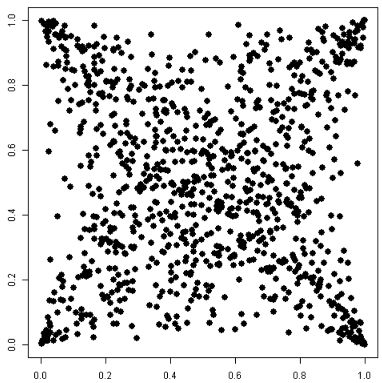
STAD70 W23 Name (f,l): ________________

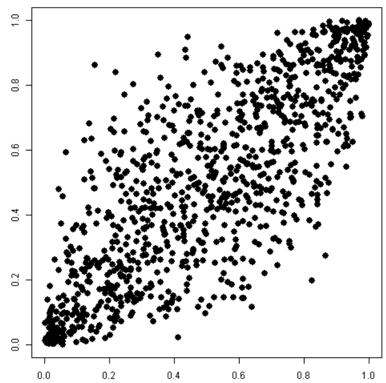
Quiz 2 Student #: ________________

1. (8 points) Below are 3 random samples of size $n = 1000$ from six possible elliptical bivariate copulas: Gaussian with correlation parameter $\rho = -0.75, 0, +0.75$, or $t(df = 1)$ with correlation parameter $\rho = -0.75, 0, +0.75$. Identify the distribution and ρ parameter that generated of each sample (e.g., t with $\rho = -.75$).

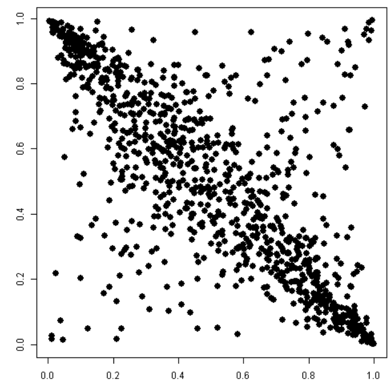
Copula:



Copula:



Copula:



Solution:

Copula: t with $\rho = 0$ Copula: Gaussian with $\rho = +0.75$ Copula: t with $\rho = -0.75$

2. Consider a market consisting of 2 assets with bivariate Normal net returns:

$$\boldsymbol{R} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \sim \mathrm{N}\left(\boldsymbol{\mu} = \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix}, \boldsymbol{\Sigma} = 0.04 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)$$

Note that the returns have the same variance and are perfectly positively correlated.

(a) (6 points) Show that *any* portfolio consisting of the two assets will have variance $\mathbb{V}[R_p] = \mathbb{V}[wR_1 + (1-w)R_2] = 0.04, \;\; \forall w \in \mathbb{R}$.

(b) (6 points) Now restrict the set of feasible portfolios to those *without short-selling* (i.e., $w \in [0, 1]$). Assuming the risk-free interest rate is $\mu_f = 0.02$, find the Sharpe ratio of the tangency/market portfolio for this restricted model.
(Hint: the answer can be found geometrically from the (σ_p, μ_p) risk-return diagram.)

Solution:

(a)

$$
\begin{aligned}
\mathbb{V}[R_p] &= \boldsymbol{w}^\top \Sigma \boldsymbol{w} \\
&= 0.04 \begin{bmatrix} w & (1-w) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ (1-w) \end{bmatrix} \\
&= 0.04 \begin{bmatrix} w + (1-w) & w + (1-w) \end{bmatrix} \begin{bmatrix} w \\ (1-w) \end{bmatrix} \\
&= 0.04[w^2 + 2w(1-w) + (1-w)^2] \\
&= 0.04[\cancel{w^2} + (\cancel{2w} - \cancel{2w^2}) + (1 - \cancel{2w} + \cancel{w^2})] = 0.04
\end{aligned}
$$

(b) Since every portfolio has the same variance, the feasible set *without short-selling* is the vertical line at $\sigma_p = \sqrt{0.04} = 0.2$, from returns $\mu_p \in [0.05, 0.1]$. Obviously, the only optimal portfolio, which is also the tangency portfolio, consists of only the higher-return asset (R_2). The slope of the line with the risk-free return, i.e., the Sharpe ratio, is thus

$$
\frac{\mu_2 - \mu_f}{\sigma_2} = \frac{0.1 - 0.02}{0.2} = \frac{0.08}{0.2} = 0.4
$$

The following plot illustrates the situation: