3

Numbers and Operations

The SAT Math section relies heavily on you knowledge of the real numbers and their properties. The real numbers can be broken up into two categories: rational numbers and irrational numbers. *Rational numbers* are all numbers that can be expressed as any whole number divided by any other non-zero whole number. Some examples are $-1, 0.75, \frac{2}{3}$, and $-1.\overline{125}$. *Irrational numbers* are all numbers that cannot be expressed as a fraction of whole numbers. They are non-repeating, never ending decimals. For example, $\sqrt{2}, 1.2345\ldots$ are all irrational numbers. The properties of the real numbers to bare in mind are:

Sample

Properties of the Real Numbers

For all real numbers a, b, and c,

$a+b=b+a$ $ab=ba$	Commutative Property
$a+(b+c)=(a+b)+c$ $a(bc)=(ab)c$	Associative Property
$a(b+c)=ab+ac$	Distributive Property
$a\cdot 1=a$	Multiplicative Identity
$a+0=a$	Additive Identity
$a+(-a)=0$	Additive Inverse
$a\cdot\frac{1}{a}=1$	Multiplicative Inverse

1 Arithmetic Word Problems

General Equation

Fraction Rules

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b} \qquad \frac{a}{b} \cdot \frac{b}{d} = \frac{ab}{cd} \qquad \frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$$

$$a \cdot \frac{b}{c} = \frac{ab}{c} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \qquad \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

Example 1: If 3 gallons of water fill 2 fish bowls, how many gallons will fill 9 fish bowls?

Sample

Example 2: If y varies directly as x, and $y = 6$ when $x = 9$, what is the value of y when $x = 12$?

Example 3: Terri rides her bike every day on her way to school. On a sunny day, Terri can make the 2.6 mile trip in half of an hour at a constant pace. When it rains, Terri's speed drops by 1 mile per hour. If Terri's trip is the same distance in the rain, what total time it takes her to get to school?

MEDIUM

1. If the side length, s, of a square is increased by n, what is the ratio of the area of the new square to the old?

   Equation/Strategy: ________________

   Solve:

   (a) $n^2 : 1$

   (b) $n^2 : s^2$

   (c) $s^2 : n^2$

   (d) $s^2 : (s+n)^2$

   (e) $(s+n)^2 : s^2$

2. From January to March, coats drop in price by 10% each month from the month before. Which ratio represents the cost of the jacket from January to March?

   Equation/Strategy: ________________

   Solve:

   (a) 100 : 81

   (b) 10 : 81

   (c) 10 : 9

   (d) 9 : 1

   (e) 11 : 9

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3. If P varies jointly as T and inversely as V, and $P = 5$ when $V = 3$, what is the value of V when P doubles and T remains unchanged?

   Equation/Strategy: ________________

   Solve:

   (a) 1.5

   (b) 2.5

   (c) 6

   (d) 10

   (e) 12

4. A jar contains D jellybeans. If there are A red jellybeans, B green jellybeans, and C orange jellybeans. What proportion of the jellybeans are red or orange?

   Equation/Strategy: ________________

   Solve:

   (a) $\dfrac{A+C}{A+B+C}$

   (b) $\dfrac{A+C}{D}$

   (c) $\dfrac{D-(A+C)}{D}$

   (d) $\dfrac{D-B}{A+B+C}$

   (e) $\dfrac{D-B}{D}$

Sample

2 Rational Numbers

General Equation

Rational Number

Definition: A ***rational number*** is any number of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. All terminating (ending) decimals and repeating decimals can be expressed as a rational number.

Example 1: If $\frac{x}{3} = \frac{y}{5}$, write an inequality stating the relationship between x and y.

Sample

Example 2: If the number of microbes in a petri dish is reduced by a factor of one-third after each day, how many days will it take for the population to be less than 10% of the original amount?

Example 3: The batting average of a baseball player is given by the proportion of hits versus the number of times at bat. If Stanley has a batting average of 0.32 and had 24 hits in one season, what is the number of times Stanley was at bat?

