Electromagnetism

Richard Robinson

September 15, 2018

Chapter 1

Electrostatics

1.1 Electric Forces

IN ELECTRODYNAMICS, there is typically a *source point* $\mathbf{r}^{0}$ where a charge is located and a *field point* $\mathbf{r}$ where a field is calculated at. The *seperation vector* is defined as

$$\boldsymbol{\mathscr{r}} \quad \mathbf{r} \quad \mathbf{r}^{0} \qquad \hat{\boldsymbol{\mathscr{r}}} = \boldsymbol{\mathscr{r}} / \mathscr{r} \tag{1.1}$$

upon definition of a coordinate system. Coulomb's law expresses the force of charges q_i on another charge q_0, given by

$$\mathbf{F} \quad K \sum \frac{q_0 q_i}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} = q_0 \mathbf{E} \tag{1.2}$$

When calculating the force via E, the charge density must be replaced by the respective equation. The charge differential is defined as

$$dq \; 7! \;\; l\, dx \quad s\, dA \quad r\, dV \tag{1.3}$$

When evaluating the results of the integration, the limiting cases such as $a \quad b$ can be found by evaluating the expression for $b = 0$.

1.2 Electric Field

The electric field at a point P which acts like a positive test charge of a set of source charges is defined as

$$\mathbf{E}(\mathbf{r}) \quad K \sum \frac{q_i}{\mathscr{r}_i^2} \hat{\boldsymbol{\mathscr{r}}}_i = K \int \frac{1}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}}\, dq = \; r\, V \tag{1.4}$$

For most cases, symmetry can be utilized such that

$$\mathbf{E} = E_x \;\; ! \;\; \hat{\boldsymbol{\mathscr{r}}} = \cos q \tag{1.5}$$

An electric dipole describes the configuration of two opposite charges q a distance d apart. The electric dipole moment is defined as $p = qd$ towards $+q$, which means for $x \quad d$, then $E = 2Kp/r^3$.

1.3 Torque

The torque of an electric dipole is defined to be

$$t = (qE)d \sin q = \mathbf{p} \quad \mathbf{E} \tag{1.6}$$

assuming its direction is perpendicular to and into the page. The work done by the external field in turning a dipole is thus

$$W = \quad \int_q t \; dq = pE(\Delta \cos q) \tag{1.7}$$

which is related to the change in potential energy via

$$U = \quad W = \quad \mathbf{p} \;\; \mathbf{E} \tag{1.8}$$

1.4 Gauss' Law

Gauss' Law states the flux is the rate of change of an electric field of a Gaussian surface; that is,

$$\Phi_E = \oint \mathbf{E} \;\; d\mathbf{A} = q/e_0 \tag{1.9}$$

meaning for each infinitesimal point for a given surface, **E** is in the direction of the filed lines, and **A** is normal to the surface. Note the "edge" of a Gaussian surface is equivalent to a single point P a distance r from E.

This results in $\Phi_E = 0$ if such closed surface does not enclose any charges and/or $\sum q = 0$. Because of this, E for regions of infinite dimensions can be calculated via

$$\sum EA = q/e_0 \qquad q \; 7! \quad lx \qquad sA \qquad rV \tag{1.10}$$

For a conductor, the field $E = s/e_0$.

Chapter 2

Electrodynamics

2.1 Potential Energy

The difference in potential energy is defined to be

$$\Delta U = \quad \int_a^b \mathbf{F} \quad d\mathbf{s} = K q_1 q_2 \quad \frac{1}{r_b} \quad \frac{1}{r_a} \tag{2.1}$$

Incidentally, the total potential energy of a system is

$$U = K \sum \sum \frac{q_i q_j}{r_{ij}} \tag{2.2}$$

These are not to be confused with the potential difference, defined as

$$\Delta V = \quad \int_a^b \mathbf{E} \quad d\mathbf{s} = \Delta U / q_o = Ed \;\; \mathrm{V} \tag{2.3}$$

which is analogous to the potential energy for a point. Lastly, the electric potential is defined to be

$$V = K \sum \frac{q_i}{r_i} = K \int \frac{dq}{r} \qquad \frac{Kp \cos q}{r^2} \tag{2.4}$$

2.2 Electrical Properties

The electric current i is defined as the net charge flowing through a surface,

$$i = \frac{dq}{dt} = \int \mathbf{j} \quad d\mathbf{A} \tag{2.5}$$

where j is the current density and is in the direction of $+q$ which may also be found via

$$\mathbf{j} = \quad en\mathbf{v}_d = s\mathbf{E} \tag{2.6}$$

The resistance of a material relates the potential difference to the current through

$$R = \frac{\Delta V}{i} = r\frac{L}{A} \ \Omega \tag{2.7}$$

2.3 Capacitance

The capacitance of a circuit relates the potential difference with the charge via

$$C = q / \Delta V \ \text{F} \tag{2.8}$$

and is only a geometrical factor. This means for a capacitive sphere, $C = 4pe_0 r$. For a parallel plate capacitor, capacitance is found via

$$\Delta V = \int_{+} E\, ds = \frac{sd}{e_0} \quad (\ ) \quad C = \frac{e_0 A}{d} \tag{2.9}$$

as the electric field for a plate $E = s/2e_0$ and $s = q/A$ and $C = q/\Delta V$. Likewise, the capacitance of spherical and cylindrical capacitors can be found from the result of their respective potential difference equations.

2.4 Capacitors

In a parallel circuit, the following equations hold:

$$q = \sum q_i = C_{eq}\Delta V \quad (\ ) \quad C_{eq} = \sum C_i \tag{2.10}$$

and in series the converse is true:

$$\Delta V = \sum \Delta V_i = q / C_{eq} \quad (\ ) \quad C_{eq}^{\ 1} = \sum C_i^{\ 1} \tag{2.11}$$

The total potential energy stored in such capacitor is defined as

$$U = C^{\ 1} \int_0^q q\, dq = \tfrac{1}{2} C (\Delta V)^2 \tag{2.12}$$

More specifically, the energy itself is stored in the field present in such region. Similarly, the energy density is the stored energy per unit given as

$$u = U / Ad = 0.5 e_0 E^2 \tag{2.13}$$

For a capacitor with a dielectric of constant k, Gauss' law gives

$$e_0 \oint \mathbf{E} \ \ d\mathbf{A} = \frac{1}{k} \quad (\ ) \quad e_0 \oint k\mathbf{E} \ \ d\mathbf{A} = q \tag{2.14}$$

Chapter 3

DC Circuits

Please see Electric Circuits which entirely comprises this chapter itself, available with other titles at http://bit.ly/eecsbooks.