Dependent and independent events

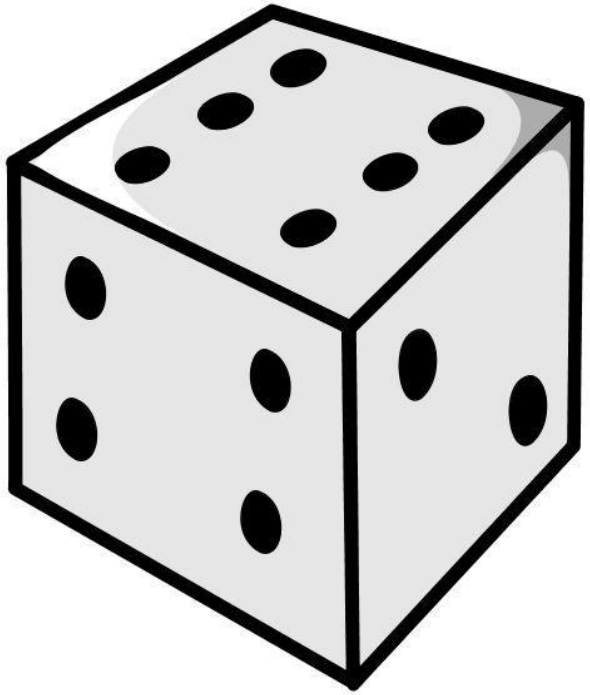
https://jose2008.github.io/ucspdatasciencegroup/

Events

Event Subset of sample space Ω

Die Ω={1,2,3,4,5,6}

Event	Name
{1,2,3,4,5,6}	Ω(certain)
{2,4,6}	Even
{1,4}	Square
{5,6}	> 4, ≥ 5

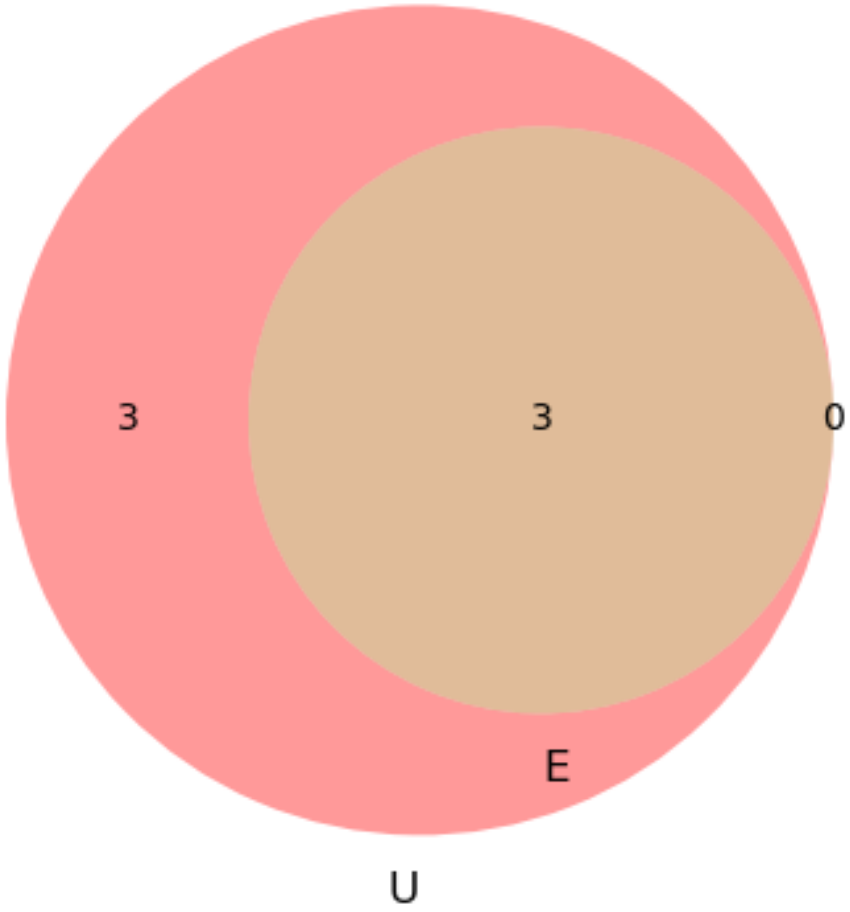


```python
import matplotlib.pyplot as plt
import matplotlib_venn as venn

U = {1, 2, 3, 4, 5, 6}
E = {1, 2, 3}

venn.venn2([U,E], set_labels=('U','E'))
plt.show()

```



Two events are independent if the result of the second event is not affected by the result of the first event. If *A* and *B* are independent events, the probability of both events occurring is the product of the probabilities of the individual events.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Exemplo

