Probability Theory and Statistics Inference

Lesson 1

Contents

Sample Espace

- Experiment: a repeatable procedure with well-defined possible outcomes.
- Examples:
    - Toss the coin, report if it lands heads or tails.
    - Toss the coin 3 times, list the results.
    - Measure the mass of a proton.
    - Count the number of taxis that pass for and avenue someday.

Sample Espace

- Set of all possible outcomes. We usually denote the sample space by Ω.
- Examples:
  - $\Omega = \{H, T\}$
  - $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.
  - $\Omega = [0, \infty)$
  - $\Omega = \{0, 1, 2, 3, 4, \ldots\}$.

Events

- A subset of the sample space.
- Let $\Omega = \{ \omega_1, \omega_2, \ldots, \omega_n \}$ be a sample space, then:
  - Impossible event: $\{\}$
  - Unitary event: $\{ \omega_i \}$
  - Safe event: Ω
- Satisfies all set theory.

Probability function

- For a sample space Ω a probability function P assigns to each outcome ω a number $P(\omega)$ called the probability of ω.

  $\Omega = \{ \omega_1 , \omega_2 , \ldots , \omega_n \}$

  - $0 \leq P(\omega) \leq 1$
  - The sum of the probabilities of all possible outcomes is 1

$$\sum_{j=1}^{n} P(\omega_j) = 1$$

Some rules of Probability

- For events A, L and R contained in a sample space Ω.
-
  - $P(A') = 1 - P(A)$.
  - If L and R are disjoint then: $P(L \cup R) = P(L) + P(R)$
  - If L and R are not disjoint, we have the inclusion-exclusion principle:

$$P(L \cup R) = P(L) + P(R) - P(L \cap R)$$

$$P(A) = \frac{\text{Number or favorable outcomes to A}}{\text{Total number of outcomes}}$$

Examples and Code

Example Consider the 2 events, A: 'X is a multiple of 2'; B: 'X is odd and less than 10'. Suppose $P(A) = .6$ and $P(B) = .25$.
(i) What is $A \cap B$?
(ii) What is the probability of $A \cup B$?

Example Let A, B and C be the events X is a multiple of 2, 3 and 6 respectively. If $P(A) = .6$, $P(B) = .3$ and $P(C) = .2$ what is $P(A \text{ or } B)$?

EJEMPLO

La demanda de dos productos A y B varía aleatoriamente en un rango de 1000 a 5000 kilogramos. El distribuidor decide bajar el precio de venta de ambos productos si la suma de sus demandas varía de 3000 a 5000 Kg. Calcular la probabilidad de que el precio de venta de ambos productos baje.

Examples and Code

Problem 3. (20 pts.) **Birthdays: counting and simulation.**
Ignoring leap days, the days of the year can be numbered 1 to 365. Assume that birthdays are equally likely to fall on any day of the year. Consider a group of n people, of which you are not a member. An element of the sample space Ω will be a sequence of n birthdays (one for each person).

(a) Define the probability function P for Ω..

(b) Consider the following events:
A: "someone in the group shares *your* birthday"
B: "some two people in the group share a birthday"
C: "some three people in the group share a birthday"

Carefully describe the subset of Ω that corresponds to each event.

(c) Find an exact formula for $P(A)$. What is the smallest n such that $P(A) > .5$?

(d) Justify why n is greater than $\frac{365}{2}$ without doing any computation. (We are looking for a short answer giving a heuristic sense of why this is so.)

(e) Use R simulation to estimate the smallest n for which $P(B) > .9$.

(f) Find an exact formula for $P(B)$.

