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STAT 433/833 Course Notes

Stochastic Processes

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Table of Contents

Abstract

These notes are intended as a resource for myself; past, present, or future students of this course, and anyone interested in the material. The goal is to provide an end-to-end resource that covers all material discussed in the course displayed in an organized manner. These notes are my interpretation and transcription of the content covered in lectures. The instructor has not verified or confirmed the accuracy of these notes, and any discrepancies, misunderstandings, typos, etc. as these notes relate to course's content is not the responsibility of the instructor. If you spot any errors or would like to contribute, please contact me directly.

1 September 6, 2018

1.1 Example 1.2 solution

Use the definition of the Markov property to show that

$$\begin{aligned} &P(X_{n+1} = x_{n+1} \mid X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_{n-k+1} = x_{n-k+1}, X_{n-k-1} = x_{n-k-1}, \dots, X_0 = x_0) \\ &\qquad = P(X_{n+1} = x_{n+1} \mid X_n = x_n), \quad k = 1, 2, \dots, n \end{aligned}$$

(i.e. we are missing one past observation).

Solution. Applying the definition of conditional probability, our expression is equivalent to

$$\frac{P(X_{n+1} = x_{n+1}, X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_{n-k+1} = x_{n-k+1}, X_{n-k-1} = x_{n-k-1}, \dots, X_0 = x_0)}{P(X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_{n-k+1} = x_{n-k+1}, X_{n-k-1} = x_{n-k-1}, \dots, X_0 = x_0)} = \frac{N}{D}$$

By the law of total probability

$$\begin{aligned} N &= \sum_{x_{n-k} \in S} P(X_{n+1} = x_{n+1}, \dots, X_{n-k} = x_{n-k}, \dots, X_0 = x_0) \\ &= \sum_{x_{n-k} \in S} P(X_{n+1} = x_{n+1} \mid X_n = x_n, \dots, X_{n-k} = x_{n-k}, \dots, X_0 = x_0) \times P(X_n = x_n, \dots, X_{n-k} = x_{n-k}, \dots, X_0 = x_0) \end{aligned}$$

By the Markov property

$$\begin{aligned} &= P(X_{n+1} = x_{n+1} \mid X_n = x_n) \sum_{x_{n-k} \in S} P(X_n = x_n, \dots, X_{n-k} = x_{n-k}, \dots, X_0 = x_0) \\ &= P(X_{n+1} = x_{n+1} \mid X_n = x_n) P(X_n = x_n, \dots, X_{n-k} \in S, \dots, X_0 = x_0) \end{aligned}$$

Since $X_{n-k} \in S$ is an event with probability 1

$$\begin{aligned} &= P(X_{n+1} = x_{n+1} \mid X_n = x_n) P(X_n = x_n, \dots, X_{n-k+1} = x_{n-k+1}, X_{n-k-1} = x_{n-k-1}, \dots, X_0 = x_0) \\ &= P(X_{n+1} = x_{n+1} \mid X_n = x_n) \cdot D \end{aligned}$$

The results follow.