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ELECTROMAGNETISM

Electrostatics

Electric Forces

In Electrodynamics, there is typically a **source point** $\mathbf{r}'$ where a charge is located and a **field point** $\mathbf{r}$ where a field is calculated at. The **seperation vector** is defined as

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}' \qquad \hat{\boldsymbol{\mathscr{r}}} = \boldsymbol{\mathscr{r}}/\mathscr{r} \tag{1}$$

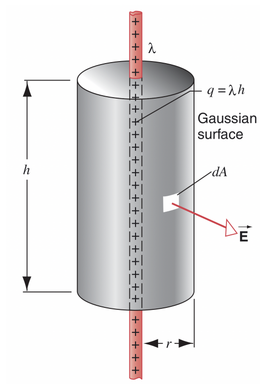

Figure 1: A Gaussian surface as a cylinder.

The force acting on a charge is given by **Coulomb's Law**,

$$\mathbf{F} = k \sum \frac{q_1 q_2}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} = \iiint d\mathbf{F} \tag{2}$$

The **electric field** exerted on a positive test charge $+q_0$ at a point is defined as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = k \sum \frac{q}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} \tag{3}$$

Field lines point outwards from $+q$ and towards $-q$, and are parallel or tangential of an electric field. For a continuous charge distribution,

$$\mathbf{E} = k \iiint \frac{dq}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} = -\nabla V \tag{4}$$

where

$$dq \sim \lambda\, d\ell \sim \sigma\, dA \sim \rho\, dV \sim \lambda R\, d\phi \sim \sigma 2\pi w\, dw \tag{5}$$

The **electric dipole** is a configuration of two equal and opposite charges q separated by a distance d, in which the electric dipole moment $p = qd$ in the direction towards $+q$. If a dipole is in an external field, the torque is

$$\text{ø} = \mathbf{p} \times \mathbf{E} \tag{6}$$

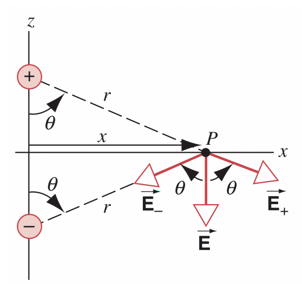

Figure 2: The field at any is the vector sum of the charges.

such that the work done by the field is

$$W = -\int_{\theta_0}^{\theta} \tau\, d\theta = pE(\Delta \cos\theta) \tag{7}$$

Consequently, the potential energy is defined as

$$U = -pE\cos\theta = -\mathbf{p} \cdot \mathbf{E} \tag{8}$$

Potentials

Gauss' Law states the electric **flux** through a field is

$$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \tag{9}$$

such that $\|\Phi\| = EA\cos\theta$ where θ is the angle between the field and field lines. The electric field outside a conductor is given by

$$E = \frac{\sigma}{\varepsilon_0} \iff q = \iint \sigma \, dA \tag{10}$$

The change in electric potential is defined as

$$\Delta U = kq_1q_2(\Delta r^{-1}) \tag{11}$$

For a system of charge, the **potential energy** is

$$U = k\sum_i \sum_{j\neq i} \frac{q_iq_j}{\mathscr{r}} \tag{12}$$

Consequently, the change in **electric potential** is

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\int \mathbf{E} \cdot d\mathbf{s} \tag{13}$$

The electric potential at a point is thus defined as

$$V = k\iiint_a^b \frac{dq}{\mathscr{r}} = k\sum \frac{q_i}{\mathscr{r}} \sim k\frac{p\cos\theta}{\mathscr{r}^2} \tag{14}$$

where the latter equivalence holds for dipoles. A surface on which the potential has the same value everywhere is **equipotential** such that $\Delta V = W = 0$. The field lines must everywhere be perpendicular to the equipotential surfaces, which implies all conductors are equipotential.

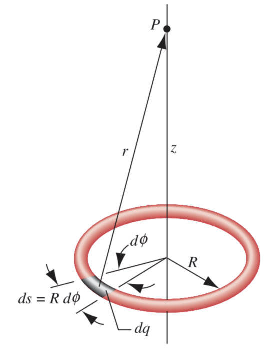

Figure 3: A uniformly charged ring with a potential P and charge element dq.

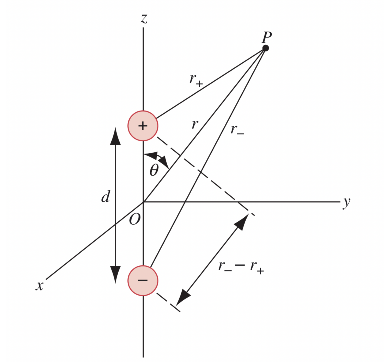

Figure 4: The geometry for calculating V at P for a dipole.

Electrodynamics

Current

THE ELECTRIC current is defined as

$$i = \frac{dq}{dt} = \int \mathbf{j} \cdot d\mathbf{A} \tag{15}$$

where j is the current density and is opposite the motion of electrons. The net charge passing through is therefore

$$q = \int i \, dt \tag{16}$$

The current density is defined as

$$\mathbf{j} = q/At = -en\mathbf{v} = \sigma \mathbf{E} \tag{17}$$

where n is the electron density and $\sigma = q/A$ is the conductivity. Furthermore, the net charge passing through the surface $q = enAL$. The resistance of the material is thus $R = L/\sigma A = \Delta V/i$.

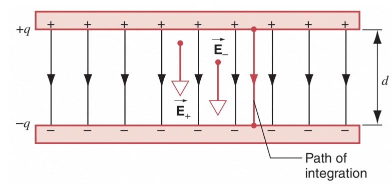

Figure 5: A parallel-plate capacitor.

Capacitance

Capacitance is defined as

$$C = \frac{q}{\Delta V} = \frac{\varepsilon_0 A}{d} \tag{18}$$

In a PPC, SC, and CC, the potential is derived via

$$\Delta V_{ppc} = \frac{qd}{\epsilon A} \qquad \Delta V_{sc} = kq\left(\frac{1}{a} - \frac{1}{b}\right) \qquad \Delta V_{ss} = \frac{q\ln(b-a)}{2\pi\epsilon L} \tag{19}$$

In a parallel combination of capacitors,

$$q = \sum q = C_{eq}\Delta V \iff C_{eq} = \sum C \tag{20}$$

and in a series combination,

$$\Delta V = \sum \Delta V = q/C_{eq} \iff C_{eq}^{-1} = \sum C^{-1} \tag{21}$$

As well, the potential energy in a capacitor is

$$dU = \Delta V \, dq \iff U = \int_0^q dU = \frac{q^2}{2C} \tag{22}$$

with $q = C\Delta V$. Specifically, the energy U is stored in E in the region. Furthermore, the **energy density** u is defined as

$$u = \frac{U}{Ad} = \tfrac{1}{2}\varepsilon_0 E^2 \tag{23}$$

by Eqns 18 & 22. Using **voltage division**, the voltage of any impedance Z including capacitance may be found via $q_0 = \sum q$ by

$$V = \frac{Z_1}{Z_{eq}} V_0 \tag{24}$$

For a **dielectric capacitor**, the charge is defined as

$$q' = q(1 - 1/\kappa) \iff E = E_0/\kappa \tag{25}$$

DC Circuits