### Final Project: Forecasting NG Price after Ukraine Crisis

#### Yu Hai, Richard Zhang, Yiyan Ma

```
## Warning: package 'readxl' was built under R version 4.1.2
##
## Attaching package: 'lubridate'
## The following objects are masked from 'package:base':
##
      date, intersect, setdiff, union
## Warning: package 'forecast' was built under R version 4.1.2
## Registered S3 method overwritten by 'quantmod':
    method
    as.zoo.data.frame zoo
## Warning: package 'Kendall' was built under R version 4.1.2
## Warning: package 'tseries' was built under R version 4.1.2
## Warning: package 'outliers' was built under R version 4.1.2
## -- Attaching packages ------ tidyverse 1.3.1 --
## v tibble 3.1.6
                  v dplyr 1.0.8
## v tidyr 1.2.0
                  v stringr 1.4.0
## v readr 2.1.2
                   v forcats 0.5.1
## v purrr 0.3.4
## Warning: package 'tidyr' was built under R version 4.1.2
## Warning: package 'readr' was built under R version 4.1.2
## Warning: package 'dplyr' was built under R version 4.1.2
## -- Conflicts ----- tidyverse_conflicts() --
## x lubridate::as.difftime() masks base::as.difftime()
## x lubridate::date() masks base::date()
## x dplyr::filter()
                         masks stats::filter()
## x lubridate::intersect() masks base::intersect()
## x dplyr::lag()
                         masks stats::lag()
## x lubridate::setdiff() masks base::setdiff()
## x lubridate::union()
                          masks base::union()
```

```
## Warning: package 'smooth' was built under R version 4.1.2
## Loading required package: greybox
## Warning: package 'greybox' was built under R version 4.1.2
## Package "greybox", v1.0.5 loaded.
##
## Attaching package: 'greybox'
## The following object is masked from 'package:tidyr':
##
##
       spread
## The following object is masked from 'package:forecast':
##
       forecast
## The following object is masked from 'package:lubridate':
##
##
       hm
## This is package "smooth", v3.1.5
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
##
## Attaching package: 'kableExtra'
## The following object is masked from 'package:dplyr':
##
##
       group_rows
## Warning: package 'psych' was built under R version 4.1.2
##
## Attaching package: 'psych'
## The following object is masked from 'package:outliers':
##
##
       outlier
## The following objects are masked from 'package:ggplot2':
##
       %+%, alpha
##
```

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                                 GDP2
                                           GDP3
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## 549 2005/11/1
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## 562 2006/12/1 101.54653 101.54653 101.54653
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## 572 2007/10/1 102.60742 102.60742 102.60742
## 573 2007/11/1 102.69883 102.69883 102.69883
## 574 2007/12/1 102.77774 102.77774 102.77774
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## 617
        2011/8/1 101.05195 101.05195 101.05195
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## 620 2011/10/1 100.89154 100.89154 100.89154
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## 647 2014/1/1 99.50753
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## 658 2014/12/1
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  661
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## 672
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## 673
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##
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   682 2016/12/1 100.11350 100.11350 100.11350
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        2017/9/1 100.90942 100.90942 100.90942
   691
  692 2017/10/1 100.96294 100.96294 100.96294
## 693 2017/11/1 100.99008 100.99008 100.99008
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  695
        2018/1/1 100.96188 100.96188 100.96188
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        2018/4/1 100.84677 100.84677 100.84677
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  698
## 699
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## 700
        2018/6/1 100.77225 100.77225 100.77225
## 701
        2018/7/1 100.74489 100.74489 100.74489
```

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2018/8/1 100.73484 100.73484 100.73484
        2018/9/1 100.74814 100.74814 100.74814
## 703
## 704 2018/10/1 100.78328 100.78328 100.78328
## 705 2018/11/1 100.83431 100.83431 100.83431
  706 2018/12/1 100.89254 100.89254 100.89254
        2019/1/1 100.94713 100.94713 100.94713
  707
        2019/2/1 100.98754 100.98754 100.98754
        2019/3/1 101.00718 101.00718 101.00718
## 709
## 710
        2019/4/1 101.00674 101.00674 101.00674
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## 712
        2019/6/1 100.95965 100.95965 100.95965
        2019/7/1 100.91735 100.91735 100.91735
## 713
## 714
        2019/8/1 100.86284 100.86284 100.86284
## 715
        2019/9/1 100.79692 100.79692 100.79692
## 716 2019/10/1 100.72420 100.72420 100.72420
## 717 2019/11/1 100.64996 100.64996 100.64996
## 718 2019/12/1 100.57745 100.57745 100.57745
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        2020/2/1 100.43086 100.43086 100.43086
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## 722
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## 729 2020/11/1
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                                       95.71599
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## 730 2020/12/1
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## 731
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## 732
        2021/2/1
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## 733
        2021/3/1
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## 734
        2021/4/1
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                                       96.24900
                             96.77051
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## 746
        2022/4/1
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        2022/5/1 100.29505
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                                       99.15533
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                                       99.23131
## 749
        2022/7/1 100.90290 100.37103
                                       99.30729
## 750
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                                       99.38328
## 751
        2022/9/1 101.51075 100.82692
                                       99.45926
## 752 2022/10/1 101.81467 101.05486
                                       99.53524
## 753 2022/11/1 102.11860 101.28280
                                       99.61122
## 754 2022/12/1 102.42252 101.51075
                                       99.68720
## 755 2023/1/1 102.72644 101.73869
                                       99.76318
```

```
## New names:
        -> `...3`
        -> `...7`
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            ...17
           `...19`
     `` -> `...21`
        -> `...23`
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       -> `...69`
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     `` -> `...73`
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            ...77
     `` -> `...79`
```

Introduction, Motivation, Relevance, and Objectives Since the start of the Russian invasion of Ukraine, the volume of Russian gas delivered to the EU through Ukraine did not actually decrease, but natural gas spot prices and futures prices rose considerably (Reuters, 2022). For example, "in the UK market, the March price was up 58.6% at 321.97 pence per therm and the winter 2022 price had risen 37.85% to 105 p/therm." One reason is that Russia's overall natural gas supply decreased compared to previous years (Reuters, 2022). Another reason is that the demand for natural gas in the EU is rising due to the economic recovery from the pandemic. Arguably, the most important reason that EU natural gas price increases is due to market speculation in response to the political uncertainties. This gives us the motivation for our project: to forecast the EU natural gas price after the Ukraine crisis.

First, let's take a step back and provide some background on the global natural gas market. The global natural gas market is composed of regional markets that are grouped based on either transoceanic shipping (i.e. the Atlantic and Pacific Basins). "In recent years, roughly 70% of natural gas flows across the globe are

transported to market destinations within the country of production, while an additional 20% flows cross international borders through pipelines, and nearly 10% is moved to market destinations as liquefied natural gas (LNG)." The limitation in transporting natural gas creates different prices among the major regions globally. For example, East Asia countries usually have high natural gas prices, followed by EU countries; the US tends to have a lower natural gas price.

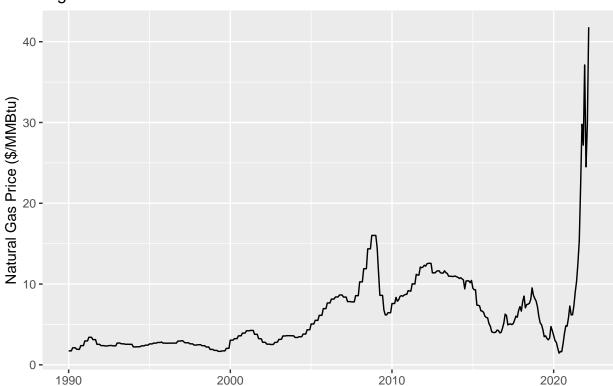
In a 2010 study, "Russia is the world's largest exporter of pipeline gas, accounting for 26% of global exports". Many EU countries are among the top 10 gas importers, including Germany, Italy, France, Spain, and the UK (formal EU member). According to IEA, the EU imported 155 bcm of natural gas from Russia in 2021, accounting for around 45% of the EU's total gas imports and 40% of its total gas consumption. Considering the EU countries plan to reduce or even embargo the natural gas imports from Russia after the Russian invasion of Ukraine, the natural gas price in Europe is expected to experience the most direct impact.

