Safe and near optimal controller synthesis for Stochastic Hybrid Games

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To my parents, Mario and Maria, who never stop giving of themselves in countless ways. To my brothers and sister, who encourage and support me.

Chapter 1

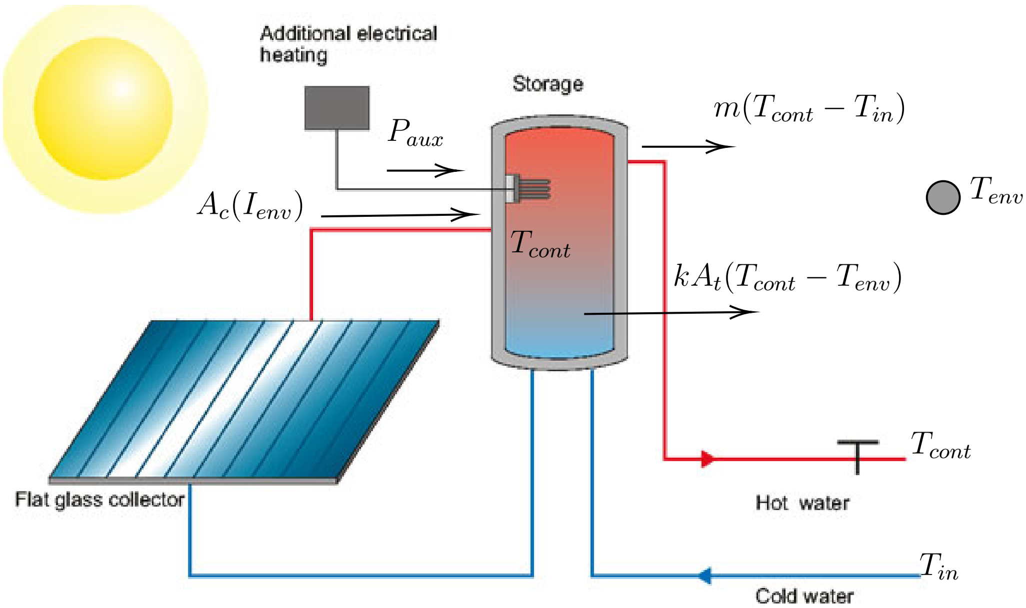
Safe and near optimal controller synthesis

In this part, we choose as a case of study an *Hybrid Solar Water Heating*, because this system link up important condition to implement the methodology described in this work. Firstly we have agents, who interact with the system, adding uncontrollables modes of operations also some controllable variables in the system that allow to model as stochastic hybrid game.

The first part is about mathematical modelling using heat transfer theory and fluids dynamics to get temperature dinamics and then to define the parameters that determine the system with some assumptions to facilate the analysis.

1.1 Case study

In this section, I test the proposed methodology for solving safe and near optimal controller synthesis for stochastic hybrid games.



1.1.1 Setup system

$$C_e M_c \frac{d}{dt} T(t) = Q_{input,1} + Q_{input,2} - Q_{loss,1} - Q_{loss,2} \tag{1.1}$$

$$Q_{input,1} = A_c \cdot I_{env}(t) \tag{1.2}$$

$$Q_{input,2} = P_{aux} \tag{1.3}$$

$$Q_{loss,1} = C_e \dot{m}(T(t) - T_{in}(t)) \tag{1.4}$$

$$Q_{loss,2} = k_c A_t (T(t) - T_{env}(t)) \tag{1.5}$$

1.1.2 State Variables

- $T(t)$ the temperature of the container in °C

- $V(t)$ the volume of the container in m^3

1.1.3 Constants

- C_e The factor heat of the water in $\mathrm{J\,°C^{-1}\,kg^{-1}}$
- $\dot{m}$ Mass flow rate input/output $\mathrm{kg\,s^{-1}}$
- M_c the mass of the container in kg
- A_c Area of colector in m^2
- A_t Area of total of surface in m^2
- k_c Conduction coeficient $\mathrm{W\,m^{-1}\,°C^{-1}}$
- P_{aux} Auxiliary heat power in W

1.1.4 Input Variables

- Valve for output water state $= \{on, off\}$
- Volume states $= \{1, 2, 3\}$
- Auxiliary heat state $= \{on, off\}$

1.1.5 Disturbance Variables

- $T_{in}(t)$ the temperature of water input/output in °C
- $I_{env}(t)$ the irradiance in $\mathrm{W\,m^{-2}}$
- $T_{env}(t)$ the outside temperature in °C

Table 1.1: Parameter values.

