Chapter 1

Background

For better understanding we introduce some topics before to analyze the problem. Each sections describe a general idea about it.

1.1 Switched Systems

Hybrid Systems are loosely defined as dynamical system whose state has two components, one of which evolves in a continuous set such as $\mathbb{R}$ while the other evolves in a discrete set such as $\mathbb{N}$ according to some transition logic based rule. The simplest model of a hybrid systems is given by: (Le Coënt et al., 2017)

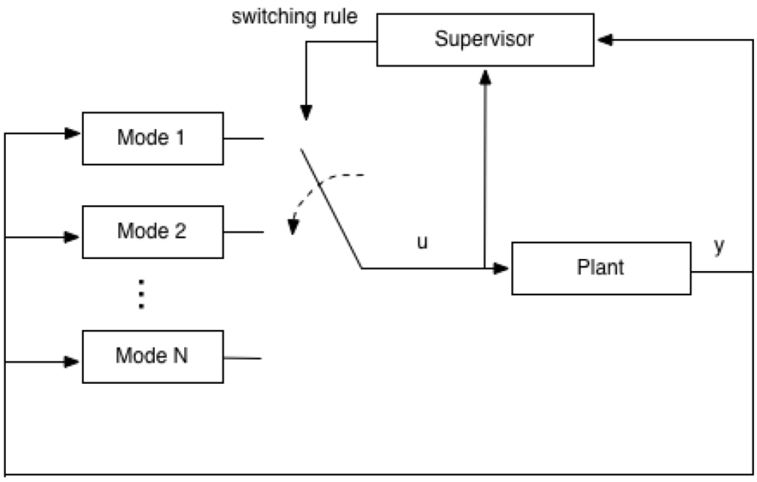

Figure 1.1: atched System Schematic

The figure above can be expressed as mathematical equation like this:

$$\dot{x} = f_{\sigma(t)}(x(t)), x \in \mathbb{R}^n,$$

$$\sigma(t) = lim\phi(x(\tau), \sigma(\tau)), \sigma \in \mathbb{N},$$

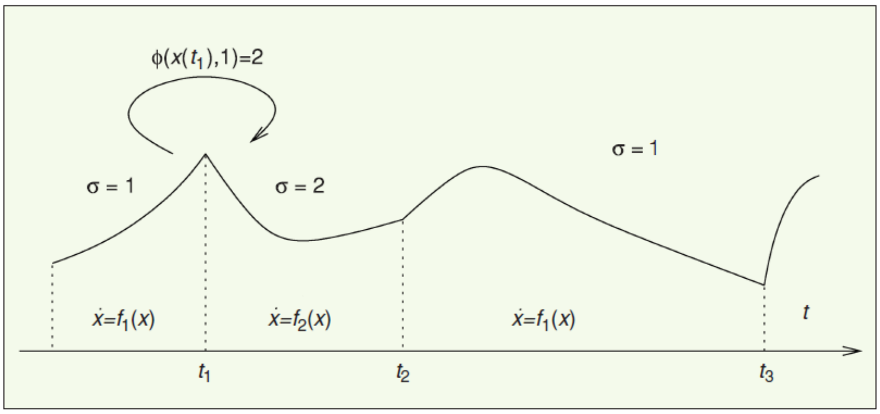

Figure 1.2: Trajectory of a hybrid system. The switching signal $\sigma(t)$ takes on integer values that change at discrete-time instances.(Liberzon, 2003)

1.2 Safety and Reachability

In this part is presented a method based on correction by design of discrete linear switched system in the time. the method consist of given a objective region R of state space, the method built a set S and a control that guide any element from S a R. This method works in an iterative way to back to reach the region R. The method can also be used for synthesize a stability control that is keep inside of R, whole states start in R. Le Coënt et al. (2016)

Problem 1 *((R,S) - Stability Problem).* Given a switched system as shown in figure before, a set of recurrence $\mathbb{R}^n$ and a safe set S

$\subset \mathbb{R}^n$,find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$ the following holds:

- *Recurrence in* R*:there are a monotonically strictly increasing sequence of (positive) integers* $k_t, t \in \mathbb{N}$ *such that for all* $t \in \mathbb{R}^n, \phi(k_l\tau; t_0, x^0, \sigma, w) \in \mathbb{R}$.
- *Stability in* S*: for all* $t \in \mathbb{R}^n, \phi(t; t_0, x^0, \sigma, w) \in S$.

Problem 2 *((R_1, R_2, S) - Reachability proglem). Given a switched system of the form shown above, two sets $R_1 \subset \mathbb{R}^n$ and $R_2 \subset \mathbb{R}^n$ and a safety set $S \subset \mathbb{R}^n$, find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$, the following holds:*

- *Reachability from R_1 to R_2: there exists an integer $k \in \mathbb{N}$ such that we have $\phi(k_l\tau; t_0, x^0, \sigma, w) \in R_2$.*
- *Stability in S: for all $t \in \mathbb{R}^+, \phi(t; t_0, x^0, \sigma, w) \in S$.*

1.3 Switched Controller synthesis

Definition 1*(Sthocastic Hybrid Game).* A stochastic hybrid game

Problem 3*(Control Synthesis Problem).* Let us consider a sampled switched system. Given three sets R,S and B, with $R \cup B \in S$ and $R \cap B = \varnothing$ find a rule $\sigma(.)$ such that, for any $x(0) \in R$.

- *τ-stability: $x(t)$ return in R infinitely often, at some multiples of sampling time τ.*
- *safety: $x(t)$ always stays in S/B.*

Bibliography

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Liberzon, D. (2003). *Switching in systems and control*. Springer Science & Business Media.