Safe and Near Optimal Controller Synthesis for Stochastic Hybrid Systems

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***Abstract*—Stochastic hybrid systems allow to model the interaction between continuos dynamics, discrete dynamics and probabilistic uncertainty. Because of their versatility, stochastic hybrid systems have emerged as a powerful framework for capturing the intricacies of complex systems. Motivated by this, considerable research effort has been devoted to the development of modeling analysis and control methods for stochastic hybrid systems.**

***Index Terms*—SHG, formatting, style, styling, insert**

I. Introduction

Hybrid systems are widely used in engineering applications and its importance has grown up considerably these last years, because of their ease of implementation for controlling cyber-physical systems. A switched systems is a set of dynamical systems, each with its own dynamical behaviour controlled by a parameter mode u whose values are in a finite set. However, due to the composition of many switched systems together, the global switched systems has a number of modes and dynamics which increases exponentially. Switched systems have numerous applications in control of mechanical systems, the automotive industry, and many other fields.

II. Preliminaries

In this section we summaries the basic definition of stochastic hybrid games, their concrete and symbolic semantics and the safety system implemented underlying the currently distributed version of UPPAAL. [1]

A. Sthocastic Hybrid Game

Definitiom 1*(Stochastic Hybrid Game)*. A stochastic hybrid game $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$ where:

- *$\mathcal{C}$ is a controller with a finite set of(controllable) modes C .*
- *$\mathcal{U}$ is a controller with a finite set of(uncontrollable) modes U .*
- *$X = \{x_1, ..., x_n\}$ is a finite set of continuos (real-valued) varibales,.*
- *for each $c \in C$ and $u \in U$, $\mathcal{F}_{c,u} : \mathbb{R}_{>0} \times \mathbb{R}^X \to \mathbb{R}^X$ is the flow-function that describe the evolution of the continuos variables over time in the combined mode (c,u), and*
- *δ is a family of density functions, $\delta_\gamma(\tau, u')$ is the density that $\mathcal{U}$ in the global configuration $\gamma = (c, u, v)$ will change to the uncontrollable mode u' after a delay of τ^4.*

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B. Sthocastic Hybrid Systems Safety Definitions

In this part is presented a method based on correction by design of discrete linear switched system in the time. the method consist of given a objective region *R* of state space, the method built a set *S* and a control that guide any element from *S* a *R*. This method works in an iterative way to back to reach the region *R*. The method can also be used for synthesize a stability control that is keep inside of R, whole states start in *R*.

In order to find safety pattern to our system we use a decomposition algorithm, which explore in a binary way regions which guarantee a beviouring inside to restrictions. [2]

Problem 1*(Control Synthesis Problem)*. Let us consider a sampled Hybrid System. Given three sets R,S and B, with $R \cup B \in S$ and $R \cap B = \varnothing$ find a rule $\sigma(.)$ such that, for any $x(0) \in R$.

- *τ-stability: $x(t)$ return in R infinitely often, at some multiples of sampling time τ.*
- *safety: $x(t)$ always stays in S/B.*

Problem 2 *((R,S) - Stability Problem)*. Given a switched system, a set of recurrence $\mathbb{R}^n$ and a safe set $S \subset \mathbb{R}^n$,find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$ the following holds:

- *Recurrence in* R*:there are a monotonically strictly increasing sequence of (positive) integers $k_t, t \in \mathbb{N}$ such that for all $t \in \mathbb{R}^n, \phi(k_l\tau; t_0, x^0, \sigma, w) \in \mathbb{R}$.*
- *Stability in* S*: for all $t \in \mathbb{R}^n, \phi(t; t_0, x^0, \sigma, w) \in S$.*

Problem 3 *((R_1, R_2, S) - Reachability problem). Given a switched system of the form shown above, two sets $R_1 \subset \mathbb{R}^n$*

and $R_2 \subset \mathbb{R}^n$ and a safety set $S \subset \mathbb{R}^n$, find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$, the following holds:

- *Reachability from R_1 to R_2: there exists an integer $k \in \mathbb{N}$ such that we have $\phi(k_l\tau; t_0, x^0, \sigma, w) \in R_2$.*
- *Stability in S: for all $t \in \mathbb{R}^+, \phi(t; t_0, x^0, \sigma, w) \in S$.*

III. Case Study: Solar Water Heating

A. Solar Water Heating as Sthocastic Hybrid Game

The hybrid solar water heating scenario with 12 modes of operations is defined like this: $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$, where the controller $\mathcal{C}$ has a finite set of controllable modes, given by resistance state $r \in \mathbb{B} = \{0, 1\}$ and piston movement $p \in \{-1, 0, 1\}$. The environment $\mathcal{U}$ has a finite set of uncontrollable modes $v \in \mathbb{B}$, that means the valve state for opening/closing water aperture. We assume that $\mathcal{U}$ given δ can switch among modes with equal probability at every period τ. The state variables in $\mathcal{X}_{(t)}$ are given by $\{T_{(t)}, V_{(t)}, E_{(t)}\}$, tank temperature, tank volumen and energy consumption respectively. Also Disturbance effect is considered in the dynamical system such as environment temperature, water input temperature and irradiance as a uncontrollable continuos environment variables.

$$\frac{d}{dt}T_{(t)} = -\frac{1}{V_{(t)}}c_1(T_{(t)} - T_{env(t)}) - \frac{\mathbf{v}}{V_{(t)}}c_2(T_{(t)} - T_{in(t)}) + \\ + \mathbf{r}\frac{c_3 Q_{aux}}{V_{(t)}} + \frac{c_4 I_{env(t)}}{V_{(t)}} \quad (1)$$

$$\frac{d}{dt}V_{(t)} = -k\mathbf{p}; \quad (2)$$

$$\frac{d}{dt}E_{(t)} = \mathbf{r}Q_{aux}; \quad (3)$$

In equation 1. the paremeters, $\{c_1, c_2, c_3, c_4\}$ remains constant in time. In equation 2, *k* is the piston velocity.

