



# Safe and near optimal controller synthesis for Stochastic Hybrid Games

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*To my parents, Mario and Maria,  
who never stop giving of themselves in  
countless ways. To my brothers and sis-  
ter, who encourage and support me.*

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# Abbreviations

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# Acknowledgments

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# Abstract

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Switched systems are used in hybrid systems that are most common in engineering applications(*e.g.*, logic-dynamic controllers, internet congestion even physical systems with impact,*etc.*). Hybrid systems have been used to model several cyber-physical systems.

In This work, we propose a pipeline to solve switched systems, providing safety and optimal functions to different cyber-physical systems, we propose a solar water heating as a case of study. We evaluate our results comparing with traditional controllers. In addition, we present a system called UPPAL STRATEGO that will be used in this work.

**Keywords:** Control Theory, Game Theory, Machine Learning, Model Checking.

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# Resumen

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Los sistemas conmutables son ampliamente usados en sistemas híbridos que a su vez son muy comunes en aplicaciones de ingeniería (ejm: controladores lógicos-dinámicos, congestión de internet, sistemas con impacto físico, etc). Los sistemas híbridos han sido usados para modelar varios sistemas ciberfísicos.

En este documento, proponemos un *Solar Water Heating* como nuestro caso de estudio. Evaluamos nuestros resultados comparando con controladores tradicionales. Además, presentamos un sistema llamado *UPPAL STRATEGO* que será usado en este trabajo.

**Palabras clave:** Teoría de juegos, Aprendizaje automático, Verificación de modelos, Teoría de control.

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# Contents

<b>List of Tables</b>	<b>XV</b>
-----------------------	-----------

<b>List of Figures</b>	<b>XVII</b>
------------------------	-------------

<b>1 Introduction</b>	<b>1</b>
1.1 Motivation and Context . . . . .	1
1.2 Problem Statement . . . . .	1
1.3 Objectives . . . . .	2
1.4 Contributions . . . . .	2
1.5 Outline . . . . .	3
<b>2 Related Works</b>	<b>5</b>
2.1 Online and Compositional Learning of Controllers with Application to Floor Heating . . . . .	5
2.2 An Improved Algorithm for the Control Synthesis of Nonlinear Sam- pled Switched Systems . . . . .	5
2.3 Distributed Synthesis of State-Dependent Switching Control . . . . .	6
2.4 Final Considerations . . . . .	6



# List of Tables





# List of Figures

# Chapter 1

## Introduction

### 1.1 Motivation and Context

Switched systems are widely used in engineering applications and its importance has grown up considerably these last years, because of their ease of implementation for controlling cyber-physical systems. A switched systems is a set of dynamical systems, each with its own dynamical behaviour controlled by a parameter mode  $u$  whose values are in a finite set  $U$  (See [Liberzon \(2003\)](#)). However, due to the composition of many switched systems together, the global switched systems has a number of modes and dynamics which increases exponentially. Switched systems have numerous applications in control of mechanical systems, the automotive industry, and many other fields.

### 1.2 Problem Statement

Nowadays, there is a large number of methods to solve switched systems; however, it does not have guarantee in safety. For that reason, we propose a new approach to solve switched systems.

## 1.3 Objectives

### General Objective

Our main objective is to propose a pipeline to solve switched systems guarantee safety and optimal synthesis controllers.

### Specific Objectives

To achieve our main objective, we have the following specific objectives:

- Define a case of study and get its mathematical model.
- Implement a safety controller for switched systems.
- Optimize the switched systems using model checking techniques.
- Evaluate each step of the pipeline and compare with traditional approaches.

## 1.4 Contributions

This thesis proposes a novel approach to solve switched systems guaranteeing safety and optimal controller synthesis. This procedure to consider stochastic variables as an input for the system. Our contributions are related to each part and are detailed below.

- Define
  - We propose a method to guarantee safe controller. This methods consider three regions to have reachability and safety in the system.
  - We also demonstrate the utility of the proposed method comparing with traditional methods.
- Analyze the stability for our case of study study.
  - analyze the zeros and poles for the system.

## 1.5 Outline



# Chapter 2

## Related Works

Our work consist on optimize and guarantee safety over controllers based on switched systems.

### 2.1 Online and Compositional Learning of Controllers with Application to Floor Heating

This work has proposed one method to perform *optimal controller synthesis for stochastic hybrid switched systems* e.g. a floor heating system in a house [Larsen et al. \(2016\)](#). Is proposed a general and scalable methodology for controller synthesis for such systems. Instead of off-line synthesis of a controller for all possible input temperatures and an arbitrary time horizon, is proposed an on-line synthesis methodology, where it periodically compute the controller only for the near future based on the current sensor readings.

### 2.2 An Improved Algorithm for the Control Synthesis of Nonlinear Sampled Switched Systems

In this paper is presented and algorithm for the control synthesis for nonlinear switched systems using and existing procedure of state-space and made available

for nonlinear systems with the help of guaranteed integration, the algorithm has been improved to be able to consider longer patterns of modes with a better running approach.

This approach permits to deal with stability, reachability, safety and forbidden region constraints. The approach was numerically validated on several examples taken from the literature. [Le Coënt et al. \(2017\)](#)

## 2.3 Distributed Synthesis of State-Dependent Switching Control

In this part is presented a correct-by-design method of state-dependent control synthesis for linear discrete-time switching systems. Given an objective region  $R$  of the state space, the method builds a region  $S$ . [Le Coënt et al. \(2016\)](#)

## 2.4 Final Considerations

This chapter presented some recent proposals related to our thesis work. Some research works have been focused on analyzing safety controller over switched systems. On the other hand, other works have focused on optimize controller synthesis using some techniques related to model checking.

The next chapter will present some concepts needed to understand better our work, those concepts are related to the modelling physical system over our case of study and represent as a state space.

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