Dark matter and fundamental fields

Richard Brito
CENTRA, Instituto Superior Técnico, Lisboa

See also https://github.com/richbrito/Corfu_school

Plan

1. Dark matter: a short and (biased) introduction

2. Black holes & ultralight bosons: superradiant instabilities

3. Impact of dark matter on binary systems

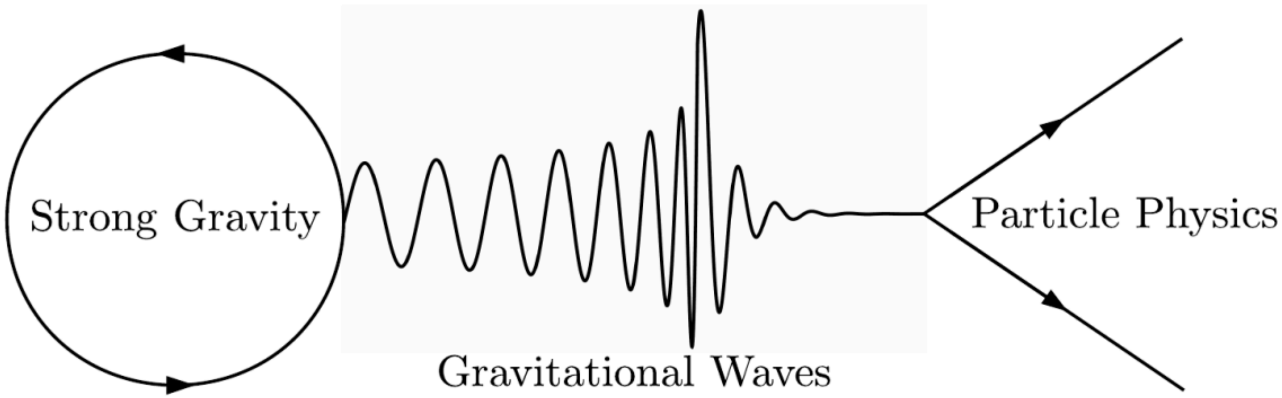
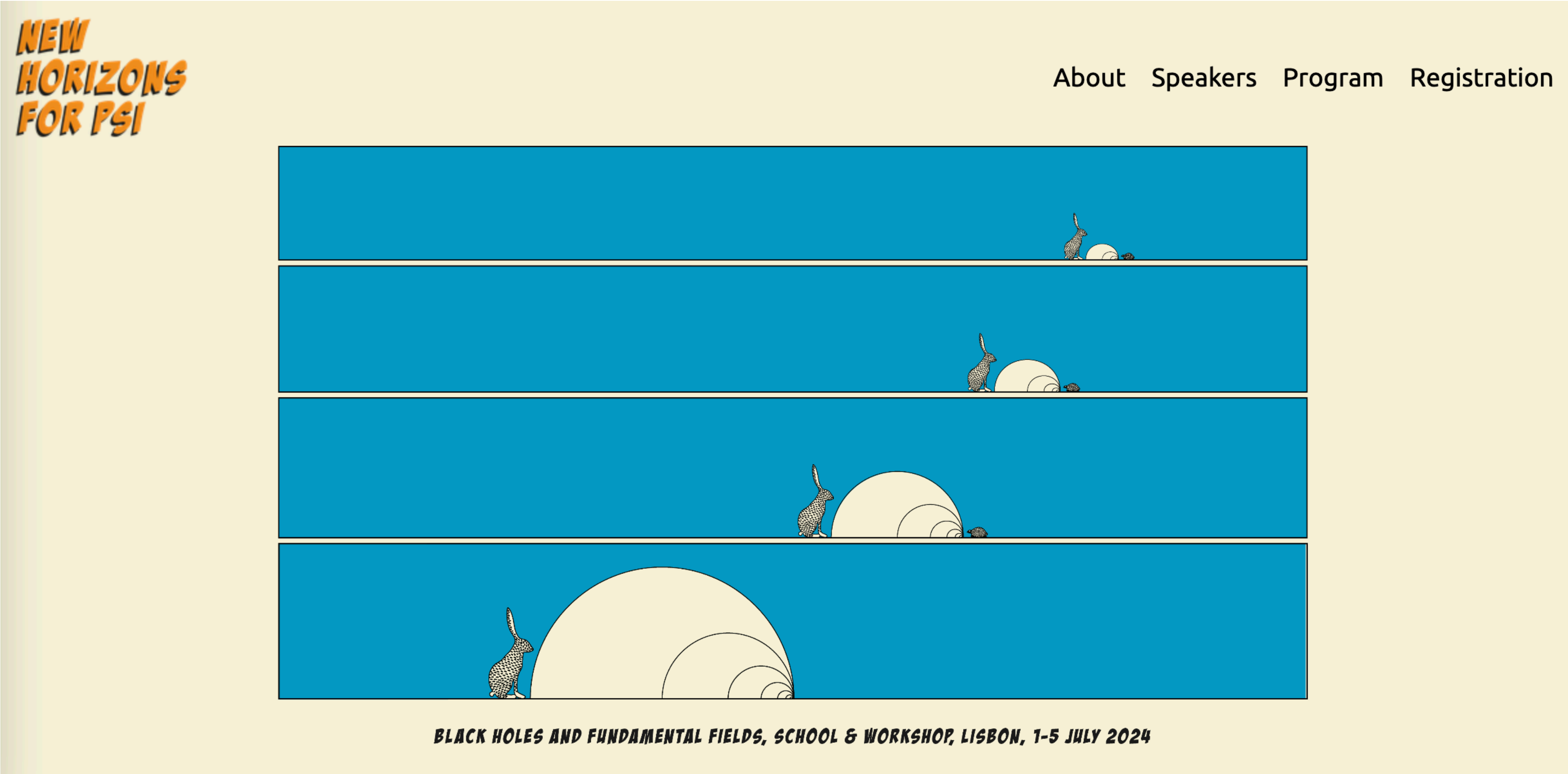
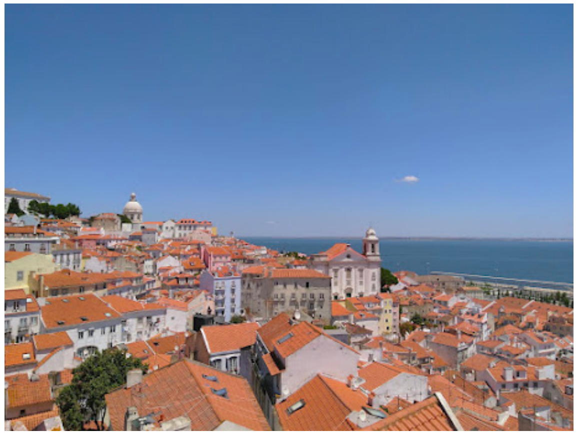


Image credit: V. Cardoso & P. Pani - CERN Courier (2017)





https://strong-gr.com/new-horizons-for-psi/

I
Dark matter:
a short and (biased) introduction

Some suggestions for further reading:

- G. Bertone & D. Hooper, “A history of dark matter”, Rev. Mod. Phys. 90, 45002 (2018), arXiv:1605.04909

- G. Bertone, D. Hooper & J. Silk, Phys. Rept. 405:279-390 (2005), “Particle Dark Matter: Evidence, Candidates and Constraints”, arXiv:hep-ph/0404175

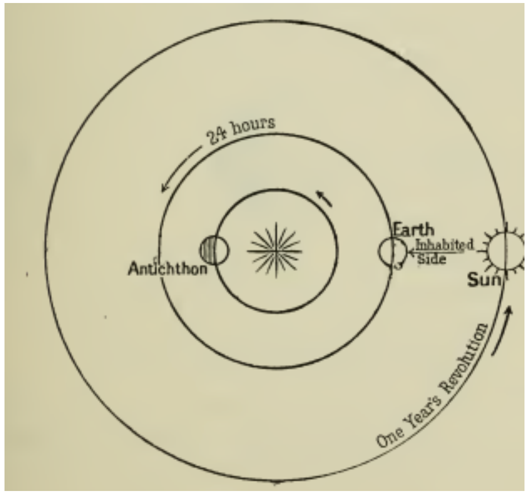
- D. F. J. Kimball, K. van Bibber, (editors) “The search for ultralight bosonic dark matter” Springer Cham, 2022 (open access)

- L. Hui, “Wave dark matter”, arXiv:2101.11735

- G. Bertone *et al*, “Gravitational wave probes of dark matter”, SciPost Phys. Core 3, 007 (2020), arXiv:1907.106010

Before the 1930s

- The idea of matter that is **invisible** to us is not new:
  - Ancient greeks already speculated about this possibility, e.g. Philolaus (c. 5th century BCE) hypothesised the existence of a "counter-earth" (*Antichthon*), which revolved in the opposite side of a "central fire" with respect to the Earth
  - **1783:** J. Michell hypothesised that if light is affected by gravity, then there could potentially exist objects that are massive enough such that not even light would be able to escape them…



Source: M. A. Orr, 1913 "Dante and the Early Astronomers"

42 *Mr.* MICHELL *on the Means of discovering the*

16. Hence, according to article 10, if the semi-diameter of a sphære of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiæ, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

Image credit: taken from a slide from V. Cardoso

Before the 1930s

- **1844:** F. Bessel predicted that the stars Sirius and Procyon had an "invisible companion" due to their the anomalous motion
- **1846:** Based on anomalies in Uranus' motion, U. Le Verrier and J. C. Adams predicted the existence of Neptune (discovered shortly after Le Verrier's predictions).
- **1859:** Le Verrier reports anomalies in the slow precession of Mercury's orbit, suggesting the existence of another planet (Vulcan)...
- **c. 1900:** Lord Kelvin attempts to estimate the amount of "dark bodies" in the Milky Way using theory of gases. He concludes that "many of our stars, perhaps a great majority of them, may be dark bodies.".
- **1906:** Based on Kelvin's work, Poincaré introduces the term ***"matière obscure"***, but concludes, based on observations of the velocity dispersions of stars, that "there is no dark matter, or at least not so much as there is shining matter".
- **1920s/(early) '30s:** Estimates of the *local* dark matter density by Kapteyn, Oort, Jeans and Lindbad, all pointing towards that "this mass cannot be excessive".

Dark matter evidence from galaxy clusters

- **1933:** Based on measurements of the velocity dispersion in the Coma galaxy cluster, and applying the virial theorem to estimate the total mass of the cluster, Fritz Zwicky concluded "that **dark matter is present** in much greater amount than luminous matter".

Virial theorem:

For a N-body system held together by gravity: $\langle K \rangle = -\frac{1}{2}\langle U \rangle$

Zwicky's argument:

Approximate cluster as a sphere of radius R and total mass M_T, formed by N galaxies of roughly equal mass m, i.e. $M_T \approx Nm$.

Assume density roughly constant: $\rho \approx \frac{M_T}{(4/3)\pi R^3}$, $M(r) \approx \frac{4\pi\rho r^3}{3}$

$$\langle U \rangle \approx -4\pi G \int_0^R \frac{M(r)\rho}{r} r^2 dr = -\frac{3GM_T^2}{5R}, \quad \langle K \rangle \approx \frac{M_T\langle v^2 \rangle}{2} \implies M_T \approx \frac{5R\langle v^2 \rangle}{3G}$$

Dark matter evidence from galaxy clusters

Via redshift one can measure velocities parallel to the line of sight, v_r .
Assuming equipartition of the kinetic energy:

$$\langle v^2 \rangle \approx 3 \langle v_r^2 \rangle \implies M_T \approx \frac{5R\langle v_r^2 \rangle}{G}$$

Coma cluster:

$$\sqrt{\langle v_r^2 \rangle} \approx 10^3\,\mathrm{km/s}\,, \quad R \approx 10^6\,\mathrm{lyr} \approx 10^{22}\,\mathrm{m} \implies M_T \approx 4 \times 10^{14} M_\odot$$

Assuming around $N \approx 10^3$ observed galaxies with estimated mass $m \approx 10^9 M_\odot$

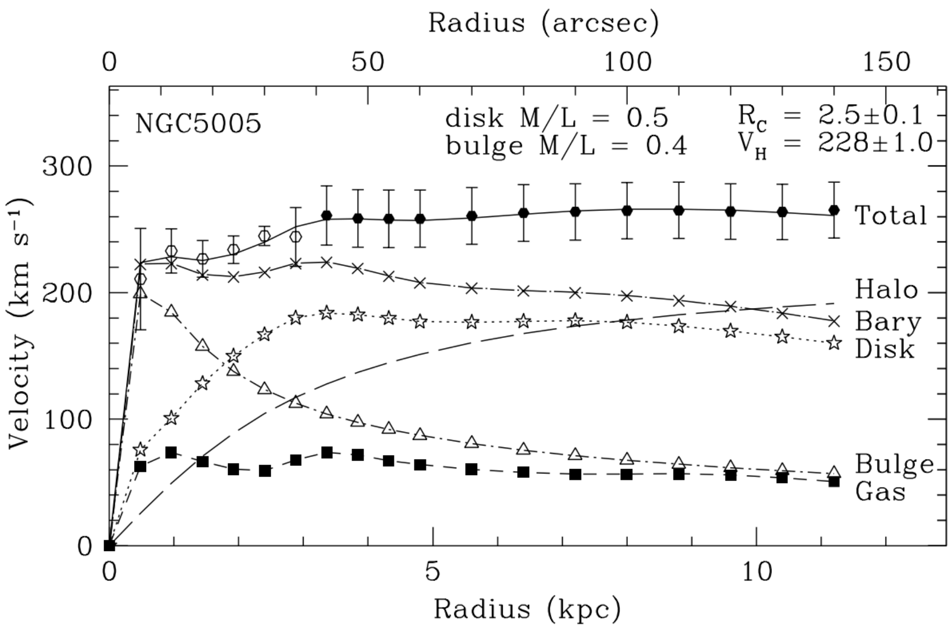
$$\implies M_{\mathrm{obs}} \approx 10^{12} M_\odot \ldots$$

"If this would be confirmed, we would get the surprising result that dark matter is present in much greater amount than luminous matter." - (translation to English) F. Zwicky, Helv. Phys. Acta 6 (1933) 110-127

More accurate estimates, taking into account the intergalactic gas in the cluster, give a total mass-to-light 10 times smaller. But still, there seems to be **much more mass than what we see**...

Dark matter evidence from galactic rotation curves

- **1970s:** Measurement of "flat rotation curves" for galaxies, using optical and 21cm-line observations (starting with seminal work by Vera Rubin and Kent Ford in 1970)



From: E.E. Richards+, arXiv:1503.05981

$$\frac{v_{\text{rot}}^2}{r} = \frac{GM(r)}{r^2} \implies v_{\text{rot}} = \sqrt{\frac{GM(r)}{r}}$$

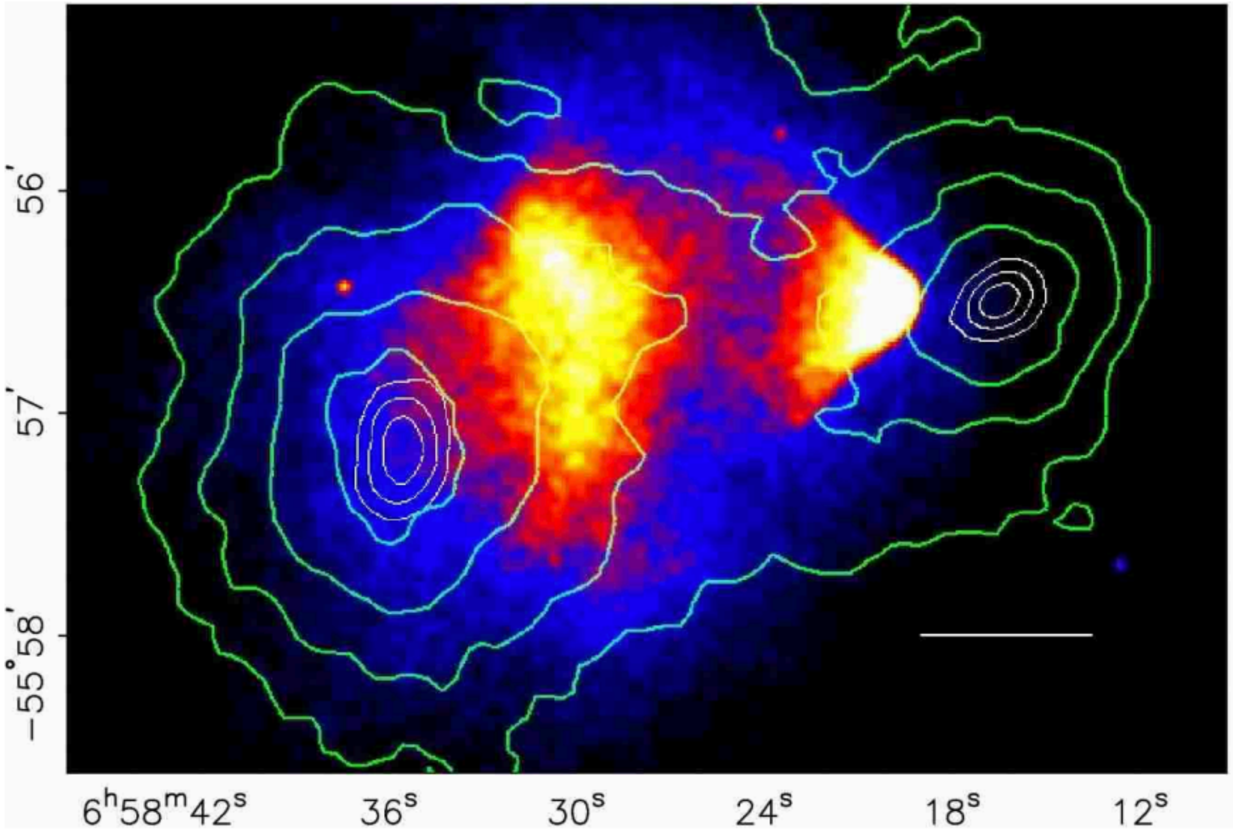
$$M(r) = 4\pi \int_0^r r'^2 \rho(r') dr'$$

If visible matter explains most of the mass of the galaxy, then in the outskirts of the galaxy one should have $v_{\text{rot}} \propto r^{-1/2}$, however that's not what see...