B. Discrete Model

We use uppaal stratego to model our system:

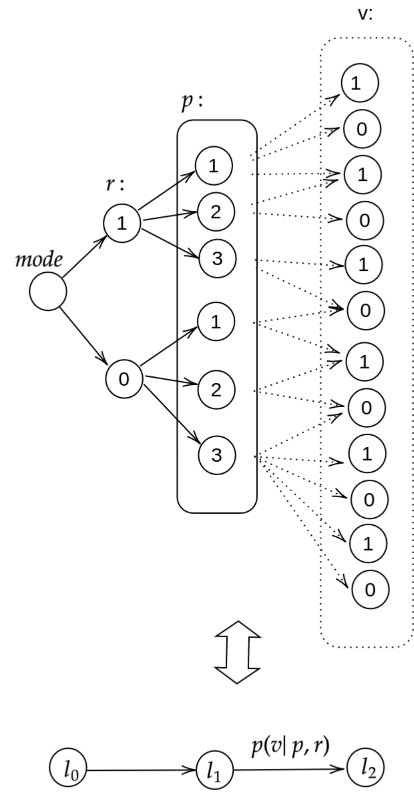

Fig. 1. Explotion Transition system

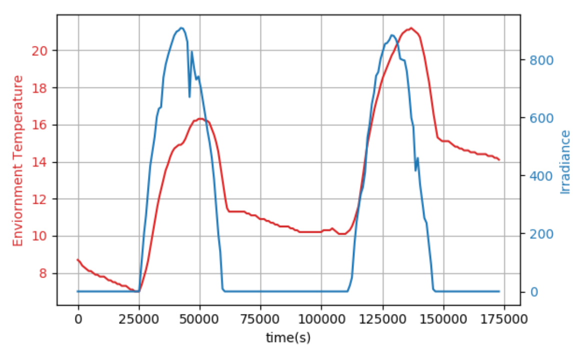

Fig. 2. Controllable and uncontrollable variables signals

IV. Experiments

A. Enviornment data

In our system we can notice some variables wich are influenced by the enviorment, in order to be more realistic, we have used data from the spain sity named sevilla. Data such as temperature and irradiance is got during 1 year which a timestep of 15 minutes.

B. Solar Water Heating simulation

The is managed by discrete controllable and uncontrollable variables which are defined by our controller and probabilities events.

Fig. 3. Controllable and uncontrollable variables signals

Considering the disturbance,controllable modes and uncontrollable modes we get asd a result:

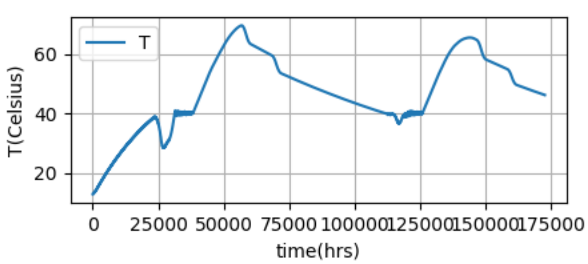

Fig. 4. Container tempeature state in time

Each controller has its performance, a simple controller is implemented as a reference to our controller synthesis. As it is shown we can notice what kind of performance is provided over our case of study.

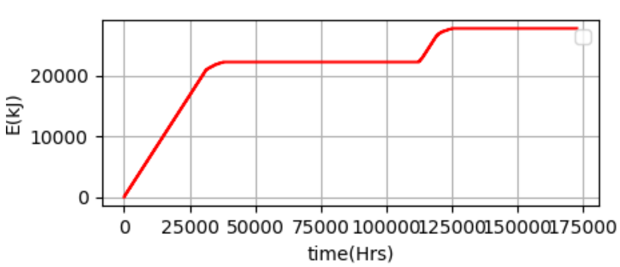

Fig. 5. Energy consumption in time

As decribed we need to guarantee a safety system in that sense we import some functionalities through Ibex C++ it imples to work with intervals in order to find a safety patterns. In this case it is basically restricted by simple constraints.

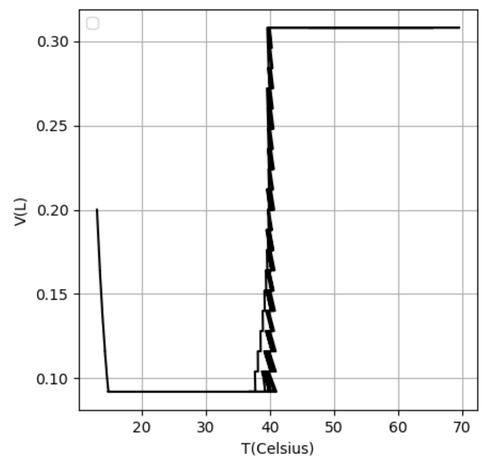

Fig. 6. Safety state space temperature-Volumen

C. *Strategy Controller Synthesis*

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D. *Figures and Tables*

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TABLE TYPE STYLES

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V. CONCLUSIONS AND FUTURE WORK

VI. REFERENCES

REFERENCES

[1] Kim G Larsen, Marius Mikučionis, Marco Muniz, Jiří Srba, and Jakob Haahr Taankvist. Online and compositional learning of controllers with application to floor heating. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 244–259. Springer, 2016.

[2] Adrien Le Coënt, Julien Alexandre dit Sandretto, Alexandre Chapoutot, and Laurent Fribourg. An improved algorithm for the control synthesis of nonlinear sampled switched systems. *Formal Methods in System Design*, pages 1–21, 2017.