



# Safe and near optimal controller synthesis for Stochastic Hybrid Games

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*To my parents, Mario and Maria,  
who never stop giving of themselves in  
countless ways. To my brothers and sis-  
ter, who encourage and support me.*

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# Chapter 1

## Safe and near optimal controller synthesis

In this part, we choose as a case of study an *Hybrid Solar Water Heating*, because this system link up important condition to implement the methodology described in this work. Firstly we have agents, who interact with the system, adding uncontrollable modes of operations also some controllable variables in the system that allow to model as stochastic hybrid game.

The first part is about mathematical modelling using heat transfer theory and fluids dynamics to get temperature dynamics and then to define the parameters that determine the system with some assumptions to facilitate the analysis.

The diagram illustrates a solar water heating system. On the left, a yellow sun icon represents solar radiation. Below it is a blue rectangular 'Flat glass collector'. A red line connects the collector to a cylindrical 'Storage' tank. Above the tank, a grey box labeled 'Additional electrical heating' has an arrow pointing into the tank labeled  $P_a = c_3 r$ . Another arrow labeled  $A_c I_e$  points from the collector to the tank. The tank is divided into three horizontal sections labeled 1, 2, and 3 from bottom to top. Section 1 contains blue liquid, section 2 contains red liquid, and section 3 is empty. A vertical arrow in section 1 points upwards, labeled  $\frac{dV_c(t)}{dt} = pV_{min} - V_c(t)$ . A horizontal arrow points from section 2 to the right, labeled  $kA_t(T_c - T_e)$ . A vertical arrow points from section 2 to the right, labeled  $vm(T_c - T_i)$ . On the right, a grey circle is labeled  $T_e$ . At the bottom, a red line exits the tank labeled 'Hot water' with temperature  $T_c$ , and a blue line enters the tank labeled 'Cold water' with temperature  $T_i$ .

$$C_e M_c \frac{d}{dt} T(t) = A_c \cdot I_{env}(t) + P_{aux} - C_e \dot{m} (T(t) - T_{in}(t)) - k_c A_t (T(t) - T_{env}(t)) \quad (1.1)$$

States			
Variable	Description	Initial Value	Units
E	Energy consumption	0.0	J
V	Container Volumen	0.13	m <sup>3</sup>
T	Container Temperature	50.0	°C
Constants			
Variable	Description	Value	Units
$C_e$	The factor heat of the water in	4186	J °C <sup>-1</sup> kg <sup>-1</sup>
$\dot{m}$	Mass flow rate input/output	0.1	kg s <sup>-1</sup>
$M_c$	the mass of the container in	100	kg
$A_c$	Area of colector in	1	m <sup>2</sup>
$A_t$	Area of total of surface in	5.56	m <sup>2</sup>
$k_c$	Conduction coeficient	16	W m <sup>-1</sup> °C <sup>-1</sup>
$P_{aux}$	Auxiliary heat power in	1000	W
Input values			
Variable	Description	Space values	
$v$	Controllable action to release water	{0, 1}	
$r$	Controllable action to heat	{0, 1}	
$p$	Uncontrollable action to change capacity water	{1, 2, 3}	
Disturbances			
Variable	Description	Range	Units
$T_{in}(t)$	the temperature of water input/output in	[20 – 25]	°C
$I_{env}(t)$	the irradiance in	[0 – 920]	W m <sup>-2</sup>
$T_{env}(t)$	the outside temperature in	[0 – 16.5]	°C

Table 1.1: Parameter values.

**Hybrid Solar Water Heating as a Sthocastic Hybrid Game.** The hybrid solar water heating scenario with 12 modes of operations is defined like this:  $\mathcal{G}_{n,m} = (\mathcal{C}, \mathcal{U}, \mathcal{X}, \mathcal{F}, \delta)$ , where the controller  $\mathcal{C}$  has a finite set of controllable modes, given by resistance state  $r \in \mathbb{B}$  and piston position  $p \in \{1, 2, 3\}$ . The environment  $\mathcal{U}$  has a finite set of uncontrollable modes  $v \in \mathbb{B}$ , that means the valve state for opening/closing water aperture. We assume that  $\mathcal{U}$  given  $\delta$  can switch among modes with equal probability at every period. The state variables in  $\mathcal{X}$  are given by  $\{T, E, V\}$ , container temperature, energy used and container volumen respectively.

