Continuous Random Variables and Continuous probability distributions

Continuous random variable

Is a continuous random variable where the data can take infinitely many values.

It is not possible to talk about the probability of the random variable assuming a particular value

Instead, we talk about the probability of the random variable assuming a value within a given interval.

Why Continuous

Anything physics

Time

Mass

Space

Flight

height

Temperature

Nearly continuous variables

Cost

stock

Rates

Comparison to Discrete

	Discrete	Continuous
Probability function	mass (pmf)	density (pdf)
≥ 0	$p(x) \geq 0$	$f(x) \geq 0$
∑ = 1	$\sum_x p(x) = 1$	$\int_{-\infty}^{\infty} f(x)dx = 1$

Which of the following are continuous random variables?

(1) The sum of numbers on a pair of two dice.

(2) The possible sets of outcomes from flipping ten coins.

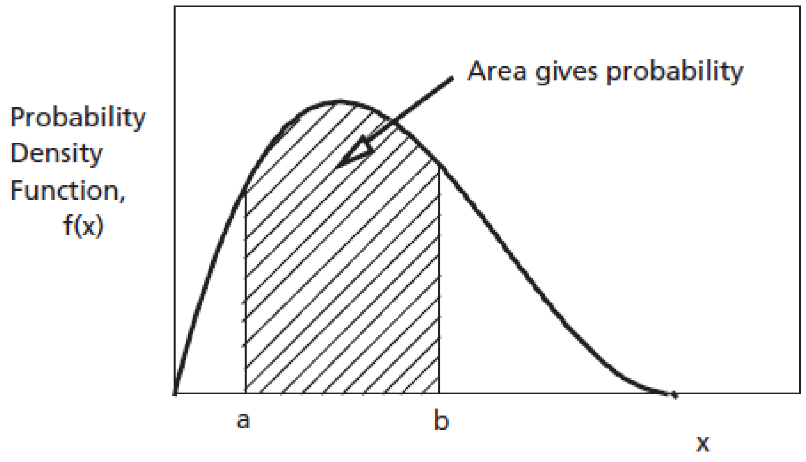
(3) The possible sets of outcomes from flipping (countably) infinite coins.

(4) The possible values of the temperature outside on any given day.

(5) The possible times that a person arrives at a restaurant.

Continuous Probability Distributions

The probability of the random variable assuming a value within some given interval from x1 to x2 is defined to be the area under the graph of the probability density function between x1 and x2.



Cumulative Distribution Function (CDF)

	Discrete	Continuous
PF → CDF	$\sum_{u \leq x} p(u)$	$\int_{-\infty}^{x} f(u)du$
CDF → PF	$p(x) = F(x) - F(x^*)$	$f(x) = F'(x)$

x^* - element preceding x

Cumulative Distribution Functions

The cumulative distribution function gives the cumulative value from negative infinity up to a random variable X and is defined by the following notation:

$$F(x) = P(X \leq x).$$

Exemplo

Una variable aleatoria X tiene la función de densidad $f(x) = c/(x^2 + 1)$, donde $-\infty < x < \infty$. (*a*) Hallar el valor de la constante *c*. (*b*) Hallar la probabilidad de que X^2 esté entre 1/3 y 1.

Hallar la función de distribución correspondiente a la función de densidad del Problema