Need $M(r) \propto r \implies \rho(r) \propto r^{-2}$ to explain $v_{\text{rot}} \approx$ const.

"Missing mass" problem

Further evidence for dark matter: the bullet cluster



Green contours: reconstructed lensing signal, proportional to the mass in the system

Coloured map: X-ray image of the merging clusters

White line: 200 kpc scale

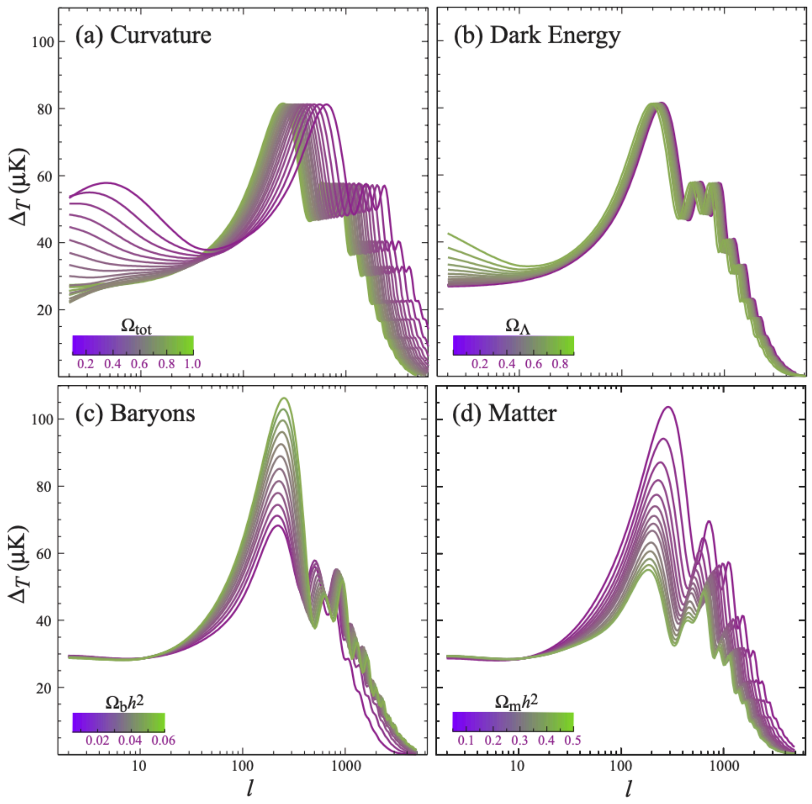
From: D. Clowe+ ApJ 648 L109 (2006)

Many more galaxy collisions studied since 2006, showing clear evidence for the presence of dark matter. [D. Harvey+, *Science* 347 (2015) 1462-1465]

Dark matter **interacts weakly** not only with other particles but also with itself [although there is still some room left here]...

Cosmic abundance

The **angular power spectrum of the CMB anisotropies** is highly sensitive to the "matter-energy" content of the Universe.



From: W. Hu & S. Dodelson, arXiv:astro-ph/0110414

Peaks in the power spectrum are a result of "**accoustic oscillations**" in the photon-baryon plasma. Peaks are sensitive to the dark matter density.

- The existence of non-baryonic matter is also confirmed when joining CMB observations with predictions from the **Big Bang nucleosynthesis (BBN)** theory, which can be used to predict the baryonic abundance.
- **CMB incredibly uniform:** need non-relativistic (*cold*) dark matter that can start clumping long before recombination to explain structure formation.

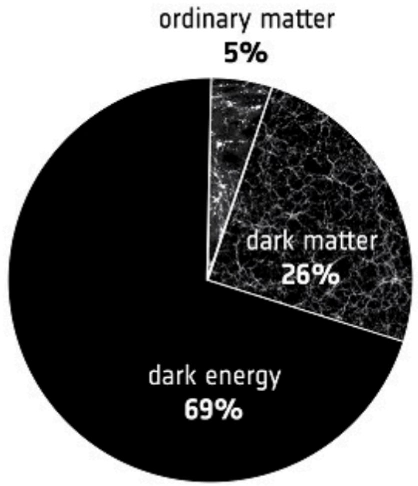


Image credit: ESA

Some known facts about dark matter

- Dark matter accounts for $\sim 26\,\%$ of the Universe's "mass-energy" budget
- Dark matter is **stable** (or extremely long-lived)
- It must have been predominantly **non-relativistic (*cold*)** at the onset of structure formation (in particular, light neutrinos cannot explain dark matter)
- It **cannot interact (non-gravitationally)** with ordinary matter or with itself **too strongly** [in particular, it is EM neutral].
- It is **not** predominantly any of the known **Standard Model particles**
- It is **distributed in halos** that extend well beyond the luminous matter of galaxies [but structure on sub-galactic scales much *less well constrained*]
- Local (average) dark matter density $\rho_{\rm DM} \approx 0.4\,{\rm GeV\,cm^{-3}} \approx 10^{-21}{\rm kg/m^3}$

This leaves room for a lot of possibilities.

What is dark matter?

Weakly interacting massive particles (WIMPs)

Adapted from: G. Bertone & T. M. P. Bait, arXiv:1810.01668

Modified gravity?

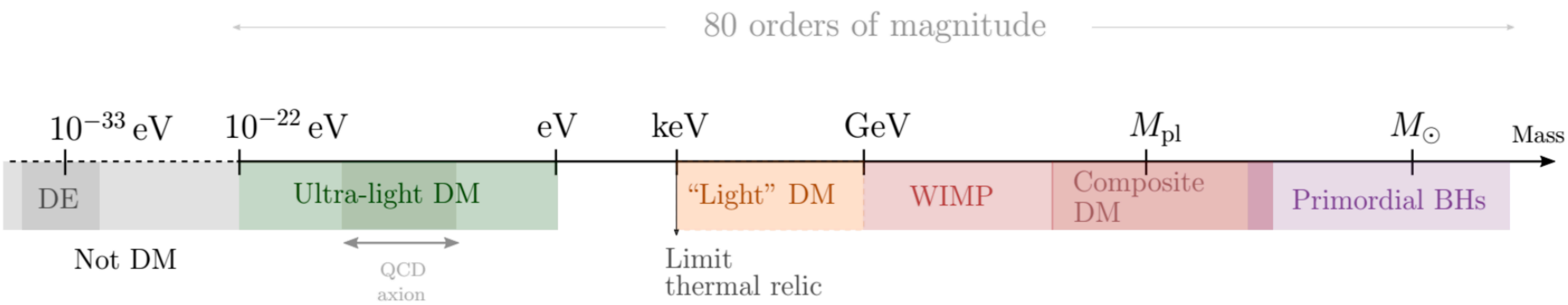
- In 1983 Mordehai Milgrom proposed that instead of introducing new particles to explain dark matter, rotation curves could be explained by **modifying Newton's 2nd law** according to [a.k.a. *Modified Newtonian Dynamics (MOND)]*:

$$\mathbf{F} = m_g \mu(a/a_0)\mathbf{a}, \qquad \mu(a/a_0 \gg 1) \approx 1, \qquad \mu(a/a_0 \ll 1) \approx a/a_0$$

$a_0 \approx 10^{-10}\,\mathrm{m\,s^{-2}}$ based on fitting MOND with rotation curves

- Relativistic theories that explain the MOND empirical law have been formulated: e.g. Tensor-Vector-Scalar gravity (TeVeS) by Bekenstein in 2004
- However, MOND has a **hard time explaining observations** such as the bullet cluster, as well as fitting the dynamics of galaxy clusters and explaining the observed CMB angular power spectrum.

Dark matter: a rough classification



From: E. Ferreira, arXiv:2005.03254

"Wave" dark matter: particles with mass $m_{\rm DM} \ll 30\,\mathrm{eV}$ [e.g. QCD axion; axion-like particles; dark photons; dilatons,…]

"Particle" dark matter: particles with mass $m_{\rm DM} \gg 30\,\mathrm{eV}$ [e.g. WIMPs such as SUSY particles; right-handed (a.k.a sterile) neutrinos,…]

"Macroscopic" dark matter: macroscopic compact objects (e.g. MACHOs; primordial black holes,…)

"Particle" vs "wave" dark matter

Velocity dispersion of the dark matter halo: $v \approx 250\,\mathrm{km/s}$

Local dark matter density: $\rho_{\mathrm{DM}} \approx 0.4\,\mathrm{GeV\,cm^{-3}} \approx 10^{-21}\mathrm{kg/m^3}$

de Broglie wavelength $\quad \Longrightarrow \lambda_{\mathrm{dB}} = \dfrac{2\pi\hbar}{m_{\mathrm{DM}} v} \approx 250\,\mathrm{km/s} \approx 1.5\,\mathrm{km}\left(\dfrac{10^{-6}\,\mathrm{eV}/c^2}{m_{\mathrm{DM}}}\right)\left(\dfrac{250\mathrm{km/s}}{v}\right)$

average number of particles occupying a volume λ_{dB}^3 $\quad \Longrightarrow N_{\mathrm{dB}} \approx \dfrac{\rho_{\mathrm{DM}}\lambda_{\mathrm{dB}}^3}{m_{\mathrm{DM}}} \approx 10^{30}\left(\dfrac{10^{-6}\,\mathrm{eV}}{m_{\mathrm{DM}}}\right)^4\left(\dfrac{250\,\mathrm{km/s}}{v}\right)^3$

Ratio of mean inter-particle distance over λ_{dB} $\quad \Longrightarrow r/\lambda_{\mathrm{dB}} \sim \left(N_{\mathrm{dB}}/\lambda_{\mathrm{dB}}^3\right)^{-1/3}/\lambda_{\mathrm{dB}} \sim 10^{-10}\left(\dfrac{m_{\mathrm{DM}}}{10^{-6}\,\mathrm{eV}/c^2}\right)^{4/3}\left(\dfrac{v}{250\,\mathrm{km/s}}\right)$

$m_{\mathrm{DM}} \gg 30\,\mathrm{eV}/c^2 \Longrightarrow N_{\mathrm{dB}} \ll 1\,, \quad r/\lambda_{\mathrm{dB}} \gg 1\ :$ well-described as individual **classical particles**

$m_{\mathrm{DM}} \ll 30\,\mathrm{eV}/c^2 \Longrightarrow N_{\mathrm{dB}} \gg 1\,, \quad r/\lambda_{\mathrm{dB}} \ll 1\ :$ well-described by a **classical wave** equation (e.g. Klein-Gordon equation for a spin-0 field)

Ultralight fermionic dark matter?

Related to the bound by S. Tremaine & J. E. Gunn'78

Pauli exclusion principle implies that:

there is a maximum density $\rho_{\max}$ that a fermion dark matter halo can have (corresponding to the case where all states are occupied up to the Fermi momentum p_F, i.e. a zero-temperature Fermi gas)

Conditions to explain dark matter:

(1) $\rho_{\max} = m_{\rm DM} n_{\max} \geq \rho_{\rm DM}$

(2) $p_F \leq m_{\rm DM} v_{\rm esc}$ ($v_{\rm esc}$ - escape velocity of a given galaxy)

Maximum number density of states of an ideal Fermi gas (assuming spherical symmetry)

$$n_{\max} = \frac{g}{(2\pi)^3} \int_0^{k_F} 4\pi k^2 \, dk = \frac{g\, p_F^3}{6\hbar^3 \pi^2} \,,$$

$$k = p/\hbar \,, \quad g = 2s + 1$$

(g - #DoF for a spin-s particle)

Assuming spin-1/2 and Milky Way-like galaxy ($\rho_{\rm DM} \approx 10^{-21} {\rm kg/m^3}$, $v_{\rm esc} \approx 550\,{\rm km/s}$):

$$\Longrightarrow \rho_{\rm DM} \leq m_{\rm DM} n_{\max} \leq \frac{g\, m_{\rm DM}^4 v_{\rm esc}^3}{6\hbar^3\pi^2} \Longrightarrow m_{\rm DM} \geq \left(\frac{6\pi^2 \hbar^3 \rho_{\rm DM}}{g\, v_{\rm esc}^3} \right)^{1/4} \sim 10\ {\rm eV}/c^2$$

Ultralight bosonic dark matter

No Pauli exclusion principle for bosons

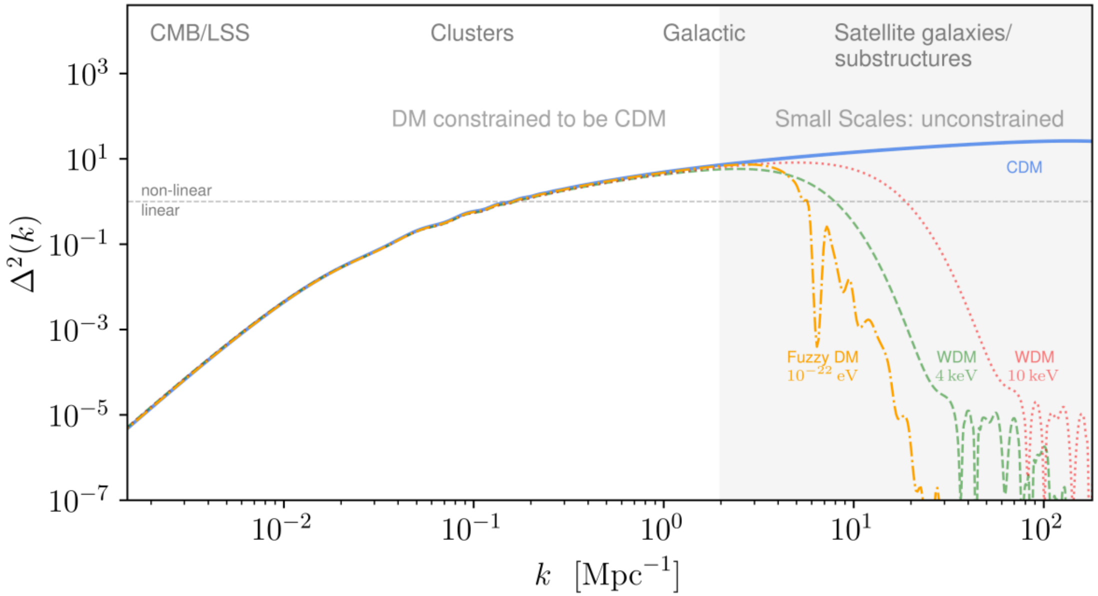
$\Longrightarrow$ dark matter bosons with $m_{\mathrm{DM}} \ll \mathrm{eV}/c^2$ not excluded by previous argument

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k)$$

(dimensionless matter power spectrum)

$$P(k) = \mathscr{F}[\xi(r)](k)$$

$$\delta(\mathbf{x}) := \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}, \; \xi(\mathbf{r}) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} - \mathbf{r}) \rangle$$



From: E. Ferreira, arXiv:2005.03254

- Observations and numerical simulations of the cold "particle"dark mater paradigm agree well on scales larger than galactic scales
- For wave dark matter, structure is suppressed at scales $\ll \lambda_{\mathrm{dB}}$. Assuming an ultralight boson explains all dark matter, observations require (at least) $m_{\mathrm{DM}} \gtrsim 10^{-22}\,\mathrm{eV}$ (corresponding to $\lambda_{\mathrm{dB}} \lesssim 0.5\,\mathrm{kpc}$ for a Milky Way-like galaxy).

QCD axion and axion-like particles

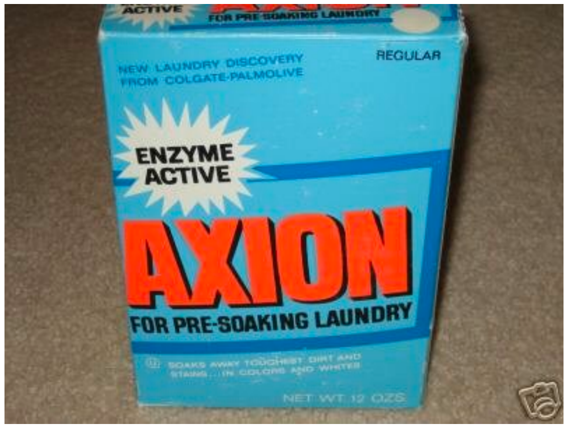
See e.g. D. Marsh, arXiv:1510.07633 for a review

Axions first introduced to solve *strong-CP problem:* why is the neutron electric dipole moment so small (or even zero)?

- A CP-violating topological term $\mathscr{L}_\theta = \dfrac{\theta}{32\pi^2}\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}$ that does not affect the classical EoM, but gives the neutron an electric dipole moment at the quantum level, is allowed in the QCD Lagrangian.

- Current experimental bounds on neutron electric dipole moment, require $|\theta| \lesssim 10^{-10}$. Why so small?

- **Rough idea:** promote θ to be a (pseudo)-scalar field [the *axion*] that dynamically relaxes to a value such that measured neutron electric dipole moment is zero (Peccei&Quinn'77; Wilczek '78; Weinberg '78)



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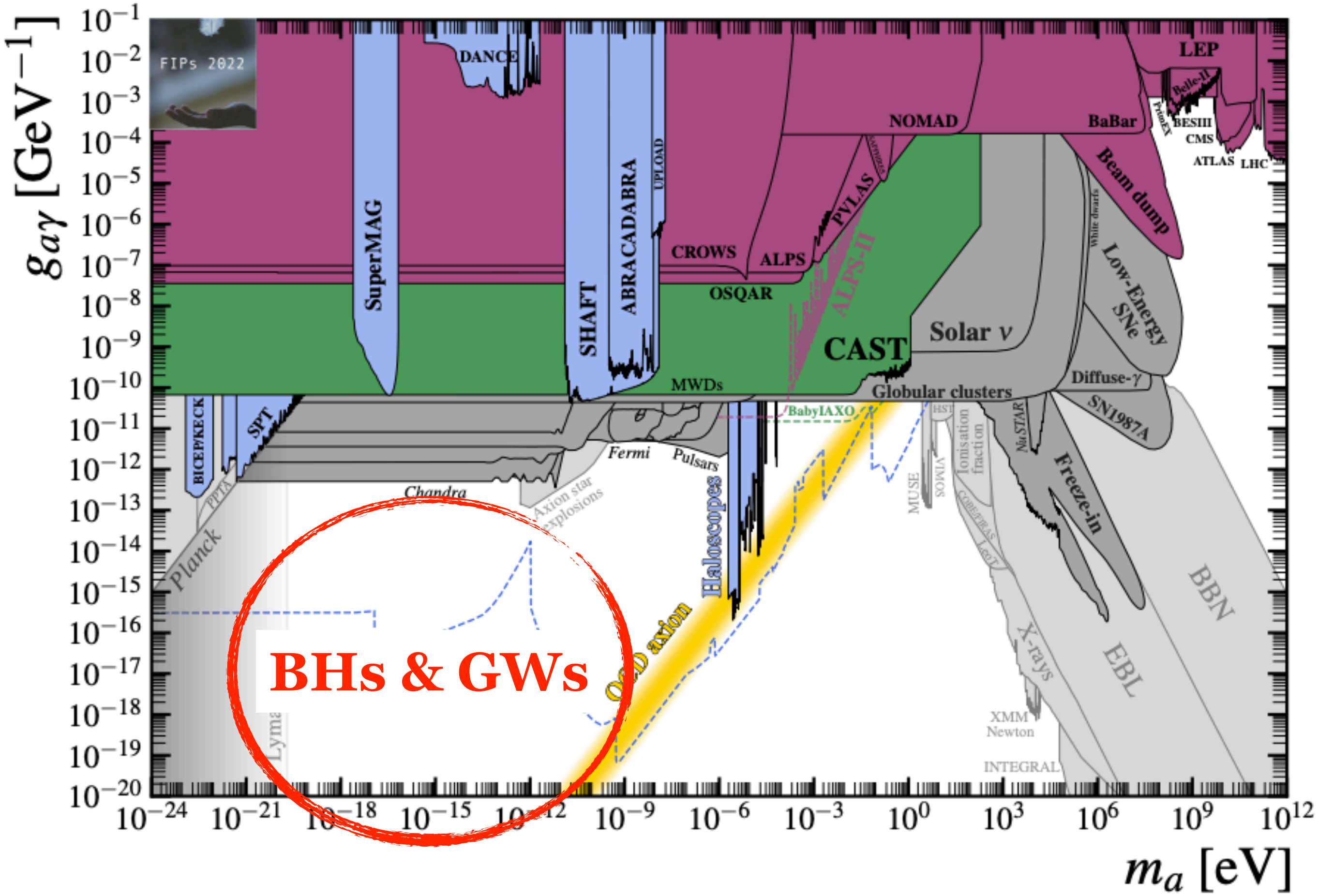
- **Rough idea:** promote θ to be a (pseudo)-scalar field [the *axion*] that dynamically relaxes to a value such that measured neutron electric dipole moment is zero (Peccei&Quinn'77; Wilczek '78; Weinberg '78)
- Axion mass depends on model, but typically expected to have masses $\ll$ eV and to couple weakly with matter: good candidate for ultralight dark matter
- Non-QCD axion-like particles have also been proposed to arise from string theory compactifications, possibly populating a large range of masses: *the string axiverse* (Arvanitaki+'09)

Axion-like particles (ALPs): current constraints

$$\mathscr{L} = -\frac{1}{2}\partial_\mu a \partial^\mu a - V(a) - \frac{g_{a\gamma}}{4} F_{\mu\nu}\tilde{F}^{\mu\nu} + \dots\,, \quad V(a) \approx \frac{m_a^2 a^2}{2} + \mathcal{O}(a^4/f_a^4)$$

For QCD axion (not required for generic ALPs): $m_a \sim 6 \times 10^{-12} \left(\frac{10^{18}\,\mathrm{GeV}}{f_a}\right)\mathrm{eV}\,, \quad g_{a\gamma} \sim 1/f_a$



Purple: lab/collider constraints

Green: lack of solar axions

Blue: direct dark matter searches

Light grey: astro/cosmo constraints that assume axions to be dark matter

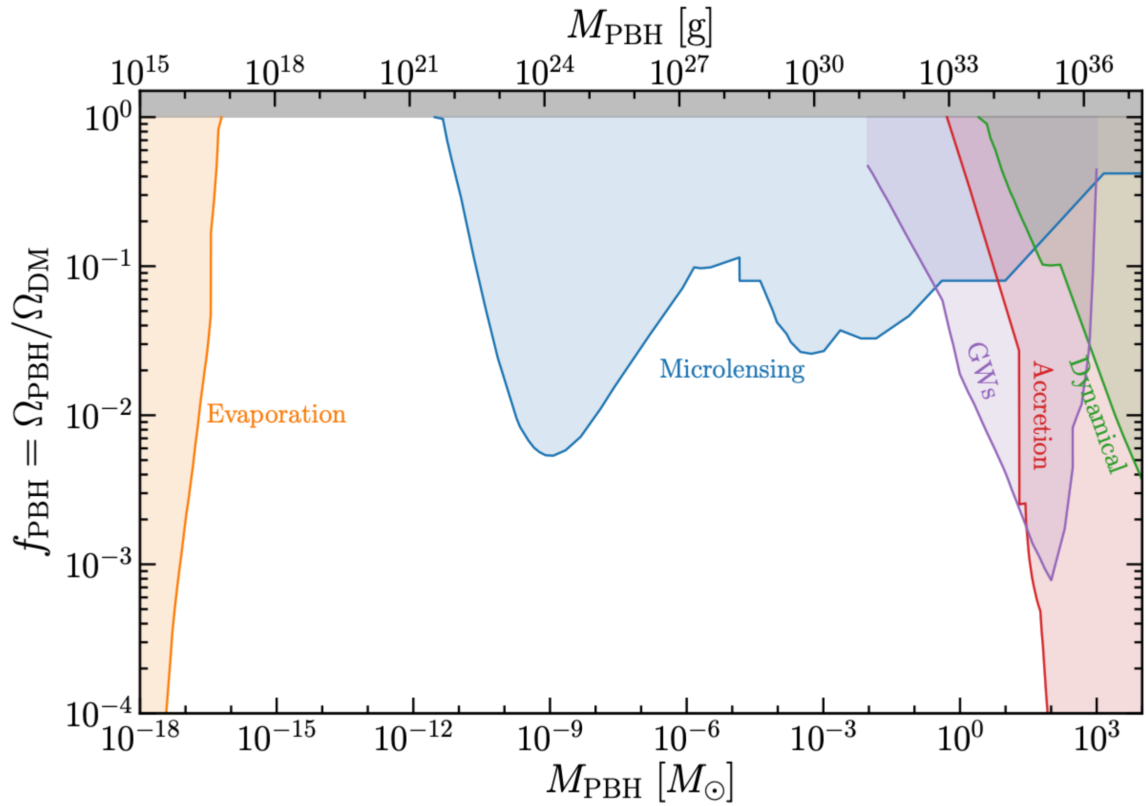
Dark grey: astro/cosmo constraints that do not assume axions to be dark matter

From: Antel+, arXiv:2305.01715;
see also https://cajohare.github.io/AxionLimits/

Primordial black holes

Massive astrophysical compact halo objects (MACHOs):

Composite baryonic objects with low luminosity, such as planets, white dwarfs, neutron stars, black holes, etc…



From: A. Green & B. J: Kavanagh, arXiv:2007.10722

- Historically, **MACHOs** were among the first candidates to explain dark matter
- Constraints on baryonic cosmic abundance and microlensing observations rules out most MACHOs
- A notable exception: **primordial black holes**, black holes formed via the collapse of large overdensities in the very early Universe which are not subject to CMB constraints on the baryonic abundance

Dark matter & Gravitational Waves

I'll only discuss these here

From: Bertone *et al*, arXiv:1907.10610

II
Black holes & ultralight bosons: superradiant instabilities

Some suggestions for further reading:

- RB, V. Cardoso & P. Pani, "Superradiance", Lecture Notes in Physics, vol 971, arXiv:1501.06570
- S. Detweiler, "Klein-Gordon equation and rotating black holes", PRD22, 2323 (1980)
- S. Dolan, "Instability of the massive Klein-Gordon field on the Kerr spacetime", PRD76, 084001 (2007), arXiv:0705.2880
- A. Arvanitaki & S. Dubovsky, "Exploring the String Axiverse with Precision Black Hole Physics", PRD83, 044026 (2011), arXiv:1004.3559
- A. Arvanitaki, M. Baryakhtar & X. Huang, "Discovering the QCD axion with black holes and gravitational waves", PRD91, 084011 (2015)

Scalar field in a curved spacetime

- Consider a simple model of a scalar field *minimally coupled* to gravity (units $G = c = 1$ from now on):

$$S_{\rm EH-KG} = \int dx^4 \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - V(\Phi) + \ldots \right)$$

$$\Downarrow$$

$$G_{\mu\nu}[g] = 8\pi T_{\mu\nu} \,, \qquad \Box^g \Phi - V'(\Phi) = 0 \,, \quad T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \left[\partial_\alpha \Phi \partial^\alpha \Phi + 2V(\Phi) \right]$$

- Neglecting self-interactions, the simplest potential is that of a *massive scalar:*

$$V(\Phi) \approx \frac{\mu^2 \Phi^2}{2} \,, \quad V'(\Phi) = \mu^2 \Phi \quad [\mu = m_b c/\hbar = 2\pi/\lambda_c]$$

- For generic situations one needs to use Numerical Relativity to solve Einstein-Klein-Gordon system, so let me consider that the scalar field influence on the curvature is *small*.

Test-field approximation

- Assume: $\Phi = \epsilon\Phi^{(1)}\,, \quad \epsilon \ll 1 \implies G_{\mu\nu} = \mathcal{O}(\epsilon^2)\,, \quad \epsilon\left(\Box^g \Phi^{(1)} - \mu^2\Phi^{(1)}\right) = 0$
- Looking at the EFEs it makes sense to expand metric as $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \mathcal{O}(\epsilon^2)$
- Up to linear order in ϵ, we then have:

$$G_{\mu\nu}[g^{(0)}] = 0\,, \; (1) \qquad \Box^{g^{(0)}}\Phi^{(1)} - \mu^2\Phi^{(1)} = 0\,, \; (2)$$

- Kerr metric is the unique stationary black hole solution of eq. (1):

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4aMr\sin^2\theta}{\Sigma}dtd\phi + \left[r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right]\sin^2\theta d\phi^2$$

$$\Sigma := r^2 + a^2\cos^2\theta\,, \quad \Delta := r^2 - 2Mr + a^2\,, \quad a := J/M$$

Event horizon: $r_+ = M + \sqrt{M^2 - a^2}$ **Ergosurface:** $r_{\text{ergo}} = M + \sqrt{M^2 - a^2\cos^2\theta}$

"Angular velocity" of the event horizon: $\Omega_H = -\lim_{r\to r_+}\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a}{2Mr_+}$

Klein-Gordon equation in a Kerr background

$$\Box^{g^{(0)}}\Phi - \mu^2\Phi = 0$$

In Kerr (using BL coordinates of previous slide) $\Downarrow$

$$-\left[\frac{\left(r^2+a^2\right)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\Phi}{\partial t^2} - \frac{4Mar}{\Delta}\frac{\partial^2\Phi}{\partial t\partial\phi} - \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial}{\partial r}\left(\Delta\frac{\partial\Phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) - \Sigma\mu^2\Phi = 0$$

(See notebook "Massive_scalar_Kerr.nb") $\Downarrow$ $\Phi = \dfrac{Y(r)}{\sqrt{r^2+a^2}}S(\theta)e^{im\phi}e^{-i\omega t}$

$$\frac{d^2}{dr_*^2}Y(r) + \left(\omega^2 - V_{\rm eff}\right)Y(r) = 0\,, \qquad dr_* = (r^2+a^2)\Delta^{-1}dr$$

$\Rightarrow$ *radial Schrödinger-like equation*

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS}{d\theta}\right) + \left[a^2(\omega^2-\mu^2)\cos^2\theta - \frac{m^2}{\sin^2\theta} + \Lambda_{lm}\right]S = 0$$

$\Rightarrow$ *Spheroidal harmonics* (spherical if $a = 0$)

$$V_{\rm eff} = \frac{\Delta\mu^2}{a^2+r^2} + \frac{\Delta\left(a^2\left(\omega^2-\mu^2\right)+\Lambda_{lm}\right) - a^2m^2 + 4amMr\omega}{\left(a^2+r^2\right)^2} + \frac{\Delta\left(a^2-4Mr+3r^2\right)}{\left(a^2+r^2\right)^3} - \frac{3\Delta^2r^2}{\left(a^2+r^2\right)^4}$$

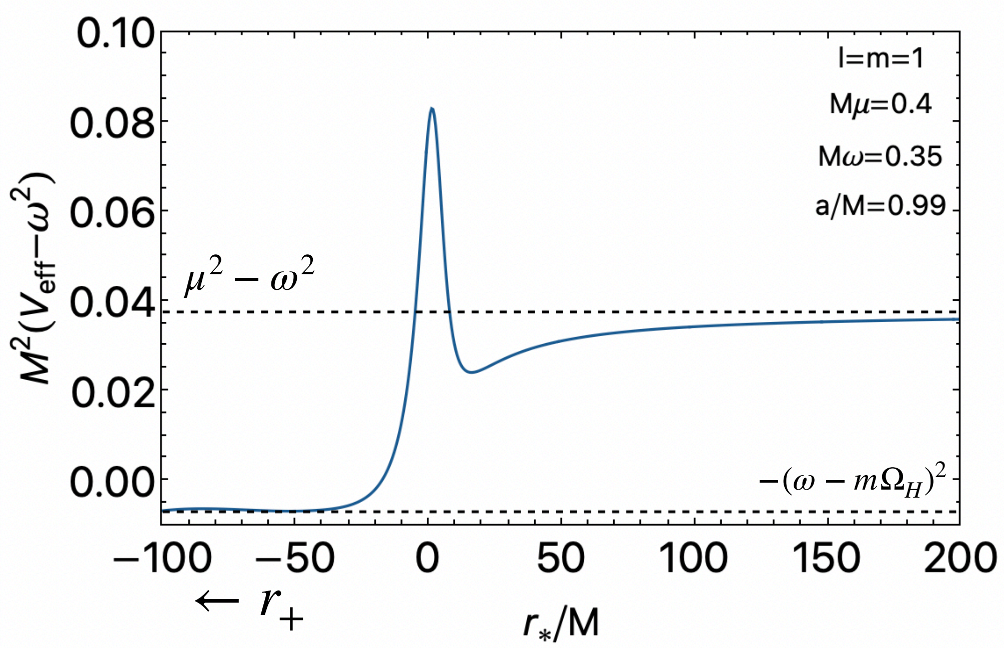
Quasi-bound states

See e.g. S. Dolan, arXiv:0705.2880

- For $\mu \neq 0$, effective potential has a potential well: existence of **bound states**?
- Problem analogous to hydrogen atom: find eigenfrequencies by imposing appropriate boundary conditions

Ingoing waves at the horizon $\lim_{r \to r_+} Y(r) \sim e^{-i(\omega - m\Omega_H) r_*}$

Decaying at infinity [Notice this requires $\mu^2 > \omega^2$] $\lim_{r \to \infty} Y(r) \sim e^{-\sqrt{\mu^2 - \omega^2} r_*}$



- Main difference with hydrogen atom problem: boundary conditions at the horizon (i.e. system is dissipative):

$$\Longrightarrow \quad \omega_{nlm} \equiv \Re(\omega_{nlm}) + i\Im(\omega_{nlm})$$

Spectrum of quasi-bound states

Review: RB, Cardoso & Pani "Superradiance" Lect. Notes Phys. 971 (2020), 2nd ed.

