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# ELECTROMAGNETISM





# Electrostatics

## Electric Forces

IN ELECTRODYNAMICS, there is typically a *source point*  $\mathbf{r}'$  where a charge is located and a *field point*  $\mathbf{r}$  where a field is calculated at. The *separation vector* is defined as

$$\mathbf{r} \equiv \mathbf{r} - \mathbf{r}' \quad \hat{\mathbf{r}} = \mathbf{r}/r \quad (1)$$

The force acting on a charge is given by *Coulomb's Law*,

$$\mathbf{F} = k \sum \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} = \iiint d\mathbf{F} \quad (2)$$

The *electric field* exerted on a positive test charge  $+q_0$  at a point is defined as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = k \sum \frac{q}{r^2} \hat{\mathbf{r}} \quad (3)$$

*Field lines* point outwards from  $+q$  and towards  $-q$ , and are parallel or tangential of an electric field. For a continuous charge distribution,

$$\mathbf{E} = k \iiint \frac{dq}{r^2} \hat{\mathbf{r}} = -\nabla V \quad (4)$$

where

$$dq \sim \lambda d\ell \sim \sigma dA \sim \rho dV \sim \lambda R d\phi \sim \sigma 2\pi w dw \quad (5)$$

The *electric dipole* is a configuration of two equal and opposite charges  $q$  separated by a distance  $d$ , in which the electric dipole moment  $p = qd$  in the direction towards  $+q$ . If a dipole is in an external field, the torque is

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (6)$$

such that the work done by the field is

$$W = - \int_{\theta_0}^{\theta} \tau d\theta = pE(\Delta \cos \theta) \quad (7)$$

Consequently, the potential energy is defined as

$$U = -pE \cos \theta = -\mathbf{p} \cdot \mathbf{E} \quad (8)$$

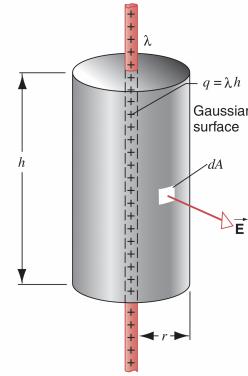


Figure 1: A Gaussian surface as a cylinder.

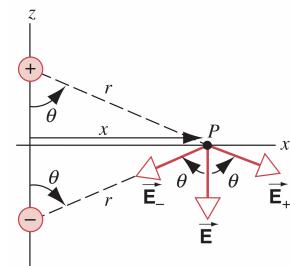


Figure 2: The field at any point is the vector sum of the charges.

### Potentials

Gauss' Law states the electric flux through a field is

$$\Phi_E = \iint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0} \quad (9)$$

such that  $\|\Phi\| = EA \cos \theta$  where  $\theta$  is the angle between the field and field lines. The electric field outside a conductor is given by

$$E = \frac{\sigma}{\epsilon_0} \iff q = \iint \sigma dA \quad (10)$$

The change in electric potential is defined as

$$\Delta U = kq_1q_2(\Delta r^{-1}) \quad (11)$$

For a system of charge, the *potential energy* is

$$U = k \sum_i \sum_{j \neq i} \frac{q_i q_j}{r} \quad (12)$$

Consequently, the change in *electric potential* is

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\int \mathbf{E} \cdot d\mathbf{s} \quad (13)$$

The electric potential at a point is thus defined as

$$V = k \iiint_a^b \frac{dq}{r} = k \sum_i \frac{q_i}{r} \sim k \frac{p \cos \theta}{r^2} \quad (14)$$

where the latter equivalence holds for dipoles. A surface on which the potential has the same value everywhere is *equipotential* such that  $\Delta V = W = 0$ . The field lines must everywhere be perpendicular to the equipotential surfaces, which implies all conductors are equipotential.

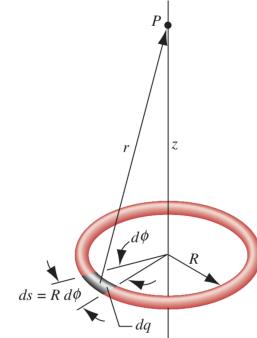


Figure 3: A uniformly charged ring with a potential  $P$  and charge element  $dq$ .

