Safe and Near Optimal Controller Synthesis for Stochastic Hybrid Systems

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***Abstract*—Stochastic hybrid systems allow to model the interaction between continuos dynamics, discrete dynamics and probabilistic uncertainty. Because of their versatility, stochastic hybrid systems have emerged as a powerful framework for capturing the intricacies of complex systems. Motivated by this, considerable research effort has been devoted to the development of modeling analysis and control methods for stochastic hybrid systems.**

***Index Terms*—SHG, formatting, style, styling, insert**

I. Introduction

Hybrid systems are widely used in engineering applications and its importance has grown up considerably these last years, because of their ease of implementation for controlling cyber-physical systems. A switched systems is a set of dynamical systems, each with its own dynamical behaviour controlled by a parameter mode u whose values are in a finite set. However, due to the composition of many switched systems together, the global switched systems has a number of modes and dynamics which increases exponentially. Switched systems have numerous applications in control of mechanical systems, the automotive industry, and many other fields. [1].

II. Hybrid Systems

In this part is presented a method based on correction by design of discrete linear switched system in the time. the method consist of given a objective region *R* of state space, the method built a set *S* and a control that guide any element from *S* a *R*. This method works in an iterative way to back to reach the region *R*. The method can also be used for synthesize a stability control that is keep inside of R, whole states start in *R*.

A. Sthocastic Hybrid Systems Modeling

B. Sthocastic Hybrid Systems Safety Definitions

Problem 1*(Control Synthesis Problem)*. Let us consider a sampled Hybrid System. Given three sets R,S and B, with $R \cup B \in S$ and $R \cap B = \varnothing$ find a rule $\sigma(.)$ such that, for any $x(0) \in R$.

- *τ-stability: $x(t)$ return in R infinitely often, at some multiples of sampling time τ.*
- *safety: $x(t)$ always stays in S/B.*

Problem 2 *((R,S) - Stability Problem)*. Given a switched system, a set of recurrence $\mathbb{R}^n$ and a safe set $S \subset \mathbb{R}^n$,find a control rule $\sigma : \mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$ the following holds:

- *Recurrence in* R*:there are a monotonically strictly increasing sequence of (positive) integers $k_t, t \in \mathbb{N}$ such that for all $t \in \mathbb{R}^n, \phi(k_l\tau; t_0, x^0, \sigma, w) \in \mathbb{R}$.*
- *Stability in* S*: for all $t \in \mathbb{R}^n, \phi(t; t_0, x^0, \sigma, w) \in S$.*

Problem 3 *((R_1, R_2, S) - Reachability problem). Given a switched system of the form shown above, two sets $R_1 \subset \mathbb{R}^n$ and $R_2 \subset \mathbb{R}^n$ and a safety set $S \subset \mathbb{R}^n$, find a control rule σ : $\mathbb{R}^+ \to U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \to U$, the following holds:*

- *Reachability from R_1 to R_2: there exists an integer $k \in \mathbb{N}$ such that we have $\phi(k_l\tau; t_0, x^0, \sigma, w) \in R_2$.*
- *Stability in S: for all $t \in \mathbb{R}^+, \phi(t; t_0, x^0, \sigma, w) \in S$.*

III. Case Study: Solar Water Heating

A. Solar Water Heating as Sthocastic Hybrid Game

The hybrid solar water heating scenario with 12 modes of operations is defined like this: $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$, where the controller $\mathcal{C}$ has a finite set of controllable modes, given by resistance state $r \in \mathbb{B} = \{0, 1\}$ and piston movement $p \in \{-1, 0, 1\}$. The environment $\mathcal{U}$ has a finite set of uncontrollable modes $v \in \mathbb{B}$, that means the valve state for opening/closing water aperture. We assume that $\mathcal{U}$ given δ can switch among modes with equal probability at every period τ. The state variables in $\mathcal{X}_{(t)}$ are given by $\{T_{(t)}, V_{(t)}, E_{(t)}\}$, tank temperature, tank volumen and energy consumption respectively. Also Disturbance effect is considered in the dynamical system such as environment temperature, water input temperature and irradiance as a uncontrollable continuos environment variables.