MEDIUM

1. If the probability of choosing a red marble from a bag of marbles is 0.3. If the probability of choosing n red marbles is 0.027, what is the value of n?

   Equation/Strategy: ____________

   Solve:

   (a) 2
   (b) 3
   (c) 6
   (d) 7
   (e) 9

2. One-third of Ms. Boyd's class takes French while four-fifths takes Spanish. How many students are taking both French and Spanish if all students are taking a language class?

   Equation/Strategy: ____________

   Solve:

   (a) 1/15
   (b) 2/15
   (c) 1/5
   (d) 1/3
   (e) 2/3

ADVANCED

3. Bob is two-thirds the age of his sister Sally. If Sally is four-thirds times the age of their sister, Patricia. If Patricia is 3 years older than Bob, what is Sally's age?

   Equation/Strategy: ____________

   Solve:

   (a) 20
   (b) 24
   (c) 28
   (d) 32
   (e) 36

4. From 1970 to 2010, the population of US living on the coast has increased by approximately 40%. If approximately 40% of the total population of the USA lives on the coast in 2010 and the population in 2010 is 308 million people, what is the population (in millions) of the US in 1970?

   Equation/Strategy: ____________

   Solve:

   (a) 49
   (b) 77
   (c) 88
   (d) 193
   (e) 220

Sample

3 Sequences and Series

General Equation

Sequence and Series Formulas

Arithmetic Sequence

$$a_n = a_1 + d(n-1)$$

Geometric Sequence

$$a_n = a_1 r^{n-1}$$

Arithmetic Series

$$S = \frac{n(a_1 + a_n)}{2}$$

Geometric Series

$$S = \frac{a_1(1-r^n)}{1-r}$$

where a_1 is the first term of the sequence, n is the number of terms of a given sequence, a_n is the nth term of the sequence, d is the common difference between consecutive terms $(a_n - a_{n-1})$, and r is the common ratio between consecutive terms $\left(\frac{a_n}{a_{n-1}}\right)$.

Sample

Example 1: Shelly is preparing to run for a marathon. If Shelly starts running on the first week with 1 kilometer, and doubles the number of kilometers every week, how many miles will she run on the sixth week?

Example 2: Jenny receives a weekly allowance of \$15 and is saving up to buy a new laptop that costs \$450. If she has \$100 saved up already, how many weeks will it take her to have enough for the laptop?

Example 3: A ball is dropped from a height of 1 meter and bounces a height of two-thirds of the previous bounce with every consecutive bounce. What is the total distance the ball has traveled after 5 bounces?

MEDIUM

1. Tom is doing a cross country bike ride. On the first day, Tom rode 5 miles. If he decides to ride an additional 20% every day, approximately how long will he have ridden on day 8?

Equation/Strategy: ____________

(a) 13 miles
(b) 14 miles
(c) 18 miles
(d) 19 miles
(e) 20 miles

Month	2	5	7
Height	5	9.5	11.5

2. Eduardo purchases a potted plant to grow at home. The height is recorded (in inches) every month. If the rate of growth is constant, at what height did Eduardo buy the plant?

Equation/Strategy: ____________

(a) 2 in
(b) 3 in
(c) 3.5 in
(d) 4 in
(e) 4.5 in

ADVANCED

3. A spherical balloon deflates at a rate such that the volume is cut in half every 2 minutes. If r is the initial radius, which expression represents the radius after 8 minutes?

Equation/Strategy: ____________

Solve:

(a) $r/2$
(b) $r/4$
(c) $r/8$
(d) $r/16$
(e) $r/32$

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4. A guitar string is plucked and the distance between the highest point and the lowest point of the first oscillation is 256 millimeters. If the distance the string travels is reduced by one-quarter with each oscillation, how many oscillations will it take for the string to have traveled a total distance of 700 millimeters?

