

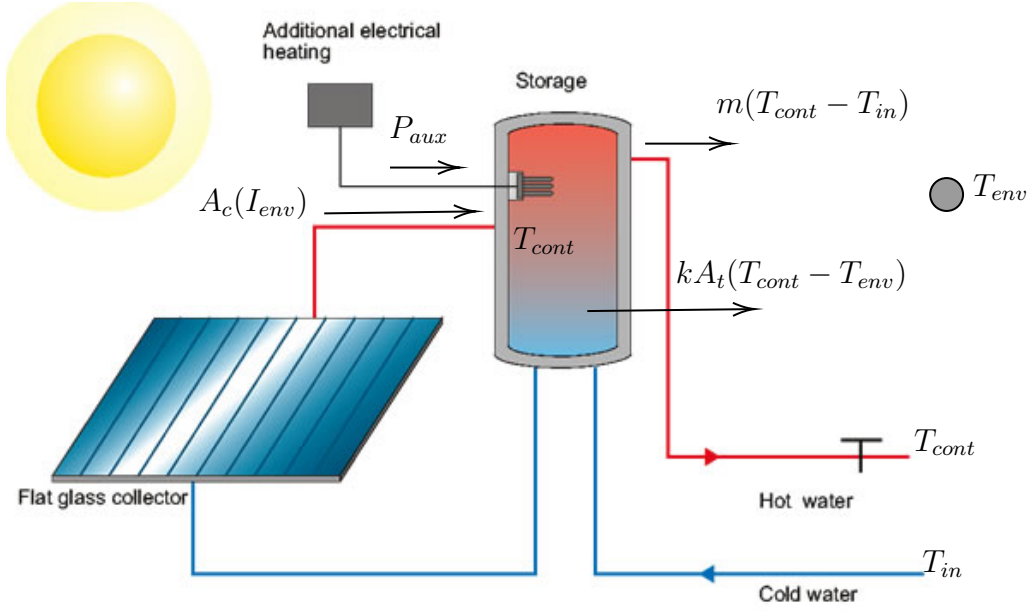
# Chapter 1

## Hybrid Solar Water heating

In this part, we choose as a case of study an *Hybrid Solar Water Heating*, because this system link up important condition to implement the methodology described in this work. Firstly we have agents, who interact with the system, adding uncontrollable modes of operations also some controllable variables in the system that allow to model as stochastic hybrid game.

The first part is about mathematical modelling using heat transfer theory and fluids dynamics to get temperature dynamics and then to define the parameters that determine the system with some assumptions to facilitate the analysis.

## 1.1 System Setup



### 1.1.1 General Equation

$$C_e M_c \frac{d}{dt} T(t) = Q_{input,1} + Q_{input,2} - Q_{loss,1} - Q_{loss,2} \quad (1.1)$$

$$Q_{input,1} = A_c \cdot I_{env}(t) \quad (1.2)$$

$$Q_{input,2} = P_{aux} \quad (1.3)$$

$$Q_{loss,1} = C_e \dot{m} (T(t) - T_{in}(t)) \quad (1.4)$$

$$Q_{loss,2} = k_c A_t (T(t) - T_{env}(t)) \quad (1.5)$$

### 1.1.2 State Variables

- $T(t)$  the temperature of the container in  $^{\circ}\text{C}$
- $V(t)$  the volume of the container in  $\text{m}^3$

### 1.1.3 Constants

- $C_e$  The factor heat of the water in  $\text{J } ^{\circ}\text{C}^{-1} \text{kg}^{-1}$
- $\dot{m}$  Mass flow rate input/output  $\text{kg s}^{-1}$
- $M_c$  the mass of the container in  $\text{kg}$
- $A_c$  Area of collector in  $\text{m}^2$
- $A_t$  Area of total of surface in  $\text{m}^2$
- $k_c$  Conduction coefficient  $\text{W m}^{-1} ^{\circ}\text{C}^{-1}$
- $P_{aux}$  Auxiliary heat power in  $\text{W}$

### 1.1.4 Input Variables

- Valve for output water state =  $\{on, off\}$
- Volume states =  $\{1, 2, 3\}$
- Auxiliary heat state =  $\{on, off\}$

### 1.1.5 Disturbance Variables

- $T_{in}(t)$  the temperature of water input/output in  $^{\circ}\text{C}$
- $I_{env}(t)$  the irradiance in  $\text{W m}^{-2}$
- $T_{env}(t)$  the outside temperature in  $^{\circ}\text{C}$

Table 1.1: Parameter values.

