# \*\*\*Supplementary materials of the article\*\*\*

# Appendix A – From formal Petri nets to their NetLogo implementations

We here briefly discuss how we developed the Petri net model and its implementation in NetLogo. By way of illustration, we focus on the sourcing system. Figure A1 shows the formal Petri net we established, based on Jensen and Kristensen (2009), to model the operations of this part of the production system in our stochastic PQ problem.

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*Figure A1*. Formal representation of the sourcing module in the Petri net.

In this formal representation, *n* identifies the raw materials and the purchased part (we use *n*=1 for RM1, *n*=2 for RM2, *n*=3 for RM3, and *n*=4 for PP4). This is used to establish a difference between the different purchase orders in the ‘PO’ place, which emerge from the last manufacturing module (that of workstation D). In this sense, the ‘sourcing’ transition fires, after a delay that represents the shipping time, once there is a token in the ‘truck’ place. This transition generates tokens in the ‘on-hand’ places of the workstation where it is first needed, according to the specific sequence of operations of each final product (see Figure 1 in Section 2.1). Figure A1 also shows that the way through which the different modules are interconnected is via ports: when manufacturing performs backflush to build a finished good, tokens are sent back to incoming according to the Drum-Buffer-Rope mechanism.

In this manner, we note that our token taxonomy is composed by a type of tokens that represents parts (used to represent the material flow: raw materials / purchased parts, work-in-progress and finished goods), and other types of tokens that represent orders (specifically: PO = sourcing orders; WO = manufacturing orders; SO = sales orders) and other events (including truck arrival and demand interarrival period, among others). This taxonomy is the base to make expressions (signature), which is needed to manage the enabling and firing of transitions (binding, guards). For instance, there is always a ‘truck’ token placed in the ‘truck’ place; a guard manages the truck interarrival delay; once the ‘sourcing’ transition enables, it fires and places a new ‘truck’ token in the ‘truck’ place.

The formal representation of Figure A1 leads to the NetLogo implementation that can be seen in Figure 4 (Section 3.2.1), which is particularised for RM3. We can see there (Figure 4) the information flow, based on orders, coming to the port (‘PO’ place) in the sourcing module that comes from downstream in the production system. Tokens of the ‘PO’ type wait in this place until the truck interarrival delay is due; the ‘sourcing’ transition guard is responsible of this process. The firing of this transition casts each ‘PO’ token and routes it to the ‘on-hand’ place that corresponds to the part (1, 2, 3, 4) by using a expression that identifies the specific place destination for each token.

# Appendix B – Site acceptance tests

Table B1 presents the set of site acceptance tests (SATs) that we performed to validate and verify our agent-based model of the PQ production system.

*Table B1*. Collection of site acceptance tests.

