



Practice Task 2

Course: Simulation of Robotics System
SRS 2025

Task 2: Equation of Motion for Mass-spring damper system

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Analytical and Numerical Report: Mass-Spring and Damper System

1. Objective

This report presents the analytical derivation of the equation of motion for a horizontal Mass-Spring Damper system using the Lagrangian Method, provides the analytical solution, and compares it with the results obtained from the provided numerical (code) implementation.

2. Mathematical Formation

2.1 Analytical Derivation using the Lagrangian Method

The mass spring damper system is modeled as a 1 degree of freedom system with displacement $x(t)$ from equilibrium. The system consists of a mass (m), a spring (stiffness k), and a viscous damper (damping coefficient b).

A. Energy and Dissipation Functions

The derivation uses the generalized Lagrange equation with the **Rayleigh Dissipation Function** to account for the non-conservative damping force. The generalized coordinate is x .

- **Kinetic Energy (T):**

$$T = \frac{1}{2} m \dot{x}^2$$

- **Potential Energy (V):**

$$V = \frac{1}{2} k x^2$$

- **Lagrangian (L):**

$$L = T - V$$

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

- **Dissipation Function (R):**

$$R = \frac{1}{2}b\dot{x}^2$$

B. The Equation of Motion (EOM)

The generalized Lagrange equation for non-conservative systems is:

$$\frac{d}{dt} \left(\frac{\partial l}{\partial \dot{x}} \right) - \frac{\partial l}{\partial x} = \frac{\partial R}{\partial \dot{x}}$$

Where partial derivative are:

$$\frac{\partial l}{\partial \dot{x}} = m\dot{x}$$

$$\frac{\partial l}{\partial x} = -kx$$

$$\frac{\partial R}{\partial \dot{x}} = b\dot{x}$$

Substitution of these values in the equation will be:

$$m\ddot{x} + kx + b\dot{x} = 0$$

This is the derived equation of motion using Lagrangian method

3. Analytical Solution of the EOM

The analytical solution to the derived homogeneous ODE, is determined by the given parameters which are:

- **M = 1kg**
- **K = 13.6 N/m**
- **B = 0.02 Ns/m**

Using above parameters the EOM will be:

$$\ddot{x} + 13.6x + 0.02\dot{x} = 0$$

By solving above equation the general solution of the ODE will be:

$$x(t) = e^{-0.01t} (C1\cos(3.6878t) + C2 \sin(3.6878t))$$

The initial conditions given are:

$$x(0) = 0.47$$

$$\dot{x}(0) = 0$$

By using above Initial conditions the particular solution of the ODE will be:

$$x(t) = e^{-0.01t} (0.47\cos(3.6878t) + 0.001356 \sin 3.6878t))$$

4. Discussion and Conclusion:

1. **Method Validity:** The Lagrangian Method successfully derived the correct EOM, $\lambda = -0.01$ and $w_d = 3.6878 \text{ rad/s}$
2. **Accuracy of Analytical Forms:** The simplified solution given in the prompt is an excellent approximation of the true analytical solution because the damping ratio is very close to zero, making the sine term almost negligible.
3. In summary, the mass-spring-damper system is extremely lightly damped, and both the simplified analytical solution and the numerical simulation accurately model the long-lasting, decaying oscillation behavior.