- For generic $M\mu$, eigenfrequencies can be found numerically [S.Dolan, 0705.2880 and notebook "QuasiBSs_Kerr.nb"]
- For $M\mu \ll 1$, equations can be solved analytically using a "matched asymptotic expansion" [Detweiler PRD**22**, (1980) 2323; Baumann+, 1908.10370]

For $r \gg M\,, M\mu \sim M\omega \ll 1$:

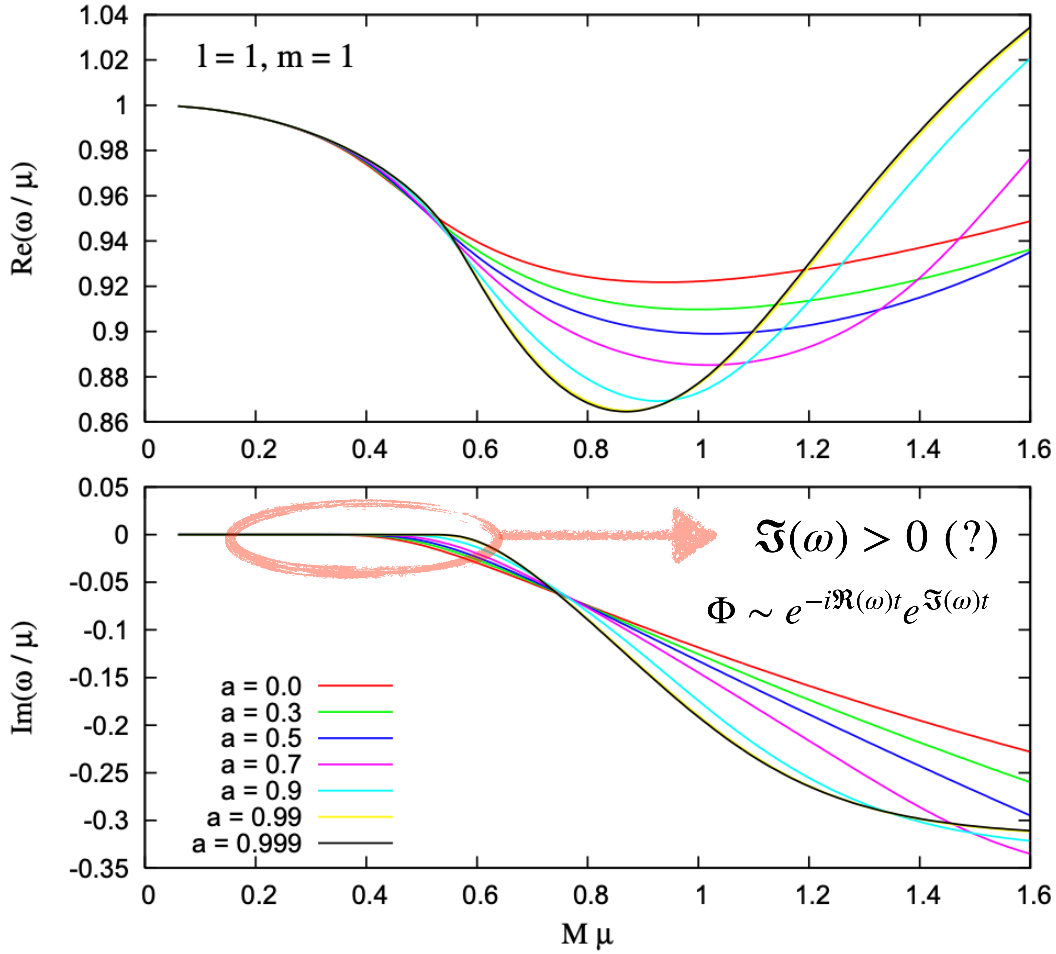
$$\frac{d^2}{dr^2}Y(r) + \left(\omega^2 - \mu^2 + \frac{2M\mu^2}{r} - \frac{l(l+1)}{r^2}\right)Y(r) = 0$$

Same as radial equation for hydrogen atom with:

$$\alpha := M\mu = 2\pi R_G/\lambda_c\,, \quad r_{\mathrm{Bohr}} = 1/(\mu\alpha)$$

$$2m_b E := \hbar^2(\omega^2 - \mu^2)$$

$$\Longrightarrow \quad \omega \approx \omega_{\mathrm{hidrogen}} = \mu\left(1 - \frac{\alpha^2}{2n^2}\right) + \ldots$$



From: S. Dolan, arXiv:0705.2880

Instability of bosonic fields in a Kerr BH

Review: RB, Cardoso & Pani "Superradiance" Lect. Notes Phys. 971 (2020), 2nd ed.

- System can actually become unstable for some values of $M\mu$, because $\Im(\omega_{nlm}) > 0$.
- Strongest instability for $M\mu \sim 0.42\,,\ a/M = 0.999\,, l = m = 1, n = 2$:

$$\tau_{\rm inst} := 1/\Im(\omega) \approx 5\,\mathrm{min}\left(\frac{M}{10\,M_\odot}\right)$$

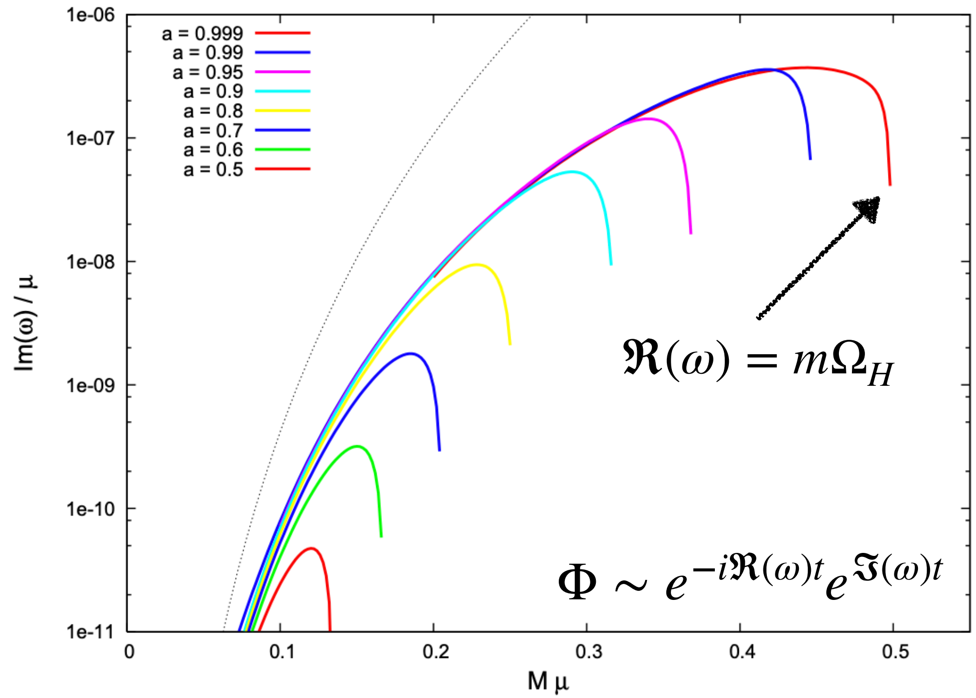
For reference: $$M\mu \approx 0.4\left(\frac{M}{55 M_\odot}\right)\left(\frac{m_b c^2}{10^{-12}\mathrm{eV}}\right)$$

For $\alpha := M\mu \ll 1$:

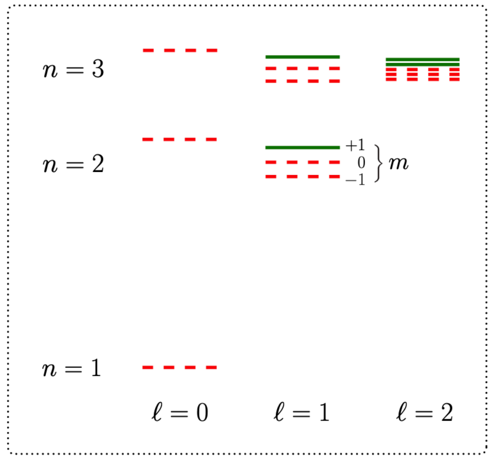
$$\Re(\omega_{nlm}) \approx \mu\left[1 - \frac{\alpha^2}{2n^2} + \mathcal{O}\left(\alpha^4\right)\right]$$

$$\Im(\omega_{nlm}) \propto \left(m\Omega_H - \Re(\omega_{nlm})\right)\alpha^{4l+5}$$

$$\boxed{\Re(\omega_{nlm}) < m\Omega_H \implies \Im(\omega_{nlm}) > 0}$$



From: Dolan, arXiv:0705.2880



Dashed: stable

Solid: unstable

From: Baumann+, arXiv:1804.03208

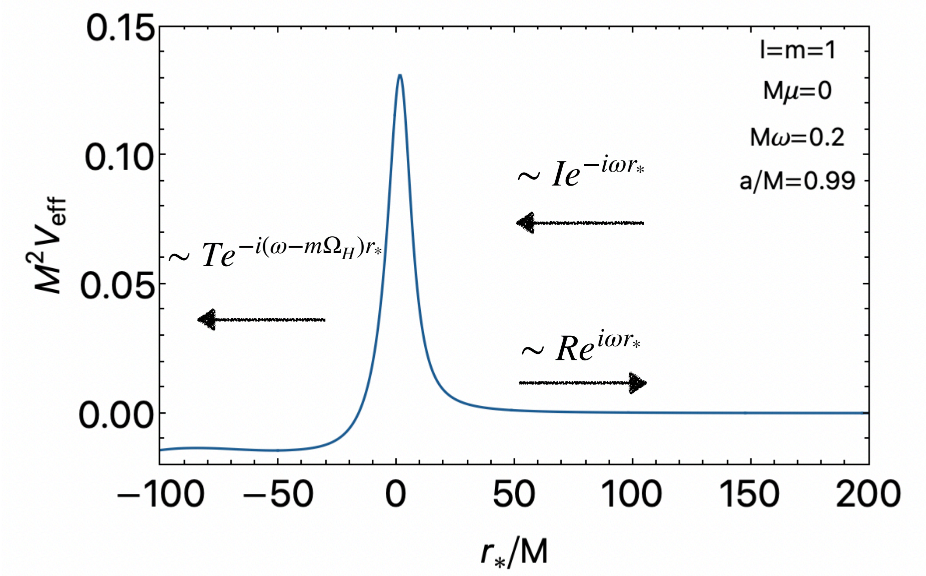
Understanding the instability: BH superradiance

Zel'dovich, '71; Misner '72; Press and Teukolsky ,'72-74;
Review: RB, Cardoso & Pani "Superradiance" Lect. Notes Phys. 971 (2020), 2nd ed.
(see also notebooks @ https://centra.tecnico.ulisboa.pt/network/grit/)

Scattering experiment (taking $\mu = 0$):

$$\lim_{r\to\infty} Y(r) \sim R\, e^{i\omega r_*} + I\, e^{-i\omega r_*}$$

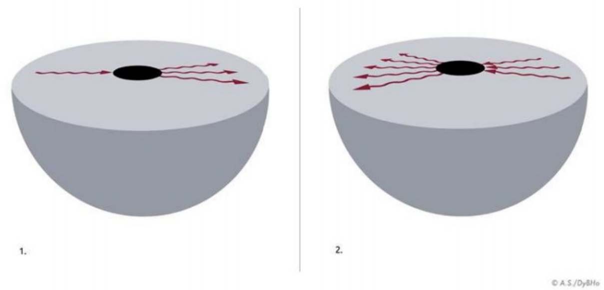
$$\lim_{r\to r_+} Y(r) \sim T e^{-i(\omega - m\Omega_H) r_*}$$



- Radial equation implies: $W = \frac{d\bar{Y}}{dr_*} Y - \frac{dY}{dr_*}\bar{Y} = \text{const} \implies |R|^2 = |I|^2 - \frac{\omega - m\Omega_H}{\omega}|T|^2$

If $\omega/m < \Omega_H \implies |R|^2 > |I|^2 \implies$ Waves can be amplified by **extracting** energy & angular momentum from the BH

- By "confining" radiation, extraction process can turn into an **instability**: this is essentially what is happening for (quasi)bound-states

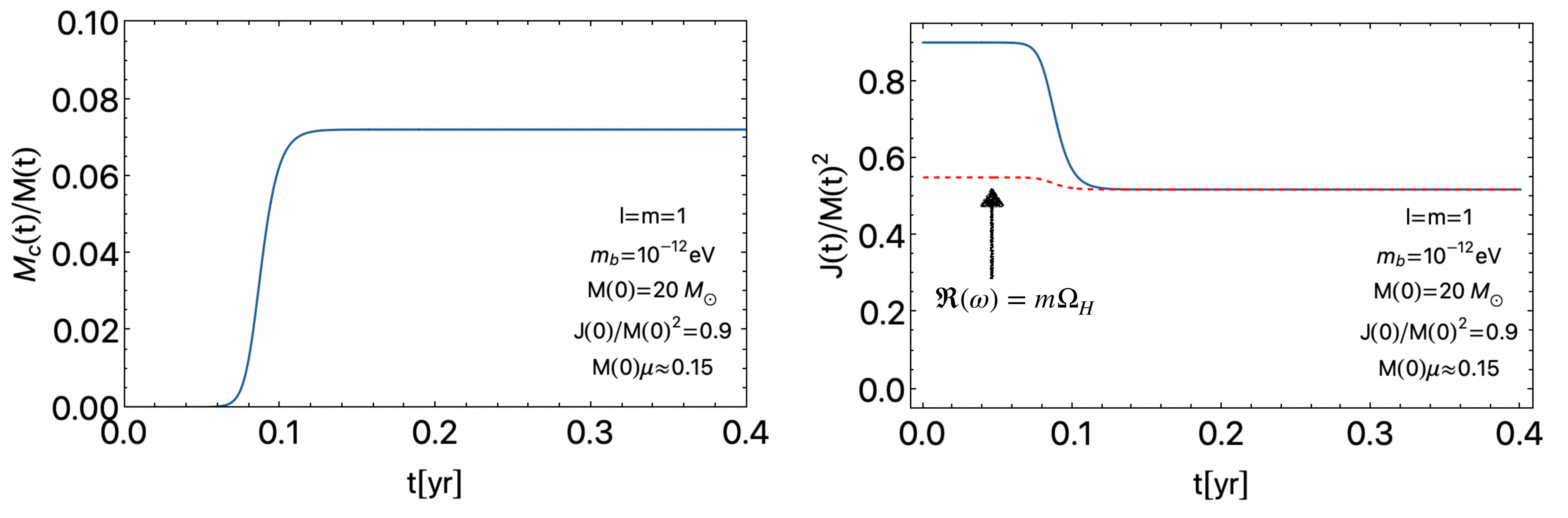


Evolution of the superradiant instability I

- Instability is “slow” (in the sense that $\tau_{\rm inst} \gg M$): to a good degree, evolution of the instability can be understood evolving the system *adiabatically* (assuming a single unstable mode for simplicity):

$$\dot{M} = -2\Im(\omega) M_c\,, \quad \dot{M}_c = 2\Im(\omega) M_c\,, \quad \dot{J} = -2\frac{m}{\Re(\omega)}\Im(\omega) M_c$$

$$M_c \text{ - “boson cloud” mass, } M_c = \int T^0_0 dV \propto e^{2\Im(\omega)t}$$



For $t \gg |\log(M_c(0))|/\Im(\omega)$ instability saturates and a “boson cloud” forms.

