Safe and near optimal controller synthesis for Switched Systems

Richard Valentín Yantas Alcantara

Advisor: PhD. Marco Muñiz

Committee Members:

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To my parents, Mario and Maria, who never stop giving of themselves in countless ways. To my brothers and sister, who encourage and support me.

Abbreviations

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Abstract

Switched systems are used in hybrid systems that are most common in engineering applications(*e.g.*, logic-dynamic controllers, internet congestion even physical systems with impact,*etc.*). Hybrid systems have been used to model several cyber-physical systems.

In This work, we propose a pipeline to solve switched systems, providing safety and optimal functions to different cyber-physical systems, we propose a solar water heating as a case of study.We evaluate our results comparing with traditional controllers. In addition, we present a system called UPPAL STRATEGO that will be used in this work.

Keywords: Control Theory, Game Theory, Machine Learning, Model Checking.

Resumen

Los sistemas commutables son ampliamente usados en sistemas híbridos que a su vez son muy comunes en aplicaciones de ingeniería(ejm: controladores logicos-dinamicos, congestión de internet, sistemas con impacto físico, etc). Los sistemas hibridos han sido usados para modelar varios sistemas ciber-físicos.

En este documento, proponemos un *Solar Water Heating* como nuestro caso de estudio. Evaluamos nuestros resultados comparando con controladores tradicionales. Ademas, presentamos un sistema llamado *UPPAL STRATEGO* que será usado en este trabajo.

Palabras clave: Teoría de juegos, Aprendizaje automático, Verificación de modelos, Teoría de control.

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Chapter 1

Introduction

1.1 Motivation and Context

Switched systems are widely used in engineering applications and its importance has grown up considerably these last years, because of their ease of implementation for controlling cyber-physical systems.A switched systems is a set of dynamical systems, each with its own dynamical behaviour controlled by a parameter mode u whose values are in a finite set U(See Liberzon (2003)). However, due to the composition of many switched systems together, the global switched systems has a number of modes and dynamics which increases exponentially.

Switched systems have numerous applications in control of mechanical systems, the automotive industry, and many other fields.

1.2 Problem Statement

Nowadays, there is a large number of methods to solve switched systems; however, it does not have guarantee in safety. For that reason, we propose a new approach to solve switched systems.

1.3 Objectives

General Objective

Our main objective is to propose a pipeline to solve switched systems guarantee safety and optimal synthesis controllers.

Specific Objectives

To achieve our main objective, we have the following specific objectives:

- Define a case of study and get its mathematical model.
- Implement a safety controller for switched systems.
- Optimize the switched systems using model checking techniques.
- Evaluate each step of the pipeline and compare with traditional approaches.

1.4 Contributions

This thesis proposes a novel approach to solve switched systems guaranteeing safety and optimal controller synthesis. This procedure to consider stochastic variables as an input for the system. Our contributions are related to each part and are detailed below.

- Define a synthesis controller with the next:.
  - We propose a method to guarantee safe controller. This methods consider three regions to have reachability and safety in the system.
  - We also demonstrate the utility of the proposed method comparing with traditional methods.
- Analyze the stability for our case of study study.
  - analyze the zeros and poles for the system.

1.5 Outline

This thesis document is divided into six chapters. After this introduction and problem formulation, in Chapter 2 we survey the literature about the recent research in safety controllers and optimal controller synthesis. Chapter 3 presents some basic concepts about the dynamical systems, traditional controllers, switched systems,stability criteria and optimal controller technique. Next, in Chapter x we describe in detail the corpus, techniques used by our pipeline and their evaluation results. Finally, the limitations, future works, and conclusions of this work are presented in Chapter z.

Chapter 2

Related Works

Our work consist on optimize and guarantee safety over controllers based on switched systems.

2.1 Online and Compositional Learning of Controllers with Application to Floor Heating

This work has proposed one method to perform *optimal controller synthesis for stochastic hybrid switched systems e.g.*, a floor heating system in a house (Larsen et al., 2016). Is proposed a general and scalable methodology for controller synthesis for such systems. Instead of off-line synthesis of a controller for all possible input temperatures and an arbitrary time horizon, is proposed an on-line synthesis methodology, where it periodically compute the controller only for the near future based on the current sensor readings.