The objective of this project is to forecast the EU natural gas price after the Ukraine crisis. The motivation is to see which model (if any), can predict natural gas prices in a period of high volatility caused by political events. We want to explore whether the following exogenous variables can capture some of the impacts of the Russia-Ukraine war, thus improving our model performance.

Specifically, we think these exogenous variables could have a close relationship with NG price: Consumption: it reflects the seasonal variations with NG price (higher in the winter when heating need is higher) GDP: it is a proxy of economic activities (high economic activities will need more energy inputs and more NG). Imports: Due to EU's high dependence on imported NG from Russia.

We also came up with four hypotheses to better understand how models work under disruptions in general: 1) Forecasting in a period of high uncertainty is difficult, as the historical observations are not very applicable to the testing period 2) Because we observe a non-linear increase in NG price from 2021, we hypothesize that a feed-forward neural networks model can better capture this trend and give a relatively accurate forecast result. 3) Adding exogenous variables can improve the model performance as they can explain some of the variations in NG price. 4) Because we observed a stronger correlation between NG price and import than with GDP, we hypothesized that adding import data can improve the model performance more than adding GDP data.

#### **Dataset Information**



Time

Figure 1. Historical EU Natural Gas Price

```
##
##
   Pettitt's test for single change-point detection
##
## data: ts_EU_price
## U* = 34916, p-value < 2.2e-16
## alternative hypothesis: two.sided
## sample estimates:
## probable change point at time K
##
                               177
##
   Pettitt's test for single change-point detection
##
##
## data: ts_EU_price_shorten_200409_202203
## U* = 6404, p-value = 9.52e-12
## alternative hypothesis: two.sided
## sample estimates:
## probable change point at time K
##
                               127
```

By using the pettitt.test function, we detected two points of inflection in our historical NG price data (p<0.05), one in September 2004 and the other one in July 2014, so we trim the data before July 2014, and take NG historical data from July 2014 to March 2022 to forecast the NG price until December 2022. Because Russia started mobilizing its troops near Ukraine at the beginning of 2022, we assume NG price since January 2022 are impacted by this event.

```
## Warning in cbind(EU_price$Price[295:372], EU_GDP$GDP, Consumption$Consumption, :
## number of rows of result is not a multiple of vector length (arg 1)

## Warning in cbind(EU_price$Price[295:387], Import$Import1): number of rows of
## result is not a multiple of vector length (arg 1)

## [,1] [,2]
## [1,] 1.00000000 -0.04584427
## [2,] -0.04584427 1.00000000
```

Table 1: Table 1. Correlation Coefficient Matrice

	Price	GDP	Consumption	Import
Price	1.0000000	0.0314424	-0.0051395	-0.0420958
GDP	0.0314424	1.0000000	-0.0545164	0.0517559
Consumption	-0.0051395	-0.0545164	1.0000000	0.1633462
Import	-0.0420958	0.0517559	0.1633462	1.0000000

To better perform the forecast, we introduce three exogenous variables: consumption, import, and GDP (as a proxy of economic status) in the EU. Historical monthly natural gas price and the monthly normalized GDP in Europe are from the Federal Reserve Banks of St.Louis' website, and the monthly natural gas consumption and import in the EU are from the Eurostat data browser. Through exploring the correlations between natural gas price and the other factors, we found that all variables have relatively weak correlations, in which GDP shows a positive correlation to NG price and import shows a negative correlation, and consumption with a really weak correlation (Table 1). Therefore, we decided to include GDP and natural gas imports as exogenous variables in our forecast. As GDP and import show the strongest positive correlation of 0.05, we also analyze models that include both variables to explore how their interaction can impact the forecast result.

```
## Warning in cbind(EU_price$Price[295:372], EU_GDP$GDP, Consumption$Consumption, :
## number of rows of result is not a multiple of vector length (arg 1)
```

Table 2: Table 2. First ten rows of data

Price (\$/MMBtu)	GDP(Normalized)	Consumption(Million Cubic Metres)	Import(Million Cubic Metres)
9.40	99.66307	43998	53575
10.38	99.70633	36881	44978
10.40	99.75215	32070	48792
10.40	99.79922	23731	47342
10.16	99.84206	23110	50724
10.45	99.87506	19391	46070
9.50	99.89666	20871	47634
9.29	99.91446	18581	42248
9.29	99.93804	21662	43615
7.39	99.97215	26536	47023

Table 3: Table 3. Data Descriptions

	n	mean	$\operatorname{sd}$	mediar	ntrimm	emlad	min	max	range	skew	kurtosi	s se	source
Price	755	5.92	2.26	5.81	5.84	2.34e+	-0045	10.4	9.0	0.268	-	0.0823	FRED
(\$/MMBtu)											0.628		
GDP(Normalized)	755	100.00	1.37	100.00	100.00	8.78e-	86.70	103.0	16.2	_	28.800	0.0498	FRED
						01				3.680			
Consumption(Milli	o <b>n</b> 55	32200.0	M0600.	.0 <b>3</b> 00200.	OBO 1400.	010.27e+	<b>-0\$</b> 400	.007600.	Ø9200.	00.464	-	385.00	(Hourosta)
Cubic Metres)											1.080		
Import(Million	755	51500.0	<b>3</b> 0460.0	051700.	OSO 1700.	06024e+	<b>-33</b> 600	.0603100.	<b>27500</b> .	0 -	-	199.00	(Hourosta)
Cubic Metres)										0.314	0.110		

```
## [1] "Results for ADF test on EU NG Price data"

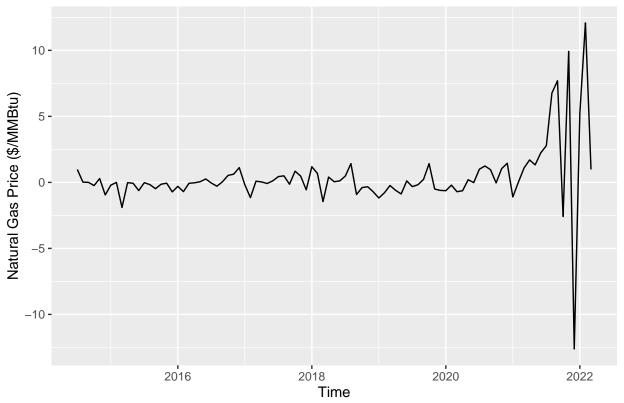
## Warning in adf.test(ts_EU_price_shorten_201407_202203, alternative =
## "stationary"): p-value greater than printed p-value

##

## Augmented Dickey-Fuller Test
##

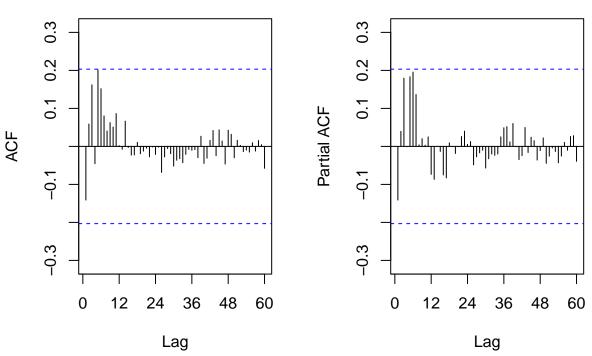
## data: ts_EU_price_shorten_201407_202203
## Dickey-Fuller = 2.6572, Lag order = 4, p-value = 0.99
## alternative hypothesis: stationary
```

Figure 2.Differenced EU Natural Gas Price





#### Natural Gas Price (\$/MMBtu)



The ADF test indicates that the original series contains a unit root (i.e. there is a stochastic trend)(p>0.99). After removing the trend by differencing, the detrended series doesn't show a constant pattern within each year. Instead, it looks like white noise with a mean around zero and fluctuating up and down randomly over time. This situation indicates that the original time series doesn't contain a seasonality component. Neither ACF nor PACF of the differenced series show significant values, which could be caused by the superimposing of autoregression and moving average terms.