GW emission from a boson cloud

Yoshino & Kodama '14; Arvanitaki, Baryakhtar & Huang, '15; RB *et al '17;* Baryakthar, Lasenby & Teo '17; Siemonsen & East '20; Siemonsen, May&East '22...

$$G_{\mu\nu}[g] = 8\pi T_{\mu\nu}[\Phi,\Phi,g]\,, \qquad \Box^g \Phi - \mu^2\Phi = 0\,, \quad T_{\mu\nu}[\Phi,\Phi,g] = \partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\left[\partial_\alpha\Phi\partial^\alpha\Phi + \mu^2\Phi^2\right]$$

$$\Phi = \epsilon\Phi^{(1)} + \ldots\,, \qquad g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon^2 h_{\mu\nu} + \ldots$$

$$\Longrightarrow\; G_{\mu\nu}[g^{(0)}] + \epsilon^2\delta G_{\mu\nu}[h] = \epsilon^2\delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}] + \ldots$$

[defining $\lambda = \epsilon^2$] $\qquad \delta G_{\mu\nu}[h] := \frac{d}{d\lambda}G_{\mu\nu}[g^{(0)} + \lambda h]\,|_{\lambda=0}\,, \quad \delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}] := \frac{d}{d\lambda}T_{\mu\nu}[\Phi,\Phi,g]\,|_{\lambda=0} = T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}]$

$$\boxed{\mathcal{O}(\epsilon^0): G_{\mu\nu}[g^{(0)}] = 0\,, \qquad \mathcal{O}(\epsilon^1): \Box^{g^{(0)}}\Phi^{(1)} - \mu^2\Phi^{(1)} = 0\,, \qquad \mathcal{O}(\epsilon^2): \delta G_{\mu\nu}[h] = \delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}]}$$

- Linear gravitational perturbations of a Kerr BH black hole simpler to compute using the *Teukolsky formalism* [Teukolsky '72,'73], which reduces the problem to the study of a single **separable** wave-like equation [**the Teukolsky equation**, see London's & Pound's lectures]:

Radial Teukolsky equation

$$\mathcal{O}[\rho^{-4}\psi_4] = \mathcal{S}[\delta T] \quad\Longrightarrow\quad \mathcal{D}_{-2}R_{\tilde{l}\tilde{m}\tilde{\omega}}(r) = T_{\tilde{l}\tilde{m}\tilde{\omega}}(r)$$

$$\mathcal{L}\;{}_{-2}S_{\tilde{l}\tilde{m}\tilde{\omega}}(\theta; a\tilde{\omega}) = 0$$

$$\rho^{-4}\psi_4 = \int_{-\infty}^{\infty}\sum_{\tilde{l}=2}^{\infty}\sum_{\tilde{m}=-\tilde{l}}^{\tilde{m}=\tilde{l}} {}_{-2}R_{\tilde{l}\tilde{m}\tilde{\omega}}(r)\,{}_{-2}S_{\tilde{l}\tilde{m}}(\theta; a\tilde{\omega})e^{i\tilde{m}\phi}e^{-i\tilde{\omega}t}d\tilde{\omega}$$

spin-weighted spheroidal harmonics

GW emission from a boson cloud

Yoshino & Kodama ’14; Arvanitaki, Baryakhtar & Huang, ’15; RB *et al* ’17; Baryakthar, Lasenby & Teo ’17; Siemonsen & East ’20; Siemonsen, May&East ’22…

$$G_{\mu\nu}[g] = 8\pi T_{\mu\nu}[\Phi,\Phi,g]\,, \qquad \Box^g\,\Phi - \mu^2\Phi = 0\,, \quad T_{\mu\nu}[\Phi,\Phi,g] = \partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\left[\partial_\alpha\Phi\partial^\alpha\Phi + \mu^2\Phi^2\right]$$

$$\Phi = \epsilon\Phi^{(1)} + \ldots\,, \qquad g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon^2 h_{\mu\nu} + \ldots$$

$$\Longrightarrow\ G_{\mu\nu}[g^{(0)}] + \epsilon^2\delta G_{\mu\nu}[h] = \epsilon^2\delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}] + \ldots$$

[defining $\lambda = \epsilon^2$] $\qquad \delta G_{\mu\nu}[h] := \frac{d}{d\lambda}G_{\mu\nu}[g^{(0)} + \lambda h]\,|_{\lambda=0}\,, \qquad \delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}] := \frac{d}{d\lambda}T_{\mu\nu}[\Phi,\Phi,g]\,|_{\lambda=0} = T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}]$

$$\boxed{\mathcal{O}(\epsilon^0) : G_{\mu\nu}[g^{(0)}] = 0\,, \qquad \mathcal{O}(\epsilon^1) : \Box^{g^{(0)}}\,\Phi^{(1)} - \mu^2\Phi^{(1)} = 0\,, \qquad \mathcal{O}(\epsilon^2) : \delta G_{\mu\nu}[h] = \delta T_{\mu\nu}[\Phi^{(1)},\Phi^{(1)},g^{(0)}]}$$

- Linear gravitational perturbations of a Kerr BH black hole simpler to compute using the *Teukolsky formalism* [Teukolsky ’72,’73], which reduces the problem to the study of a single **separable** wave-like equation [**the Teukolsky equation**, see London’s & Pound’s lectures]:

$$T_{\tilde{l}\tilde{m}\tilde{\omega}}(r) \propto \int dt \int d\Omega\, {}_{-2}S^*_{\tilde{l}\tilde{m}}(\theta; a\tilde{\omega})\, e^{i\tilde{\omega}t - i\tilde{m}\phi}\, \mathcal{T}[\delta T] \xrightarrow[\delta T \sim e^{\pm 2i\Re(\omega)t \pm 2im\phi}]{} \tilde{\omega} = 2\Re(\omega)\,,\ |\tilde{m}| = 2m$$

For a cloud in a given state (n,l,m), GWs are emitted with frequency $\tilde{\omega} = 2\Re(\omega) \sim 2\mu$.

GW emission from a boson cloud

Yoshino & Kodama '14; Arvanitaki, Baryakhtar & Huang, '15; RB *et al '17;* Baryakthar, Lasenby & Teo '17; Siemonsen & East '20; Siemonsen, May&East '22...

- Radial Teukolsky equation can be solved using a Green's function approach. For details regarding computation see RB+ arXiv:1706.0633; Siemonsen&East arXiv.1910.09476.
- Once radial equation is solved one can compute GW polarisations:

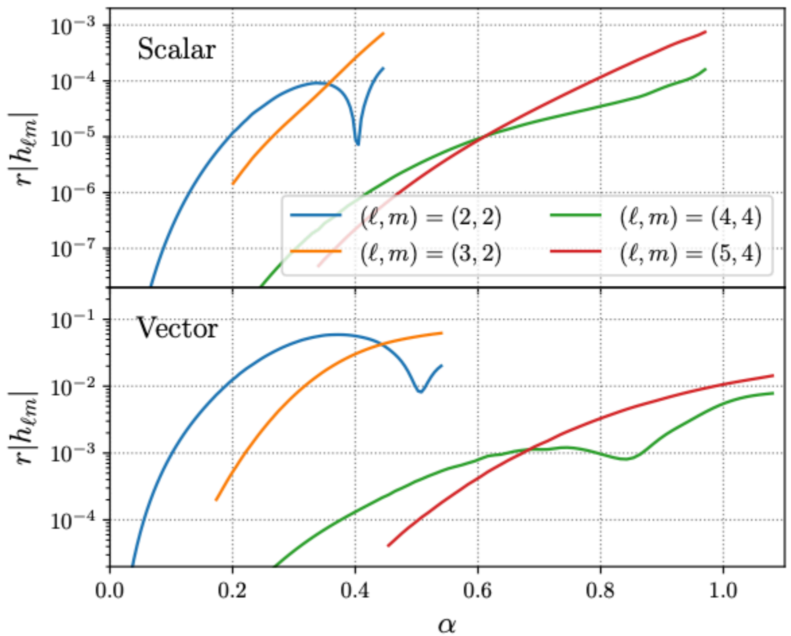
$$\lim_{r\to\infty} \psi_4 = -\frac{1}{2}\left(\ddot{h}_+ - i\ddot{h}_\times\right) \propto M_c \sum_{\tilde{l}\geq\tilde{m}} \frac{e^{i\tilde{\omega}(r-t)-i\tilde{m}\phi}\,{}_{-2}S_{\tilde{l}\tilde{m}}(\theta)}{r} \quad \Longrightarrow$$

$$h_+ \approx \frac{1}{r}\sum_{\tilde{l}\geq\tilde{m}} h^{\tilde{l}\tilde{m}}\left[{}_{-2}S_{\tilde{l}\tilde{m}} + (-1)^{\ell}\,{}_{-2}S_{\tilde{l}\tilde{m}}\right]\cos(\tilde{\omega}t + \tilde{\phi}_{\tilde{l}\tilde{m}})$$

$$h_\times \approx -\frac{1}{r}\sum_{\tilde{l}\geq\tilde{m}} h^{\tilde{l}\tilde{m}}\left[{}_{-2}S_{\tilde{l}\tilde{m}} - (-1)^{\ell}\,{}_{-2}S_{\tilde{l}\tilde{m}}\right]\sin(\tilde{\omega}t + \tilde{\phi}_{\tilde{l}\tilde{m}})$$

Useful tools:

- Mathematica notebook to solve radial Teukolsky equation @ https://github.com/richbrito/gw_superradiance/
- Python codes to compute waveforms & many other useful quantities:
  - SuperRad,(developers: Nils Siemonsen, Taillte May & Will East, arXiv:2211.03845);
  - gwaxion, (main developer: Max Isi + contributions RB)



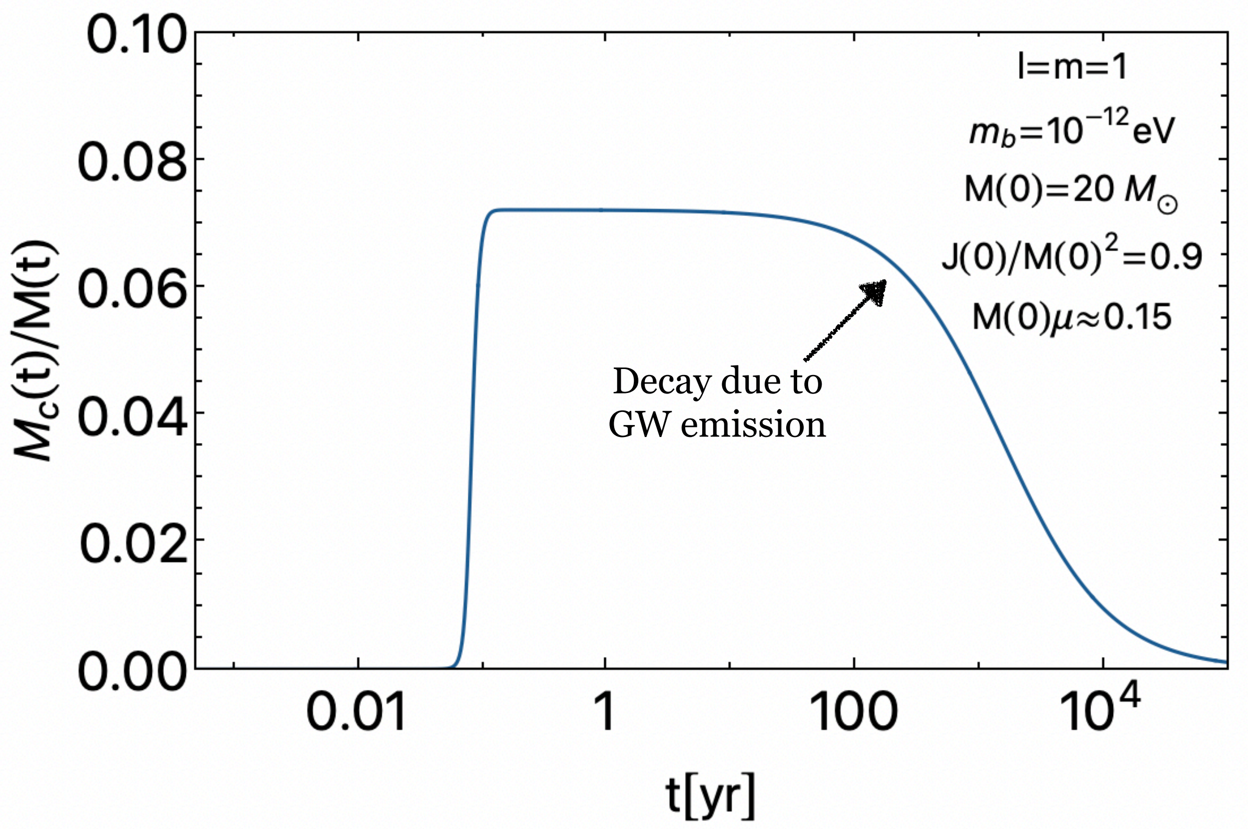
From: Siemonsen, May & East, arXiv:2211.03845

Evolution of the superradiant instability II

- Cloud dissipates due to emission of gravitational waves, on timescale typically much larger than the instability timescale.

$$\dot{M} = -2\Im(\omega)M_c\,, \quad \dot{M}_c = 2\Im(\omega)M_c - \dot{E}_{\rm GW}\,, \quad \dot{J} = -2\frac{m}{\Re(\omega)}\Im(\omega)M_c$$

$$\dot{E}_{\rm GW} \propto (M_c/M)^2 \implies M_c(t > t_{\rm sat}) \approx \frac{M_c^{\rm sat}}{1 + (t - t_{\rm sat})/\tau_{\rm gw}} \qquad \tau_{\rm GW} := M_c^{\rm sat}/\dot{E}_{\rm GW}^{\rm sat}$$



$$\dot{E}_{\rm GW} = \frac{r^2}{16\pi}\int d\Omega\left(\dot{h}_+^2 + \dot{h}_\times^2\right)$$

For $M\mu \ll 1$:

$$\dot{E}_{\rm GW}^{\rm scalar} \propto \frac{M_c^2}{M^2}(M\mu)^{4m+10}$$

$$\dot{E}_{\rm GW}^{\rm vector} \propto \frac{M_c^2}{M^2}(M\mu)^{4m+6}$$

For scalar clouds: Yoshino & Kodama '14; RB, Cardoso & Pani '15; RB *et al* '17

For vector clouds: Baryakthar *et al* '17; Siemonsen & East '20

See notebook "adiabatic_evolution_superrad.nb"

Superradiant instability: summary

Damour '76; Gaina '78; Zouros & Eardley '79; Detweiler '80; Cardoso&Yoshida '05; Dolan '07; Arvanitaki+ '10, Rosa & Dolan '12; Pani+ '12; RB, Cardoso & Pani '13; Baryakthar, Lasenby & Teo '17; East '17; Cardoso+ '18; Frolov+ '18; Dolan '18; Baumann et al '19; RB, Grillo & Pani '20; Dias+ '23,...

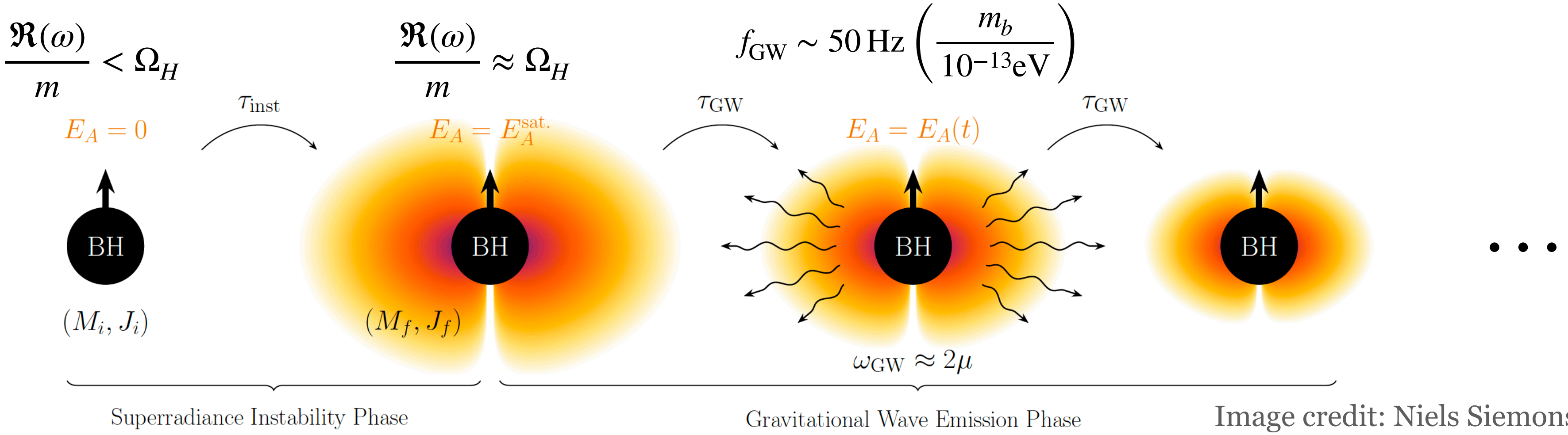
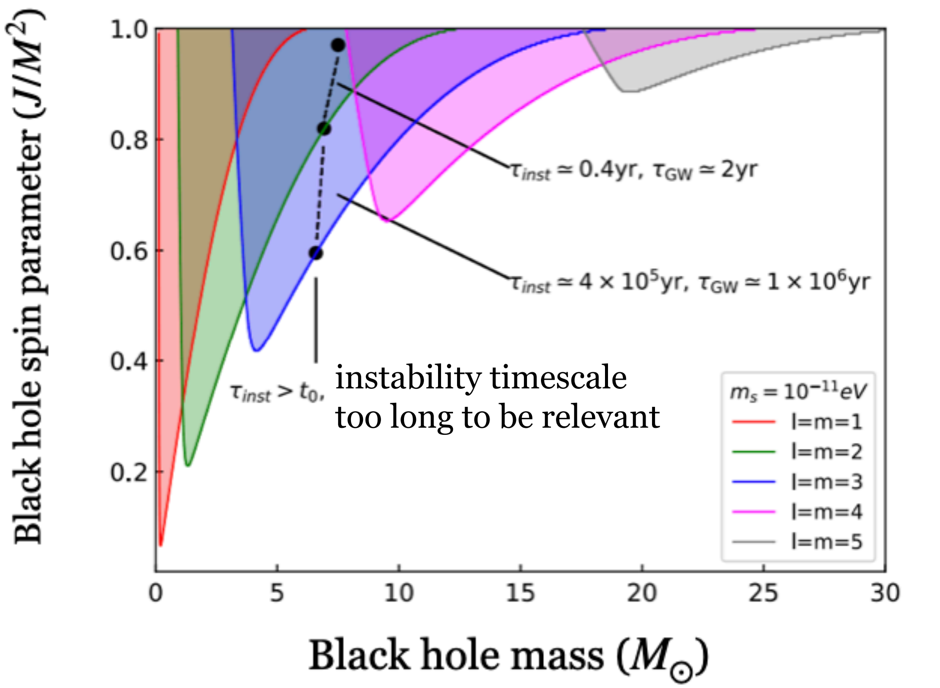