2.2 An Improved Algorithm for the Control Synthesis of Nonlinear Sampled Switched Systems

In this paper is presented and algorithm for the control synthesis for nonlinear switched systems using and existing procedure of state-space and made available for nonlinear systems with the help of guaranteed integration, the algorithm has been improved to be able to consider longer patterns of modes with a better running approach.

This approach permits to deal with stability, reachability, safety and forbidden region constraints. The approach was numerically validated on several examples taken from the literature.(Le Coënt et al., 2017)

2.3 Distributed Synthesis of State-Dependent Switching Control

In this part is presented a correct-by-design method of state-dependent control synthesis for linear discrete-time switching systems. Given an objective region R of the state space, the method builds a region S. (Le Coënt et al., 2016)

2.4 Final Considerations

This chapter presented some recent proposals related to our thesis work. Some research works have been focused on analyzing safety controller over switched systems. On the other hand, other works have focused on optimize controller synthesis using some techniques related to model checking.

The next chapter will present some concepts needed to understand better our work, those concepts are related to the modelling physical system over our case of study and represent as a state space.

Chapter 3

Background

For better understanding we introduce some topics before to analyze the problem. Each sections describe a general idea about it.

3.1 Switched Systems

Hybrid Systems are loosely defined as dynamical system whose state has two components, one of which evolves in a continuous set such as $\mathbb{R}$ while the other evolves in a discrete set such as $\mathbb{N}$ according to some transition logic based rule. The simplest model of a hybrid systems is given by: (Le Coënt et al., 2017)

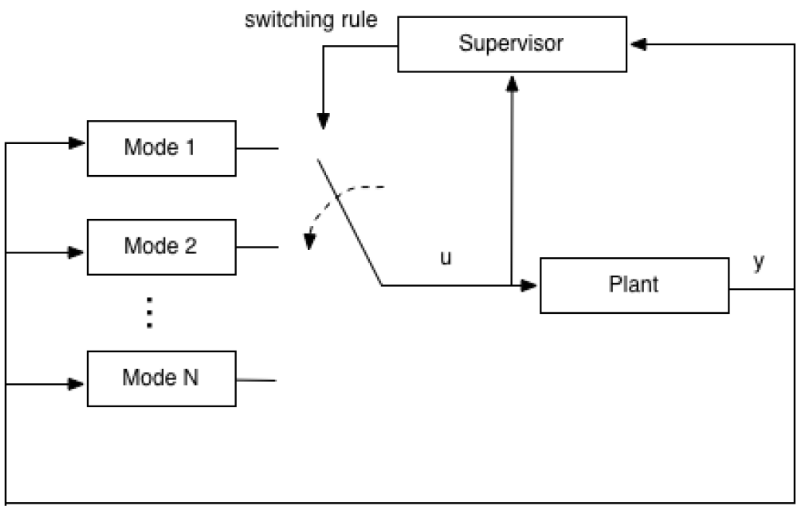

Figure 3.1: Switched System Schematic

The figure above can be expressed as mathematical equation like this:

$$\dot{x} = f_{\sigma(t)}(x(t)), x \in \mathbb{R}^n,$$

$$\sigma(t) = lim\phi(x(\tau), \sigma(\tau)), \sigma \in \mathbb{N},$$

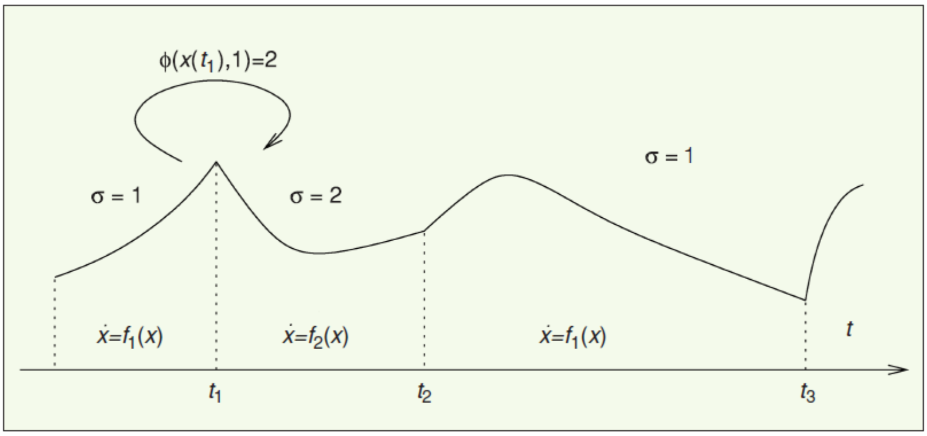

Figure 3.2: Trajectory of a hybrid system. The switching signal $\sigma(t)$ takes on integer values that change at discrete-time instances.(Liberzon, 2003)