|  |  |  |  |
| --- | --- | --- | --- |
| ***SAT #1****:* Analysis of Goldratt’s setting in the deterministic scenario  *Goal:* Checking that the system performs as intended in the traditional PQ problem | | | |
| ***Test reference and definition*** | ***Specific conditions of the test*** | ***Definition of done (DoD) / Expectations*** | ***Fulfilment of expectations?*** |
| ***SAT #1-a****:* Prioritisation of product P | * bu-prio: prio-P * bu-gral-config: Goldratt * bu-var: deterministic | * = 100 units,   = 30 units (per week)1   * = $6,300,   = $300 (per week)1,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #1-b****:* Prioritisation of product Q | * bu-prio: prio-Q * bu-gral-config: Goldratt * bu-var: deterministic | * = 60 units,   = 50 units (per week)1   * = $5,700,   = -$300 (per week)1,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #2****:* Analysis of variant settings in the deterministic scenario  *Goal:* Checking that the system is able to adapt to alternative settings | | | |
| ***Test reference and definition*** | ***Specific conditions of the test*** | ***Definition of done (DoD) / Expectations*** | ***Fulfilment of expectations?*** |
| ***SAT #2-a****:* Prioritisation of product P in the first alternative setting | * bu-prio: prio-P * bu-gral-config: Variant #14 * bu-var: deterministic | * = 80 units,   = 40 units (per week)5,6   * = $6,000,   = $0 (per week)5,6,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #2-b****:* Prioritisation of product Q in the first alternative setting | * bu-prio: prio-Q * bu-gral-config: Variant #14 * bu-var: deterministic | * = 80 units,   = 40 units (per week)5,6   * = $6,000,   = $0 (per week)5,6,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #2-c****:* Prioritisation of product P in the second alternative setting | * bu-prio: prio-P * bu-gral-config: Variant #27 * bu-var: deterministic | * = 100 units,   = 30 units (per week)5,8   * = $6,300,   = $300 (per week)5,8,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #2-d****:* Prioritisation of product Q in the second alternative setting | * bu-prio: prio-Q * bu-gral-config: Variant #27 * bu-var: deterministic | * = 20 units,   = 70 units (per week)5,9   * = $5,100,   = -$900 (per week)5,9,2   * 100% C/U in the bottleneck (works. B)3 | ☑16 |
| ***SAT #3****:* Analysis of the stochastic scenario  *Goal:* Checking that the system performs as intended when uncertainties are introduced | | | |
| ***Test reference and definition*** | ***Specific conditions of the test*** | ***Definition of done (DoD) / Expectations*** | ***Fulfilment of expectations?*** |
| ***SAT #3-a****:* Prioritisation of product P in the presence of the variability in the mix of demand | * bu-prio: prio-P * bu-gral-config: Goldratt * bu-var: variability\_mixdemand10 * Sum of demand is constant: 15011 | * Mix of demand varies as expected12 * < $300 (per week)13 | ☑17 |
| ***SAT #3-b****:* Prioritisation of product Q in the presence of the variability in the mix of demand | * bu-prio: prio-Q * bu-gral-config: Goldratt * bu-var: variability\_mixdemand10 * Sum of demand is constant: 15011 | * Mix of demand varies as expected12 * < -$300 (per week)13 | ☑17 |
| ***SAT #3-c****:* Prioritisation of product P in the presence of the variability in the volume of demand | * bu-prio: prio-P * bu-gral-config: Goldratt * bu-var: variability\_volumedemand14 * Mix of demand is constant: 2/315 | * Sum of demand varies as expected12 * < $300 (per week)13 | ☑17 |
| ***SAT #3-d****:* Prioritisation of product Q in the presence of the variability in the volume of demand | * bu-prio: prio-Q * bu-gral-config: Goldratt * bu-var: variability\_volumedemand14 * Mix of demand is constant: 2/315 | * Sum of demand varies as expected12 * < -$300 (per week)13 | ☑17 |

Notes: 1For more detail, see Goldratt (1990a); 2, as holding extra stock at the end of each week is not necessary for the deterministic scenario, where the production system is able to anticipate future demand. It would entail assuming unnecessary inventory-related costs; 3C/U refers to capacity utilisation, which allows one to make sure that the production system is appropriately regulated; 4In this case, the demand for products P and Q is reduced respectively to 80 and 40 units per week; 5This result can be obtained through exact optimisation techniques. We used the simplex algorithm for linear programming; 6In the configuration ‘variant #1’, the demand of customers can be completely satisfied. For this reason, the result is the same regardless of what product is prioritised; 7In this case, the demand of product P is kept at 100 units per week (like in Goldratt’s setting), while the demand for product Q is increased from 60 to 70 units per week; 8As can be expected, this solution is the same as in SAT #1-a; 9As can be expected, this solution is different to that in SAT #1-a. This occurs as the demand for product Q, which is being prioritised, grows in the configuration ‘variant #2’; 10Here, we used the following parameters for defining the statistical distribution: . In addition, the unit holding cost has been set to $1, $4, and $10; 11There is no variability in the demand volume in this case; 12According to the statistical distribution defined above; 13The net profit will always decrease due to the effect of uncertainties. The decrease may stem from the reduction in the sales (i.e. throughput decrease) and/or from the increase in inventory costs (i.e. operating expense increase). Note that, unlike in the deterministic scenario, end-product inventories are now required to optimise customer satisfaction; 14Here, we used the following parameters for defining the statistical distribution: . In addition, the holding cost has been set to $1, $4, and $10. 15There is no variability in the demand mix in this case. 16In a deterministic scenario, after the warm-up period, the results obtained with the simulation model are always the same. These results perfectly match the DoD. 17The goodness-of-fit test of the law of variability in the demand mix (SAT #3-a, b) and the demand volume (SAT #3-c,d) cannot be rejected for a significance level of 5% (alpha-error). In addition, the one-sided sample T-test allows us to reject the null hypothesis. Therefore, there is sufficient evidence supporting the evidence of the alternative hypothesis (H1: E(NP)<+300, SAT #3-a,c; H1: E(NP)<-300, SAT #3-b,d) for a significance level of 5%. Considering both tests, we confirm that the results of the runs match the DoD.