Image credit: Niels Siemonsen



Adapted from: Chen, RB, Cardoso, arXiv:2106.00021

For most unstable mode:

$$\tau_{\rm inst}^{\rm scalar} \approx 30\ {\rm days}\left(\frac{M}{10\,M_\odot}\right)\left(\frac{0.1}{M\mu}\right)^9\left(\frac{0.9}{J/M^2}\right)$$

$$\tau_{\rm GW}^{\rm scalar} \approx 10^5\ {\rm yr}\left(\frac{M}{10\,M_\odot}\right)\left(\frac{0.1}{M\mu}\right)^{15}\left(\frac{0.5}{\Delta(J/M^2)}\right)$$

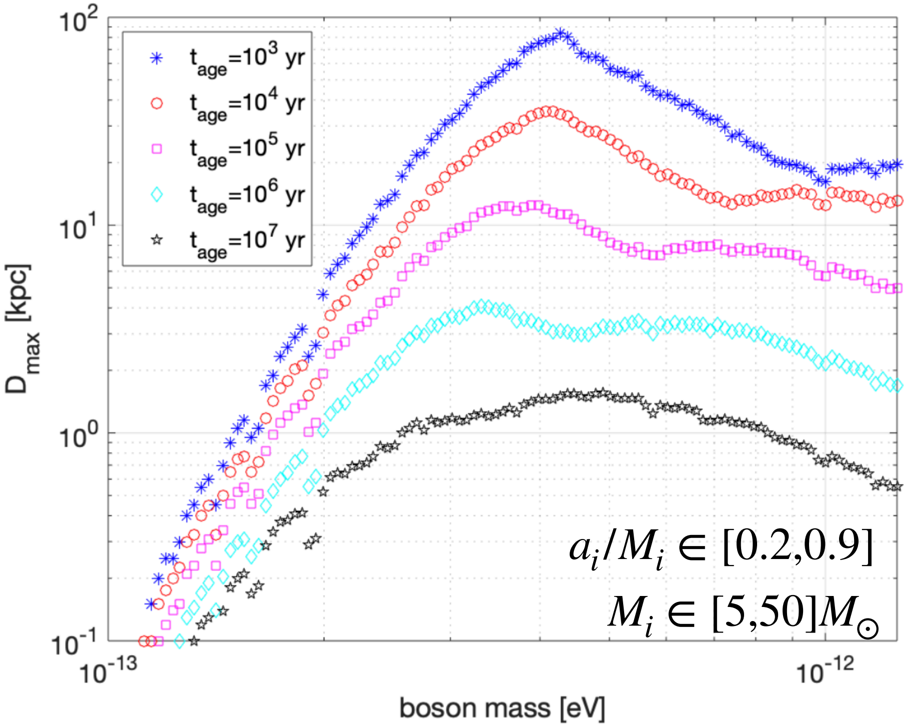
$$\tau_{\rm inst}^{\rm vector} \approx 280\ {\rm s}\left(\frac{M}{10\,M_\odot}\right)\left(\frac{0.1}{M\mu}\right)^7\left(\frac{0.9}{J/M^2}\right)$$

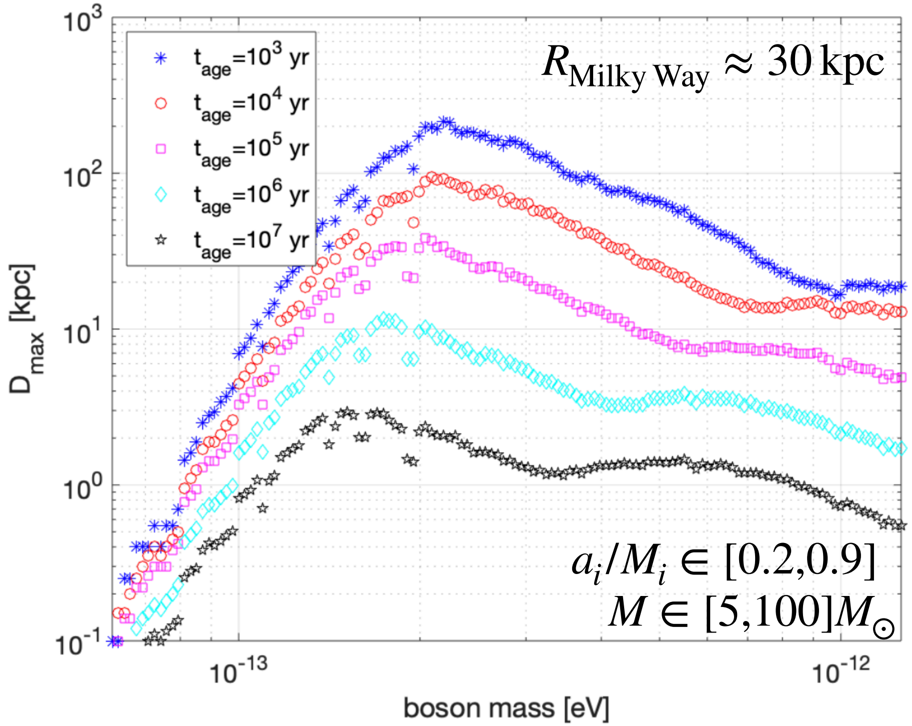
$$\tau_{\rm GW}^{\rm vector} \approx 2\ {\rm days}\left(\frac{M}{10\,M_\odot}\right)\left(\frac{0.1}{M\mu}\right)^{11}\left(\frac{0.5}{\Delta(J/M^2)}\right)$$

(even smaller instability timescale for massive spin-2 case, Dias+ '23)

LIGO blind all-sky searches

- The LVK collaboration performed a **“all-sky” search** for long-duration monochromatic GW signals from scalar clouds using **O3 data** in the frequency band [20,610] Hz.
- They did not find evidence for any such signals in the data, but results can be used to impose **constraints** on the existence of such systems.





From: LVK Collaboration, arXiv: 2111.15507

- Plots show **maximum distance** for which at least 5% of a simulated population of BHs with a scalar cloud would produce a detectable GW signal.
- Assumed a uniform distribution in BH natal spin and a mass distribution $p(m) \propto m^{-2.3}$ for a total of $\sim 10^8$ BHs in the Milky Way. Notice that there are **large uncertainties** associated with the choice of such distributions.

Future detectors

"Quick and dirty" approximation for (angle-averaged) optimal signal-to-noise ratio (SNR):

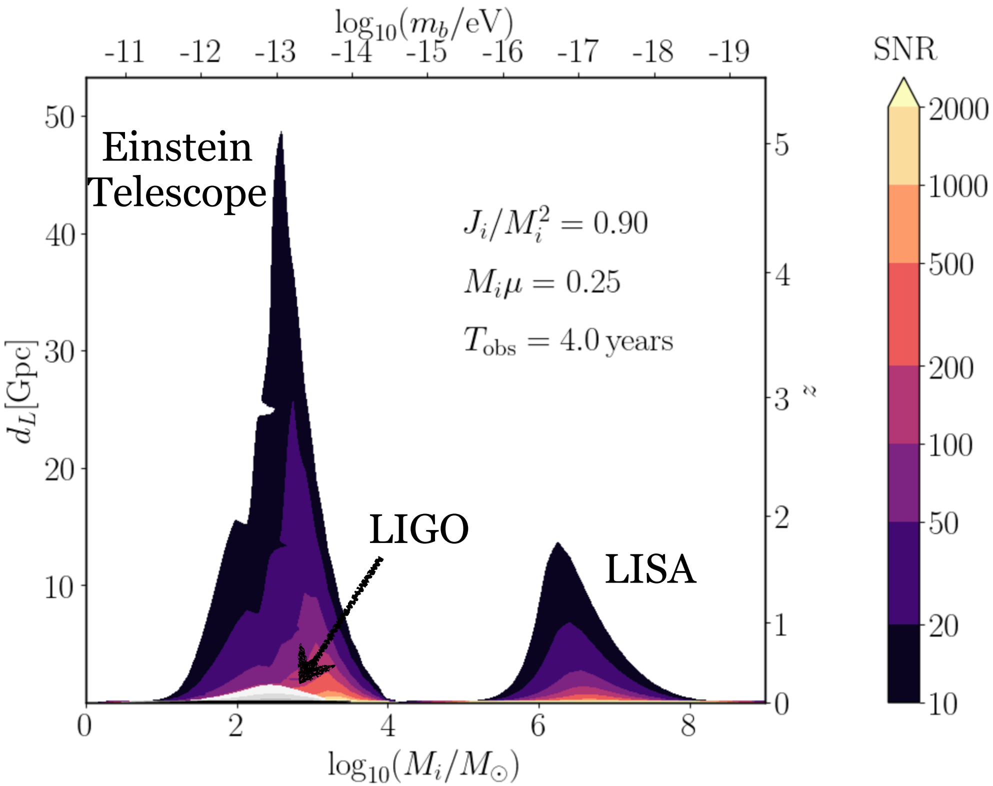
$$\mathrm{SNR} \propto h_0 \sqrt{\frac{N_{\rm cycles}}{f S_n(f)}}$$

$$N_{\rm cycles} \approx f T_{\rm obs}, \quad \text{if} \quad T_{\rm obs} \ll \tau_{\rm GW}$$

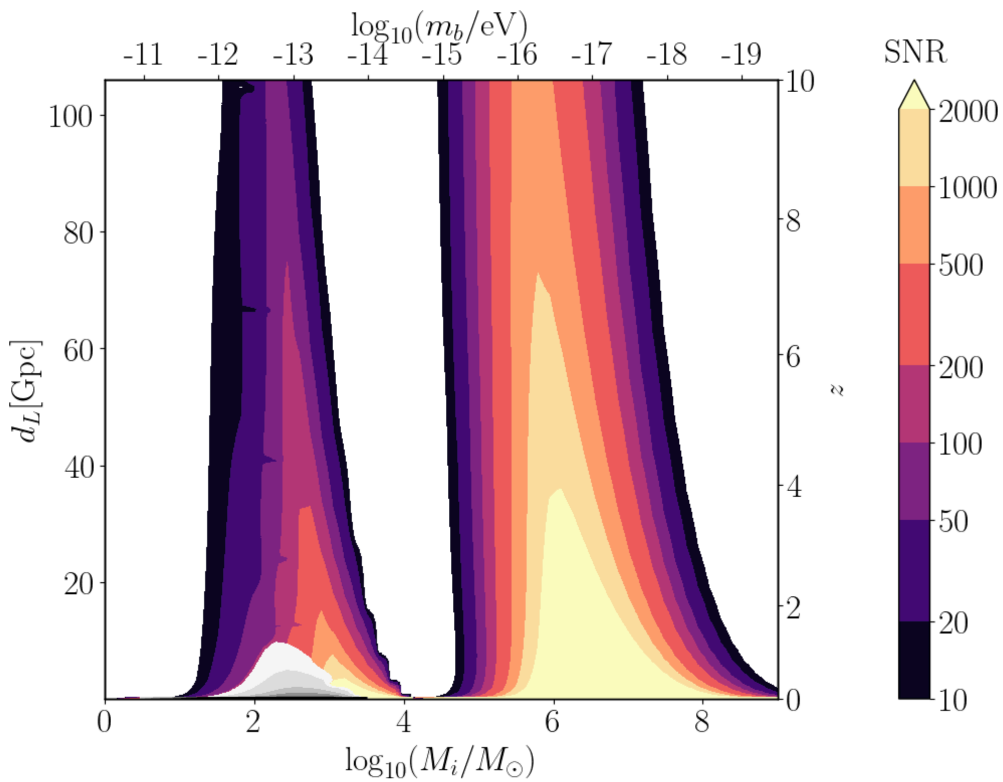
$$N_{\rm cycles} \approx f \tau_{\rm GW}, \quad \text{if} \quad T_{\rm obs} \gg \tau_{\rm GW}$$

Jupyter notebook to do plots below @ https://tinyurl.com/2twpwjku
(**Note:** I'm assuming GW signal is at maximum amplitude at the start of the observation, so is this is a best case scenario)

Scalar field



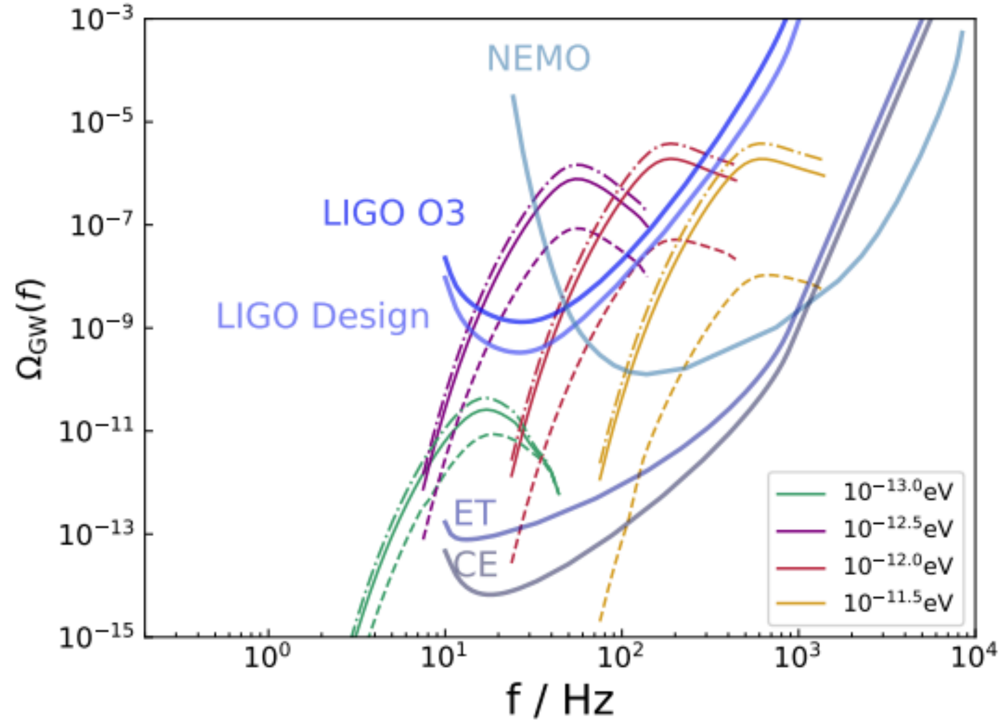
Vector field



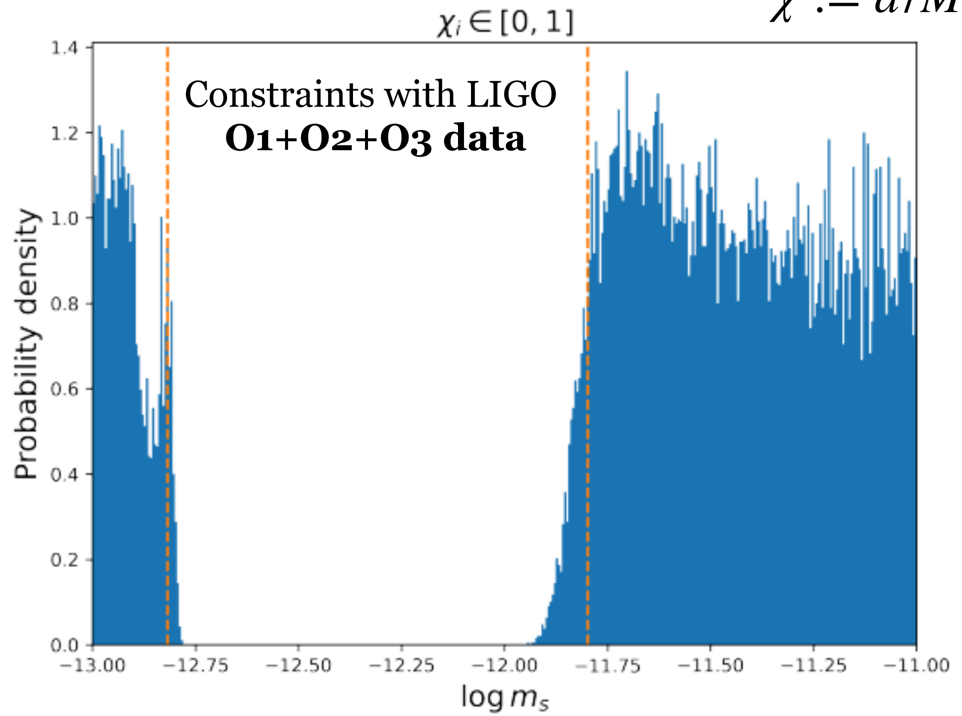
Stochastic GW background from boson clouds

RB *et al*, '17; Tsukada *et al* '18-20; Chen, RB & Cardoso '21; Chen, Jiang & Q.-G. Huang '22

- The incoherent superposition of the many GW signals too weak to be detected can produce a stochastic background, that can detected by cross-correlating data from different detectors.



