



Safe and near optimal controller synthesis for Switched Systems

Richard Valentín Yantas Alcantara

Advisor: PhD. Marco Muñiz

Committee Members:

*Thesis submitted to the
Department of Computer Science
in partial fulfillment of the requirements for the degree of
Master in Computer Science.*

Universidad Católica San Pablo – UCSP
December of 2018 – Arequipa – Peru

*To my parents, Mario and Maria,
who never stop giving of themselves in
countless ways. To my brothers and sis-
ter, who encourage and support me.*

Abbreviations

Acknowledgments

First and foremost, I want to thank God for having guided me throughout these two years of study.

I would like to thank my family for their continuous support and encouragement during these years of study.

I would like to thank in a special way to the National Council for Science, Technology and Technological Innovation (CONCYTEC-PERU) and to the National Fund for Scientific Development, Technological and Technological Innovation (FONDECYT-CIENCIACTIVA), which through the Management Agreement 234-2015-FONDECYT have allowed the grant and financing of my studies in the Master Program in Computer Science at Universidad Católica San Pablo (UCSP).

I would like to express my gratitude and appreciation to my advisor Jorge Poco for giving his guidance and support throughout the Master program and the preparation of this thesis.

Finally, I thank all the people who directly or indirectly helped me in the preparation and presentation of this work, either by exchanging ideas, giving me advice and recommendations or by encouraging me to continue.

Abstract

Switched systems are used in hybrid systems that are most common in engineering applications(*e.g.*, logic-dynamic controllers, internet congestion even physical systems with impact,*etc.*). Hybrid systems have been used to model several cyber-physical systems.

In This work, we propose a pipeline to solve switched systems, providing safety and optimal functions to different cyber-physical systems, we propose a solar water heating as a case of study. We evaluate our results comparing with traditional controllers. In addition, we present a system called UPPAL STRATEGO that will be used in this work.

Keywords: Control Theory, Game Theory, Machine Learning, Model Checking.

Resumen

Los sistemas conmutables son ampliamente usados en sistemas híbridos que a su vez son muy comunes en aplicaciones de ingeniería (ejm: controladores lógicos-dinámicos, congestión de internet, sistemas con impacto físico, etc). Los sistemas híbridos han sido usados para modelar varios sistemas ciberfísicos.

En este documento, proponemos un *Solar Water Heating* como nuestro caso de estudio. Evaluamos nuestros resultados comparando con controladores tradicionales. Además, presentamos un sistema llamado *UPPAL STRATEGO* que será usado en este trabajo.

Palabras clave: Teoría de juegos, Aprendizaje automático, Verificación de modelos, Teoría de control.

Contents

List of Tables	XV
-----------------------	-----------

List of Figures	XVII
------------------------	-------------

1 Introduction	1
1.1 Motivation and Context	1
1.2 Problem Statement	1
1.3 Objectives	1
1.4 Contributions	2
1.5 Outline	2
2 Related Works	3
2.1 Online and Compositional Learning of Controllers with Application to Floor Heating	3
2.2 An Improved Algorithm for the Control Synthesis of Nonlinear Sampled Switched Systems	3
2.3 Distributed Synthesis of State-Dependent Switching Control	4
2.4 Final Considerations	4
3 Background	5
3.1 Switched Systems	5
3.2 Safety and Reachability	6
3.3 Switched Controller synthesis	7

4	Hybrid Solar Water heating	9
4.1	System Modelling	9
4.1.1	System Setup	9
4.1.2	Differential Equations	10
4.1.3	Experimental Results	12
5	Discussion and Conclusions	15
5.1	Limitations and Future Work	15
	Bibliography	17

List of Tables

4.1 Parameter values.	12
-------------------------------	----

List of Figures

3.1	Switched System Schematic	5
3.2	Trajectory of a hybrid system. The switching signal $\sigma(t)$ takes on integer values that change at discrete-time instances. (Liberzon, 2003)	6
4.1	Hybrid Solar Water Heating Diagram	9
4.2	Hybrid Solar Water Heating response	13

Chapter 1

Introduction

1.1 Motivation and Context

Switched systems are widely used in engineering applications and its importance has grown up considerably these last years, because of their ease of implementation for controlling cyber-physical systems. A switched systems is a set of dynamical systems, each with its own dynamical behaviour controlled by a parameter mode u whose values are in a finite set U (See [Liberzon \(2003\)](#)). However, due to the composition of many switched systems together, the global switched systems has a number of modes and dynamics which increases exponentially.