Parameter	Values
$C_e$	$4186 \text{J}^\circ\text{C}^{-1} \text{kg}^{-1}$
$\dot{m}$	$0.1 \text{kg s}^{-1}$
$M_c$	$100 \text{kg}$
$A_c$	$1 \text{m}^2$
$A_t$	$5.5557 \text{m}^2$
$k_c$	$16 \text{W m}^{-1}^\circ\text{C}^{-1}$
$P_{aux}$	$1000 \text{J s}^{-1}$

## 1.2 Hybrid solar water heating as a Sthocastic Hybrid Game

The hybrid solar water heating scenario with 12 modes of operations is defined like this:  $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$ , where the controller  $\mathcal{C}$  has a finite set of controllable modes, given by resistance state  $r \in \mathbb{B}$  and piston position  $p \in \{1, 2, 3\}$ . The environment  $\mathcal{U}$  has a finite set of uncontrollable modes  $v \in \mathbb{B}$ , that means the valve state for opening/closing water aperture. We assume that  $\mathcal{U}$  given  $\delta$  can switch among modes with equal probability at every period. The state variables in  $\mathcal{X}$  are given by  $\{T, E, V\}$ , container temperature, energy used and container volumen respectively.

Given the container temparature and volume, a controllable modes  $r \in \mathbb{B}$  and  $p \in \{1, 2, 3\}$  and uncontrollable mode  $v \in \mathbb{B}$  and a time delay  $\tau$ .

$$\frac{d}{dt}T(t) = \frac{1}{V(t)}C_1(T_{env}(t) - T(t)) + \frac{1}{V(t)}C_2I_{env}(t) + \frac{r}{V(t)}C_3 + \frac{v}{V()}C_4(T_{in}(t) - T(t)) \quad (1.6)$$

$$\frac{d}{dt}V(t) = \text{sgn}(100p - V(t)); \quad (1.7)$$

$$\frac{d}{dt}E_{used} = rC_3; \quad (1.8)$$

The equation 1.6 has some constants  $\{C_1, C_2, C_3, C_4\}$  this parameters are computed with the parameteres defines in table 1.1 for specific Hybrid Solar Water

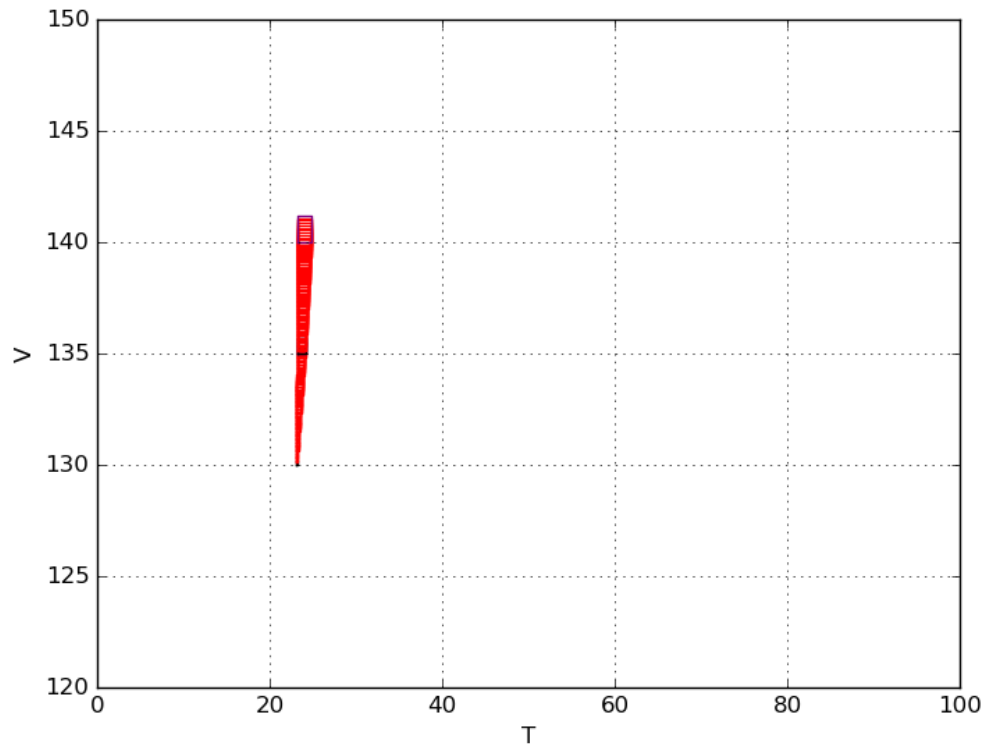


Figure 1.1: State Space.

