

Eigenvector:  $\vec{x}$  where  $A\vec{x} = \lambda \vec{x}$

eigenvalue:  $\lambda$

in upper triangular form:  $\lambda$ s are the pivots

to find:  $(a-\lambda)(d-\lambda)-bc=0$

Determinant

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Linear dependence

$$\sum_{i=1}^k \alpha_i \vec{v}_i = \vec{0}$$

any set of  $\alpha$ s

Projection (LS in 2d)

projection of  $\vec{b}$  onto  $\vec{a}$

$$x_1 \vec{a} \text{ where } x_1 = \frac{\langle \vec{b}, \vec{a} \rangle}{\langle \vec{a}, \vec{a} \rangle} = \frac{\langle \vec{b}, \vec{a} \rangle}{\|\vec{a}\|^2}$$

Least Squares

$$A\vec{x} \approx \vec{b}$$

$$\vec{x} = (A^T A)^{-1} A^T \vec{b}$$

Orthonormal Matrix w/ columns  $\vec{a}_i$

$$\text{Orthogonal: } \langle \vec{a}_i, \vec{a}_j \rangle = 0 \quad A^T A = I$$

$$\text{Normal: } \langle \vec{a}_i, \vec{a}_i \rangle = 1$$

$$A^T = A^{-1}$$

OMP

$$M\vec{x} \approx \vec{y} \text{ where } M \text{ is fat}$$

① initialize  $\vec{e}$  to  $\vec{y}$ ,  $j$  to 1,  $k$  to sparsity  $1/k$ ,  $A = []$

② while  $J \leq k$

i. compute inner product for each vector in the set,  $\vec{m}_i$ , with  $\vec{e}$ :  $\angle_i = \langle \vec{m}_i, \vec{e} \rangle$

ii. Column concatenate  $A$  with col. vector that had max inner product with  $\vec{e}$ :  $A = [A \mid \vec{m}_i]$

iii. Use LS to compute  $\vec{x}$  given this iteration:  $\vec{x} = (A^T A)^{-1} A^T \vec{y}$

iv. Update error vector:  $\vec{e} = \vec{y} - A\vec{x}$

v.  $j = j + 1$

Steady state:

$$A\vec{x} = I\vec{x}$$

$$(A-I)\vec{x} = \vec{0}$$

cols should also sum to 1

free variables;

find in terms of other variable

[Richard 2hr]

Norm

$$\|\vec{x}\|^2 = \sqrt{x^2 + y^2 + z^2 + \dots}$$

$$\vec{v}^T \vec{v} = \|\vec{v}\|^2$$

Null Space / Column Span

$$\text{Nul}(A) = \text{set of } \vec{x} \text{ where } A\vec{x} = \vec{0}$$

$$\text{Col}(A) = \text{Span of column vectors of } A$$

Rank: Max (independent row vectors, independent columns)

# Op Amps

Xuse NVA

Amplifier

IDEAL: Golden Rules: Assume Ideal Op Amp unless otherwise specified

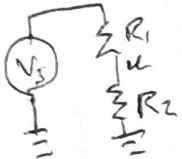
$$I^+ \& I^- = 0$$

Negative Feedback:  
golden rules apply

$$u_+ = u_-$$

## Circuits

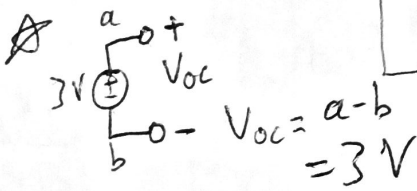
Voltage divider



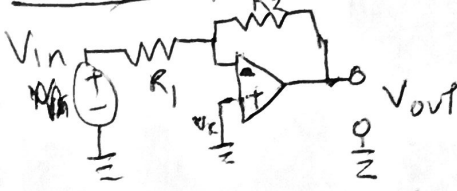
$$u = \frac{R_2 V_s}{R_1 + R_2}$$

Passive Sign Convention

i flows into +



## Inverting Amplifier



$$V_{out} = -\frac{R_2}{R_1} V_{in} + V_R \left( \frac{R_2}{R_1} + 1 \right)$$

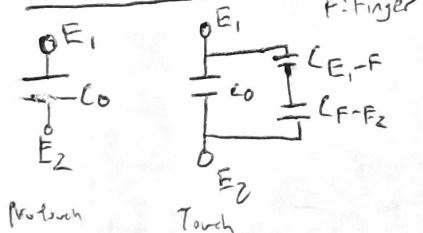
## Super position

turn off  $I_s \rightarrow$  open circuit

turn off  $V_s \rightarrow$  short wire

compute Voltages/Currents and sum them each one (source)