$$\frac{d}{dt}T_{(t)} = -\frac{1}{V_{(t)}}c_1(T_{(t)} - T_{env(t)}) - \frac{\mathbf{v}}{V_{(t)}}c_2(T_{(t)} - T_{in(t)}) + \\ + \mathbf{r}\frac{c_3 Q_{aux}}{V_{(t)}} + \frac{c_4 I_{env(t)}}{V_{(t)}} \tag{1}$$

$$\frac{d}{dt}V_{(t)} = -k\mathbf{p}; \tag{2}$$

$$\frac{d}{dt}E_{(t)} = \mathbf{r}Q_{aux}; \tag{3}$$

In equation 1. the paremeters, $\{c_1, c_2, c_3, c_4\}$ remains constant in time. In equation 2, *k* is the piston velocity.

Identify applicable funding agency here. If none, delete this.

B. Solar Water Heating simulation

Considering the disturbance,controllable modes and uncontrollable modes we get as a result:

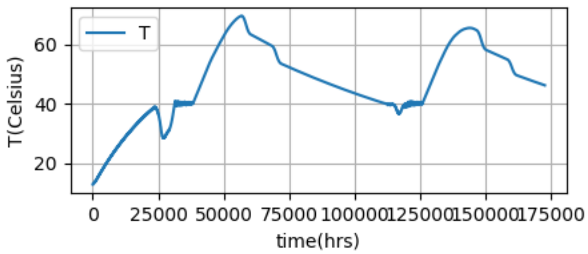

Fig. 1. Container tempeature state in time

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As mention before, safety behaviour consist in guaranty behaviour inside of the box as limits of state variables, as a consequence we define a pattern as a sequence of operations modes.

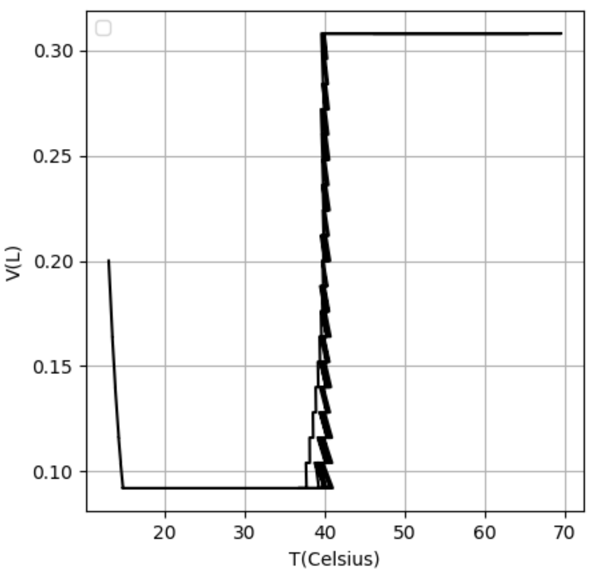

Fig. 2. State Space volumen-temperature

C. Strategy Controller Synthesis

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive”.
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: “Wb/m^2” or “webers per square meter”, not “webers/m^2”. Spell out units when they appear in text: “. . . a few henries”, not “. . . a few H”.
- Use a zero before decimal points: “0.25”, not “.25”. Use “cm^3”, not “cc”.)

D. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{4}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(4)”, not “Eq. (4)” or “equation (4)”, except at the beginning of a sentence: “Equation (4) is . . .”

E. Some Common Mistakes

- The word “data” is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- Do not use the word “essentially” to mean “approximately” or “effectively”.
- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
- Do not confuse “imply” and “infer”.
- The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the “et” in the Latin abbreviation “et al.”.

- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

F. Authors and Affiliations

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

G. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

H. Figures and Tables

a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. **??**", even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

IV. EXPERIMENTS AND RESULTS

RESULTS ANS EXPERIMENTS SECTIONS, are equal to $\left\{2.44e^{-5}, 4.77e^{-6}, 0.0024, 0.01\right\}$ respectively.

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page. [1]

V. REFERENCES

REFERENCES

[1] Kim G Larsen, Marius Mikučionis, Marco Muniz, Jiří Srba, and Jakob Haahr Taankvist. Online and compositional learning of controllers with application to floor heating. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 244–259. Springer, 2016.