Equation/Strategy: ____________

Solve:

(a) 2.5
(b) 3
(c) 3.5
(d) 4
(e) 5

Sample

4 Elementary Number Theory

General Equation

Quotient and Divisor Theorem

For all integers a and b,

$$a = q \cdot b + r$$

where q is the quotient and r is the remainder such that $r < b$.

Example 1: Gumdrops come in bags of 60. Mr. Lee wants to buy enough bags so that he has a perfect square number of gumdrops. What is the least number of bags Mr. Lee will need to buy?

Example 2: The sum of the angles of a regular polygon is given by the equation $S(n) = 180(n-2)$ where n is the number of sides. How many sides does the smallest regular polygon have if is the smallest regular polygon possible such that each individual angle is over $120°$?

Example 3: A group of 10 students participate in a math competition. The students are required to shake hands with all other participates in the competition. If each handshake between two people is counted only once, how many distinct handshakes are there overall?

Sample

MEDIUM

1. There are n jellybeans in a jar. The number of jellybeans can be separated into groups of 24 or groups of 42. What is the least possible number of jellybeans in the jar?

   Equation/Strategy: ______________

   Solve:

   (a) 126
   (b) 168
   (c) 252
   (d) 504
   (e) 1008

2. A full revolution of the hour hand around the face of a clock corresponds to 12 hours. If the hour hand starts at 12:00 AM and completes 6.75 revolutions, what is the current time?

   Equation/Strategy: ______________

   Solve:

   (a) 3:00 AM
   (b) 9:00 AM
   (c) 3:00 PM
   (d) 6:00 PM
   (e) 9:00 PM

ADVANCED

3. If f is the number of faces of a cube, v is the number of vertices, and e is the number of edges, what is the value of $v - e + f$?

   Equation/Strategy: ______________

   Solve:

   (a) 0
   (b) 1
   (c) 2
   (d) 6
   (e) 12

4. Ms. Rizzo has a box of sand in her classroom. If the value of the volume is a positive integer greater than 1 in^3 that is both a perfect square and a perfect cube, what is the surface area of the closed box?

   Equation/Strategy: ______________

   Solve:

   (a) 6
   (b) 32
   (c) 64
   (d) 72
   (e) 96

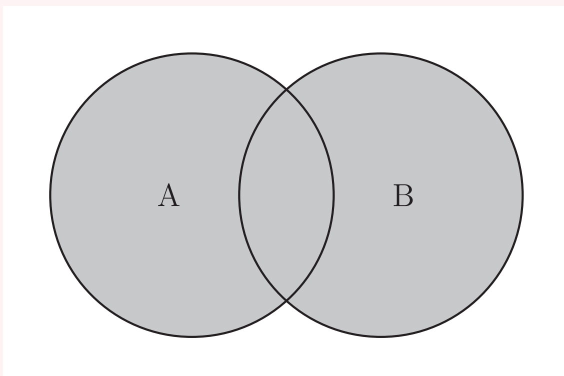
Sample

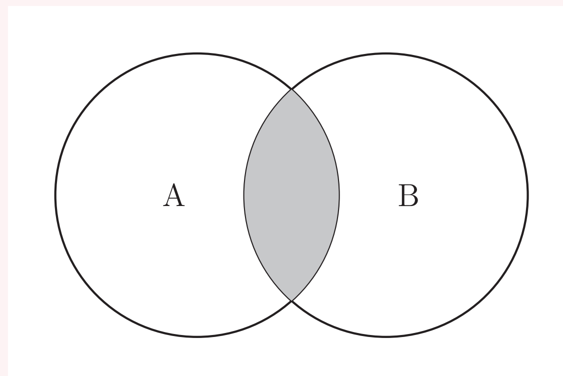
5 Sets

General Equation

Sets

Union	**Intersection**
$P \cup Q$	$P \cap Q$
All elements in either set P *or* Q	All elements in both P *and* Q





Sample

Example 1: Martha's has 15 cupcakes that are vanilla frosted and 10 cupcakes that are chocolate frosted. If two-thirds of Martha's cupcakes are both vanilla and chocolate frosted, how many cupcakes have vanilla frosting?

