RICHARD ROBINSON

ECONOMICS & PROJECT MANAGEMENT

Time Value of Money

Interest Rates

The dimension for an interest rate i is $c_1/c_2/T$ where c is a currency and T is the interest period. If an amount P is borrowed for N periods at i, the final amount is

$$F = P(1+i)^N = P(1+i_s)^m = P + I_c \tag{1}$$

which is known as compounding where the total interest on such loan I_c is the compound interest. Consequently, the simple interest $I_s = PiN$ is such interest not compounded.

For example, a NIR of i / year compounded monthly is $i/12$ per month.

The *nominal interest rate* (NIR) is the conventional annual interest rate. Suppose a period is divided by m. If r is the NIR for the full period, the interest rate is

$$i_s = r/m \iff r = i_s m \tag{2}$$

The *effective interest rate* is the actual interest rate given by

$$i_e = \frac{F}{P} - 1 = \left(1 + \frac{r}{m}\right)^m - 1 \sim e^r - 1 \tag{3}$$

A cash flow diagram is a visualization of cash flows and interest over time. Note that N years from time t is the end of period N and beginning of $N+1$.

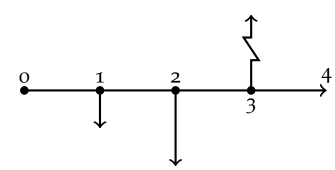

Figure 1: A cash flow diagram. The broken line at $t = 3$ indicates the net sum of the cash flow at that period.

Cash Flow Analysis

A *cash flow event* (CFE) is defined as a disbursement (paid) or receipt (received). The discrete cash flow patterns for compound interest factors (CIFs) are:

1. *Single*: A single CFE;
2. *Annuity*: A set of CFEs over a sequence of periods;
3. *Arithmetic Series*: A set of CFEs that change by a constant amount from one period to the next;

4. *Geometric Series*: A set of CFEs that change by a constant proportion from one period to the next;

The principle of discrete compounding assumes each CFE occurs at the end of a period such that a payment at t occurs at the end of period $t-1$.

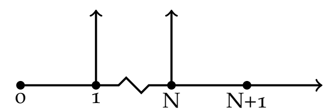

Figure 2: A cash flow diagram for an annuity over N periods

The compound amount factor is used for single cash flows, given by

$$(F/P) = (1+i_e)^N = (1+r/m)^{mt} \tag{4}$$

The present worth factor is the inverse, (P/F). The sinking fund factor gives the size A of a CFE equivalent to a future amount F, given by

$$(A/F) = i/[F/P-1] \tag{5}$$

The uniform series compound amount factor is the inverse, (F/A). The *capital recovery factor* gives the value A of such equal periodic CFEs, given by

$$(A/P) = (F/P)(A/F) \tag{6}$$

with the inverse being the *series present worth factor*, (P/A). The capital recovery formula accounts for the salvage value S, given by

$$A = P(A/P) - S(A/F) = (P-S)(A/P) + Si \tag{7}$$

Annuity Due: The amount owed from a mortgage is $P = F[(F/P) - (A/P)_{N=12T}(F/A)]$ where N the term and T is the amortization period.

The *annuity due* is the amount lent requiring N monthly payments of A starting today at an annual rate of i, such that $P = A + A(P/A)_{N-1}$. The *arithmetic gradient to annuity factor* is given by

$$(A/G) = i^{-1} - N/[F/P-1] \iff A_{tot} = A' + G(A/G) \tag{8}$$

Geometric Series

The present worth of a geometric series is

$$P = \sum \frac{A(1+g)^{j-1}}{F/P} \tag{9}$$

Let the growth-adjust rate $i^\circ = (1+i)/(1+g) - 1$. The geometric gradient series to present worth factor is thus

$$(P/A)_g = (P/A)_{i^\circ}/(1+g) \tag{10}$$

There are four possible scenarios for gradients series:

1. $i > g > 0$ or $g > i > 0$; the basic scenarios.
2. $g = i > 0$; then $i^\circ = 0$ and $P = NA/(1+g)$
3. $g < 0$; series id decreasing and $i^\circ > 0$.