

1. (10 points) Consider a bet with equal odds (i.e., win/lose \$1 for each \$1 you bet) and probability of winning  $p = 0.52$ .
- (a) What is the Kelly-optimal proportion ( $f^*$ ) of wealth to wager at each bet?
- (b) Starting with initial wealth  $V_0 = 100$ , write a formula for your wealth after winning 5 and losing 4 bets with the above strategy.

**Solution:**

(a) For even odds, the Kelly-optimal fraction is equal to the magnitude of the “edge”, i.e.  $f^* = p - q = 2p - 1 = 0.52 - 0.48 = 0.04 = 4\%$ .

(b) We have:

$$\begin{aligned} V_9 &= V_0(1 + f^*)^5(1 - f^*)^4 \\ &= 100(1.04)^5(0.96)^4 \end{aligned}$$

2. (10 points) Consider the following prices of two assets on different days:

stock \ day	1	2	3	4	5
$P$	35.63	35.59	35.49	35.43	35.41
$S$	33.74	32.72	31.69	32.45	34.98

Calculate the pairs-trading strategy profit of going long \$100 of  $S$  & short \$100 of  $P$  on day 1, and unwinding the position on day 5.

**Solution:** On day 1, we buy/long  $100/S_1$  shares of  $S$  and short-sell  $100/P_1$  shares of  $P$ . Since the strategy has 0 set-up cost (long & short \$100), the profit is just the payoff when you unwind the position:

$$\begin{aligned} \text{profit} &= S_5 \times (\text{shares long}) - P_5 \times (\text{shares short}) \\ &= S_5 \times 100/S_1 - P_5 \times 100/P_1 \\ &= 100 \times (34.98/33.74 - 35.41/35.63) \quad (= 4.29262) \end{aligned}$$