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ELECTROMAGNETISM

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Electrostatics

Electric Forces

IN ELECTRODYNAMICS, there is typically a **source point** \mathbf{r}' where a charge is located and a **field point** \mathbf{r} where a field is calculated at. The **separation vector** is defined as

$$\mathbf{r} \equiv \mathbf{r} - \mathbf{r}' \quad \hat{\mathbf{r}} = \mathbf{r}/r \quad (1)$$

The force acting on a charge is given by **Coulomb's Law**,

$$\mathbf{F} = k \sum \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} = \iiint d\mathbf{F} \quad (2)$$

The **electric field** exerted on a positive test charge $+q_0$ at a point is defined as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = k \sum \frac{q}{r^2} \hat{\mathbf{r}} \quad (3)$$

Field lines point outwards from $+q$ and towards $-q$, and are parallel or tangential of an electric field. For a continuous charge distribution,

$$\mathbf{E} = k \iiint \frac{dq}{r^2} \hat{\mathbf{r}} = -\nabla V \quad (4)$$

where

$$dq \sim \lambda d\ell \sim \sigma dA \sim \rho dV \sim \lambda R d\phi \sim \sigma 2\pi w dw \quad (5)$$

The **electric dipole** is a configuration of two equal and opposite charges q separated by a distance d , in which the electric dipole moment $p = qd$ in the direction towards $+q$. If a dipole is in an external field, the torque is

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (6)$$

such that the work done by the field is

$$W = - \int_{\theta_0}^{\theta} \tau d\theta = pE(\Delta \cos \theta) \quad (7)$$

Consequently, the potential energy is defined as

$$U = -pE \cos \theta = -\mathbf{p} \cdot \mathbf{E} \quad (8)$$

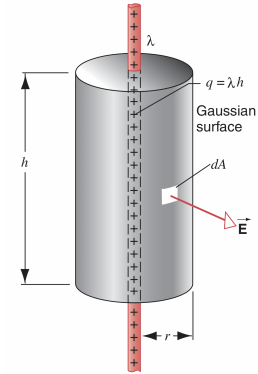


Figure 1: A Gaussian surface as a cylinder.

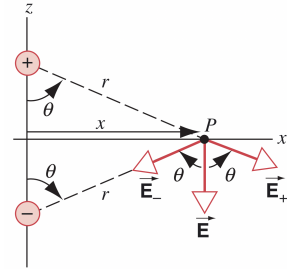


Figure 2: The field at any point is the vector sum of the charges.

Potentials

Gauss' Law states the electric **flux** through a field is

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0} \quad (9)$$

such that $\|\Phi\| = EA \cos \theta$ where θ is the angle between the field and field lines. The electric field outside a conductor is given by

$$E = \frac{\sigma}{\epsilon_0} \iff q = \iint \sigma dA \quad (10)$$

The change in electric potential is defined as

$$\Delta U = kq_1q_2(\Delta r^{-1}) \quad (11)$$

For a system of charge, the **potential energy** is

$$U = k \sum_i \sum_{j \neq i} \frac{q_i q_j}{r} \quad (12)$$

Consequently, the change in **electric potential** is

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\int \mathbf{E} \cdot d\mathbf{s} \quad (13)$$

The electric potential at a point is thus defined as

$$V = k \iiint_a^b \frac{dq}{r} = k \sum \frac{q_i}{r} \sim k \frac{p \cos \theta}{r^2} \quad (14)$$

where the latter equivalence holds for dipoles. A surface on which the potential has the same value everywhere is **equipotential** such that $\Delta V = W = 0$. The field lines must everywhere be perpendicular to the equipotential surfaces, which implies all conductors are equipotential.

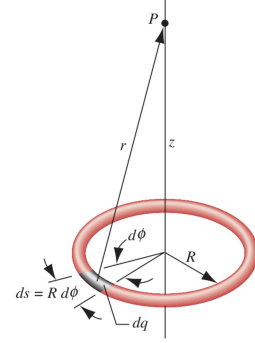


Figure 3: A uniformly charged ring with a potential P and charge element dq .

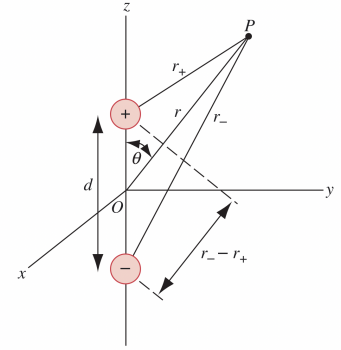


Figure 4: The geometry for calculating V at P for a dipole.

Electrodynamics

Current

THE ELECTRIC current is defined as

$$i = \frac{dq}{dt} = \int \mathbf{j} \cdot d\mathbf{A} \quad (15)$$

where j is the current density and is opposite the motion of electrons. The net charge passing through is therefore

$$q = \int i dt \quad (16)$$

The current density is defined as

$$\mathbf{j} = q/At = -en\mathbf{v} = \sigma\mathbf{E} \quad (17)$$

where n is the electron density and $\sigma = q/A$ is the conductivity.

Furthermore, the net charge passing through the surface $q = enAL$.

The resistance of the material is thus $R = L/\sigma A = \Delta V/i$.

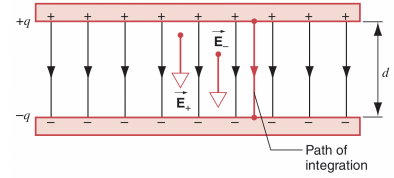


Figure 5: A parallel-plate capacitor.

Capacitance

Capacitance is defined as

$$C = q/\Delta V \quad (18)$$

In a PPC, SC, and CC, the potential is derived via

$$\Delta V_{ppc} = \frac{qd}{\epsilon A} \quad \Delta V_{sc} = kq \left(\frac{1}{a} - \frac{1}{b} \right) \quad \Delta V_{ss} = \frac{q \ln(b-a)}{2\pi\epsilon L} \quad (19)$$

In a parallel combination of capacitors,

$$q = \sum q = C_{eq}\Delta V \iff C_{eq} = \sum C \quad (20)$$

and in a series combination,

$$\Delta V = \sum \Delta V = q/C_{eq} \iff C_{eq}^{-1} = \sum C^{-1} \quad (21)$$

As well, the potential energy in a capacitor is

$$dU = \Delta V dq \iff U = \int_0^q dU = \frac{q^2}{2C} \quad (22)$$

with $q = C\Delta V$. Specifically, the energy U is stored in E in the region.