RICHARD ROBINSON

ELECTRIC CIRCUITS

Analysis Methods

Definitions

THE PASSIVE SIGN CONVENTION states that an impedance's voltage and current are positive if its reference current is in the direction of the voltage drop; that is, from positive to negative. For resistive circuits, **power** is defined as

$$p = dw/dt = vi \tag{1}$$

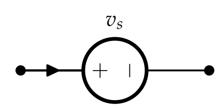

Figure 1: An independent voltage source.

For dependent sources the voltage or current values s are denoted by diamonds and given by

$$x_s = \alpha y_x \tag{2}$$

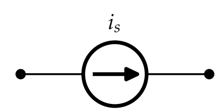

Figure 2: An independent current source.

where y_x is the controlling sources. Voltage and current are related through **Ohm's law**,

$$\mathbf{V} = Z\mathbf{I} \quad \text{for} \quad Z = \{R, j\omega L, -j/\omega C\} \tag{3}$$

where Z is the **impedance.**

Kirchoff's Laws for circuit analysis state that the sum of all currents at a node equals zero (KCL) as does the sum of all voltages across a loop (KVL).

Simple Analysis Methods

The voltage across branches in parallel are equal, as are currents across branches in series. Otherwise, **voltage** and **current division** must be used, in which:

$$v_x = \frac{Z_x}{Z_{eq}} v_s \quad \text{and} \quad i_x = \frac{Z_t}{Z_x + Z_t} i_s \tag{4}$$

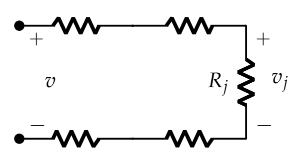

Figure 3: A circuit for voltage division.

Furthermore, the Δ**-to-Y transformation** transforms an interconnection of 3 impedances between the two types in Figures 5 & 6, converted via:

$$Z_i = \frac{Z'_{jk}}{\Sigma Z'} \quad \text{and} \quad Z'_i = \frac{P_{jk}}{Z_i} \tag{5}$$

where $P_{jk} = Z_i Z_j + Z_i Z_k + Z_j Z_k$.

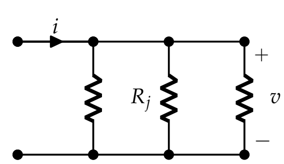

Figure 4: A circuit for current division.

Nodal Analysis

The method of **nodal analysis** is most commonly used when no node in the circuit connects more than three branches. It is done procedurally as follows:

1. Select the node with the most branches as the reference node, typically the bottom–most one.

2. Define the node voltages v_i, which are the voltage rises across a branch or branches from the reference node to another node i.

3. For each nonreference node, generate a KCL equation:

$$v_a : \sum i_a = \sum \frac{v_a - v_s}{Z_i} = 0 \tag{6}$$

   and solve for v_i.

4. If a voltage source is between two nodes, a supernode exists in which the nodes are related by $v_i = v_j + \alpha y_x$.

Mesh Analysis

Consequently, the **mesh analysis** method is most commonly used when there is a node connecting more than three branches. It is also done procedurally as follows:

1. Assign mesh current directions around each loop, typically clockwise.

2. For all nodes, develop individual KCL equations.

3. For each mesh, generate a KVL equation:

$$v_a : \sum v_a = \sum (i_a - i_i) Z_i = 0 \tag{7}$$

   where i_i is any other mesh current passing through Z.

4. Solve for the branch currents, letting $i_a = i_\alpha$; if a branch is shared, $i_\alpha = i_a - i_b$. Then solve the system of equations.

Source Transformations

The **source transformation** technique states a voltage $v_s = i_s Z$ source in series with an impedance Z (Thévenin equivalent) is equivalent to a current source i_s in parallel with the same impedance (Norton equivalent). As well, an impedance in parallel with v_s or in series with i_s has no effect and may be directly removed.

Any linear circuit N which is converted to a Thévenin equivalent between a pair of terminals is known as a **Thévenin** circuit, shown below.

Thévenin's Theorem

This circuit is especially useful for calculating the current and voltage of the terminals of the load Z_L. To calculate V_T, replace Z_L with an open circuit and find the voltage drop across the open circuit, typically via voltage division. There are several methods to find Z_T:

Method 1 (All Sources): The current i_{sc} is found by replacing Z_L with a short circuit and calculating the resulting current. Therefore, the Thévenin impedance is

$$Z_T = V_T / i_{sc} \tag{8}$$

Method 2 (Independent Sources): Deactivate all independent sources, then calculate the equivalent impedance $Z_{eq} = Z_T$ for the network.

Method 3 (Dependent Sources): Deactivate all independent sources, then apply a test source between the terminals. The Thévenin impedance is then

$$Z_T = V_{sc} / i_{sc} \tag{9}$$

Superposition

The principle of **superposition** states suppressing all but one source sequentially and summing the values is equivalent to the original circuit. To suppress a voltage course is to replace it with a short circuit, and an open circuit for a current source.

Specifically, for a circuit with n independent sources, a maximum of n superposition circuits may be created with one source activated per circuit. Therefore, the total current or voltage $x = \sum x_i$.