# Appendix C - Dashboard of the decision support system

The NetLogo environment provides experimenters with an interface window through which they can control the set-up, launch, and observe the progress of simulation runs. Figure C1 shows a screenshot of the interface that we developed in NetLogo in one of the simulation runs. This has been constructed for the purpose of user-friendliness, with the aim of helping non-experts in agent-based techniques easily explore the dynamics of the stochastic PQ problem. This interface may be interpreted as the dashboard of our model-driven decision support system, offering at-a-glance views of the behaviour of the production system over time and the performance metrics.

The interface is made up of three main areas. To the left, experimenters need to set up the simulation through three choosers. One (*bu-prio*) allows them to select the product that will be prioritised, either P or Q. The second one (*bu-gral-config)* configures the system by defining the processing times, costs and prices, and capacity constraints. The option “Goldratt”, selected in Figure B1, sets up the PQ scenario as it was defined in Section 2. The third one (*bu-var*) establishes the scenario of uncertainties by selecting one of four self-explanatory options: (i) deterministic; (ii) variability\_mixdemand; (iii) variability\_volumedemand; and (iv) variability\_volume&mixdemand. The specific parameters associated with each scenario —those related to the statistical distributions and the unit holding cost, except in option (i)— are requested through pop-up windows at the beginning of each run. In addition, the reader can see a slider that defines the time horizon of the run, together with the ‘setup’, ‘go’, and ‘step’ buttons that allow experiments to control the progress of the simulation.

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*Figure C1*. The interface of the agent-based decision support system in one of the simulation runs.

In the middle of the interface, the interactive animation appears. In this sense, the implementation in NetLogo of the Petri net, which shapes the dynamics of the stochastic PQ production system, is structured along the six-lane layout described in Section 3.2. In this way, experimenters can click on the different elements of the animation, which allows them to get accurate data about the state production system at all times. Lastly, to the right and at the bottom, we have included monitors and plots to track the evolution of important variables and performance metrics over the run. Among many others, this includes the evolution of the demand, the position of the different inventories the accumulated throughput. In line with Section 2.3, there are two first-line metrics that allow us to understand the behaviour of the net profit. These are the mean sales (, ), which may reduce the throughput, and the mean inventory costs (), which may increase the operating expense. Also, we note that when the run finishes, a CSV file is generated with all the relevant information to facilitate the subsequent analysis.

# Appendix D – Illustration of the results of the simulations

Figures D1 and D2 display the evolution of the average net profit over time in the first of the five simulation runs that were conducted in each scenario for the simulations with demand volume uncertainty (Section 4.1) and demand mix uncertainty (Section 4.2), respectively.

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*Figure D1*. Evolution of the net profit in the five different scenarios of demand volume uncertainty.

Diagrama

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*Figure D2*. Evolution of the net profit in the five different scenarios of demand mix uncertainty.

