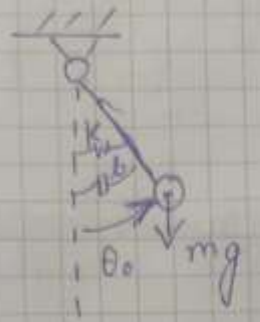
<u>Variant values :- m=0.5 kg, k=5.8 N/m, b=0.04 N*s/m, l=0.96, theta =-1.36663525 rad, x_0=0.78 m</u>

The following equation is **nonlinear** because of the $\sin\theta$ term and has a linear torque $k\theta$.

Below is my hand written solution to Variant 1: pendulum with torsional spring–damper

Variant - 1



For Kinetic energy :-

$$K(\theta,\dot\theta) = \frac{1}{2} I\dot\theta^2 = \frac{1}{2}(ml^2)\dot\theta^2$$

For potential energy :-

$$P(\theta) = P_{gravity} + P_{spring}$$

$$= -mgl\cos\theta + \frac{1}{2}k\theta^2$$

$$\therefore L = K - P$$

$$= \left(\frac{1}{2}ml^2\dot\theta^2\right) - \left(-mgl\cos\theta + \frac{1}{2}k\theta^2\right)$$

$$= \frac{1}{2}ml^2\dot\theta^2 + mgl\cos\theta - \frac{1}{2}k\theta^2$$

We know,

$$d/\left(\frac{\partial L}{\partial\dot\theta}\right)$$

We Know,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = 0$$

$$\frac{dL}{d\theta} = \cancel{ml^2\dot{\theta}} \cancel{- mgl\sin\theta - K\theta}$$
$$- mgl\sin\theta - K\theta$$

$$\frac{dL}{d\dot{\theta}} = ml^2\dot{\theta}$$

$$\frac{d\left(\frac{\partial L}{\partial \dot{\theta}}\right)}{dt} = ml^2\ddot{\theta}$$

$$\therefore ml^2\ddot{\theta} + mgl\sin\theta + K\theta + b\dot{\theta} = 0$$

$$\Rightarrow (0.5)\times(0.96)^2\ddot{\theta} + (0.5)\times(9.81)\times(0.96)\sin\theta$$
$$+ 5.8\theta \quad + 0.04\dot{\theta}$$

$$\Rightarrow 0.4608\ddot{\theta} + 0.04\dot{\theta} + 5.8\theta + 4.7088\sin(\theta) = 0$$