From the historical NG price data, we observed a huge increase starting from January 2021. Because we want to compare models' performance under disruptions, we purposely include this increasing trend in the training set. Thus, our training set includes data from July 2014 to December 2021, while our test set includes data from January 2022 to March 2022. The same time frame is used to split exogenous variables so that they will have the same length as NG price data when fitting the models and forecasting.

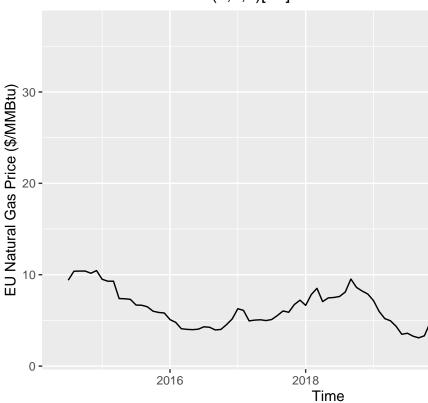
In order to accurately forecast the natural gas price in 2022, we created three scenarios for each of the two exogenous variables. For GDP growth in the EU in 2022, the EU Economic and Social committee estimated growth of 4% (GDP\_scenario\_1) before the Russian invasion of Ukraine, and dropped its estimation to 3% (GDP\_scenario\_2) after the invasion. Yet, some investment banks like Credit Suisse predicted annual GDP growth of only 1% (GDP\_scenario\_3) due to this invasion. While the level of sanctions on natural gas imports from Russia is still under debate in different countries in the EU, we expect the total natural gas import in the EU would decrease by about 20% (Import\_scenario\_2) if there is a complete embargo on Russian imports. If the EU countries failed to embargo Russia, a 10% decrease (Import\_scenario\_2) in natural gas imports is expected with some sanctions on Russia, and only a 5% decrease (Import\_scenario\_3) if the sanctions are not strictly implemented.

Method Our method is to fit ARIMA, Neural Network (NN), STL+ETS, and TBATS models on the NG price training set (including models with exogenous variables), and use the fitted models to forecast NG price from January 2022 to March 2022. Then the forecasted results are compared to the observed data and the accuracies of different models are compared. Then, the best model with the lowest RMSE score

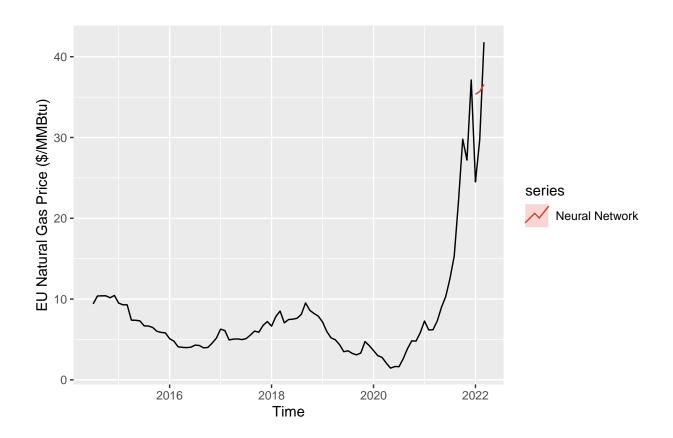
will be selected to forecast the NG price from April 2022 to December 2022. If the model allows us to add in exogenous variables, we would add in proposed scenarios for GDP and import to generate corresponding forecast results.

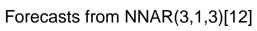
Result Graphs below show the comparison between forecasted prices from 2022/1 to 2022/3 using models

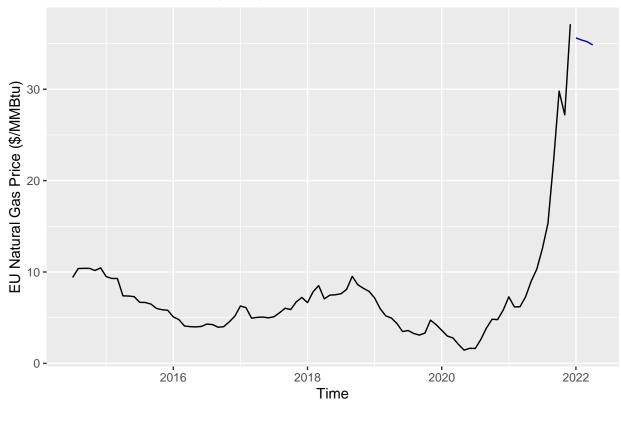


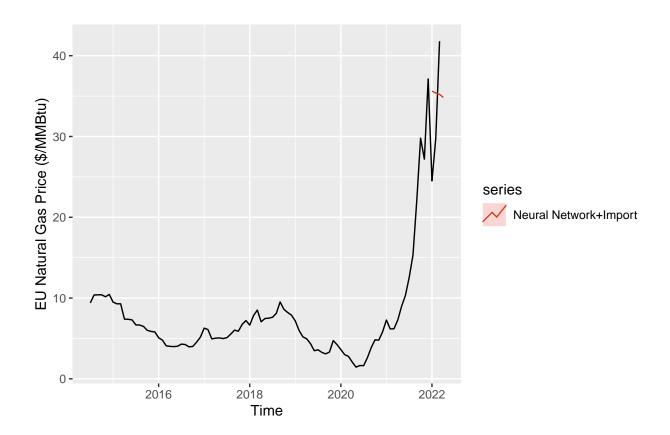


that fitted the training set and the observed data.