Expectation

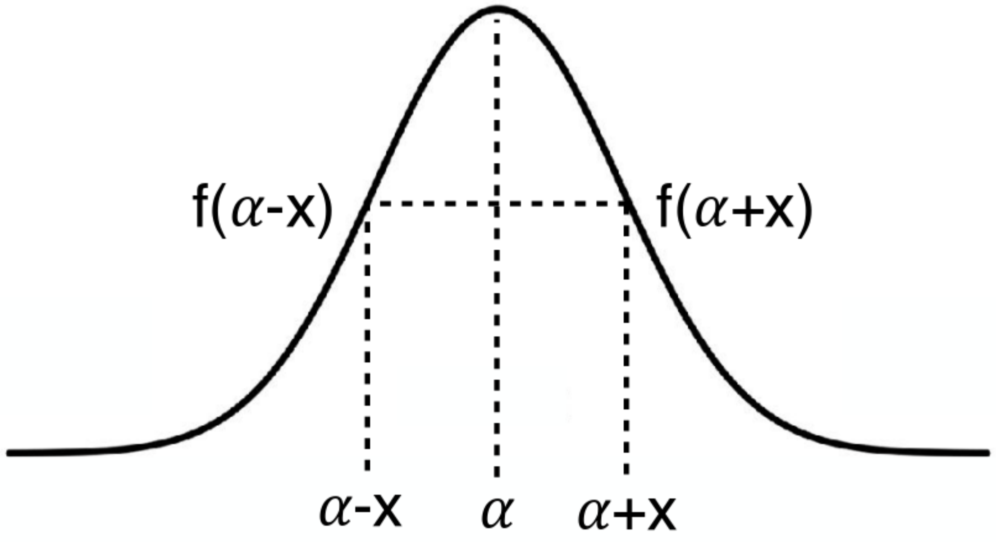
	Discrete	Continuous
EX	$\sum x \cdot p(x)$	$\int_{-\infty}^{\infty} x f(x) dx$

As discrete: Average of many samples

Properties

Support set = [a,b] $a \le EX \le b$

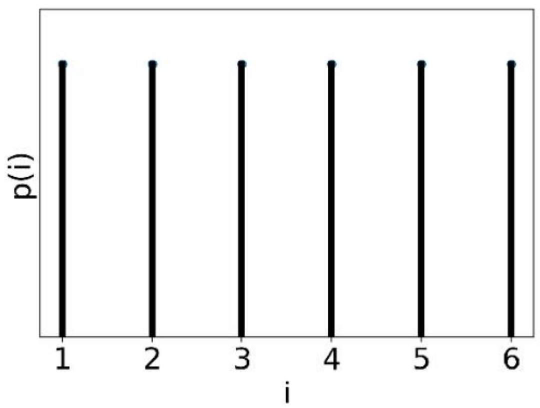
Symmetry If for some α, $f(\alpha+x)=f(\alpha-x)$ for all x then $EX = \alpha$



Exemplo: Expectation

$$
\begin{aligned}
E(X) &= \sum_{i=1}^{6} P(i) \cdot i \\
&= \sum_{i=1}^{6} \frac{1}{6} \cdot i \\
&= \frac{1+2+\ldots+6}{6} \\
&= \frac{1}{6}\frac{(1+6)\cdot 6}{2} \\
&= \frac{7}{2} = 3.5
\end{aligned}
$$ ✅