Given the container temperature and volume, a controllable modes  $r \in \mathbb{B}$  and  $p \in \{1, 2, 3\}$  and uncontrollable mode  $v \in \mathbb{B}$  and a time delay  $\tau$ .

$$\begin{aligned} \frac{d}{dt}T(t) = \frac{1}{V(t)}[-c_1(T(t) - T_{env}(t)) - pc_2(T(t) - T_{in}(t)) \\ -fc_3(pV_{min} - V(t))(T(t) - T_{in}(t)) + c_4I_{env}(t) + rc_5] \end{aligned} \quad (1.2)$$

$$\frac{d}{dt}V(t) = pV_{min} - V(t); \quad (1.3)$$

$$\frac{d}{dt}E_{used} = rc_3; \quad (1.4)$$

The equation 1.2 has some constants  $\{c_1, c_2, c_3, c_4\}$  this parameters are computed with the parameteres defines in table **\*\* Referencia no definida thesis-doc.tex:148\*** for specific Hybrid Solar Water Heating are equal to  $\{2.44e^{-5}, 4.77e^{-6}, 0.0024, 0.01\}$  respectively.

## 1.2 Simulations and Results

### 1.2.1 Solaris Data and prediction data

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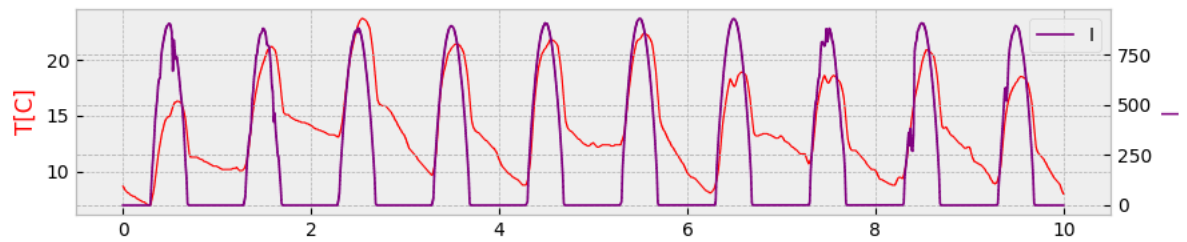


Figure 1.2: An example image not including a Wombat

**Data preparation.** The Figure environment “floats” to find an optimal position on the page. When you begin a figure, you can use the options [htbp!] to specify whether the figure should go “here”, the “top” of this page, the “bottom” of this page, or a special “page” reserved for floats. LaTeX, in its infinite wisdom, will sometimes prevent you from putting a figure in an “ugly” place, but you can (sort of) override its decision using the “!” option. When you haven’t used any option, the environment assumes that you provided [tbp], and it’s choosing to put the figure at the top of the page, above where your section goes.

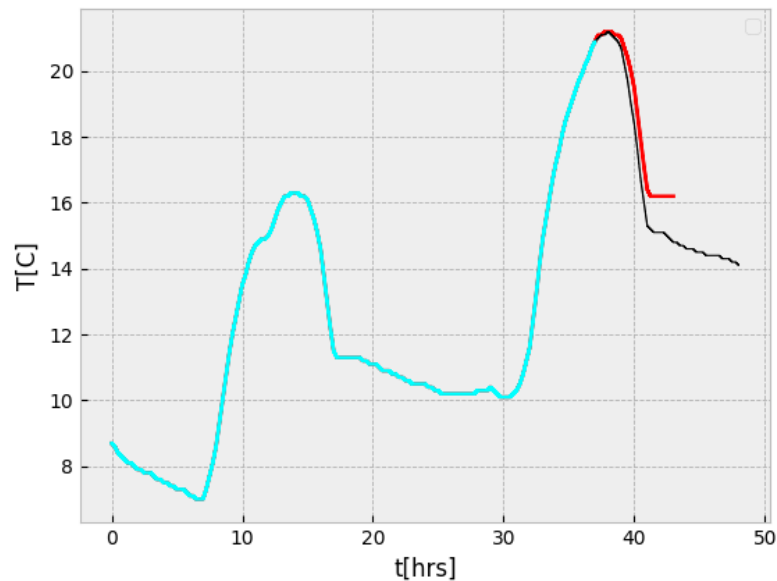


Figure 1.3: An example of prediction using ARIMA forecaster

### 1.2.2 Experiments

[floor heating] Regarding our experiments, we have two major components: a simulator written in PYTHON and a number of controllers, including the ones produced by UPPAAL STRATEGO. The simulator implements the solar water heating hybrid game  $\mathcal{G}_{n,m}$ . For our experiments, in the simulator we fix a time horizon  $H$  of 75 minutes with a period  $P$  of 5 minutes. As in the real house, every 15 minutes, the simulator outputs the current room temperatures  $T$  which are read by the controller. Subsequently, the controller inputs the control valves  $V$  which are used by the simulator for the next 5 minutes. The house has a desired temperature  $T^g$  and alpha parameters which denotes the importance to optimize either target or energy consumption. Our goal is to optimize the comfort in the water consumption and also energy consumption as a cost function. To define this cost function subject to controller (strategy)  $\sigma$  and  $\mathcal{G}_{n,m}$  of the form  $\pi = \gamma_0 \xrightarrow{t_1} \gamma_1 \xrightarrow{t_2} \dots \xrightarrow{t_{k-1}} \gamma_{k-1} \xrightarrow{t_k} \gamma_k$  where  $k = H/P$  is the number of control steps in the run  $\pi$ . Let  $T_i(\gamma_j)$  denote the container temperature  $T_i$  at configuration  $\gamma_j$ . Then the cost function is defined by

$$\text{dist}(\pi) = \alpha(E) + (1 - \alpha)(T - T_g) \quad (1.5)$$

In our experiments, we evaluate a number of different controllers. The simulator uses the distance function  $\text{dist}$  to compare the different controllers.