Switched systems have numerous applications in control of mechanical systems, the automotive industry, and many other fields.

1.2 Problem Statement

Nowadays, there is a large number of methods to solve switched systems; however, it does not have guarantee in safety. For that reason, we propose a new approach to solve switched systems.

1.3 Objectives

General Objective

Our main objective is to propose a pipeline to solve switched systems guarantee safety and optimal synthesis controllers.

Specific Objectives

To achieve our main objective, we have the following specific objectives:

- Define a case of study and get its mathematical model.
- Implement a safety controller for switched systems.
- Optimize the switched systems using model checking techniques.
- Evaluate each step of the pipeline and compare with traditional approaches.

1.4 Contributions

This thesis proposes a novel approach to solve switched systems guaranteeing safety and optimal controller synthesis. This procedure to consider stochastic variables as an input for the system. Our contributions are related to each part and are detailed below.

- Define a synthesis controller with the next:
 - We propose a method to guarantee safe controller. This methods consider three regions to have reachability and safety in the system.
 - We also demonstrate the utility of the proposed method comparing with traditional methods.
- Analyze the stability for our case of study study.
 - analyze the zeros and poles for the system.

1.5 Outline

This thesis document is divided into six chapters. After this introduction and problem formulation, in **Chapter 2** we survey the literature about the recent research in safety controllers and optimal controller synthesis. **Chapter 3** presents some basic concepts about the dynamical systems, traditional controllers, switched systems, stability criteria and optimal controller technique. Next, in Chapter x we describe in detail the corpus, techniques used by our pipeline and their evaluation results. Finally, the limitations, future works, and conclusions of this work are presented in Chapter z.

Chapter 2

Related Works

Our work consist on optimize and guarantee safety over controllers based on switched systems.

2.1 Online and Compositional Learning of Controllers with Application to Floor Heating

This work has proposed one method to perform *optimal controller synthesis for stochastic hybrid switched systems e.g.*, a floor heating system in a house ([Larsen et al., 2016](#)). Is proposed a general and scalable methodology for controller synthesis for such systems. Instead of off-line synthesis of a controller for all possible input temperatures and an arbitrary time horizon, is proposed an on-line synthesis methodology, where it periodically compute the controller only for the near future based on the current sensor readings.

2.2 An Improved Algorithm for the Control Synthesis of Nonlinear Sampled Switched Systems

In this paper is presented and algorithm for the control synthesis for nonlinear switched systems using and existing procedure of state-space and made available for nonlinear systems with the help of guaranteed integration, the algorithm has been improved to be able to consider longer patterns of modes with a better running approach.

This approach permits to deal with stability, reachability, safety and forbidden region constraints. The approach was numerically validated on several examples taken from the literature. ([Le Coënt et al., 2017](#))

2.3 Distributed Synthesis of State-Dependent Switching Control

In this part is presented a correct-by-design method of state-dependent control synthesis for linear discrete-time switching systems. Given an objective region R of the state space, the method builds a region S . ([Le Coënt et al., 2016](#))

2.4 Final Considerations

This chapter presented some recent proposals related to our thesis work. Some research works have been focused on analyzing safety controller over switched systems. On the other hand, other works have focused on optimize controller synthesis using some techniques related to model checking.

The next chapter will present some concepts needed to understand better our work, those concepts are related to the modelling physical system over our case of study and represent as a state space.

Chapter 3

Background

For better understanding we introduce some topics before to analyze the problem. Each sections describe a general idea about it.