- **Owning a dog and growing your own herb garden.**
- **Winning the lottery and running out of milk.**
- **Buying a lottery ticket and finding a penny on the floor (your odds of finding a penny does not depend on you buying a lottery ticket).**
- **Taking a cab home and finding your favorite movie on cable.**
- **Getting a parking ticket and playing craps at the casino.**

Two events are dependent if the result of the first event affects the outcome of the second event so that the probability is changed. The probability of both events occurring is the product of the probabilities of the individual events:

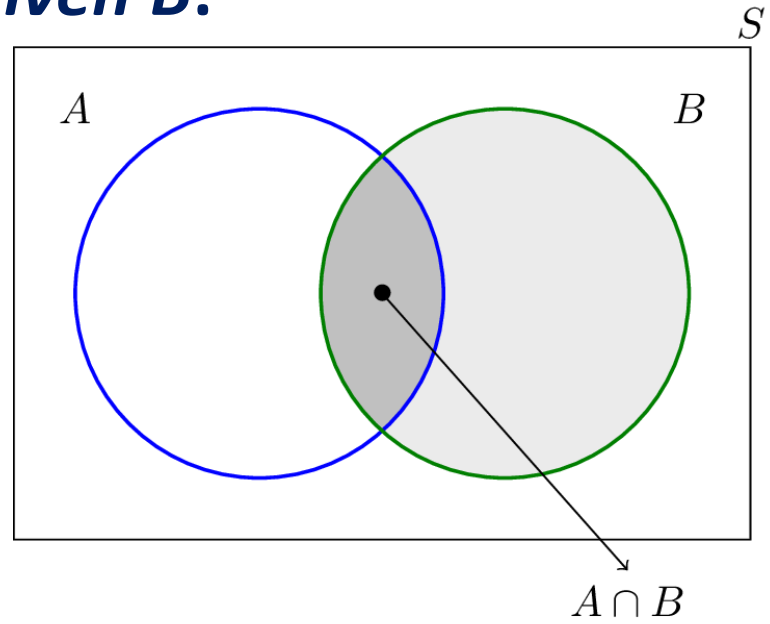
$$P(A \text{ and } B) = P(A) \cdot P(B/A)$$

Exemplo

- **Robbing a bank and going to jail.**
- **Not paying your power bill on time and having your power cut off.**
- **Boarding a plane first and finding a good seat.**
- **Buying ten lottery tickets and winning the lottery. The more tickets you buy, the greater your odds of winning.**
- **Driving a car and getting in a traffic accident.**

Marginal Probability, Conditional Probability

The *conditional probability* of an event A is the probability that the event will occur given the knowledge that an event B has already occurred. This probability is written *P(A|B)*, notation for the *probability of A given B*.



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Exemplos

EJEMPLO 1.22. Hallar la probabilidad de que en un sólo lanzamiento de un dado resulte un número menor que 4, (*a*) no se da ninguna otra información, (*b*) se da que el lanzamiento resultó en un número impar.

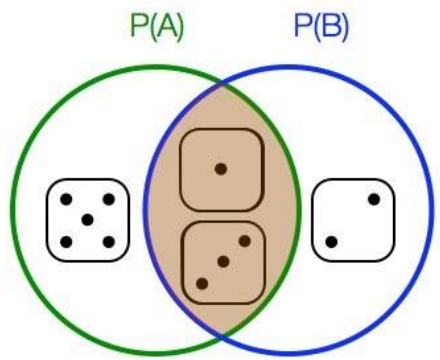
Conditional Probability

What is the Probability of

rolling a dice and it's value is less than 4

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

knowing that the value is an odd number



Exemplo

```python
import random, math

def random_kid():
    return random.choice(["boy","girl"])


both_girls = 0
older_girl = 0
either_girl = 0

random.seed(0)
for _ in range(10000):
    younger = random_kid()
    older = random_kid()
    if older == "girl":
        older_girl += 1
    if older == "girl" and younger == "girl":
        both_girls += 1


print("P(both_girls|older): " , both_girls/older_girl)
```

Properties of Conditional Probability

Because conditional probability is just a probability, it satisfies the three axioms of probability. That is, as long as $P(B) > 0$:

1. $P(A \mid B) \geq 0$
2. $P(B \mid B) = 1$
3. **If $A_1, A_2, \ldots A_k$ are mutually exclusive events, then $P(A_1 \cup A_2 \cup \ldots \cup A_k \mid B) = P(A_1 \mid B) + P(A_2 \mid B) + \ldots + P(A_k \mid B)$ and likewise for infinite unions.**

Marginal Probability

The probability of an event occurring (p(A)), it may be thought of as an unconditional probability. It is not conditioned on another event. Example: the probability that a card drawn is red (p(red) = 0.5). Another example: the probability that a card drawn is a 4 (p(four)=1/13).