**Controllers.** In the following we introduce a number of controllers which we use in our experiments. We present the current controller operating in the house, two controllers proposed by engineers and the controllers synthesized using online synthesis and UPPAAL-STRATEGO.

*Bang-Bang Controller.* The bang-bang controller is currently running in the physical house and after each reading of container temperature  $T$ , it simply switch the resistor *ON* of container where  $T < T^g$  and leaves the remaining switch *OFF*.

*Greedy controller*

*Safe Controller with Patterns.* The Figure environment "floats" to find an optimal position on the page. When you begin a figure, you can use the options [htbp!] to specify whether the figure should go "here", the "top" of this page, the "bottom" of this page, or a special "page" reserved for floats. LaTeX, in its infinite wisdom, will sometimes prevent you from putting a figure in an "ugly" place, but you can (sort of) override its decision using the "!" option. When you haven't used

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*Statego Online Controller.* The Figure environment "floats" to find an optimal position on the page. When you begin a figure, you can use the options [htbp!] to specify whether the figure should go "here", the "top" of this page, the "bottom" of this page, or a special "page" reserved for floats. LaTeX, in its infinite wisdom, will sometimes prevent you from putting a figure in an "ugly" place, but you can (sort of) override its decision using the "!" option. When you haven't used any option, the environment assumes that you provided [tbp], and it's choosing to put the figure at the top of the page, above where your section goes.

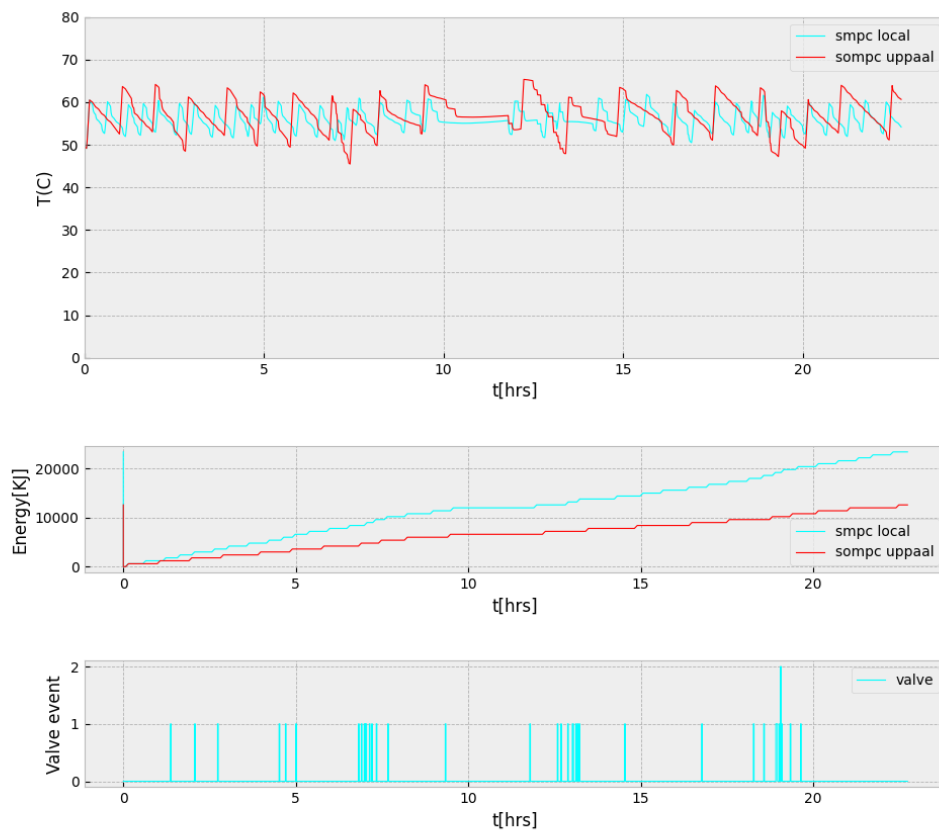


Figure 1.4: An example image not including a Wombat

*Safe and Near Optimal Controller.*

## **Evaluation Scenarios**

### **Controller Evaluation**

# Bibliography