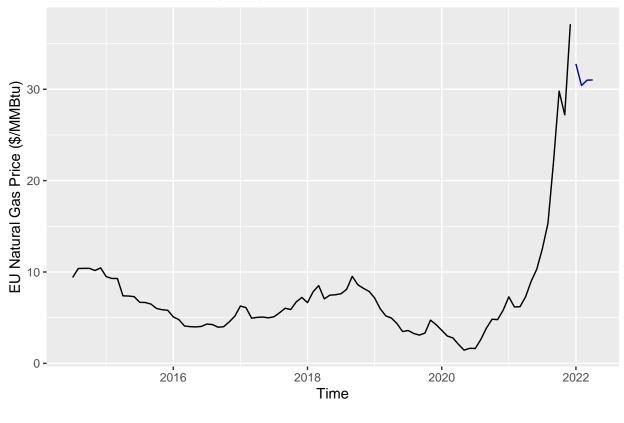


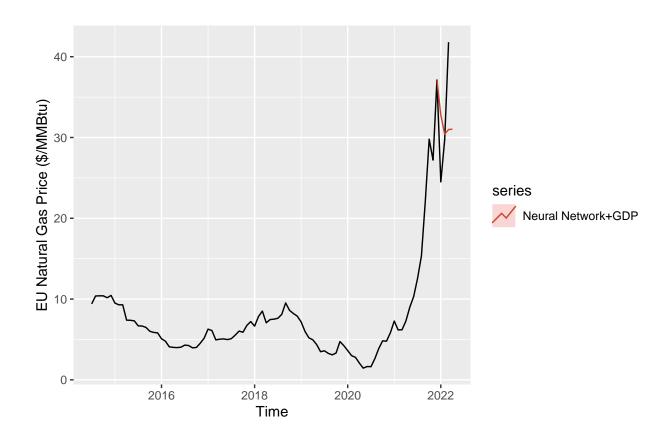






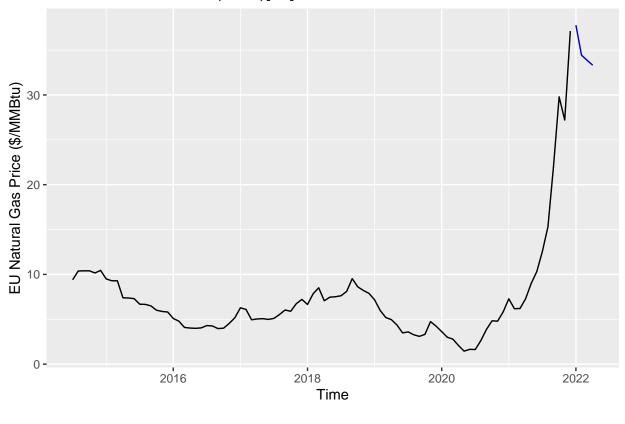


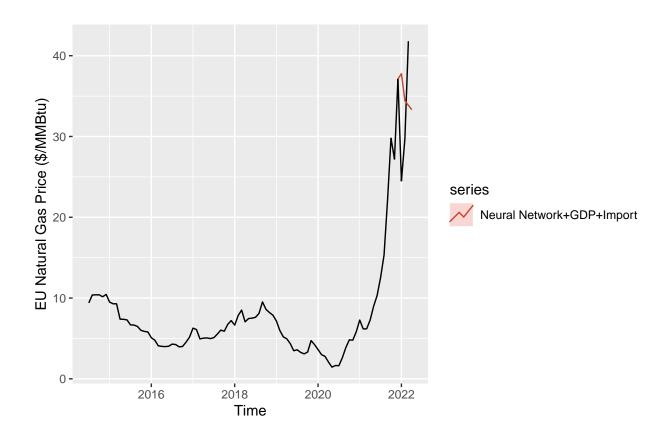




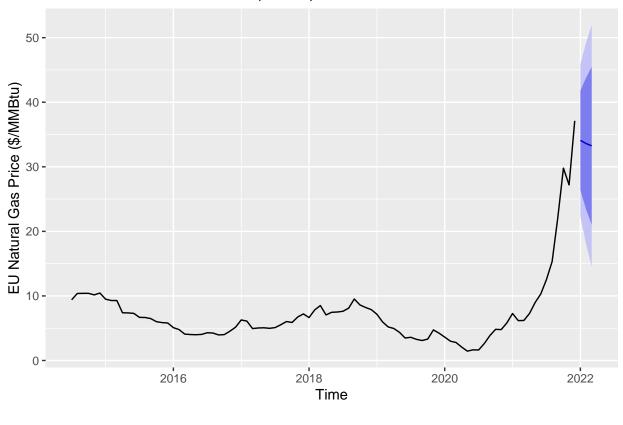
- ## Warning in forecast.nnetar(NN\_fit\_withGDPandImport, xreg =
  ## cbind(ts\_EU\_GDP\_test, : xreg contains different column names from the xreg used
- ## in training. Please check that the regressors are in the same order.

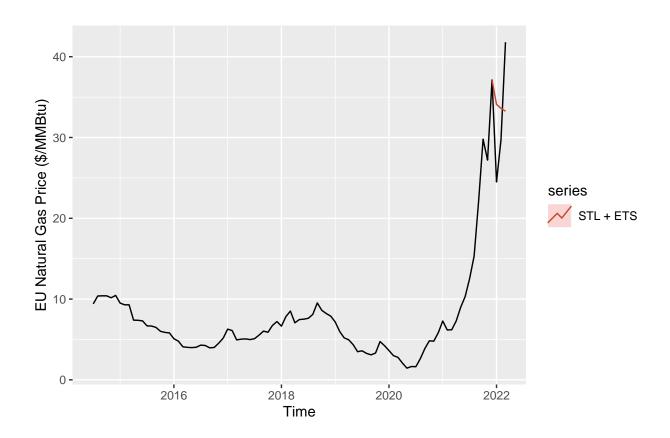
## Forecasts from NNAR(3,1,4)[12]



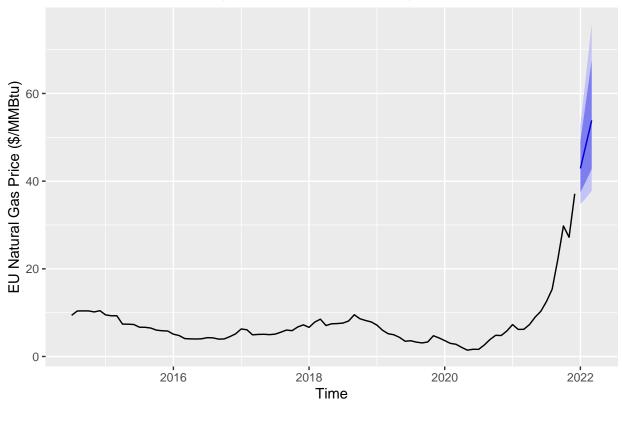


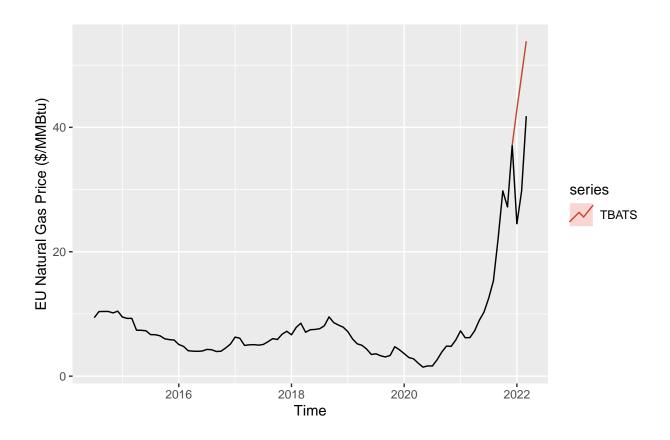
# Forecasts from STL + ETS(M,A,N)



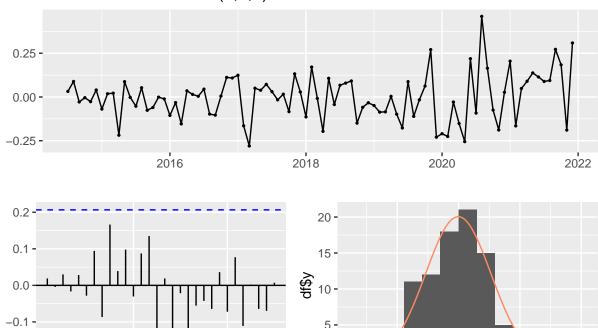


## Forecasts from TBATS(0.099, $\{0,0\}$ , 1, $\{<12,1>\}$ )





#### Residuals from ARIMA(1,0,4) with non-zero mean



5 -

0 -

-0.25 0.00 0.25

residuals

0.50

```
##
    Ljung-Box test
##
##
## data: Residuals from ARIMA(1,0,4) with non-zero mean
## Q* = 14.671, df = 12, p-value = 0.2599
## Model df: 6.
                 Total lags used: 18
```

