

# Analysis Methods



# **Definitions**

The passive sign convention states that an impedance's voltage and current are positive if its reference current is in the direction of the voltage drop; that is, from positive to negative. For resistive circuits, power is defined as

$$p = dw/dt = vi (1)$$

For dependent sources the voltage or current values s are denoted by diamonds and given by

$$x_{s} = \alpha y_{x} \tag{2}$$

where  $y_x$  is the controlling sources. Voltage and current are related through Ohm's law,

$$\mathbf{V} = Z\mathbf{I} \quad \text{for} \quad Z = \{R, j\omega L, -j/\omega C\}$$
 (3)

where Z is the **impedance**.

Kirchoff's Laws for circuit analysis state that the sum of all currents at a node equals zero (KCL) as does the sum of all voltages across a loop (KVL).

### Simple Analysis Methods

The voltage across branches in parallel are equal, as are currents across branches in series. Otherwise, voltage and current division must be used, in which:

$$v_x = \frac{Z_x}{Z_{eq}} v_s$$
 and  $i_x = \frac{Z_t}{Z_x + Z_t} i_s$  (4)

Furthermore, the  $\Delta$ -to-Y transformation transforms an interconnection of 3 impedances between the two types in Figures 5 & 6, converted via:

$$Z_i = \frac{Z'_{jk}}{\Sigma Z'}$$
 and  $Z'_i = \frac{P_{jk}}{Z_i}$  (5)

where  $P_{jk} = Z_i Z_j + Z_i Z_k + Z_j Z_k$ .

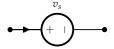


Figure 1: An independent voltage source.

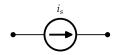


Figure 2: An independent current

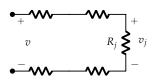


Figure 3: A circuit for voltage division.

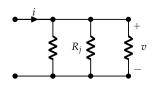


Figure 4: A circuit for current division.

## Nodal Analysis

The method of **nodal analysis** is most commonly used when no node in the circuit connects more than three branches. It is done procedurally as follows:

- 1. Select the node with the most branches as the reference node, typically the bottom–most one.
- 2. Define the node voltages  $v_i$ , which are the voltage rises across a branch or branches from the reference node to another node i.
- 3. For each nonreference node, generate a KCL equation:

$$v_a: \sum i_a = \sum \frac{v_a - v_s}{Z_i} = 0 \tag{6}$$

and solve for  $v_i$ .

4. If a voltage source is between two nodes, a supernode exists in which the nodes are related by  $v_i = v_j + \alpha y_x$ .

## Mesh Analysis

Consequently, the **mesh analysis** method is most commonly used when there is a node connecting more than three branches. It is also done procedurally as follows:

- 1. Assign mesh current directions around each loop, typically clockwise.
- 2. For all nodes, develop individual KCL equations.
- 3. For each mesh, generate a KVL equation:

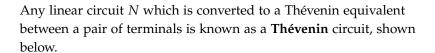
$$v_a: \sum v_a = \sum (i_a - i_i) Z_i = 0 \tag{7}$$

where  $i_i$  is any other mesh current passing through Z.

4. Solve for the branch currents, letting  $i_a = i_\alpha$ ; if a branch is shared,  $i_\alpha = i_a - i_b$ . Then solve the system of equations.

#### Source Transformations

The **source transformation** technique states a voltage  $v_s = i_s Z$  source in series with an impedance Z (Thévenin equivalent) is equivalent to a current source  $i_s$  in parallel with the same impedance (Norton equivalent). As well, an impedance in parallel with  $v_s$  or in series with  $i_s$  has no effect and may be directly removed.



#### Thévenin's Theorem

This circuit is especially useful for calculating the current and voltage of the terminals of the load  $Z_L$ . To calculate  $V_T$ , replace  $Z_L$  with an open circuit and find the voltage drop across the open circuit, typically via voltage division. There are several methods to find  $Z_T$ :

**Method 1 (All Sources)**: The current  $i_{sc}$  is found by replacing  $Z_L$  with a short circuit and calculating the resulting current. Therefore, the Thévenin impedance is

$$Z_T = V_T / i_{sc} \tag{8}$$

**Method 2 (Independent Sources)**: Deactivate all independent sources, then calculate the equivalent impedance  $Z_{eq} = Z_T$  for the network.

**Method 3 (Dependent Sources)**: Deactivate all independent sources, then apply a test source between the terminals. The Thévenin impedance is then

$$Z_T = V_{sc}/i_{sc} \tag{9}$$

#### Superposition

The principle of **superposition** states suppressing all but one source sequentially and summing the values is equivalent to the original circuit. To suppress a voltage course is to replace it with a short circuit, and an open circuit for a current source.

Specifically, for a circuit with n independent sources, a maximum of n superposition circuits may be created with one source activated per circuit. Therefore, the total current or voltage  $x = \sum x_i$ .