# Appendix E – Statistical analyses

Before performing the regression analyses, we evaluated the heteroscedasticity in the data by means of Levene’s tests of equality of variances. The results are shown in Table E1, for the demand volume uncertainty (Section 4.1), and Table E2, for the demand mix uncertainty (Section 4.2). Given that the p-value is significantly lower than 5% in both cases, the null hypothesis of equal variances (i.e. homoscedasticity) is rejected. We then conclude that the different populations, defined by different values of the shape parameters, and , respectively, have different variances. This was taken into consideration in the selection of the regression model, as discussed in Section 4.

*Table E1.* Levene’s test of equality of variances for the net profit in the analysis of the demand volume uncertainty.

|  |  |  |  |
| --- | --- | --- | --- |
| *Shape parameter,* | Mean | Standard deviation | Frequency |
| 2 | -$1,469.63 | $480.74 | 5 |
| 9 | -$876.60 | $273.85 | 5 |
| 34 | $47.07 | $145.86 | 5 |
| 120 | $219.55 | $60.33 | 5 |
| 300 | $269.11 | $17.85 | 5 |
| Total | -$362.08 | $744.51 | 25 |
| *W0* 1 = 4.5187 | | *prob > F* = 0.0092 | |

Note: 1*W0* reports Levene’s robust test statistic.

*Table E2*. Levene’s test of equality of variances for the net profit in the analysis of the demand mix uncertainty.

|  |  |  |  |
| --- | --- | --- | --- |
| *Shape parameter,* | Mean | Standard deviation | Frequency |
| 3 | -$835.0 | $248.24 | 5 |
| 7 | $84.4 | $169.45 | 5 |
| 17 | $252.8 | $46.70 | 5 |
| 34 | $291.8 | $14.92 | 5 |
| 120 | $298.6 | $16.69 | 5 |
| Total | $18.52 | $459.90 | 25 |
| *W0* 1 = 7.5502 | | *prob > F* = 0.0007 | |

Note: 1*W0* reports Levene’s robust test statistic.

Table E3 provides comprehensive information on the heteroskedastic fractional polynomial regression analysis for the scenario with demand volume uncertainty (Section 4.1). We observe that the regression model that converges is that with parameters 0 and 0, , leading to the equation, . Table E3 also confirms the significance of the model (*prob > χ²* = 0.0000) and reports on the relevant coefficients, including the confidence intervals. In addition, this table confirms heteroscedasticity through a likelihood-ratio test.

*Table E3*. Regression analysis of the relationship between the net profit and the shape parameter for volume mix.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Fractional polynomial comparisons1: | | | | | |  |
| shape | Deg. Freed. | Mod. Dev. | Dev. Diff. | P-value | Powers |  |
| omitted | 0 | 348.774 | 33.532 | 0.000 |  |  |
| linear | 1 | 340.569 | 25.328 | 0.000 | 1 |  |
| m = 1 | 2 | 321.401 | 6.159 | 0.046 | -0.5 |  |
| m = 2 | 4 | 315.552 | 0.000 | -- | 0 0 |  |
| Heteroskedastic linear regression2 | | | Number of obs. = 25 | | Wald χ²(2)=108.85 | |
| ML estimation | | | Log likelihood = -157.6209 | | *prob > χ²* = 0.0000 | |
| ***NP\_avg***3 | Coef. | Std. Err. | z-score | P>abs(z) | Min - 95% CI | Max - 95% CI |
|  | 954.9826 | 132.6398 | 7.20 | 0.000 | 695.0134 | 1214.952 |
|  | -87.94716 | 14.32665 | -6.14 | 0.000 | -116.0269 | -59.86744 |
|  | -2,318.727 | 302.3225 | -7.67 | 0.000 | -2,911.268 | -1,726.186 |
| **lnsigma2**4 | Coef. | Std. Err. | z-score | P>abs(z) | Min - 95% CI | Max - 95% CI |
| **ln\_var** | 1.320733 | 0.2165268 | 6.10 | 0.000 | 0.8963846 | 1.745118 |
| **\_cons** | -1.325178 | 1.841143 | -0.72 | 0.472 | -4.933749 | 2.283393 |
| Likelihood-ratio test of lnsigma2 | | | χ²(1)=34.23 | | *prob > F* = 0.0000 | |