3.1 Switched Systems

Hybrid Systems are loosely defined as dynamical system whose state has two components, one of which evolves in a continuous set such as \mathbb{R} while the other evolves in a discrete set such as \mathbb{N} according to some transition logic based rule. The simplest model of a hybrid systems is given by: ([Le Coënt et al., 2017](#))

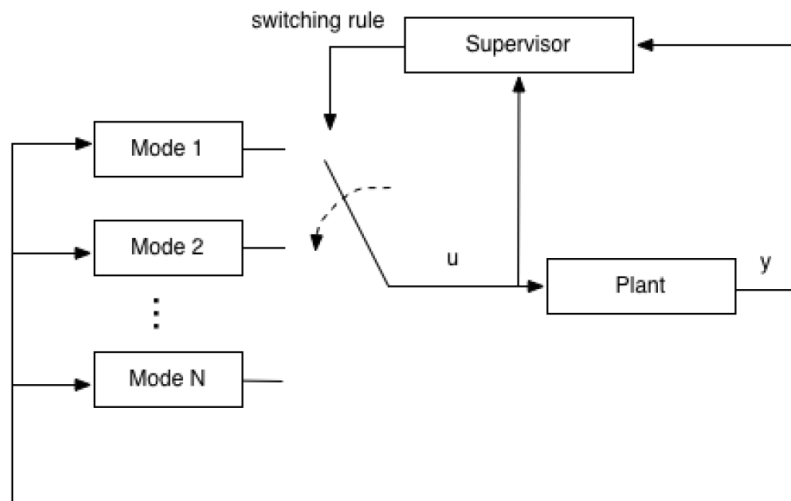


Figure 3.1: Switched System Schematic

The figure above can be expressed as mathematical equation like this:

$$\dot{x} = f_{\sigma(t)}(x(t)), x \in \mathbb{R}^n,$$

$$\sigma(t) = \lim \phi(x(\tau), \sigma(\tau)), \sigma \in \mathbb{N},$$

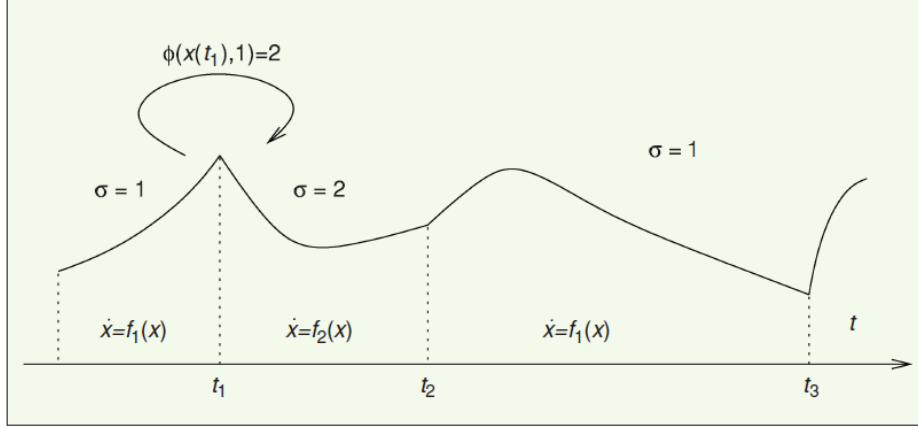


Figure 3.2: Trajectory of a hybrid system. The switching signal $\sigma(t)$ takes on integer values that change at discrete-time instances. (Liberzon, 2003)

3.2 Safety and Reachability

In this part is presented a method based on correction by design of discrete linear switched system in the time. the method consist of given a objective region R of state space, the method built a set S and a control that guide any element from S a R . This method works in an iterative way to back to reach the region R . The method can also be used for synthesize a stability control that is keep inside of R , whole states start in R . Le Coënt et al. (2016)

Problem 1 ((R, S) - *Stability Problem*). Given a switched system as shown in figure before, a set of recurrence \mathbb{R}^n and a safe set S

$\subset \mathbb{R}^n$, find a control rule $\sigma : \mathbb{R}^+ \rightarrow U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \rightarrow U$ the following holds:

- *Recurrence in R*: there are a monotonically strictly increasing sequence of (positive) integers $k_t, t \in \mathbb{N}$ such that for all $t \in \mathbb{R}^n, \phi(k_t \tau; t_0, x^0, \sigma, w) \in R$.
- *Stability in S*: for all $t \in \mathbb{R}^n, \phi(t; t_0, x^0, \sigma, w) \in S$.