3.2 Safety and Reachability

In this part is presented a method based on correction by design of discrete linear switched system in the time. the method consist of given a objective region R of state space, the method built a set S and a control that guide any element from S a R. This method works in an iterative way to back to reach the region R. The method can also be used for synthesize a stability control that is keep inside of R, whole states start in R. Le Coënt et al. (2016)

Problem 1 *((R,S) - Stability Problem).* Given a switched system as shown in figure before, a set of recurrence $\mathbb{R}^n$ and a safe set S

$\subset \mathbb{R}^n$,find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$ the following holds:

- *Recurrence in* R*:there are a monotonically strictly increasing sequence of (positive) integers* $k_t, t \in \mathbb{N}$ *such that for all* $t \in \mathbb{R}^n, \phi(k_l\tau; t_0, x^0, \sigma, w) \in \mathbb{R}$.
- *Stability in* S*: for all* $t \in \mathbb{R}^n, \phi(t; t_0, x^0, \sigma, w) \in S$.

Problem 2 *((R_1, R_2, S) - Reachability proglem). Given a switched system of the form shown above, two sets $R_1 \subset \mathbb{R}^n$ and $R_2 \subset \mathbb{R}^n$ and a safety set $S \subset \mathbb{R}^n$, find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$, the following holds:*

- *Reachability from R_1 to R_2: there exists an integer $k \in \mathbb{N}$ such that we have $\phi(k_l\tau; t_0, x^0, \sigma, w) \in R_2$.*
- *Stability in S: for all $t \in \mathbb{R}^+, \phi(t; t_0, x^0, \sigma, w) \in S$.*

3.3 Controller synthesis

Problem 3*(Control Synthesis Problem).* Let us consider a sampled switched system. Given three sets R,S and B, with $R \cup B \in S$ and $R \cap B = \varnothing$ find a rule $\sigma(.)$ such that, for any $x(0) \in R$.

- *τ-stability: $x(t)$ return in R infinitely often, at some multiples of sampling time τ.*
- *safety: $x(t)$ always stays in S/B.*

Chapter 4

Hybrid Solar Water heating

In this part is taken as case of study an *Hybrid Solar Water Heating*, we propose three parts in the pipeline.

4.1 System Modelling

4.1.1 System Setup

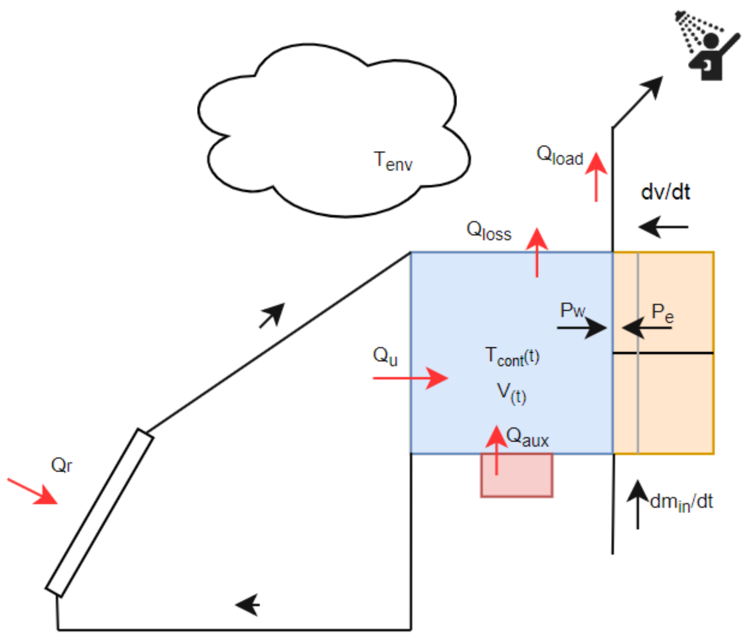

Figure 4.1: Hybrid Solar Water Heating Diagram

4.1.2 State Space Representation

$$\frac{d}{dt}\mathbf{x}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + B_w\mathbf{w}(t)$$