$\chi := a/M$



(Different line types correspond to different assumptions about black hole population)

From: Chen *et al* arXiv:2204.03482

$$\frac{dE_s}{df_s} \approx E_{\rm GW}\delta(f(1+z)-f_s)$$

$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f} = \frac{f}{\rho_c}\int d\chi dM dz \frac{dt_L}{dz}\frac{d^2\dot{n}}{dM d\chi}\frac{dE_s}{df_s}$$

$\frac{d^2\dot{n}}{dM d\chi}$ - - BH formation rate

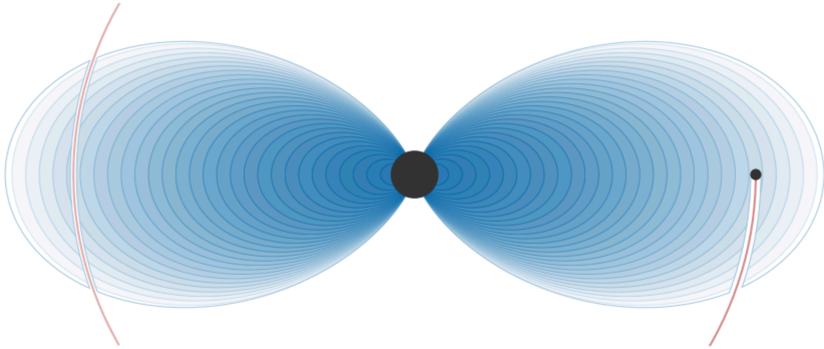
See: Phinney, arXiv:astro-ph/0100828

III
Impact of dark matter on binary systems

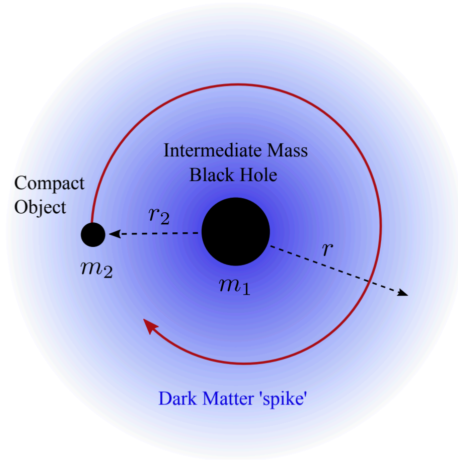
Some suggestions for further reading:

- E. Barausse, V. Cardoso & P. Pani, “Can environmental effects spoil precision gravitational-wave physics?”, PRD89 (2014) 104059, arXiv:1404.7149
- C. Macedo, V. Cardoso & P. Pani, “Into the lair: gravitational-wave signatures of dark matter”, APJ 774 48 (2013) arXiv:1302.2646
- K. Eda, Y. Itoh, S. Kuroyanagi & J. Silk, “Gravitational waves as a probe of dark matter mini-spikes”, PRD91, 044045 (2015) arXiv:1408.3534
- D. Baumann, H. S. Chia & R. A. Porto, “Probing ultralight bosons with binary black holes”; PRD99, 044001 (2019), arXiv:1804.03208
- D. Baumann, G. Bertone, J. Stout, G. M. Tomaselli, “Ionization of gravitational atoms”, PRD105, 115036 (2022), arXiv:2112.14777

Inspirals in dark matter environments



From: Baumann+, PRD105, 115036 (2022)



From: Kavanagh+, PRD102, 083006 (2020)

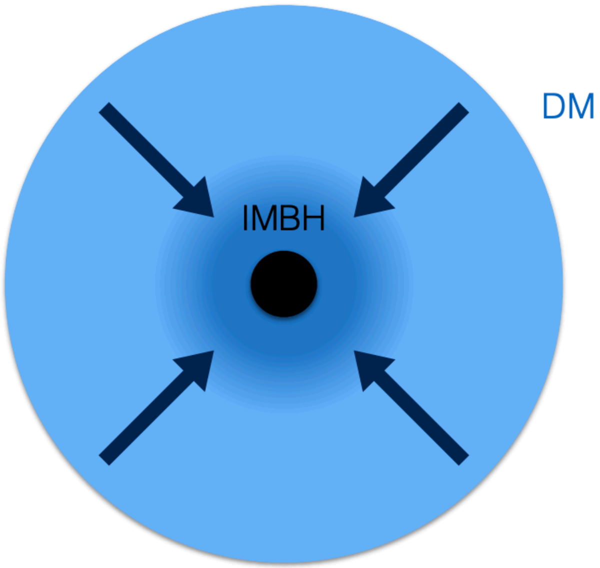
- A dense dark matter can affect the inspiral of a binary system due to:
  - **Direct gravitational pull**: mass enclosed within orbit varies during inspiral
  - **Accretion**: masses and spins of the objects can change due to accretion of the surrounding matter, which in turn translates in an effective force that changes orbital motion
  - **Dynamical friction** (a.k.a. gravitational drag): gravitational pull on the objects from perturbations in the medium excited by the moving objects
- **Other possible effects:** resonances, tidal deformations, …

[Macedo, Cardoso & Pani '13; Baumann+'18, '19; de Luca & Pani '22]

But isn’t dark matter too dilute to affect a binary system?

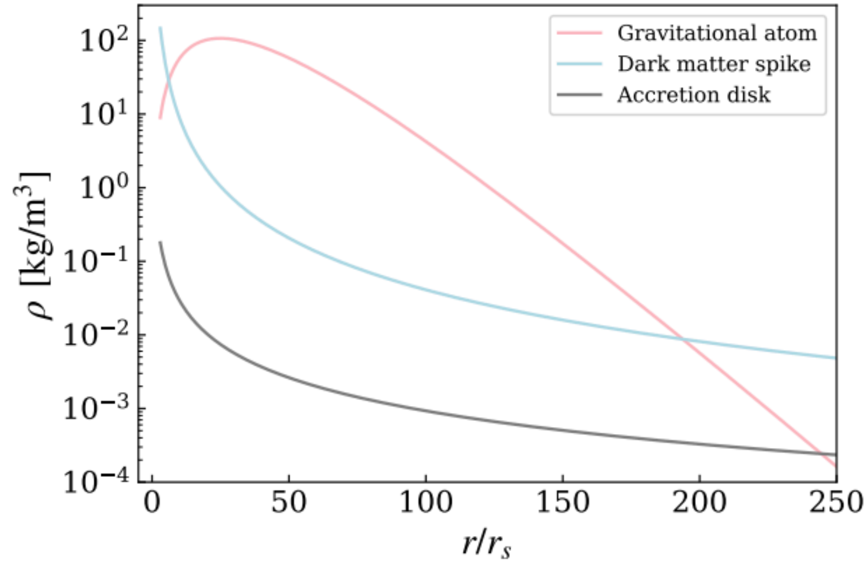
$$\rho_{\text{local DM}} \approx 10^{-21}\text{kg/m}^3$$

Dark matter overdensities close to black holes



Credit: Bradley J. Kavanagh

$\rho_{\text{local DM}} \approx 10^{-21}\text{kg/m}^3$



From: Cole+, arXiv:2211.01362

- **Dark matter spikes** formed through the adiabatic growth of a massive BH at the center of a dark matter halo [Gondolo & Silk '99]
- **Boson clouds** (a.k.a. gravitational atoms) formed through superradiant instabilities
- **Caveats:** BH hosting the overdensity need to have a "quiet" life, e.g. BH mergers could destroy spikes/clouds; non-gravitational interactions could flatten spikes…

Accretion

- Assume small black hole with mass m_p moving with constant velocity v in a medium made of collisionless dark matter particles of density $\rho_{\rm DM}$. In the non-relativistic regime, its mass will change due to accretion as:

$$\dot{m}_p \approx \sigma \rho_{\rm DM} v \approx \pi r_c^2 \rho_{\rm DM} v \approx \frac{4\pi m_p^2}{v} \rho_{\rm DM}$$

$\sigma \approx \pi r_c^2$ - cross section

$r_c \approx \frac{2m_p}{v^2}$ - "effective capture radius"

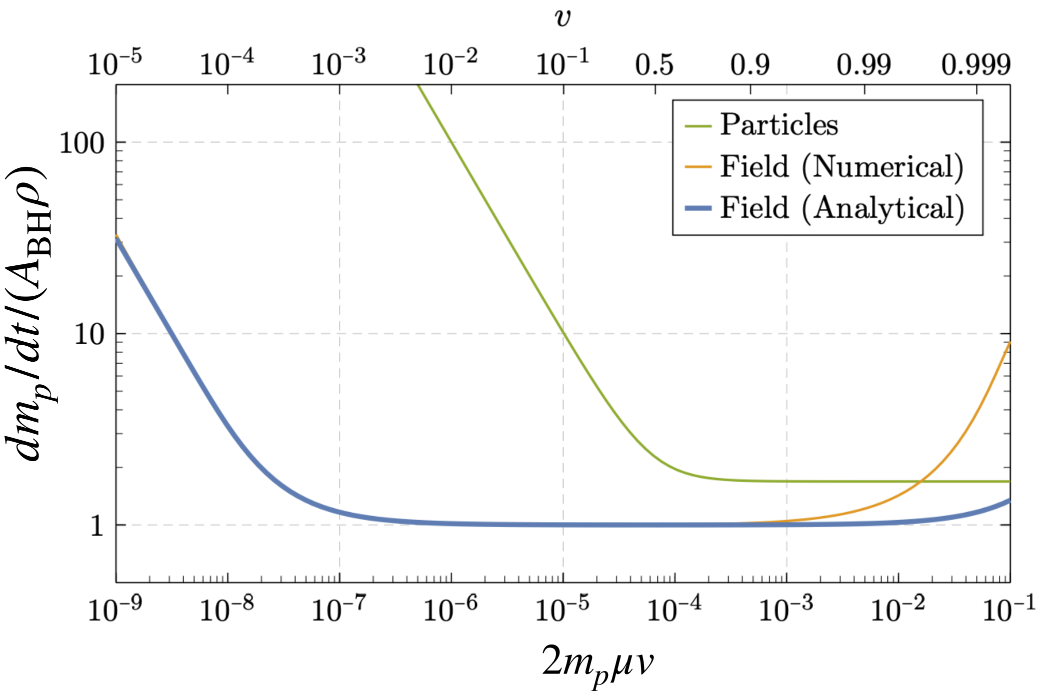
- Accretion acts on the orbit as an effective force that slows down moving object:

$$L_{\rm orb} = m_p r^2 \Omega_{\rm orb} \approx m_p r^2 \sqrt{M/r^3}$$

$$\dot{L}_{\rm orb} = 0 \implies \dot{r} \approx -2r \frac{\dot{m}_p}{m_p}$$

$$\frac{\dot{r}_{\rm acc}}{\dot{r}_{\rm RR}} \sim 3 \times 10^{-2} \left(\frac{M}{10^5 M_\odot} \right)^2 \left(\frac{r}{100M} \right)^{9/2} \left(\frac{\rho_{\rm DM}}{\rm kg/m^3} \right)$$

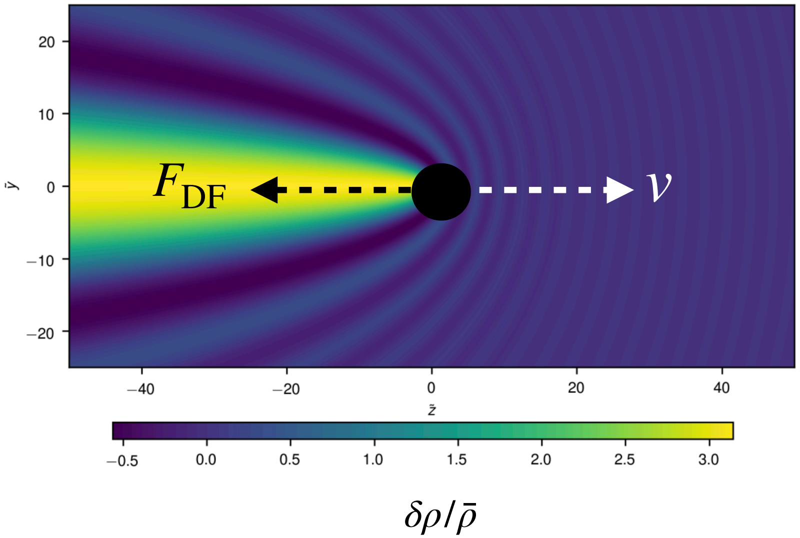
$$\dot{r}^{\rm RR} \equiv \dot{E}_{\rm GW}/(dE_{\rm orb}/dr)$$



Adapted from: Baumann+, PRD105, 115036 (2022)

Dynamical friction

S. Chandrasekhar, ’43



Adapted from: Lancaster+, JCAP01(2020)001

❖ **Physical intuition:** as the object moves through the environment, its gravitational potential induces density perturbations whose backreaction on the object cause it to slow down

❖ For non-relativistic velocities (assumes asymptotically uniform medium):

$$\dot{E}_{\rm DF} \approx \frac{4\pi m_p^2 \rho_{\rm DM}}{v}\mathcal{I}(v) \implies \dot{E}_{\rm orb} \equiv \frac{dE_{\rm orb}}{dr}\dot{r} \approx -\dot{E}_{\rm DF} + \dots$$

$\mathcal{I}(v)$ - slowly-varying function of v. Depends on whether medium is best described by an ultralight field of collisionless particle [e.g. Vicente & Cardoso, 2201.08854]

$$\frac{\dot{r}_{\rm DF}}{\dot{r}_{\rm RR}} \sim 3 \times \left(\frac{M}{10^5 M_\odot}\right)^2 \left(\frac{r}{100M}\right)^{11/2} \left(\frac{\rho_{\rm DM}}{\rm kg/m^3}\right)\mathcal{I}(v)$$

Impact of dark matter environment in GWs

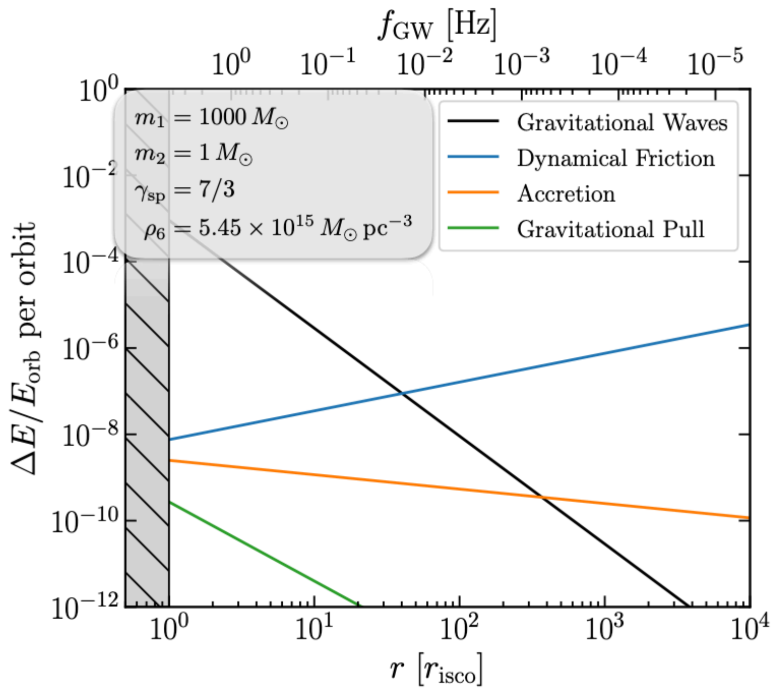
- Orbit is affected by GW radiation-reaction, dynamical friction and accretion:

$$\dot{r} = -\dot{r}_{\rm RR} - \dot{r}_{\rm DF} - \dot{r}_{\rm acc}$$

$$\dot{r}^{\rm RR} \equiv \dot{E}_{\rm GW}/(dE_{\rm orb}/dr)$$

$$E_{\rm orb} = \frac{m^2}{2}v^2 + \Phi_{\rm grav}\,, \quad \Phi_{\rm grav} = -\frac{M}{r} + \Phi_{\rm DM}(r)$$

$$\Omega_{\rm orb} = \sqrt{\Phi'_{\rm grav}(r)/r}$$



Credit: Bradley J. Kavanagh

- Assuming density profile of the form $\rho_{\rm DM} \approx \rho_0\,(R/r)^\gamma$:

$$\tilde{h}(f) \sim \mathscr{A} e^{i\Psi(f)}\,, \quad \Psi(f) \sim \Psi^{(0)}_{\rm GR}\left[1 + (\text{PN corrections}) + \delta\Psi_{\rm env}(f)\right]\,, \quad \Psi^{(0)}_{\rm GR} = 1/(138\eta x^{5/2})$$

$$\delta\Psi_{\rm DF} \propto -\rho_0 x^{\gamma - 11/2} \qquad \delta\Psi_{\rm acc} \propto -\rho_0 x^{\gamma-9/2} \qquad \delta\Psi_{\rm grav\ pull} \propto \rho_0 x^{\gamma-3}$$