Problem 2 ((R_1, R_2, S) - Reachability problem). Given a switched system of the form shown above, two sets $R_1 \subset \mathbb{R}^n$ and $R_2 \subset \mathbb{R}^n$ and a safety set $S \subset \mathbb{R}^n$, find a control rule $\sigma : \mathbb{R}^+ \rightarrow U$ such that, for any initial condition $x_0 \in R_1$ and any perturbation $\varpi : \mathbb{R}^+ \rightarrow U$, the following holds:

- *Reachability from R_1 to R_2 :* there exists an integer $k \in \mathbb{N}$ such that we have $\phi(k_l\tau; t_0, x^0, \sigma, w) \in R_2$.
- *Stability in S :* for all $t \in \mathbb{R}^+$, $\phi(t; t_0, x^0, \sigma, w) \in S$.

3.3 Switched Controller synthesis

Definition 1 (*Stochastic Hybrid Game*). A stochastic hybrid game

Problem 3 (*Control Synthesis Problem*). Let us consider a sampled switched system. Given three sets R, S and B , with $R \cup B \in S$ and $R \cap B = \emptyset$ find a rule $\sigma(\cdot)$ such that, for any $x(0) \in R$.

- *τ -stability:* $x(t)$ return in R infinitely often, at some multiples of sampling time τ .
- *safety:* $x(t)$ always stays in S/B .

Chapter 4

Hybrid Solar Water heating

In this part is taken as case of study an *Hybrid Solar Water Heating*, we propose three parts in the pipeline.

4.1 System Modelling

4.1.1 System Setup

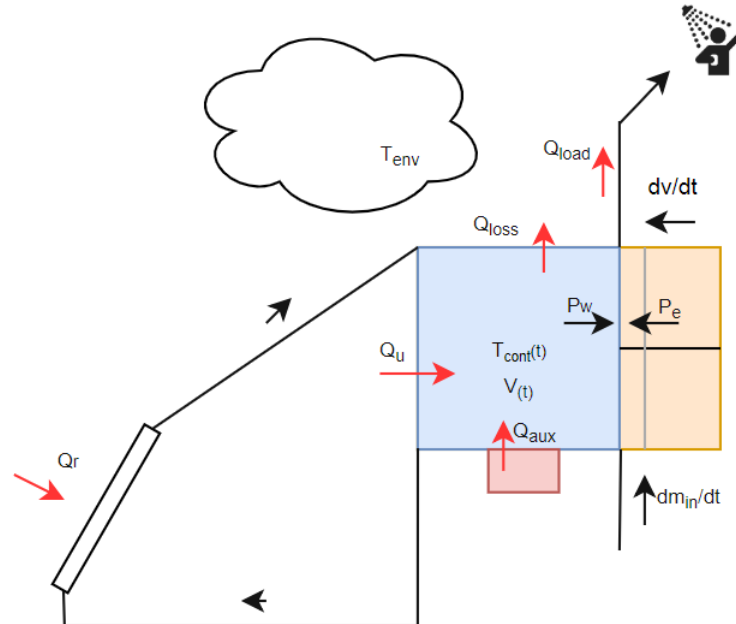


Figure 4.1: Hybrid Solar Water Heating Diagram

4.1.2 Differential Equations

General equation for temperature dynamics, we have two modes $Valve = ON, OFF$:

$$\frac{d}{dt}T_{cont}(t) = \frac{A_c \cdot I_{env}}{M_{cont}C_e} - \frac{(T_{cont} - T_{in}) \cdot \dot{m}}{M_{cont}} - \frac{k_c \cdot A_t \cdot (T_{cont} - T_{env})}{M_{cont}C_e} + \frac{\dot{Q}_a}{M_{cont} \cdot C_e} \quad (4.1)$$