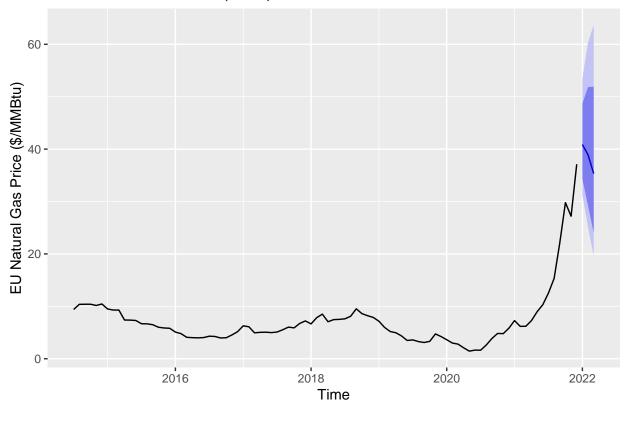
24

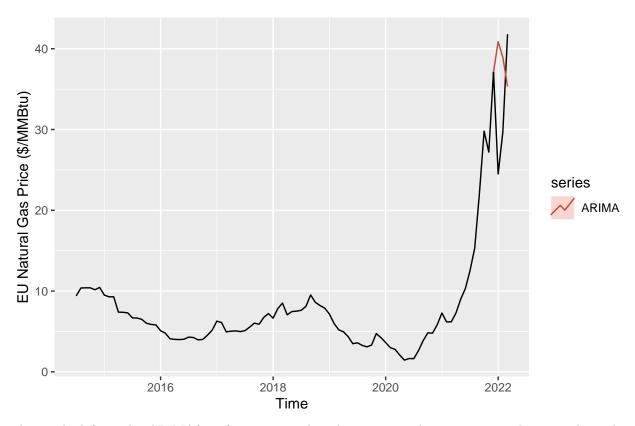
12

Lag

-0.2 **-**

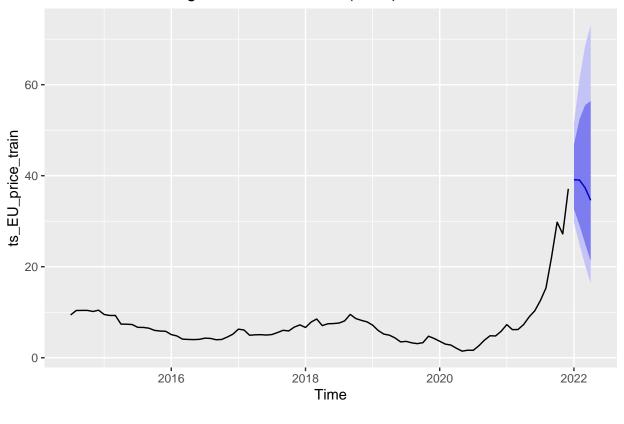
## Forecasts from ARIMA(1,0,4) with non-zero mean

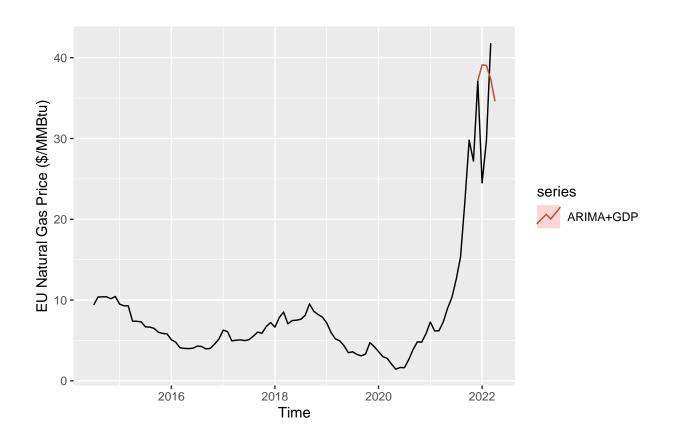




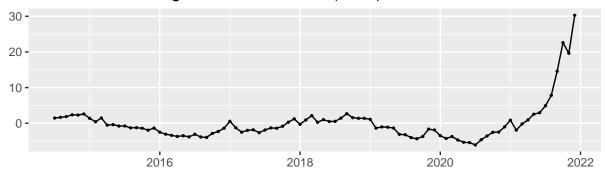
The residual from the ARIMA(1,0,4) appears to be white noise with a mean around zero and no clear trend observed. The residuals are normally distributed, and all ACF values are within the intervals (i.e. insignificant).

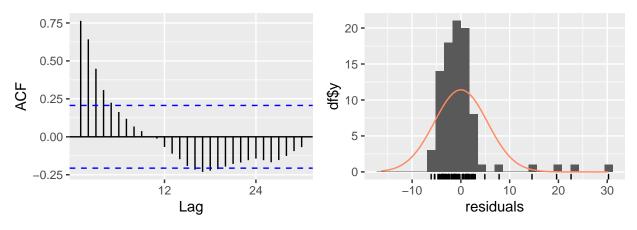
# Forecasts from Regression with ARIMA(2,0,1) errors





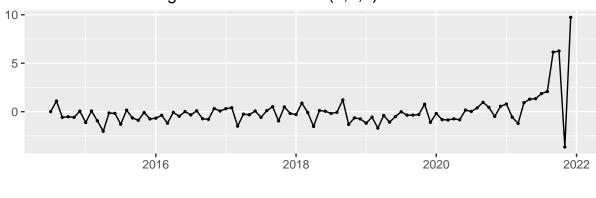
## Residuals from Regression with ARIMA(0,0,0) errors

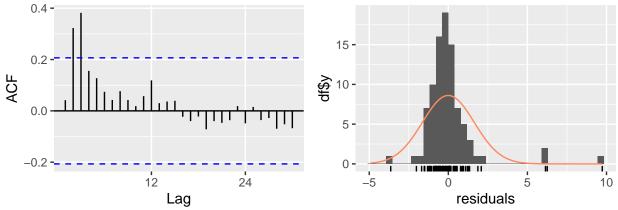




```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(0,0,0) errors
## Q* = 156.27, df = 16, p-value < 2.2e-16
##
## Model df: 2. Total lags used: 18</pre>
```

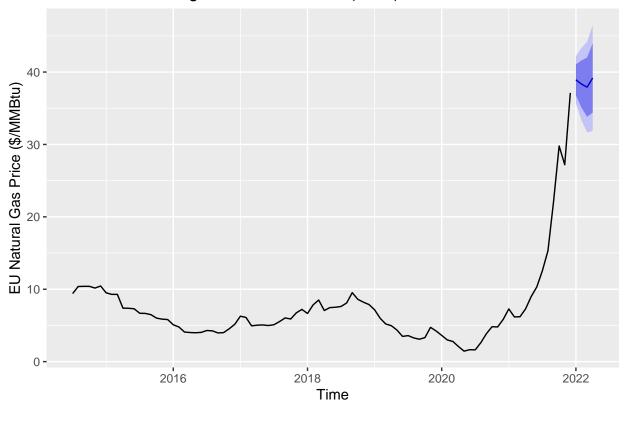
## Residuals from Regression with ARIMA(0,1,1) errors

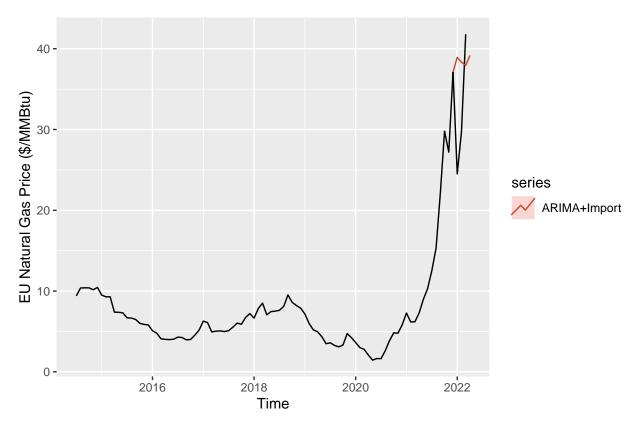




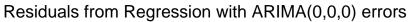
```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(0,1,1) errors
## Q* = 31.831, df = 15, p-value = 0.006789
##
## Model df: 3. Total lags used: 18
```

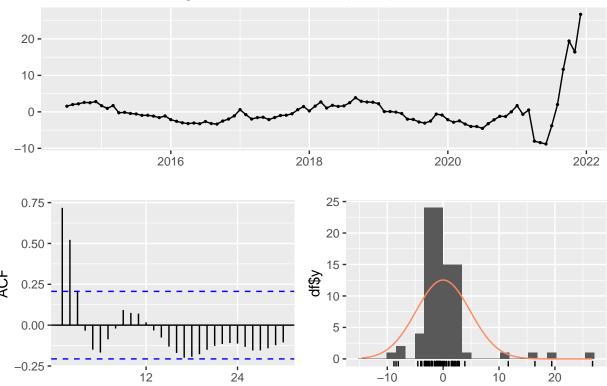
## Forecasts from Regression with ARIMA(0,1,1) errors





When fitting the training set with import as a regressor, the result is ARIMA (0,0,0), which means the series is white noise. However, when checking the residuals, a clear pattern in ACF is observed, and residuals are not normally distributed. An ARIMA (0,1,1) was manually fitted and gave a better residual result (residuals around zero and show no clear trend). Thus, ARIMA (0,1,1) was used to forecast NG price from 2022/01 to





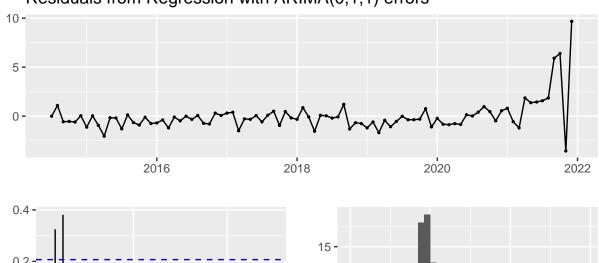
residuals

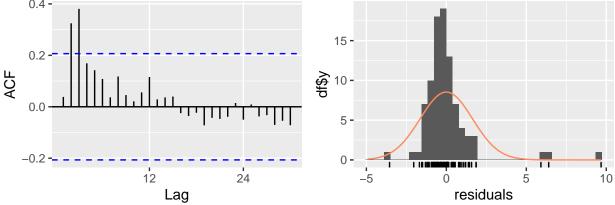
2022/03.

```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(0,0,0) errors
## Q* = 100.41, df = 15, p-value = 1.088e-14
##
## Model df: 3. Total lags used: 18
```

Lag

#### Residuals from Regression with ARIMA(0,1,1) errors

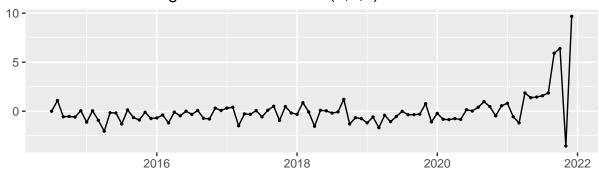


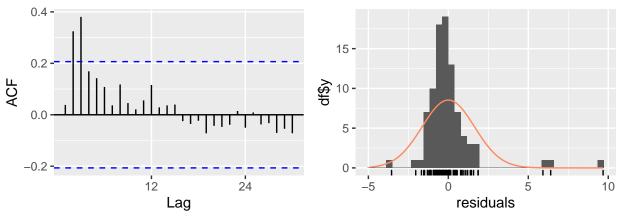


```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(0,1,1) errors
## Q* = 33.952, df = 14, p-value = 0.002096
##
## Model df: 4. Total lags used: 18

## Warning in forecast.forecast_ARIMA(ARIMA_fit_withImportandGDP, xreg =
## cbind(ts_EU_import_test, : xreg contains different column names from the xreg
## used in training. Please check that the regressors are in the same order.
```

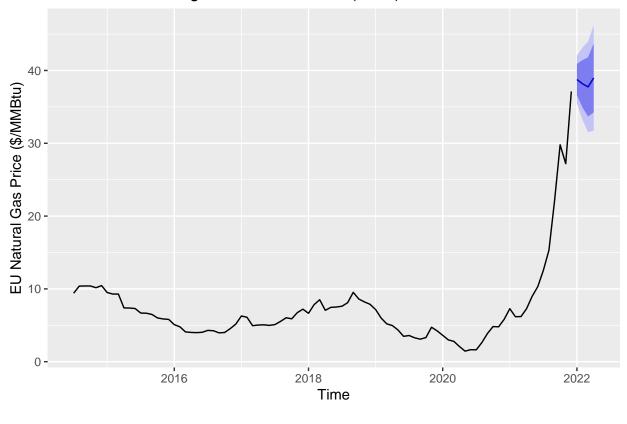
## Residuals from Regression with ARIMA(0,1,1) errors

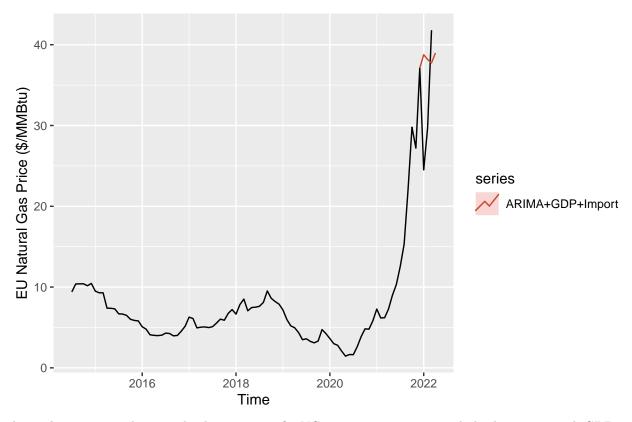




```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(0,1,1) errors
## Q* = 33.952, df = 14, p-value = 0.002096
##
## Model df: 4. Total lags used: 18
```

## Forecasts from Regression with ARIMA(0,1,1) errors





A similar situation happened when we autofit NG price training set with both import and GDP as regressors. Thus, we also manually fit an ARIMA (0,1,1) and used it to forecast NG price from 2022/01 to 2022/03.

## The best model by RMSE is: STL+ETS

Table 4: Forecast Accuracy for EU NG Price

	ME	RMSE	MAE	MPE	MAPE	ACF1	Theil's U
STL+ETS	-1.64152	7.75066	7.34184	-10.58410	24.22103	-0.02914	0.71586
ARIMA	-6.36534	11.46380	10.67543	-27.41896	37.73004	-0.02928	0.94519
TBATS	-16.36282	16.64441	16.36282	-55.53750	55.53750	-0.18540	1.87892
NN	-3.85824	7.76115	7.35176	-17.26777	25.62537	-0.03281	0.65565
ARIMA+GDP	-6.49303	10.33134	9.46206	-26.77667	33.87950	-0.04130	0.89128
ARIMA+Import	-6.38398	9.95491	8.97348	-26.18050	32.37539	-0.02838	0.81747
ARIMA+Import+GDP	-6.21720	9.85005	8.91934	-25.63472	32.09908	-0.02838	0.80810
NN+Import	-3.39198	8.14347	7.78853	-16.20111	26.71901	-0.03135	0.69836
NN+GDP	0.61816	7.86677	6.58699	-3.39698	20.63393	-0.00919	0.79498
NN+GDP+Import	-3.34505	9.33338	8.63943	-16.99620	29.66200	-0.00813	0.71727

According to the table, the model with the lowest RMSE score is NN purely dependent on historical NG price, the second-best model is STL+ETS, followed by ARIMA+Import+GDP. Thus, we choose NN as the model to fit the forecast for 2022. It is noteworthy that our choice of the test set is uncommon, because we expect NG prices to follow different trends after the Ukraine crisis. Thus, this result only applies to our data set which undergoes a huge increase and lots of fluctuations. The effect of exogenous variables depends

on different models. For example, among all ARIMA models, the best one is ARIMA with both GDP and Import, but among all NN models, the NN without any exogenous variable gives the lowest RMSE score.