Example 2: In the set of integers from 1 to 100 inclusively, how many numbers are divisible by 3 or 5 but not both?

Example 3: Mr. Dropal has 16 students in his class. There are 10 male students. Half of all students are taking only Chinese and one-quarter are taking Chinese and French. If the class must take either French or Chinese, what proportion represents the maximum number of females taking only French?

MEDIUM

1. Mr. Carter has a garden of yellow, white, and red rose bushes. The proportion of red rose bushes to all other bushes is 0.4. When he buys an additional 4 red rose bushes, the proportion increases to 0.5. How many non-red rose bushes did Mr. Carter originally have?

   Equation/Strategy: ______________

   Solve:

   (a) 8
   (b) 12
   (c) 20
   (d) 24
   (e) 36

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2. The average (arithmetic mean) of a set of 5 positive integers is 10 and the median is 10. What is the largest possible value of the largest member of the set?

   Equation/Strategy: ______________

   Solve:

   (a) 10
   (b) 50
   (c) 22
   (d) 37
   (e) 40

ADVANCED

3. Helen has a box of 40 chocolates. The proportion cherry filled or coconut chocolates is 0.3, whereas the proportion of coconut or creme filled is 0.4. If the number of creme filled is twice the number of cherry filled and there is at least one cherry filled, what proportion of the box is coconut?

   Equation/Strategy: ______________

   Solve:

   (a) 0.075
   (b) 0.1
   (c) 0.15
   (d) 0.2
   (e) 0.25

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4. When a positive integer n is divided by 4, the remainder is 3. When n is divided by 5, the remainder is 1. How many values of n are there from 1 to 40?

   Equation/Strategy: ______________

   Solve:

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

Sample

6 Counting Techniques

General Equation

Permutations and Combinations

When choosing r items from n items,

Permutations When order matters	**Combinations** When order does not matter
Without Repetition $_nP_r = \dfrac{n!}{(n-r)!}$ *With Repetition* n^r	*Without Repetition* $_nC_r = \dfrac{n!}{r!(n-r)!}$ *With Repetition* $\dfrac{(n+r-1)!}{r!(n-1)!}$

Sample

Example 1: Mandy is throwing an ice cream party and has 3 flavors of ice cream and 4 different toppings. How many different combinations can her guests make?

Example 2: Jerry is making a sundae with 3 scoops of ice cream. If he has 5 flavors of ice cream, how many different combinations can Jerry make if he can only use each flavor once?

Example 3: If a four-digit pin number contains the digits 0 to 9 where no digit can be repeated more than twice, how many different combinations for pin numbers are possible?

MEDIUM

1. If set A contains all even integers under twenty and set B contains all even prime numbers, then the set of common elements between set A and set B is

Equation/Strategy: ____________

Solve:

(a) {}
(b) {0}
(c) {2}
(d) {0, 2}
(e) All even numbers

2. If a four point star has 8 vertices, and an eight point star has 16 vertices, how many vertices does a 10 point star have?

Equation/Strategy: ____________

Solve:

(a) 12
(b) 16
(c) 20
(d) 24
(e) 32

ADVANCED

3. A number of volleyballs compete in a tournament. If each team must play one another, and there are a total of 120 matches, how many teams competed?

Equation/Strategy: ____________

Solve:

(a) 5 teams
(b) 6 teams
(c) 7 teams
(d) 8 teams
(e) 12 teams

4. An equilateral triangle is divided so that the midpoint of each line is the vertex of an inscribed triangle. If the process continues, how many triangles will there be after n divisions?

Equation/Strategy: ____________

Solve:

(a) 2^n
(b) 3^n
(c) 4^n
(d) $4n$
(e) $8n$

Sample