When the $O_{valve} = ON$, then $\dot{m} \neq 0$, organizing (4.1) we have:

$$\frac{d}{dt}T_{cont}(t) = T_{cont}(t) \cdot \left(-\frac{\dot{m}}{M_{cont}} - \frac{k_c \cdot A_t}{M_{cont}C_e}\right) + \frac{A_c \cdot I_{env}(t)}{M_{cont}C_e} + T_{in}(t) \cdot \frac{\dot{m}}{M_{cont}} + T_{env}(t) \cdot \frac{k_c \cdot A_t}{M_{cont}C_e} + \frac{\dot{Q}_a(t)}{M_{cont} \cdot C_e} \quad (4.2)$$

When the $O_{valve} = OFF$, then $\dot{m} = 0$, replacing in (4.1) we have:

$$\frac{d}{dt}T_{cont}(t) = T_{cont}(t) \cdot \left(-\frac{k_c \cdot A_t}{M_{cont}}\right) + \frac{A_c \cdot I_{env}(t)}{M_{cont}C_e} + T_{env}(t) \cdot \frac{k_c \cdot A_t}{M_{cont}} + \frac{\dot{Q}_a(t)}{M_{cont} \cdot C_e} \quad (4.3)$$

The volume dynamics has four modes of operations:

When the $O_{volume} = 0$:

$$\frac{d}{dt}V_{cont}(t) = 0 \quad (4.4)$$

When the $O_{volume} = 1$:

$$\frac{d}{dt}V_{cont}(t) = \begin{cases} 0, & \text{if } V_{cont} = 100. \\ -A_{pist} \cdot v_{pist}, & \text{if } V_{cont} > 100. \end{cases} \quad (4.5)$$

When the $O_{volume} = 2$:

$$\frac{d}{dt}V_{cont}(t) = \begin{cases} +A_{pist} \cdot v_{pist}, & \text{if } V_{cont} < 200. \\ 0, & \text{if } V_{cont} = 200. \\ -A_{pist} \cdot v_{pist}, & \text{if } V_{cont} > 200. \end{cases} \quad (4.6)$$

When the $O_{volume} = 3$:

$$\frac{d}{dt}V_{cont}(t) = \begin{cases} 0, & \text{if } V_{cont} = 300. \\ +A_{pist} \cdot v_{pist}, & \text{if } V_{cont} < 300. \end{cases} \quad (4.7)$$

The heat dynamics has two modes of operations:

$(O_{heat} = ON)$

$$Energy : \frac{d}{dt}Q(t) = 1000 \quad (4.8)$$

$(O_{heat} = OFF)$

$$Energy : \frac{d}{dt}Q(t) = 0 \quad (4.9)$$

4.1.2.1 State Variables

- T_{cont} the temperature of the container in °C
- V_{cont} the volume of the container in m³

4.1.2.2 Constants

- C_e The factor heat of the water in J °C⁻¹ kg⁻¹
- \dot{m} the rate of input/output water in kg s⁻¹
- M_{cont} the mas of the container in kg
- A_c Area of colector in m²
- A_t Area of total of surface in m²
- k_c Conduction coeficient W m⁻¹ °C⁻¹
- Q_a the auxiliary heat in J
- A_{pist} Area of piston m²
- v_{pist} Velocity of piston m s⁻¹

4.1.2.3 Input Variables

- O_{valve} mode of the valve = ON,OFF
- O_{volume} mode of the rate of change of the volume = 0,1,2,3
- O_{heat} mode of the auxiliary heat = ON,OFF

4.1.2.4 Disturbance Variables

- T_{in} the temperature of water input/output in °C
- I_{env} the irradiance in W m^{-2}
- T_{env} the outside temperature in °C

4.1.3 Experimental Results

Table 4.1: Parameter values.