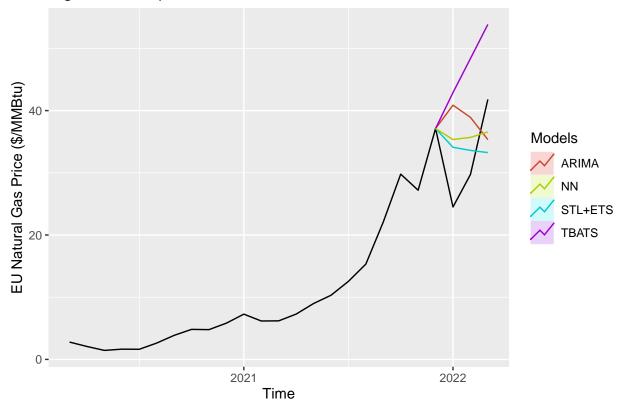


Figure 3. Comparison between Observed Data and Forecasted Values

To better analyze the differences between different models, forecasts without exogenous variables are plotted and analyzed separately below. This graph only shows a comparison between observed data and the forecasts based on the NG historical price training set. TBATS performed well in following the trend observed recently, but it failed to integrate the fluctuations, thus receiving the highest RMSE score. NN and STL+ETS forecasted the same price in January, while NN and ARIMA forecasted the same price in March.

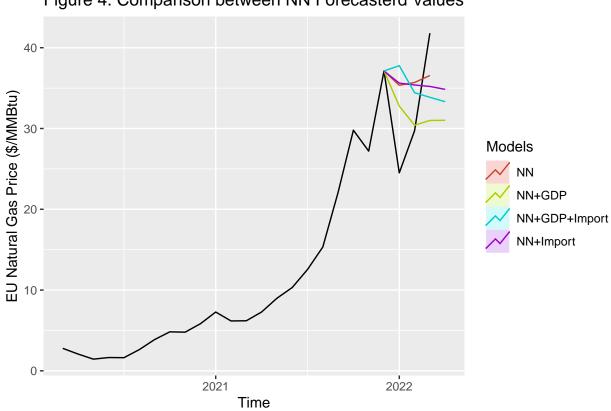


Figure 4. Comparison between NN Forecasterd Values

Among NN models results, both NN with import as a regressor and NN with both import and GDP forecast an increase in natural gas price in January 2022 and then a significant decrease in February 2022, while NN and NN with GDP forecast a decrease in January. Overall, there is a significant overlap between NN+GDP+Import and NN+import forecasts, which indicates that import has a greater impact on the forecast results. Compared to the original NN, the integration of import data increases the fluctuation of the forecast results, while GDP only brings up the level of forecast slightly and keeps the fluctuation similar as before (Figure 4).

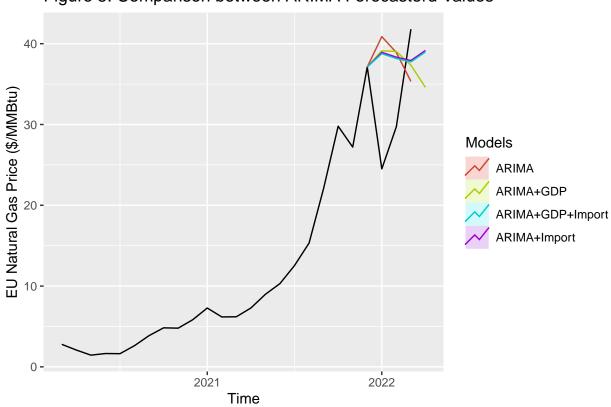


Figure 5. Comparison between ARIMA Forecasterd Values

Among ARIMA models results, all four models show an increase trend in January, and then a decrease trend in February. Compared to the ARIMA model without exogenous variables, GDP as a regressor doesn't change the overall trend of the forecast, while import shows a larger impact on the forecast result When both GDP and import data are added into the ARIMA model, the forecast almost overlaps with the one from ARIMA+import, which indicates that import has a greater impact onto the forecast result compared to GDP (Figure 5).

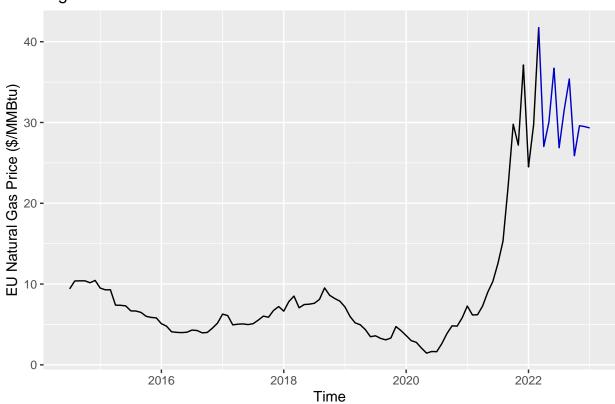


Figure 6. Forecasted EU NG Price with NN Model

According to the best-performed model, which is NN purely dependent on NG historical price, the forecast results show that EU NG price will fluctuate a lot throughout the year with an overall decreasing trend. The peak is forecasted to be in March 2022, and the price will fall back to 28 \$/MMBtu at the end of the year (Figure 6).

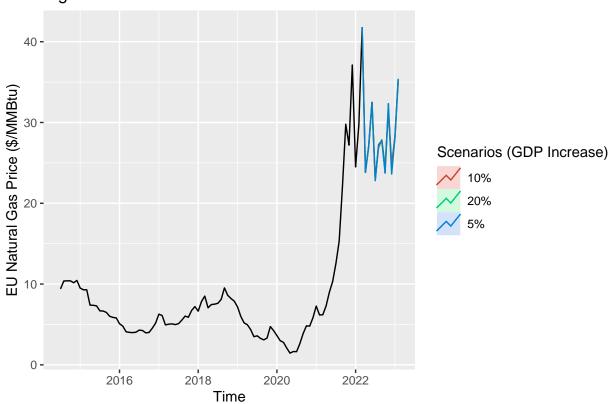


Figure 7. EU NG Price Forecast with NN under GDP scenarios

NN with GDP as a regressor shows a similar pattern compared to the original NN model, the peak is still forecasted to happen in March 2022, but overall the integration of GDP drives up the level of the price forecast to 35 \$/MMBtu at the end of the year. Three GDP scenarios don't lead to any significant effect on the forecast results (Figure 7).

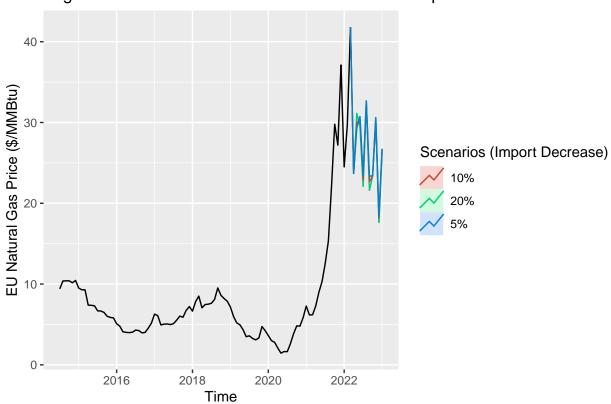


Figure 8. EU NG Price Forecast with NN under Import Scenarios

NN with import as a regressor forecasts a more significant decreasing trend of NG price over 2022, even though at the end of the year, the price is forecasted to be 30 \$/MMBtu (similar to previous NN models). The lowest price forecast reaches almost 20 \$/MMBtu. There is no significant difference between the three proposed import scenarios (Figure 8).

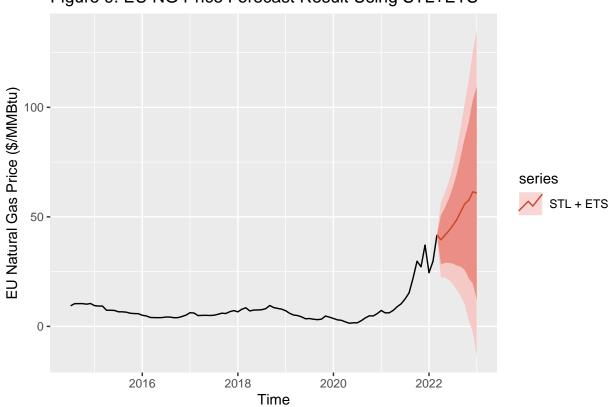


Figure 9. EU NG Price Forecast Result Using STL+ETS

Because our historical data shows a significant increase since 2021, we expect the forecast in 2022 to have a high uncertainty. This is proved by our second-best model: STL\_ETS. The forecast results show an overall increasing trend reaching an NG price of around 60 \$/MMBtu until the end of the year. There is also a wide confidence interval ranging from negative NG price value to higher than 150 \$/MMBtu (Figure 9).