Heating are equal to  $\{2.44e^{-5}, 4.77e^{-6}, 0.0024, 0.01\}$  respectively.

### 1.3 Simulations

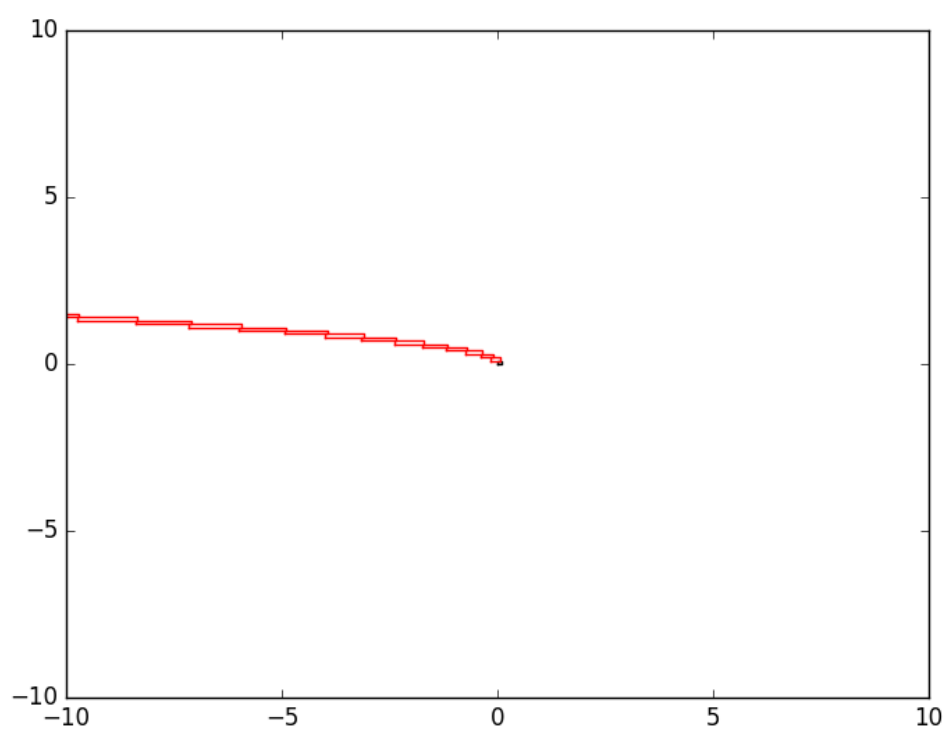


Figure 1.2: State Space 2.

