# Electromagnetism

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# Chapter 1

# Electricity

## 1.1 Electric Forces

IN ELECTRODYNAMICS, there is typically a *source point* $\mathbf{r}'$ where a charge is located and a *field point* $\mathbf{r}$ where a field is calculated at. The *seperation vector* is defined as

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}' \qquad \hat{\boldsymbol{\mathscr{r}}} = \boldsymbol{\mathscr{r}}/\mathscr{r} \tag{1.1}$$

upon definition of a coordinate system. Coulomb's law expresses the force of charges $q_i$ on another charge $q_0$, given by

$$\mathbf{F} \equiv K \sum \frac{q_0 q_i}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} = q_0 \mathbf{E} \tag{1.2}$$

When calculating the force via $E$, the charge density must be replaced by the respective equation. The charge differential is defined as

$$dq \mapsto \lambda\, dx \sim \sigma\, dA \sim \rho\, dV \tag{1.3}$$

When evaluating the results of the integration, the limiting cases such as $a \gg b$ can be found by evaluating the expression for $b = 0$.

## 1.2 Electric Field

The electric field at a point $P$ which acts like a positive test charge of a set of source charges is defined as

$$\mathbf{E}(\mathbf{r}) \equiv K \sum \frac{q_i}{\mathscr{r}_i^2} \hat{\boldsymbol{\mathscr{r}}}_i = K \int \frac{1}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}}\, dq \tag{1.4}$$

For most cases, symmetry can be utilized such that

$$\mathbf{E} = E_x \to \hat{\boldsymbol{\imath}} = \cos\theta \tag{1.5}$$

An electric dipole describes the configuration of two opposite charges $q$ a distance $d$ apart. The electric dipole moment is defined as $p = qd$, which means the rate of the field for $x \gg d$ id $1/\imath^3$.

## 1.3 Energy

The torque of an electric dipole is defined to be

$$\tau = pE\sin\theta = \mathbf{p} \times \mathbf{E} \tag{1.6}$$

assuming its direction is perpendicular to and into the page. The work done by the external field in turning a dipole is thus

$$W = -\int_\theta \tau \, d\theta = pE(\Delta\cos\theta) \tag{1.7}$$

which is related to the change in potential energy via

$$U = -W = -\mathbf{p} \cdot \mathbf{E} \tag{1.8}$$

## 1.4 Gauss' Law

Gauss' Law states the flux is the rate of change of an electric field of a Guassion surface; that is,

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0 \tag{1.9}$$

meaning for each infinitesimal point for a given surface, $\mathbf{E}$ is in the direction of the filed lines, and $\mathbf{A}$ is normal to the surface.

This results in $\Phi_E = 0$ if such closed surface does not enclose any charges and/or $\sum q = 0$. Because of this, $E$ can be calculated via

$$\sum EA = q/\epsilon_0 \qquad q \mapsto \lambda x \sim \sigma A \tag{1.10}$$

For a conductor, the field $E = \sigma/\epsilon_0$.