Switched Systems:

$$\frac{d}{dt}\mathbf{x}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t) + B_w(t)\mathbf{w}(t)$$

Where:

- $\mathbf{x}$ is the state vector
- $\mathbf{u}$ is the input vector
- A is the state matrix
- B is the input matrix

4.1.2.1 Differential Equations

$$\begin{aligned}
\tfrac{d}{dt}T_{cont}(t) &= \tfrac{k_1 \cdot I_{env}}{M_{cont}} - \tfrac{(T_{cont}-T_{in})\cdot \dot{m}}{M_{cont}} - \tfrac{k_2\cdot(T_{cont}-T_{env})}{M_{cont}} + \tfrac{Q_a}{M_{cont}\cdot C_e} \\
\tfrac{d}{dt}T_{cont}(t) &= T_{cont}(t)\cdot(-\tfrac{\dot{m}}{M_{cont}} - \tfrac{k_2}{M_{cont}}) + \tfrac{k_1\cdot I_{env}(t)}{M_{cont}} + T_{in}(t)\cdot\tfrac{\dot{m}}{M_{cont}} + T_{env}(t)\cdot\tfrac{k_2}{M_{cont}} + \tfrac{Q_a(t)}{M_{cont}\cdot C_e} \ldots\ldots (1.1) \\
\tfrac{d}{dt}T_{cont}(t) &= T_{cont}(t)\cdot(-\tfrac{k_2}{M_{cont}}) + \tfrac{k_1\cdot I_{env}(t)}{M_{cont}} + T_{env}(t)\cdot\tfrac{k_2}{M_{cont}} + \tfrac{Q_a(t)}{M_{cont}\cdot C_e} \ldots\ldots (1.2) \\
\tfrac{d^2}{dt^2}V_{cont}(t) &= \tfrac{A^2_{pist}}{M_{pist}}\cdot(p_e(t) - p_r(t)) \ldots\ldots (1.3)
\end{aligned}$$

4.1.2.2 State Variables

- T_{cont} the temperature of the container in °C
- V_{cont} the volume of the container in m^3

4.1.2.3 Constants

- C_e The factor heat of the water $\mathrm{J\,°C^{-1}\,kg^{-1}}$
- $\dot{m}$ the rate of input/output water mass by unit time $\mathrm{kg\,s^{-1}}$
- M_{cont} the mas of the container in kg
- k_1 radiation constant kg
- k_2 conduction constant kg

4.1.2.4 Input Variables

- Q_a the auxiliary heat in J

4.1.2.5 Disturbance Variables

- T_{in} the temperature of water input/output °C
- I_{env} the irradiance in $\mathrm{W\,m^{-2}}$
- T_{env} the outside temperature in °C

4.1.3 Matrix Representation

$\sigma = u_1$:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{\dot{m}-k_2}{M_{cont}} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} T_{cont} \\ V_{cont} \\ \dot{V}_{cont} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} p_e \\ p_r \\ Q_a \end{bmatrix}, \mathbf{w} = \begin{bmatrix} \frac{I_{env}(t) \cdot k_1 + T_{env}(t) \cdot k_2 + T_{in}(t) \cdot \dot{m}}{M_{cont}} \\ 0 \\ 0 \end{bmatrix}$$

$\sigma = u_2$:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{\dot{m}}{M_{cont}} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} T_{cont} \\ V_{cont} \\ \dot{V}_{cont} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} p_e \\ p_r \\ Q_a \end{bmatrix}, \mathbf{w} = \begin{bmatrix} \frac{I_{env}(t) \cdot k_1 + T_{env}(t) \cdot k_2}{M_{cont}} \\ 0 \\ 0 \end{bmatrix}$$

Chapter 5

Discussion and Conclusions

The main objective of this work is to prove our methodology in hybrid systems such as *Hybrid Solar Water Heating*, in the modelling process was necessary to simplify the equations in order to focus us in the general methodology.

5.1 Limitations and Future Work

Get an most approximated solution. As we explain in Section 4, we are assuming lineal some behaviour variables in order to avoid *Partial Differential Equations*.

Massaging Machine. An interesting applications on future would be to build a massage machine, it is associated to comfort and it has several modes of control.

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