Electric Circuits

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Chapter 1

Introduction

1.1 Definitions

The basic definitions of circuit analysis must first be thoroughly understood. The main units of electrodynamics are voltage and current, defined as

$$v = \frac{dw}{dq} \quad \text{and} \quad i = \frac{dq}{dt} \tag{1.1}$$

respectively. The voltage is precisely the potential difference across an element, and the current is the flow of charge across a branch. Additionally, the power of an element is

$$p = \frac{dw}{dt} = vi \;\; \text{W} \tag{1.2}$$

The passive sign convention (PSC) states an element's voltage is positive if its reference current is in the direction of the voltage drop; that is, from positive to negative. By convention, currents entering a node are negative, and positive if leaving.

1.2 Kirchoff's Laws

The current i in any branch of a circuit can be arbitrarily chosen unless there is a current source i_s. In this case, the branch current is in the direction of i_s. Kirchoff's current and voltage laws state for any planar circuit, then

$$\sum_{\text{junc.}} i = 0 \quad \text{and} \quad \sum_{\text{loop}} v = 0 \tag{1.3}$$

Conventionally, the direction of each voltage loop is clockwise. Typically, Kirchoff's law equations have the form

$$\sum_{\text{loop}} v_s = \sum_{\text{loop}} i_i R_i \quad \text{and} \quad \sum_{\text{to}} i = \sum_{\text{from}} i \tag{1.4}$$

1.3 Simple Analysis Methods

The voltage across branches in parallel are equal, as is the current across a branch in series. Otherwise, the voltage-series and current-parallel division equations may be used,

$$v_j = iR_j = \frac{R_j}{R_{eq}} v_s \quad \text{and} \quad i_j = v/R_j = \frac{R_{eq}}{R_j} i_s \tag{1.5}$$

where v_s is the total voltage drop of the series connection and i_s is the current entering the parallel connection. Furthermore, the Δ-Y transformation transforms an interconnection

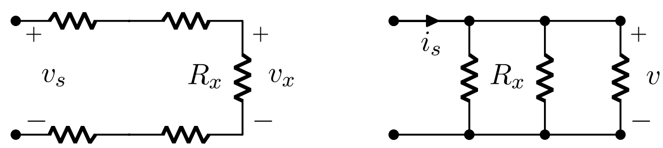

Figure 1.1: A simple voltage and current division circuits.

of 3 elements between the two types of junction, via

$$R_i = \begin{cases} (R'_j R'_k) \,/\, (R'_i + R'_b + R'_c) & \Delta \to Y \\ (R'_i R'_j + R'_j R'_k + R'_k R'_i) \,/\, R'_i & Y \to \Delta \end{cases} \tag{1.6}$$

respectively.

1.4 Nodal Analysis

The method of nodal analysis is most commonly used when no node in the circuit connects more than three branches. It is done procedurally as follows:

1. Select the node with the most branches as the reference node, typically the bottom-most node.

2. Define the node voltages v_i, which are the voltage rises across a branch from the reference node to another node i.

3. For each non-reference node, generate a KCL equation

$$i_a : \sum i_a = \sum (v_a - v_s)/R_i = 0 \tag{1.7}$$

   to solve for v_a.

4. If a voltage source is between two nodes, a supernode can be formed in which the nodes are related by $v_i = v_j + v_s$.

1.5 Mesh Analysis

Consequently, the mesh analysis method is used for nodes with > 3 branches. It is done as follows:

1. Assign mesh current i directions around each loop, conventionally clockwise.
2. For all nodes, develop individual KCL equations.
3. For each mesh, generate a KVL equation
$$v_a : \sum v_a = \sum (i_a - i_i) R_i = 0 \tag{1.8}$$
where i_i is any other mesh current passing through R_i if applicable.
4. Solve for the branch currents, letting $i_a = i_\alpha$. If a branch is shared between mesh currents, $i_\alpha = i_a - i_b$.

1.6 Source Transformations

The source transformation technique allows for a voltage source $v_s = i_s Z$ in series with an impedance Z to be transformed into an equivalent current source i_s in parallel with Z, and vice versa. These transformations are known as a Thevenin and Norton equivalent circuit, respectively.

A result of this theorem is that an impedance that is in parallel with v_s or in series with i_s has no effect at the terminals and thus can be removed from the circuit.

Figure 1.2: Thevenin and Norton equivalent circuits

1.7 Thevenin's Theorem

Thevenin's theorem is useful in finding the current or voltage of the terminals of a load Z_L. It defines the method to calculate V_T, by first replacing Z_L with an open circuit and finding the voltage drop V_T across the open circuit, typically via voltage division.

Method 1 (Mixed Sources): The current i_s is found by replacing Z_L with a short circuit and calculating the resulting current. This leads to the Thevenin impedance being

$$Z_T = V_T / i_s \tag{1.9}$$

Method 2 (Ind. Sources): Deactivate all independent sources, then calculate the impedance $Z_{eq} = Z_T$ of the resulting network.

Method 3 (Dep. Sources): Deactivate all independent sources, then apply a test source between the terminals. This gives $V_s = V_T$.

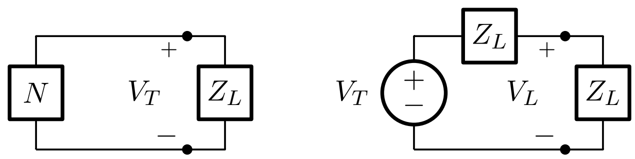

Figure 1.3: A typical circuit converted to a Thevenin circuit

1.8 Superposition

The principle of superposition states suppressing all but one source sequentially and summing the values is allowed. To suppress a voltage source is to replace it with a short circuit and a current source an open circuit.

Specifically, for a circuit with n independent sources, a maximum of n superposition circuits may be created with one source per circuit. The total current or voltage of the original circuit is thus $x = \sum x_i$.