## Capacitive Touch Screen



## Thevenin

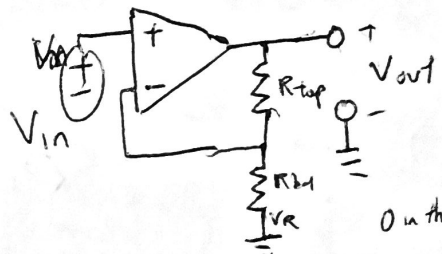
① Measure voltage across the OC + SC

This is  $V_{th}$

② Zero all independent sources and apply test voltage/current, find corresponding  $Z/V$ , and plug into  $R = \frac{V}{I}$  to get  $R_{th}$

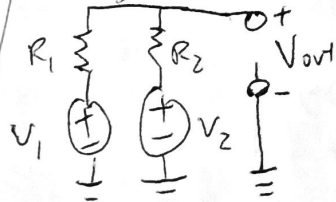
find voltage equivalent to  $R_{th}$  from point A to B

## Non-Inverting Amplifier



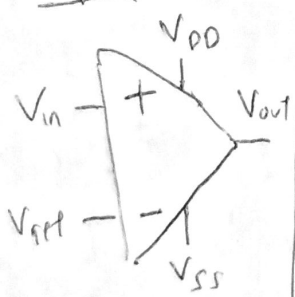
$$V_{out} = \left( 1 + \frac{R_{top}}{R_{b1}} \right) V_{in} - V_R \left( \frac{R_{top}}{R_{b1}} \right)$$

## Voltage Summer



$$V_{out} = V_1 \left( \frac{R_2}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right)$$

## Comparator



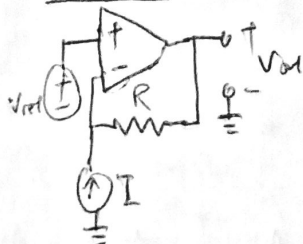
if  $V_{in} > V_{ref}$

$$V_{out} = V_{DD}$$

else  $V_{in} < V_{ref}$

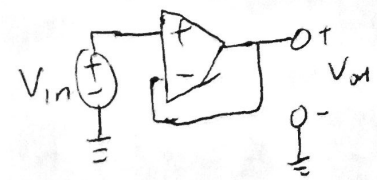
$$V_{out} = V_{SS}$$

## Transresistance Amplifier



$$V_{out} = -I R + V_{ref}$$

## Unity Gain Buffer



$$V_{out} = V_{in}$$

$$Q = CV$$

$$R = \frac{\rho L}{A}$$

$$C = \frac{\epsilon A}{d}$$

$$V = IR$$

$$I = \frac{Q}{t}$$

$$P = IV$$

Actual types tips

Look @ what they give you

\* Read the ENTIRE problem desc - Give what they ask for! FOLLOW INSTRUCTIONS

- pay attention to units/determine how to solve problems

- If you don't know what to do, just do algebra  $\rightarrow$  \* take some liberties w/ commutative/distributive property  
- use asymptotes/denoms, etc for proofs

\* Get Partial Credit!!!

- when they say in terms of  $x, y, z$ , not all have to be used

- be careful of matrix dimensions

Algebra

for matrices  
/ scalars

$$(A^2 + B^2) \vec{v} = A^2 \vec{v} + B^2 \vec{v}$$

$$A^2 \vec{x} = \lambda^2 \vec{x} \text{ if } A \vec{x} = \lambda \vec{x}$$

Problems

Beacons where we have to sacrifice one as a "reference"

- can create  $n-1$  equations with  $n$  beacons  
- not linear though

- can create  $n-2$  linear equations w/  $n$  beacons  
by subtracting 1 equation to linearize

square signals  $\rightarrow$  slope this  $\rightarrow$  comparator



LSQR/OMP work for square systems

Orthonormal:  $\vec{x}^T \vec{x} = \mathbf{I}$

And the manipulation where all the  $\vec{a}_i \vec{a}_j^T \rightarrow 0$

only  $\vec{a}_i \vec{a}_i^T \rightarrow 1$  remains