Parameter	Values
C_e	$4186\mathrm{J}\,°\mathrm{C}^{-1}\,\mathrm{kg}^{-1}$
$\dot{m}$	$0.1\mathrm{kg}\,\mathrm{s}^{-1}$
M_c	100kg
A_c	$1\mathrm{m}^2$
A_t	$5.5557\mathrm{m}^2$
k_c	$16\mathrm{W}\,\mathrm{m}^{-1}\,°\mathrm{C}^{-1}$
P_{aux}	$1000\mathrm{J}\,\mathrm{s}^{-1}$

1.2 Hybrid solar water heating as a Sthocastic Hybrid Game

The hybrid solar water heating scenario with 12 modes of operations is defined like this: $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$, where the controller $\mathcal{C}$ has a finite set of controllable modes, given by resistance state $r \in \mathbb{B}$ and piston position $p \in \{1,2,3\}$. The environment $\mathcal{U}$ has a finite set of uncontrollable modes $v \in \mathbb{B}$, that means the valve state for opening/closing water aperture. We assume that $\mathcal{U}$ given δ can switch among modes with equal probability at every period. The state variables in $\mathcal{X}$ are given by $\{T, E, V\}$, container temperature, energy used and container volumen respectively.

Given the container temparature and volume, a controllable modes $r \in \mathbb{B}$ and $p \in \{1,2,3\}$ and uncontrollable mode $v \in \mathbb{B}$ and a time delay τ.

$$\begin{aligned}\frac{d}{dt}T(t) = \frac{1}{V(t)}[&-c_1(T(t) - T_{env}(t)) - pc_2(T(t) - T_{in}(t)) \\ &-fc_3(pV_{min} - V(t))(T(t) - T_{in}(t)) + c_4 I_{env}(t) + rc_5]\end{aligned} \tag{1.6}$$

$$\frac{d}{dt}V(t) = pV_{min} - V(t); \tag{1.7}$$

$$\frac{d}{dt}E_{used} = rc_3; \tag{1.8}$$

The equation 1.6 has some constants $\{c_1, c_2, c_3, c_4\}$ this parameters are computed with the parameteres defines in table 1.1 for specific Hybrid Solar Water Heating are equal to $\{2.44e^{-5}, 4.77e^{-6}, 0.0024, 0.01\}$ respectively.

1.3 Simulations

near optimal controller near optimal controller near optimal controller near optimal controller near optimal controller

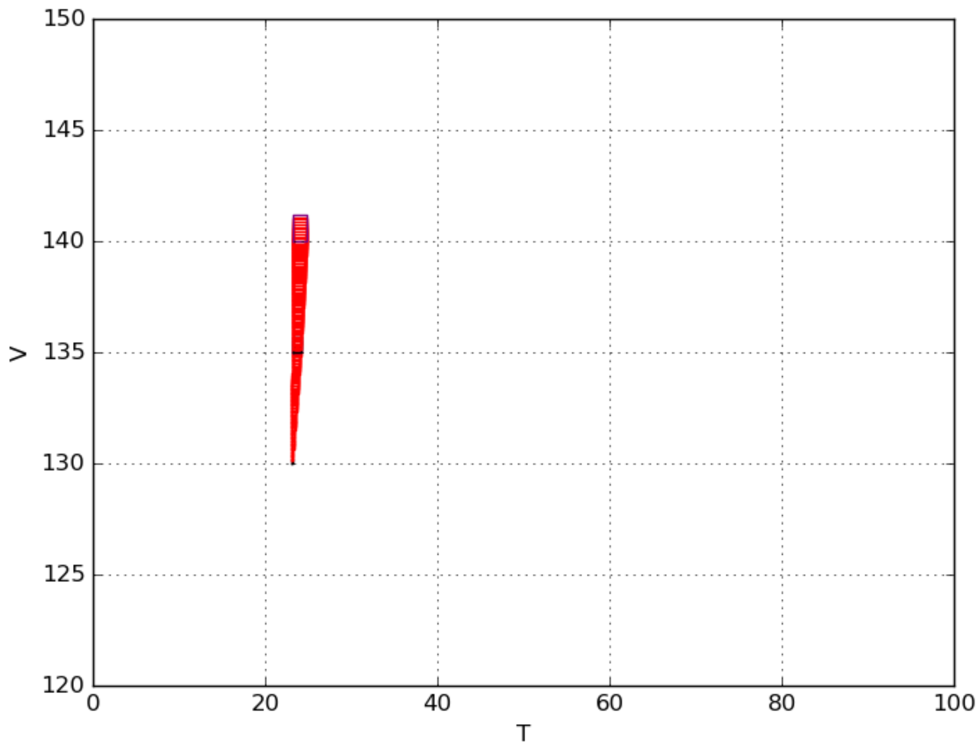


Figure 1.1: An example image not including a Wombat

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1.4 Results

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Bibliography