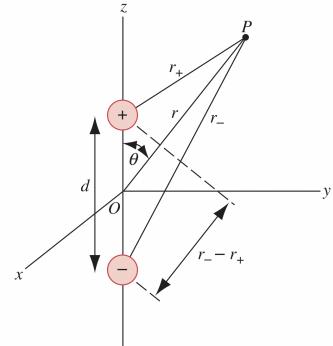


Figure 4: The geometry for calculating  $V$  at  $P$  for a dipole.

# Electrodynamics

## Current

THE ELECTRIC current is defined as

$$i = \frac{dq}{dt} = \int \mathbf{j} \cdot d\mathbf{A} \quad (15)$$

where  $j$  is the current density and is opposite the motion of electrons.  
The net charge passing through is therefore

$$q = \int i dt \quad (16)$$

The current density is defined as

$$\mathbf{j} = q/At = -env = \sigma\mathbf{E} \quad (17)$$

where  $n$  is the electron density and  $\sigma = q/A$  is the conductivity.

Furthermore, the net charge passing through the surface  $q = enAL$ .

The resistance of the material is thus  $R = L/\sigma A = \Delta V/i$ .

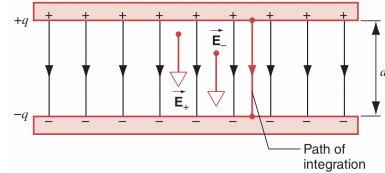


Figure 5: A parallel-plate capacitor.

## Capacitance

Capacitance is defined as

$$C = \frac{q}{\Delta V} = \frac{\epsilon_0 A}{d} \quad (18)$$

In a PPC, SC, and CC, the potential is derived via

$$\Delta V_{ppc} = \frac{qd}{\epsilon A} \quad \Delta V_{sc} = kq \left( \frac{1}{a} - \frac{1}{b} \right) \quad \Delta V_{ss} = \frac{q \ln(b-a)}{2\pi\epsilon L} \quad (19)$$

In a parallel combination of capacitors,

$$q = \sum q = C_{eq}\Delta V \iff C_{eq} = \sum C \quad (20)$$

and in a series combination,

$$\Delta V = \sum \Delta V = q/C_{eq} \iff C_{eq}^{-1} = \sum C^{-1} \quad (21)$$

As well, the potential energy in a capacitor is

$$dU = \Delta V dq \iff U = \int_0^q dU = \frac{q^2}{2C} \quad (22)$$

with  $q = C\Delta V$ . Specifically, the energy  $U$  is stored in  $E$  in the region.

Furthermore, the *energy density*  $u$  is defined as

$$u = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2 \quad (23)$$

by Eqns 18 & 22. Using *voltage division*, the voltage of any impedance  $Z$  including capacitance may be found via  $q_0 = \sum q$  by

$$V = \frac{Z_1}{Z_{eq}} V_0 \quad (24)$$

For a *dielectric capacitor*, the charge is defined as

$$q' = q(1 - 1/\kappa) \iff E = E_0/\kappa \quad (25)$$

### DC Circuits

The *emf* of a voltage source is defined as

$$\mathcal{E} = dW/dq = iR \quad (26)$$

The electric potential  $v$  decreases across a voltage source in a circuit.

*For more information, view the Electric Circuits textbook supplement.*

The emf is related to the electric field by

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} \quad (27)$$

where  $\mathbf{E} = \mathbf{F}/q = \rho j$ .

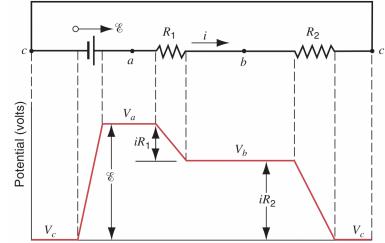


Figure 6: A simple circuit and its potential difference change.

# *Magnetic Forces*

## *Magnetic Field*

The magnetic force is defined as

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad \|\mathbf{F}\| = |q|vB \sin \phi \quad (28)$$

where  $B$  is the field and  $v$  the velocity. The combined electric and magnetic fields yield the Lorentz force,

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad (29)$$

For a velocity selector, then  $v = E/B$ , and for a particle in a circular path, then:

$$|q|vB = mv^2/r \quad \omega = v/r = |q|B/m \quad (30)$$

where  $\omega = 2\pi f = r'$  is the angular velocity, and the kinetic energy is  $K = 0.5mv^2$ .