



# **Time Series Forecasting**

Programming for Data Science

## Classical Time Series Forecasting

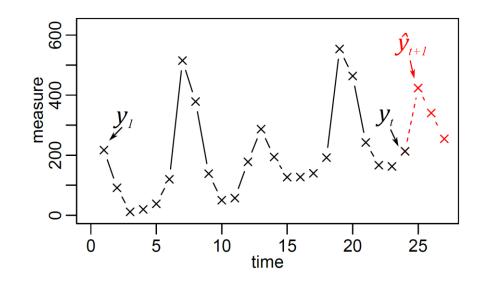


#### Time Series

- Sequence of measure values
- Equidistant
- Complete

#### Forecast Models

- One model represents one time series
  - Trend (long-term changes)
  - Season (regular reoccurring changes)
- Accuracy is the only requirement





### **ARMA Models**

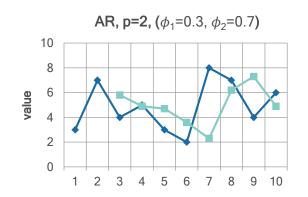


#### Box and Jenkins Methodology

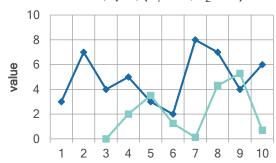
Modeling time series with AutoRegression

#### Classification and Model Hierarchy

• AR(p):  $\hat{x}_t = c + \sum_{i=1}^p \phi_i \cdot x_{t-i}$ AutoRegression



- MA(q):  $\hat{x}_t = \sum_{i=1}^q \theta_i \cdot \varepsilon_{t-i}$ Moving Average
- $\varepsilon_0$ ,  $\cdots$ ,  $\varepsilon_q=0$ ,  $\varepsilon_i=x_{t-i}-\hat{x}_{t-i}$ MA, q=2, ( $\theta_1$ =0.5,  $\theta_2$ =0.5)



• Combine AR and MA by addition:  $\hat{x}_t = \sum_{i=1}^p \phi_i \cdot x_{t-i} + \sum_{i=1}^q \theta_i \cdot \varepsilon_{t-i} + c$  The constant c also needs to be optimized.



### Model Estimation (Math Part)

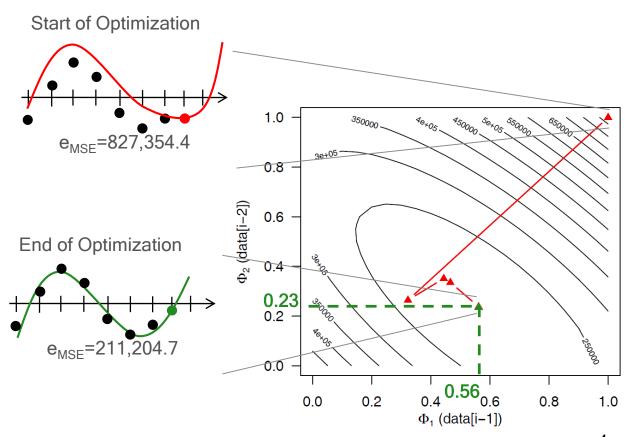


#### Problem

 Find the model parameters that minimize the error on the training data

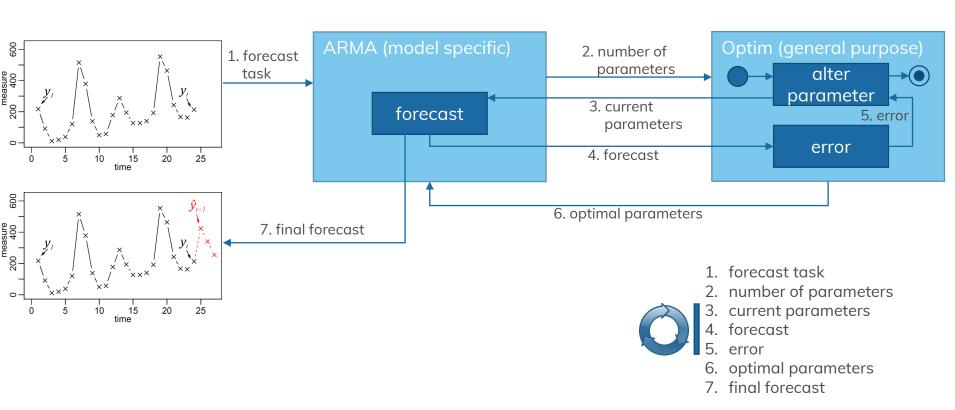
### Example

- Forecast Model Type AR(2):
  - $\hat{x}_t = \phi_1 \cdot x_{t-1} + \phi_2 \cdot x_{t-2}$
- Error Metric: MSE
  - $\frac{1}{n} \sum_{i=1}^{n} (x_i \hat{x}_i)^2$



### **Model Estimation**



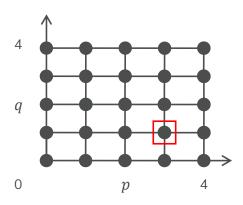


## Global Optimization Methods



#### Grid Search (Derivation Free)

- Simple but robust method: no local suboptimal
- First define a step width to sample the parameter space (Grid)
- Exponential number of combinations X<sup>D</sup> (granularity X and dimensionality D)
- Model evaluation for every combination





### Task



#### Step 0

- Load the given csv file in your language/environment.
- Explore the data and its structure.

#### Step 1

Implement the AR and MA parts of the ARMA model\*.

#### Step 2

• Use a general purpose optimizer (Nelder-Mead) to fit the model parameters to a given time series.

#### Step 3

- Search for the optimal combination of AR (p) and MA (q) components of the model for each given time series\*. Perform a grid search\* for  $p \in [0,3]$  and  $q \in [0,3]$ . (Slide 5, Steps 1-6)
- Report the forecast error (insample error).

<sup>\*</sup>use your own implementation



## Package suggestions



#### R

- (data.table)
- stats

### python3

- pandas
- numpy
- scipy.optimize
- (matplotlib)



## Modeling Results



#### Result

 Mean Squared Error of all time series using the best combination of p and q and a constant c

### Comparison

- The values for p and q can vary based on your implementation and parameter initialization
- Only compare if your MSE correlate with our solution and are in a similar range

shop_id	Mean Squared Error
145	210
260	345
315	303
375	265
548	403
560	1272
750	539
897	9458
1332	3476
1750	377



## **Exercise Appointment**



#### We compare and discuss the results

- Tuesday, 12.01.2021,
- Consultation: Please use the forum in Opal.
- Please prepare your solutions! Send us your code!

#### If you have questions, please mail us:

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