Parameter	Values
C_e	$4186 \text{J } ^\circ\text{C}^{-1} \text{kg}^{-1}$
\dot{m}	0.1kg s^{-1}
M_{cont}	100kg
A_c	1m^2
A_t	5.5557m^2
k_c	$16 \text{W m}^{-1} ^\circ\text{C}^{-1}$
Q_a	1000J s^{-1}
A_{pist}	1m^2
v_{pist}	0.1m s^{-1}

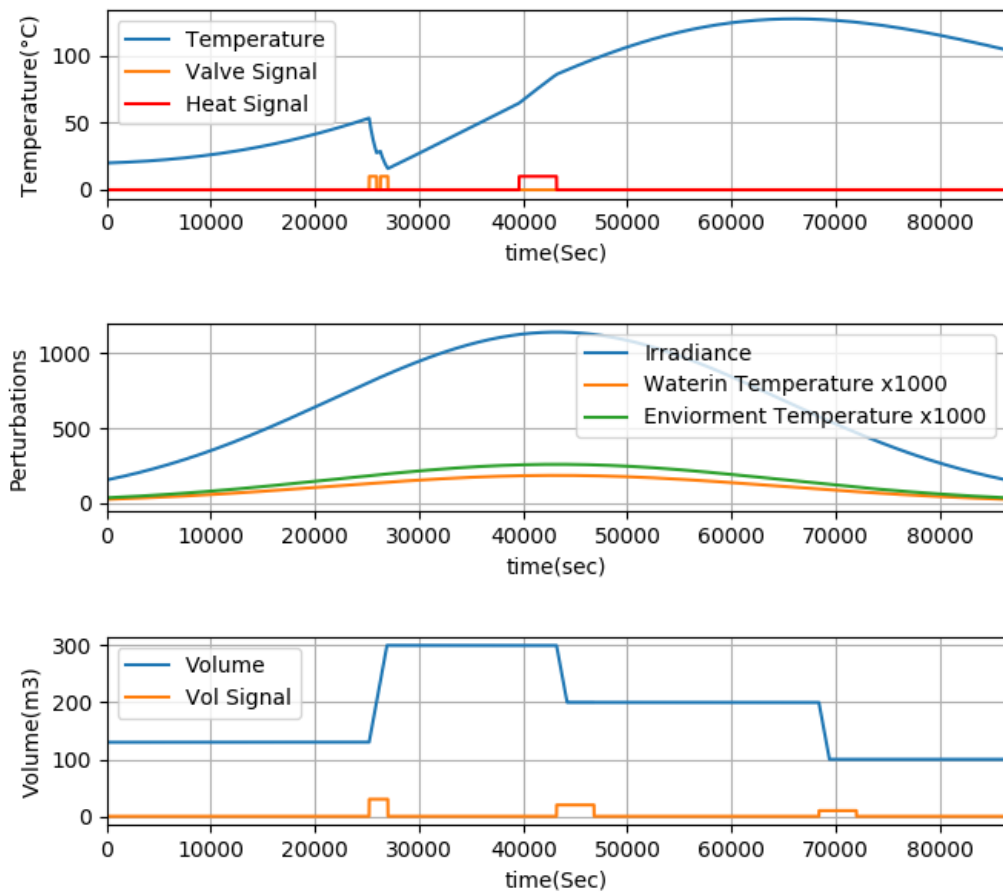


Figure 4.2: Hybrid Solar Water Heating response

Chapter 5

Discussion and Conclusions

The main objective of this work is to prove our methodology in hybrid systems such as *Hybrid Solar Water Heating*, in the modelling process was necessary to simplify the equations in order to focus us in the general methodology.

5.1 Limitations and Future Work

Get an most approximated solution. As we explain in Section 4, we are assuming lineal some behaviour variables in order to avoid *Partial Differential Equations*.

Massaging Machine. An interesting applications on future would be to build a massage machine, it is associated to comfort and it has several modes of control.

Bibliography

- Larsen, K. G., Mikučionis, M., et al. (2016). Online and compositional learning of controllers with application to floor heating. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 244–259. Springer.
- Le Coënt, A., dit Sandretto, J. A., et al. (2017). An improved algorithm for the control synthesis of nonlinear sampled switched systems. *Formal Methods in System Design*, pages 1–21.
- Le Coënt, A., Fribourg, L., et al. (2016). Distributed synthesis of state-dependent switching control. In *International Workshop on Reachability Problems*, pages 119–133. Springer.
- Liberzon, D. (2003). *Switching in systems and control*. Springer Science & Business Media.