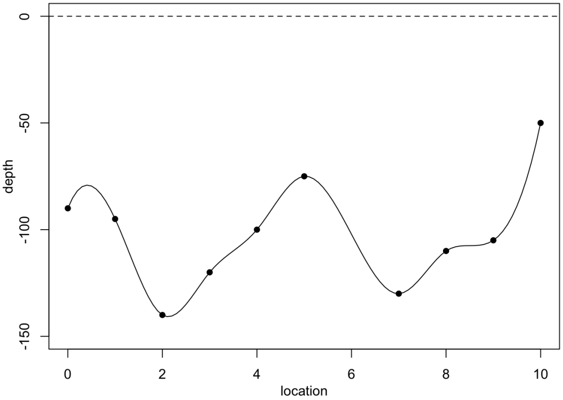
(a) Constant interpolating spline



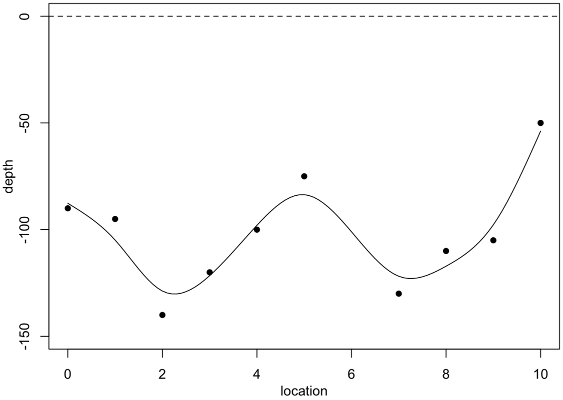
(b) Linear interpolating spline

- Constant: discontinuities at observation locations
- Linear: continuous, but gradient is discontinuous (visible kinks)

Spline Approaches: Cubic Interpolating and Smoothing



(c) Cubic interpolating spline



(d) Cubic smoothing spline

- Cubic interpolating: passes through all points, continuous gradient and curvature
- Cubic smoothing: allows measurement error, smoother curve

Interpolating vs. Smoothing

Interpolating Spline

Passes through every data point

Assumes no measurement error

noisier data →

Smoothing Spline

Passes close to data points

Allows for measurement error

Both are **piecewise polynomial** functions with smoothness constraints at join points (**knots**).

Summary

Key points:

- Parametric models (polynomials) may not capture local features of data
- Splines are piecewise polynomials joined smoothly at **knots**
- Interpolating splines pass through all data points
- Smoothing splines allow for measurement error
- Different spline orders (constant, linear, cubic) offer different smoothness guarantees

Next lecture: General modelling approaches and the formal definition of splines.