Note: 1This block provides information on the process of obtaining the regression equation; 2This block reports on the main results of the regression model obtained, confirming its significance; 3This block reports the values of the coefficients of the regression models, including confidence intervals; 4This block is a likelihood-ratio test for the parameters of the variance function. As the χ²(1) statistic is significant, we confirm that heteroscedasticity is present.

Importantly, we confirmed the predictive capacity of this model. To this end, we selected an intermediate value of the shape parameter, specifically, . For this value, the regression model results in , with the confidence interval [$156.82, $249.18]. To this end, we run a simulation with , obtaining , which is within the previous interval. Then, we performed a one-sample t-test to confirm that we cannot reject that the simulation output is consistent with the results of the model (p-value = 0.7591).

Finally, Table E4 provides detail on the regression study for the scenario with demand mix uncertainty (Section 4.3). First, it shows that the heteroskedastic fractional polynomial regression model that converges is that with parameters -1 and -1, . In this sense, the model becomes . This table also confirms that the model fits well with the data, given that *prob > χ²* = 0.0000. The coefficient analysis can also be seen in Table D4, together with a likelihood-ratio test that confirms the heteroscedasticity.

*Table E4*. Regression analysis of the relationship between the net profit and the shape parameter for demand mix.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Fractional polynomial comparisons1: | | | | | |  |
| shape | Deg. Freed. | Mod. Dev. | Dev. Diff. | P-value | Powers |  |
| omitted | 0 | 304.320 | 30.473 | 0.000 |  |  |
| linear | 1 | 303.075 | 29.228 | 0.000 | 1 |  |
| m = 1 | 2 | 289.685 | 15.838 | 0.000 | -1 |  |
| m = 2 | 4 | 273.847 | 0.000 | -- | -1 -1 |  |
| Heteroskedastic linear regression2 | | | Number of obs. = 25 | | Wald χ²(2)=129.59 | |
| ML estimation | | | Log likelihood = -136.9234 | | *prob > χ²* = 0.0000 | |
| ***NP\_avg*** | Coef. | Std. Err. | z-score | P>abs(z) | Min - 95% CI | Max - 95% CI |
|  | -4,962.083 | 582.3854 | -8.52 | 0.000 | -6,103.548 | -3,820.629 |
|  | 1,645.454 | 338.8844 | 4.86 | 0.000 | 981.2526 | 2,308.655 |
|  | 273.2012 | 12.1276 | 22.53 | 0.000 | 249.4315 | 296.9708 |
| **lnsigma2**3 | Coef. | Std. Err. | z-score | P>abs(z) | Min - 95% CI | Max - 95% CI |
| **ln\_var** | 1.707 | 0.2094597 | 8.15 | 0.000 | 1.296466 | 2.117533 |
| **\_cons** | 16.39631 | 1.054681 | 15.55 | 0.000 | 14.32918 | 18.46345 |
| LR test of lnsigma2 | | | χ²(1)=38.01 | | *prob > F* = 0.0000 | |

Note: 1This block provides information on the process of obtaining the regression equation; 2This block reports on the main results of the regression model obtained, confirming its significance; 3This block reports the values of the coefficients of the regression models, including confidence intervals; 4This block is a likelihood-ratio test for the parameters of the variance function. As the χ²(1) statistic is significant, we confirm that heteroscedasticity is present.

We also checked the predictive capacity of this model. We set up a new run with , and we obtained $317.43. In this case, the regression model results in , with the confidence interval [$280.86, $318.87]. Thus, this result is within the confidence interval. In addition, we conducted a one-sample t-test to verify that the output of the simulation run is aligned with the results of the regression model (p-value = 0.7859).