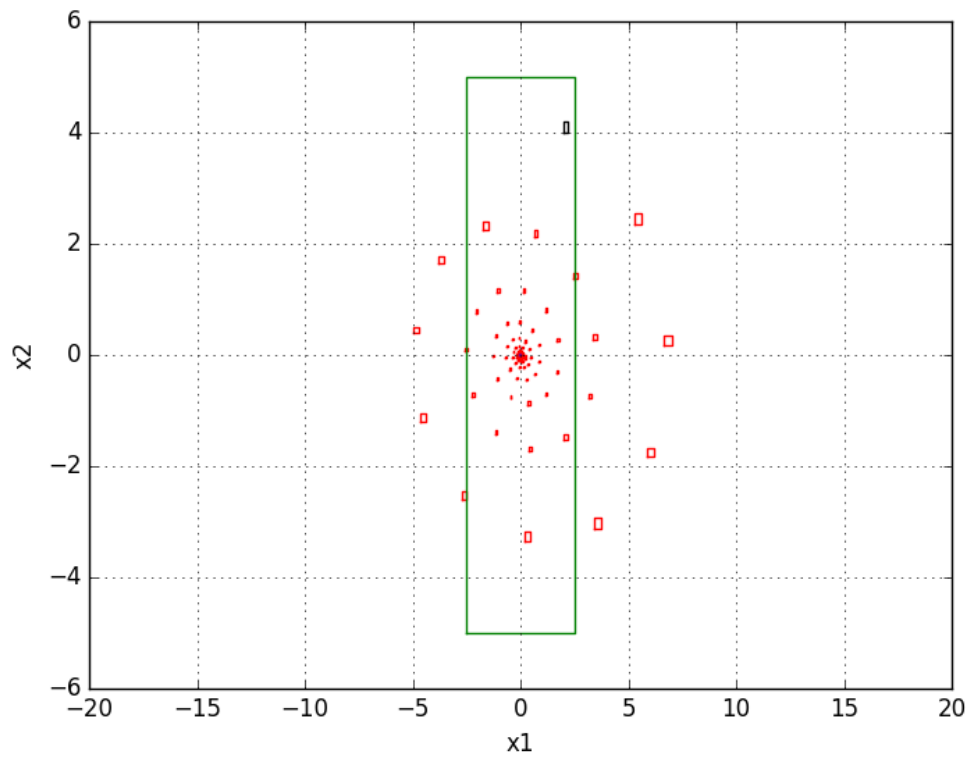


Figure 1.3: State Space 3.