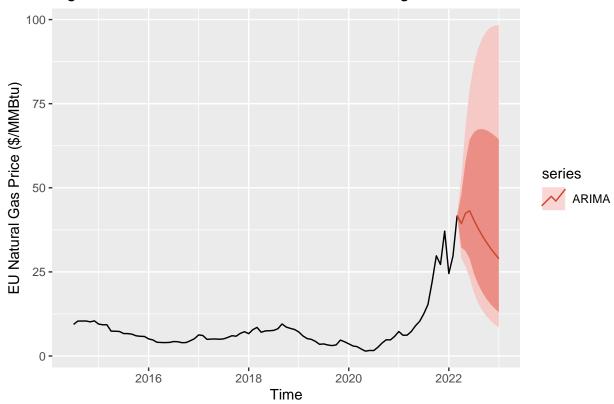


Figure 10. EU NG Price Forecast Result Using ARIMA

To have a sense of how the integration of exogenous variables affects the confidence interval change, we conducted a forecast using ARIMA models, even though their RMSE score is not as good as NN or STL+ETS. In contrast to the STL+ETS result, the ARIM forecast shows a peak in NG price around mid 2022, and a drop to around 30 \$/MMBtu at the end of the year. However, the confidence interval is still wide with a range from 10 to almost 100 \$/MMBtu (Figure 10).

```
ARIMA_Four_fit_withGDP_202212 <- auto.arima(ts_EU_price_shorten_201407_202203,
                             seasonal=FALSE,
                             lambda=0,
                             xreg=ts_EU_GDP_201407_202203
ARIMA Four for withGDP1 202212 <- forecast::forecast(ARIMA Four fit withGDP 202212,
                           xreg=ts_EU_GDP1_202203_202301,
                           h=9
ARIMA_Four_for_withGDP2_202212 <- forecast::forecast(ARIMA_Four_fit_withGDP_202212,
                           xreg=ts_EU_GDP2_202203_202301,
                           h=9
                           )
ARIMA_Four_for_withGDP3_202212 <- forecast::forecast(ARIMA_Four_fit_withGDP_202212,
                           xreg=ts_EU_GDP3_202203_202301,
                           h=9
autoplot(ts EU price shorten 201407 202203, main="Figure 11. EU NG Price Forecast Results

→ under 3 GDP Scenarios") +

  autolayer(ARIMA_Four_for_withGDP1_202212, series="4%",showgap=F) +
 autolayer(ARIMA_Four_for_withGDP2_202212, series="3% ",PI=FALSE,showgap=F) +
```

```
autolayer(ARIMA_Four_for_withGDP3_202212, series="1%",PI=FALSE,showgap=F)+
guides(colour=guide_legend(title="Scenarios (GDP Increase)"))+
ylab("EU Natural Gas Price ($/MMBtu)")
```

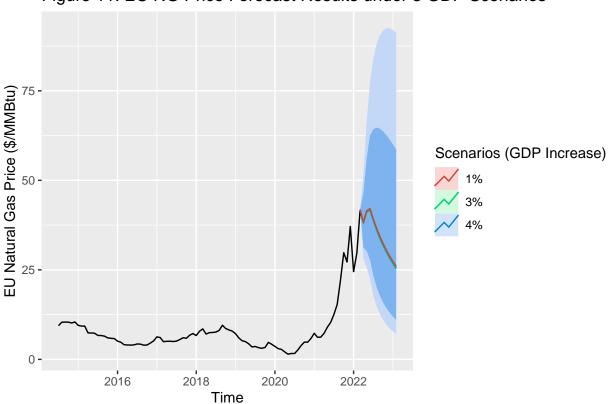


Figure 11. EU NG Price Forecast Results under 3 GDP Scenarios

The ARIMA model with GDP as a regressor shows a similar trend, leading to a slightly lower price forecast at the end of the year (25/MMbtu)thantheoriginal ARIMA model (30/MMBtu), and the forecast is still with high uncertainty (i.e. wide confidence interval). There are no significant forecast differences under the three scenarios we proposed, and neither of the three scenarios causes a significant change in the forecast result compared to the original ARIMA model (Figure 11).

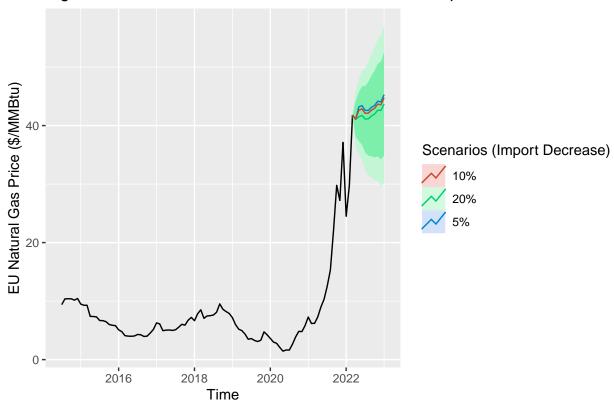


Figure 12. EU NG Price Forecast Results under 3 Import Scenarios

ARIMA models with import as a regressor show completely different results than the previous models. There are much smaller fluctuations and the overall trend is going upward, leading to an NG price forecast at around 45 \$/MMBtu at the end of 2022. The much narrower confidence interval indicates that the integration of import data improves the uncertainty of the price forecast. Three import scenarios lead to similar trends but slightly off level of forecasts. Import decrease by 5% returns the highest NG price forecast, while import decreased by 20% returns the lowest NG price forecast (Figure 12). Even though historical import and NG price data are negatively correlated, we found that the scenario with the lowest import decrease (5%) leads to the lowest NG price forecast, and the highest import decrease (20%) leads to the highest NG price forecast (Figure x). This indicates that in highly fluctuated time series, the already weak correlation between two variables might be polluted, thus the forecast results might not show the same correlation relationship shown in the historical data.

Conclusion Our project allows us to test the hypothesis at the beginning of our project: 1) Indeed, forecasting in a period of high uncertainty is a difficult exercise, even just forecasting 3 data points. In the test data, compared to Dec 2021, NG price decreased significantly in Jan 2022, increased in Feb 2022, and continued to increase in march 2022. Only two models predicted this general fluctuation correctly: NN and STL+ETS. However, even these two models misrepresented the magnitude of this fluctuation. The models' RMSE ranges from 7 to 16, confirming that predicting in a period with high volatility is difficult. 2) As expected, the Neural network with the lowest RMSE score at 7.54, turned out to be the best-performed model among all models in capturing the nonlinear trend in our time series. 3) Adding exogenous variables doesn't necessarily lead to an improvement in model performance. From our results, ARIMA models performed better with exogenous variables, while NN models performed worse with exogenous variables. Interestingly, we found that some exogenous variables can improve the certainty of the forecast while others cannot. For example, adding import data into the ARIMA model greatly narrowed the confidence level while adding GDP led to a slight shift in confidence interval but the width keeps the same. In conclusion, adding exogenous variables does not necessarily decreases the RMSE score, but may improve the certainty of the forecast. 4) The variable has a stronger correlation with NG price indeed affects the forecast results more,

but it doesn't necessarily improve the forecast performance. In both NN and ARIMA models, import has a greater impact on NG price forecast results. Import data also leads to greater change in RMSE compared to the ARIMA and NN without any exogenous variables but does not necessarily lower the RMSE score.