Examples and Code

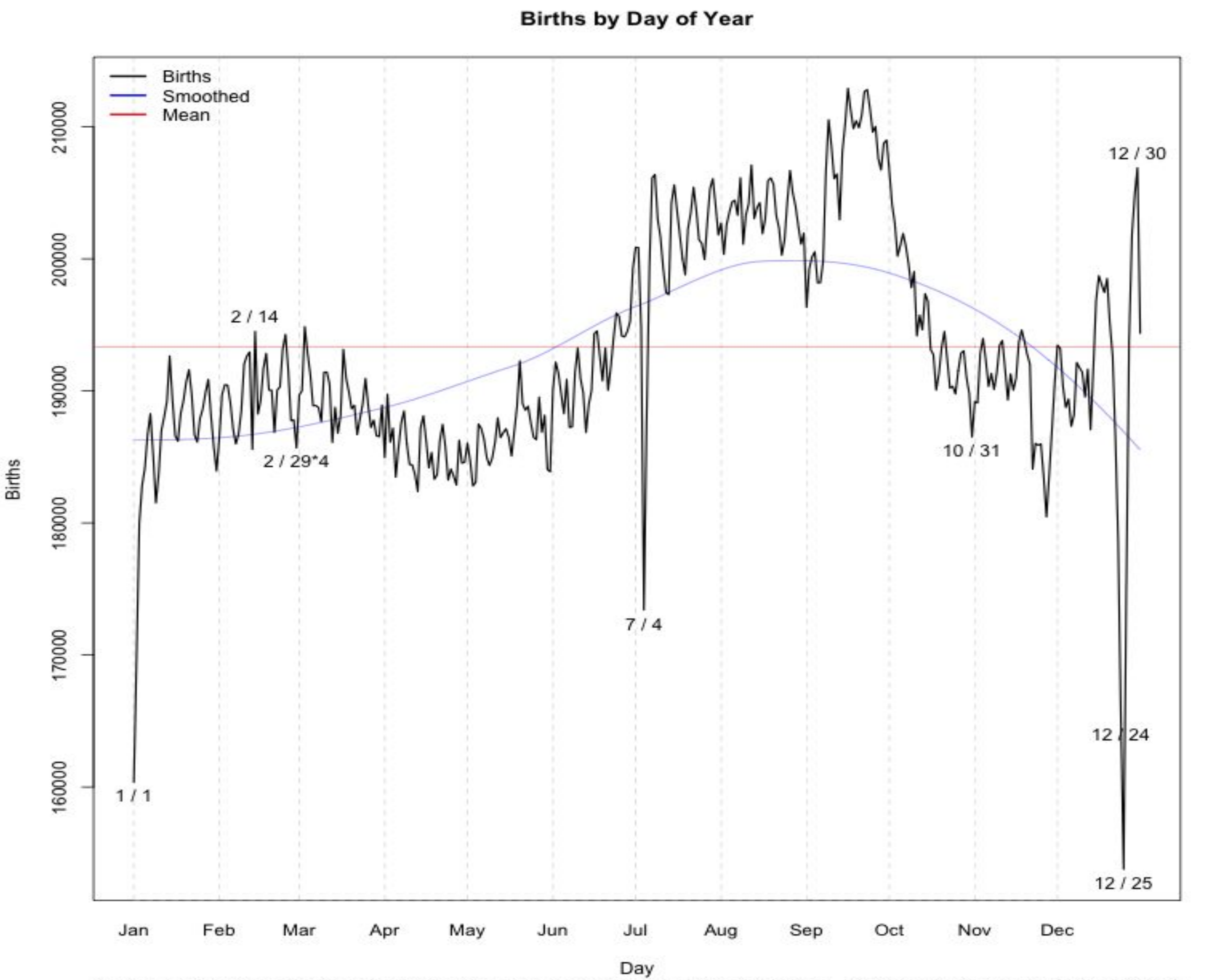
```
import sys

# main params
n = int(sys.argv[1])

def prob_B():
	total_days = 365
	decrease_days = 365
	output = 1

	for i in range(n):
		a = decrease_days / (total_days)
		decrease_days -= 1
		output *= a

	return (1.0 - output)
```



Source: National Vital Statistics System natality data, as provided by Google BigQuery. Graph by Chris Mulligan (chmullig.com)

Math Team

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