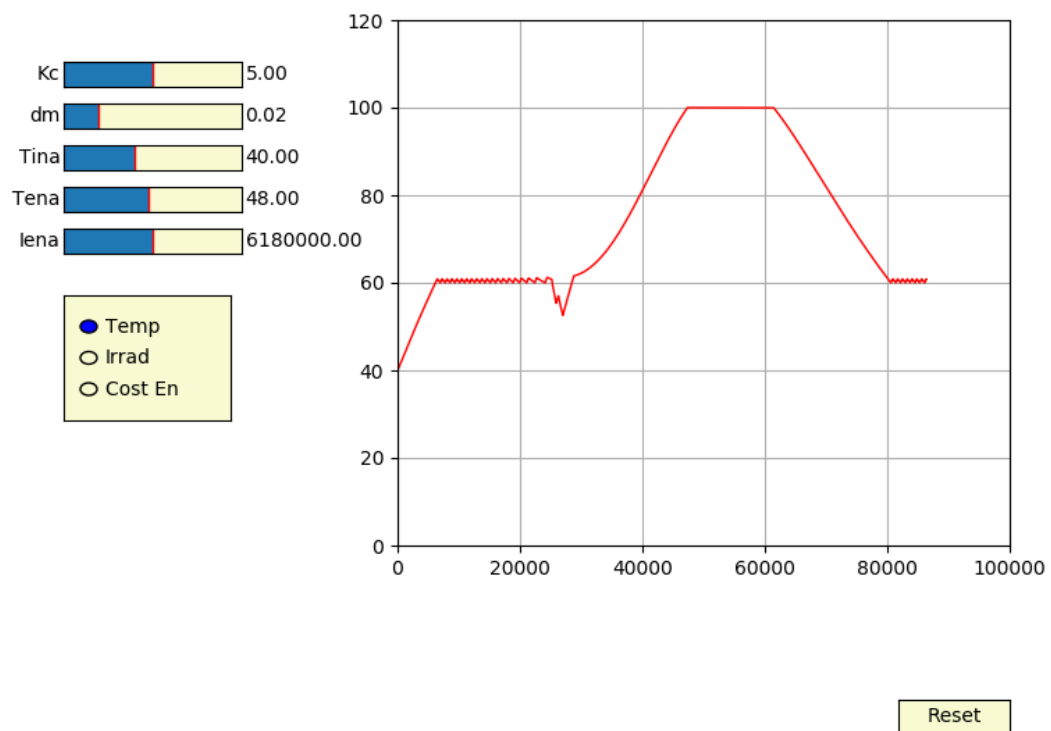


Figure 1.4: Running test for safety with Ibex



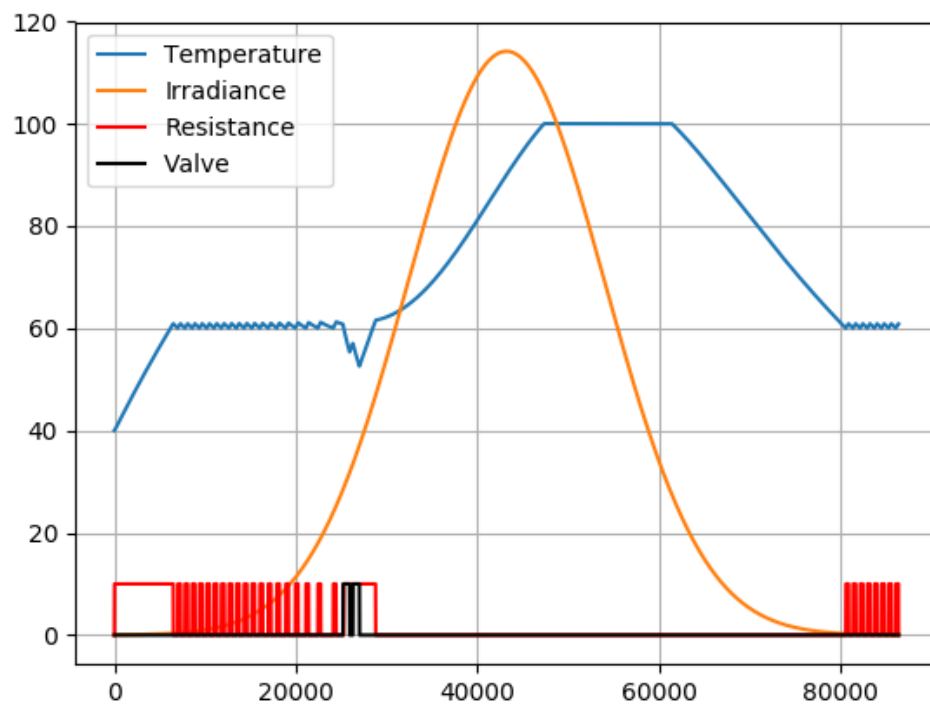


Figure 1.5: System behaviour with discrete inputs

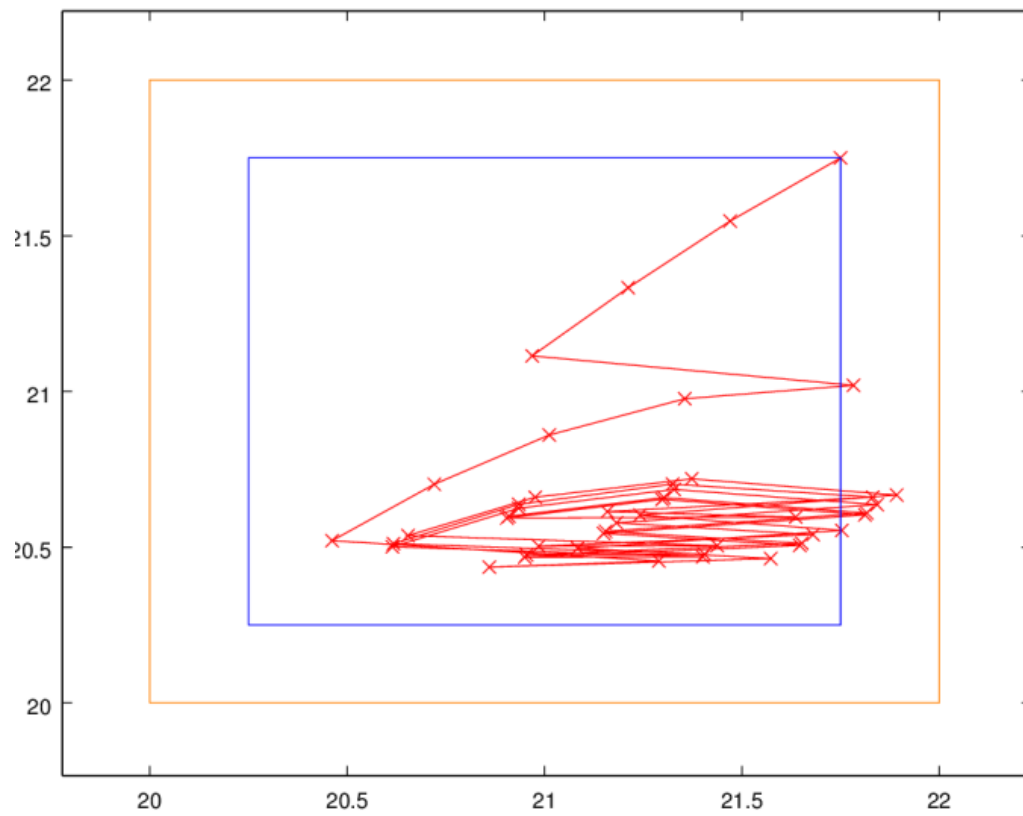


Figure 1.6: Running test for safety with Ibex