# Electromagnetism

Richard Robinson

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# Chapter 1

# Electrostatics

## 1.1 Electric Forces

IN ELECTRODYNAMICS, there is typically a *source point* $\mathbf{r}'$ where a charge is located and a *field point* $\mathbf{r}$ where a field is calculated at. The *seperation vector* is defined as

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}' \qquad \hat{\boldsymbol{\mathscr{r}}} = \boldsymbol{\mathscr{r}}/\mathscr{r} \tag{1.1}$$

upon definition of a coordinate system. Coulomb's law expresses the force of charges $q_i$ on another charge $q_0$, given by

$$\mathbf{F} \equiv K \sum \frac{q_0 q_i}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} = q_0 \mathbf{E} \tag{1.2}$$

When calculating the force via $E$, the charge density must be replaced by the respective equation. The charge differential is defined as

$$dq \mapsto \lambda\, dx \sim \sigma\, dA \sim \rho\, dV \tag{1.3}$$

When evaluating the results of the integration, the limiting cases such as $a \gg b$ can be found by evaluating the expression for $b = 0$.

## 1.2 Electric Field

The electric field at a point $P$ which acts like a positive test charge of a set of source charges is defined as

$$\mathbf{E}(\mathbf{r}) \equiv K \sum \frac{q_i}{\mathscr{r}_i^2} \hat{\boldsymbol{\mathscr{r}}}_i = K \int \frac{1}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}}\, dq = \nabla V \tag{1.4}$$

For most cases, symmetry can be utilized such that

$$\mathbf{E} = E_x \to \hat{\boldsymbol{\mathscr{r}}} = \cos\theta \tag{1.5}$$

An electric dipole describes the configuration of two opposite charges $q$ a distance $d$ apart. The electric dipole moment is defined as $p = qd$ towards $+q$, which means for $x \gg d$, then $E = 2Kp/\mathscr{r}^3$.

## 1.3 Torque

The torque of an electric dipole is defined to be

$$\tau = (qE)d\sin\theta = \mathbf{p} \times \mathbf{E} \tag{1.6}$$

assuming its direction is perpendicular to and into the page. The work done by the external field in turning a dipole is thus

$$W = -\int_\theta \tau \; d\theta = pE(\Delta\cos\theta) \tag{1.7}$$

which is related to the change in potential energy via

$$U = -W = -\mathbf{p} \cdot \mathbf{E} \tag{1.8}$$

## 1.4 Gauss' Law

Gauss' Law states the flux is the rate of change of an electric field of a Gaussian surface; that is,

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0 \tag{1.9}$$

meaning for each infinitesimal point for a given surface, $\mathbf{E}$ is in the direction of the filed lines, and $\mathbf{A}$ is normal to the surface. Note the "edge" of a Gaussian surface is equivalent to a single point $P$ a distance $r$ from $E$.

This results in $\Phi_E = 0$ if such closed surface does not enclose any charges and/or $\sum q = 0$. Because of this, $E$ for regions of infinite dimensions can be calculated via

$$\sum EA = q/\epsilon_0 \qquad q \mapsto \lambda x \sim \sigma A \sim \rho V \tag{1.10}$$

For a conductor, the field $E = \sigma/\epsilon_0$.

# Chapter 2

# Electrodynamics

## 2.1 Potential Energy

The difference in potential energy is defined to be

$$\Delta U = -\int_a^b \mathbf{F} \cdot d\mathbf{s} = K q_1 q_2 \left( \frac{1}{r_b} - \frac{1}{r_a} \right) \tag{2.1}$$

Incidentally, the total potential energy of a system is

$$U = K \sum \sum \frac{q_i q_j}{r_{ij}} \tag{2.2}$$

These are not to be confused with the potential difference, defined as

$$\Delta V = -\int_a^b \mathbf{E} \cdot d\mathbf{s} = \Delta U / q_o = Ed \ \text{ V} \tag{2.3}$$

which is analogous to the potential energy for a point. Lastly, the electric potential is defined to be

$$V = K \sum \frac{q_i}{r_i} = K \int \frac{dq}{r} \sim \frac{Kp\cos\theta}{r^2} \tag{2.4}$$

## 2.2 Electrical Properties

The electric current $i$ is defined as the net charge flowing through a surface,

$$i = \frac{dq}{dt} = \int \mathbf{j} \cdot d\mathbf{A} \tag{2.5}$$

where $j$ is the current density and is in the direction of $+q$ which may also be found via

$$\mathbf{j} = -en\mathbf{v}_d = \sigma \mathbf{E} \tag{2.6}$$

The resistance of a material relates the potential difference to the current through

$$R = \frac{\Delta V}{i} = \rho \frac{L}{A} \ \ \Omega \tag{2.7}$$

## 2.3 Capacitance

The capacitance of a circuit relates the potential difference with the charge via

$$C = q/\Delta V \ \ \mathrm{F} \tag{2.8}$$

and is only a geometrical factor. This means for a capacitive sphere, $C = 4\pi\epsilon_0 r$. For a parallel plate capacitor, capacitance is found via

$$\Delta V = \int_{+}^{-} E \ ds = \frac{\sigma d}{\epsilon_0} \iff C = \frac{\epsilon_0 A}{d} \tag{2.9}$$

as the electric field for a plate $E = \sigma/2\epsilon_0$ and $\sigma = q/A$ and $C = q/\Delta V$. Likewise, the capacitance of spherical and cylindrical capacitors can be found from the result of their respective potential difference equations.

## 2.4 Capacitors

In a parallel circuit, the following equations hold:

$$q = \sum q_i = C_{eq}\Delta V \iff C_{eq} = \sum C_i \tag{2.10}$$

and in series the converse is true:

$$\Delta V = \sum \Delta V_i = q/C_{eq} \iff C_{eq}^{-1} = \sum C_i^{-1} \tag{2.11}$$

The total potential energy stored in such capacitor is defined as

$$U = C^{-1} \int_0^q q \ dq = \tfrac{1}{2} C(\Delta V)^2 \tag{2.12}$$

More specifically, the energy itself is stored in the field present in such region. Similarly, the energy density is the stored energy per unit given as

$$u = U/Ad = 0.5\epsilon_0 E^2 \tag{2.13}$$

For a capacitor with a dielectric of constant $\kappa$, Gauss' law gives

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\kappa} \iff \epsilon_0 \oint \kappa \mathbf{E} \cdot d\mathbf{A} = q \tag{2.14}$$

# Chapter 3

# DC Circuits

Please see *Electric Circuits* which entirely comprises this chapter itself, available with other titles at `http://bit.ly/eecsbooks`.

# Chapter 4

# Magnetostatics

## 4.1 Magnetic Field

For particles in a magnetic field, the magnetic force is defined as

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = |q|vB\sin\phi \tag{4.1}$$

However, if both an electric and magnetic field act upon a charge, the Lorentz force is

$$\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \tag{4.2}$$

If both fields and the velocity vectors are tri-perpendicular in the $xyz$-plane, then the electric and magnetic force are in opposite directions, meaning

$$qE = qvB \iff v = E/B \tag{4.3}$$

This configuration is known as a velocity selector, as only particles with such velocity $v$ can pass through unaffected.