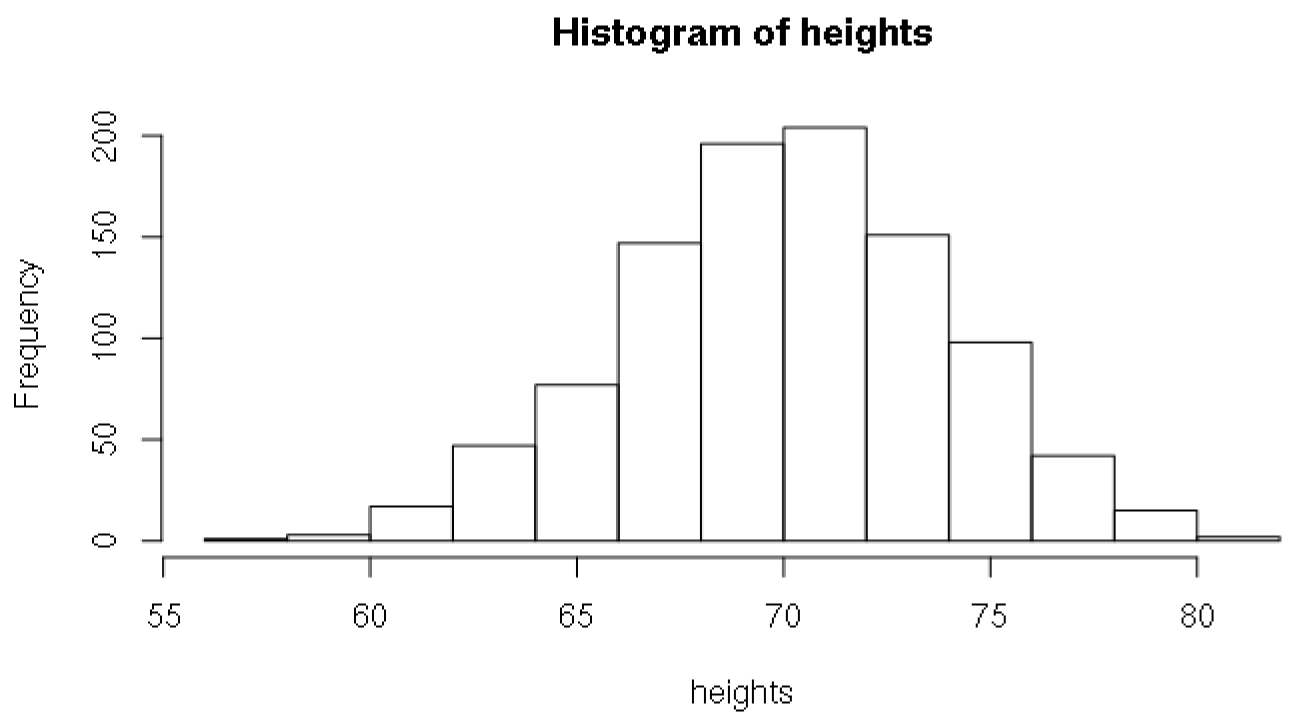
Fair Die



Variance

	Discrete	Continuous
$V(X) \triangleq E(X-\mu)^2$	$\sum_x p(x)(x-\mu)^2$	$\int_{-\infty}^{\infty} f(x)(x-\mu)^2 dx$

Exemplo: Normal Distribution



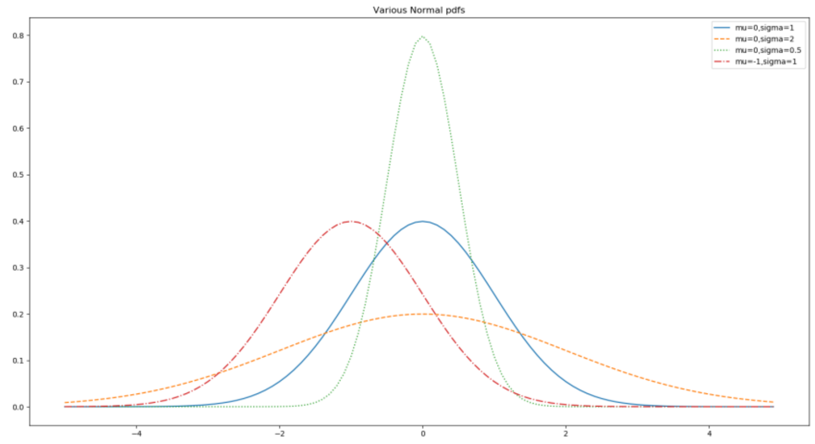
Exemplo: Normal Distribution

A normal distribution in a variate X with mean μ(mu) and variance Σ(sigma)is a statistic distribution with probability density function

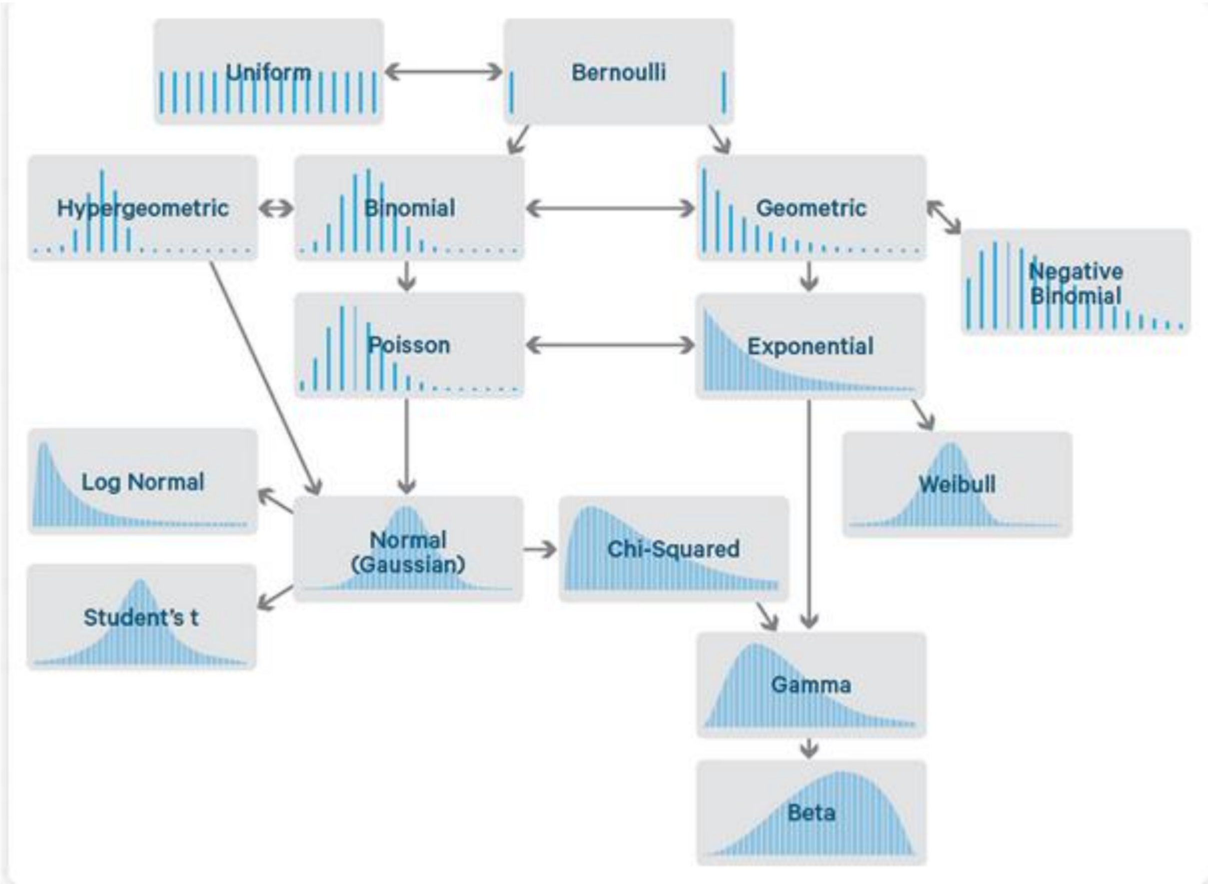
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

Exemplo: Normal Distribution

```
import math, random
import matplotlib.pyplot as plt


def normal_pdf(x, mu=0, sigma=1):
    sqrt_two_pi = math.sqrt(2*math.pi)
    return (math.exp(-(x-mu)**2/2/sigma**2)/(sqrt_two_pi*sigma))



xs  =  [x  /  10.0   for x  in  range(-50,  50)]
plt.plot(xs,[normal_pdf(x,sigma=1)  for x   in  xs],'-',label='mu=0,sigma=1')
plt.plot(xs,[normal_pdf(x,sigma=2)  for x   in  xs],'--',label='mu=0,sigma=2')
plt.plot(xs,[normal_pdf(x,sigma=0.5)    for x   in  xs],':',label='mu=0,sigma=0.5')
plt.plot(xs,[normal_pdf(x,mu=-1)           for x   in  xs],'-.',label='mu=-1,sigma
plt.legend()
plt.title("Various Normal pdfs")
plt.show()
```



Probability distribution with python



Example: The Binomial Distribution

Applications

Positive responses to a treatment

Faulty components

Rainy days in a month

Delayed flights

Example: The Binomial Distribution

The binomial distribution consists of the probabilities of each of the possible numbers of successes on N trials for independent events that each have a probability of p of occurring.

- The General Binomial Probability Formula:

$$P(k \text{ out of } n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{(n-k)}$$

Example: The Binomial Distribution

```
import numpy as np
import scipy.stats as ss
import matplotlib.pyplot as plt

X = ss.binom(25,0.5)
x = np.arange(10)

plt.plot(x,X.pmf(x),"bo")
plt.vlines(x,0,X.pmf(x),"b")
plt.show()
```