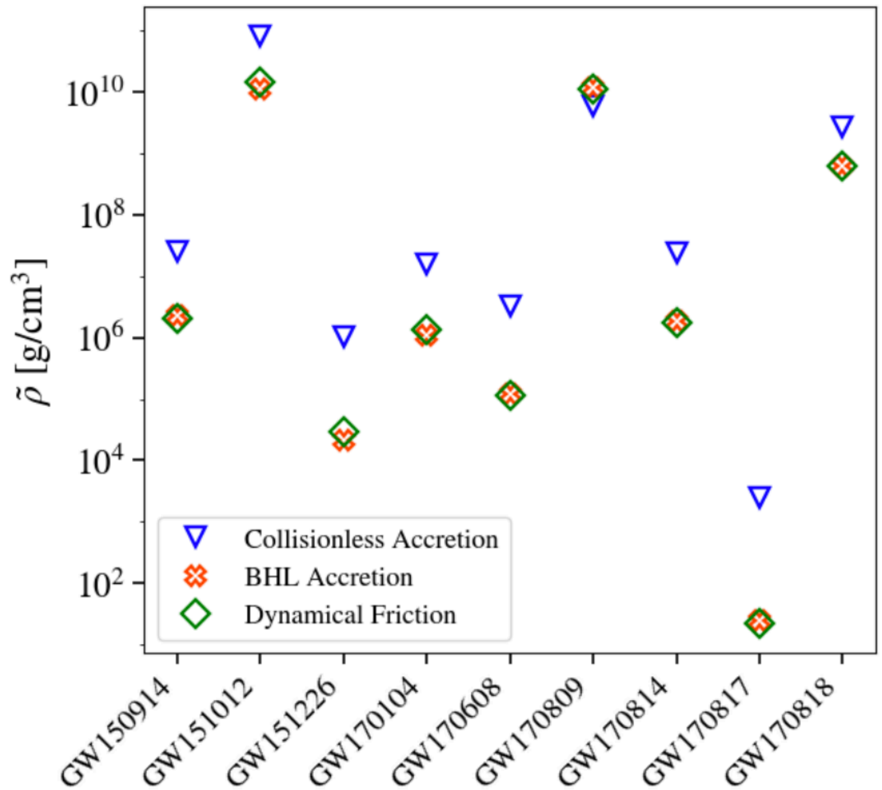
$(-5.5+\gamma)$ PN $\qquad (-4.5+\gamma)$ PN $\qquad (-3+\gamma)$ PN

$$x = (\pi f M)^{2/3}$$

[see notebook “DM_inspiral.nb”]

Constraints on environmental effects with LIGO

- Recent analysis shows that (GWTC-1) LIGO binaries are **consistent with vacuum waveforms**
- LIGO can only **constrain very dense environments** at the moment
- **Low-mass events** typically better to constrain environmental effects, because they merger at larger frequencies, so low-frequency part of their inspiral is more important in the LIGO band



Note: GW170817 - First binary neutron star

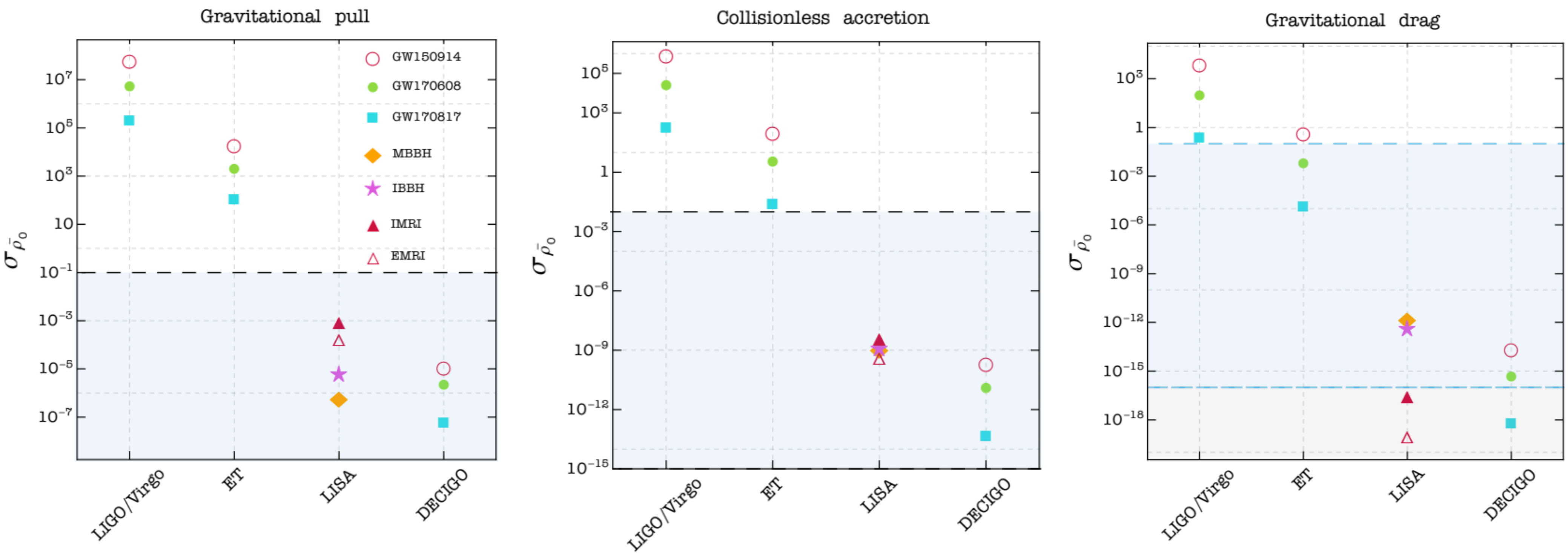
For reference:

$$\rho_{\rm gold} \approx 20 \mathrm{g/cm}^3$$

From: Santoro+, arXiv:2309.05061

Prospective constraints

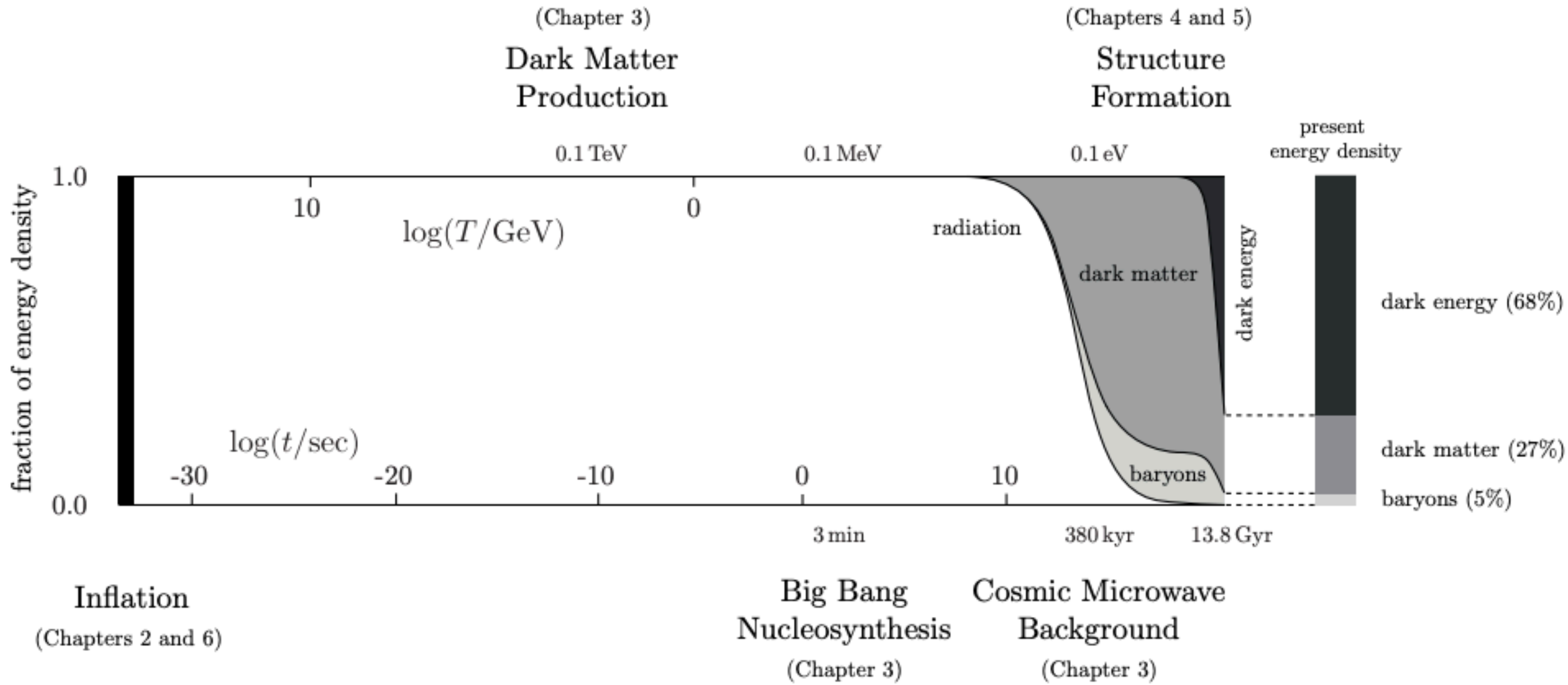
- LISA and DECIGO can, in principle, **probe densities typical of those of dark matter**
- Highly asymmetric binaries [extreme/intermediate mass-ratio inpirals (**E/IMRIs**)] especially interesting sources to probe environmental effects
- **Questions:** can we infer properties on the environment in which binaries evolve? Can environments be a source of systematic errors? Can environments limit our ability to test GR? How well do we need to model environmental effects?



From: Cardoso & Maselli, A&A 644, A147 (2020)

Backup slides

Universe history



Source:
https://cmb.wintherscoming.no/pdfs/baumann.pdf

Cold dark matter: structure formation

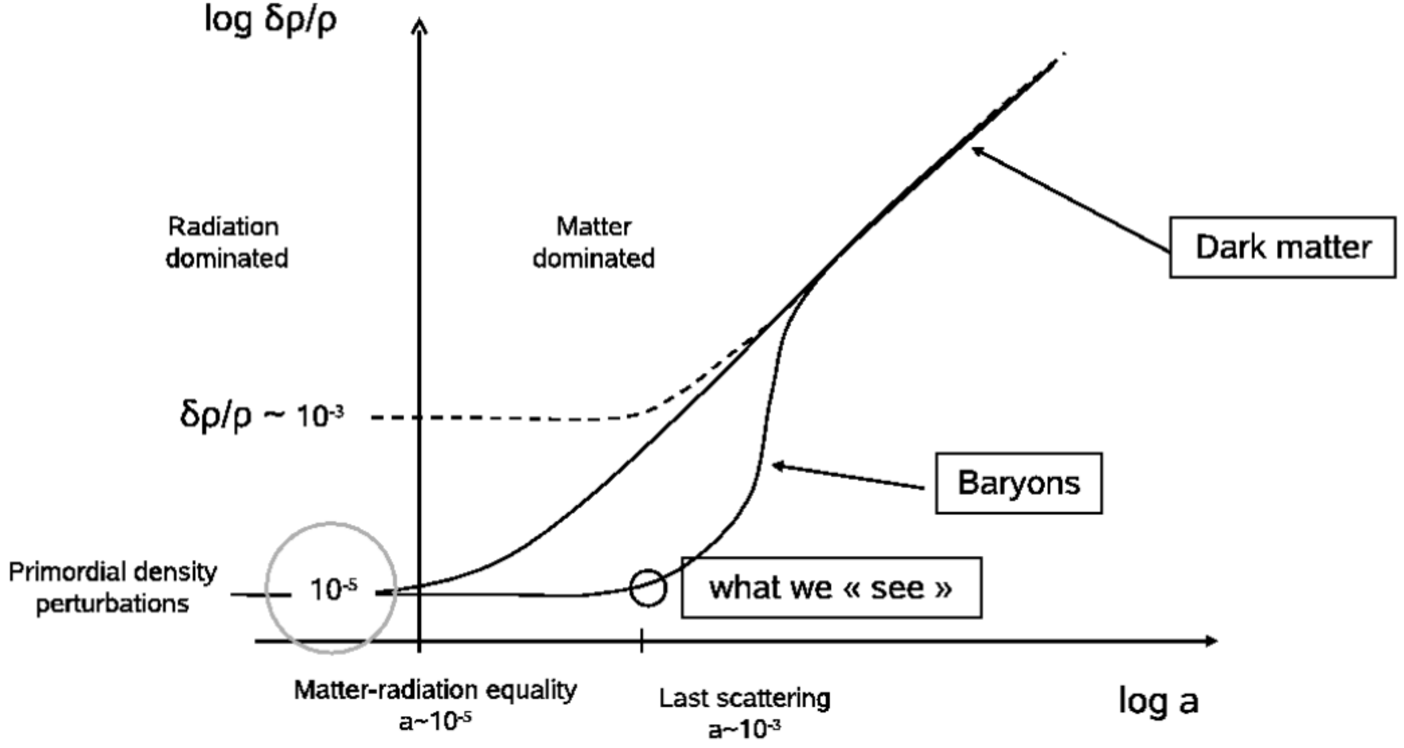


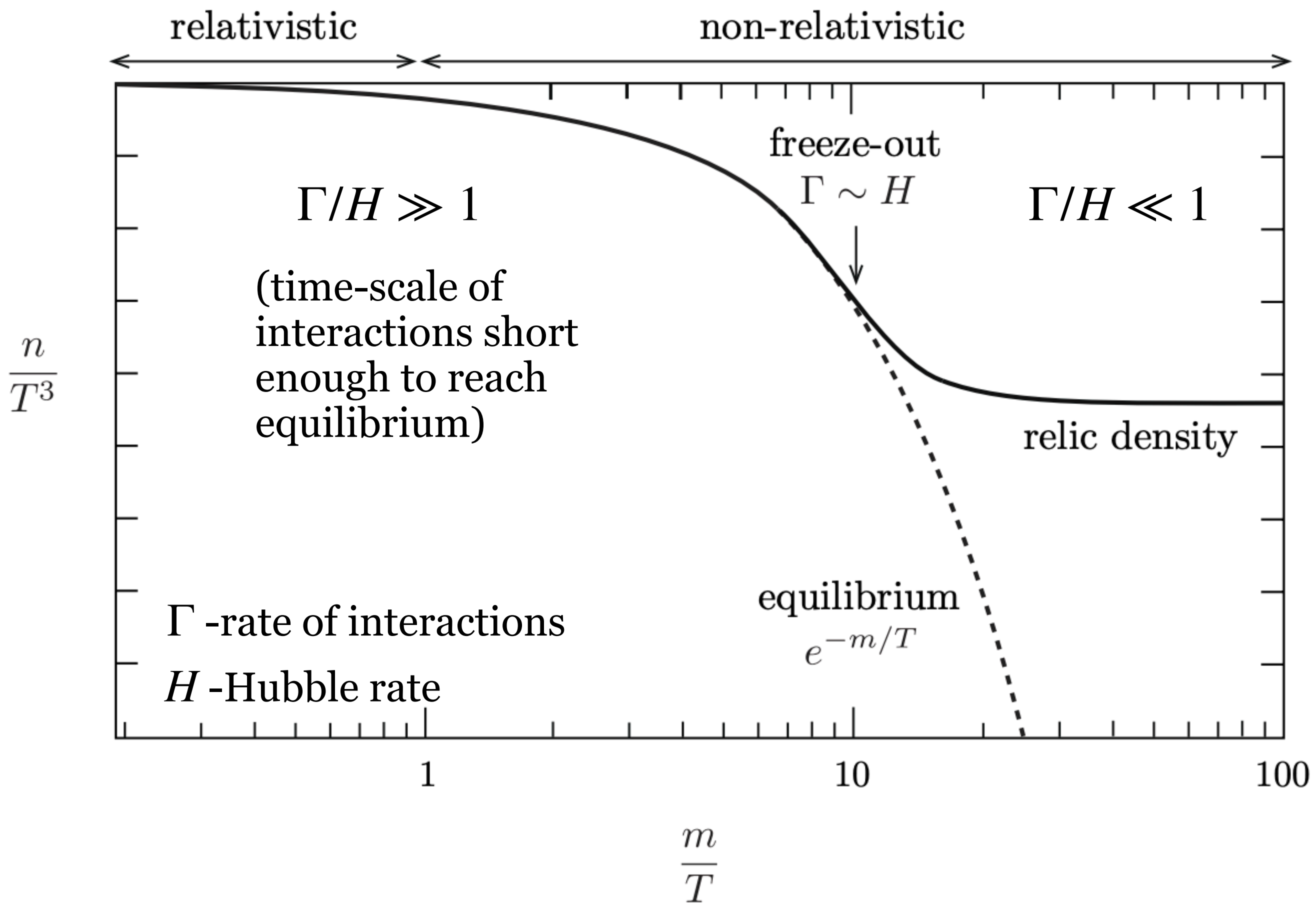
FIG. 2: Sketch of structure formation in the Cold Dark Matter (CDM) scenario for a large scale perturbation. At decoupling, the baryons, which were until then strongly coupled to the photons, fall in the gravitational potential build by the dark matter.

Source: https://indico.cern.ch/event/357886/contributions/849354/attachments/1151117/1652507/darkmatter_bnd2015_tytgat.pdf

Thermal freeze-out and WIMPs

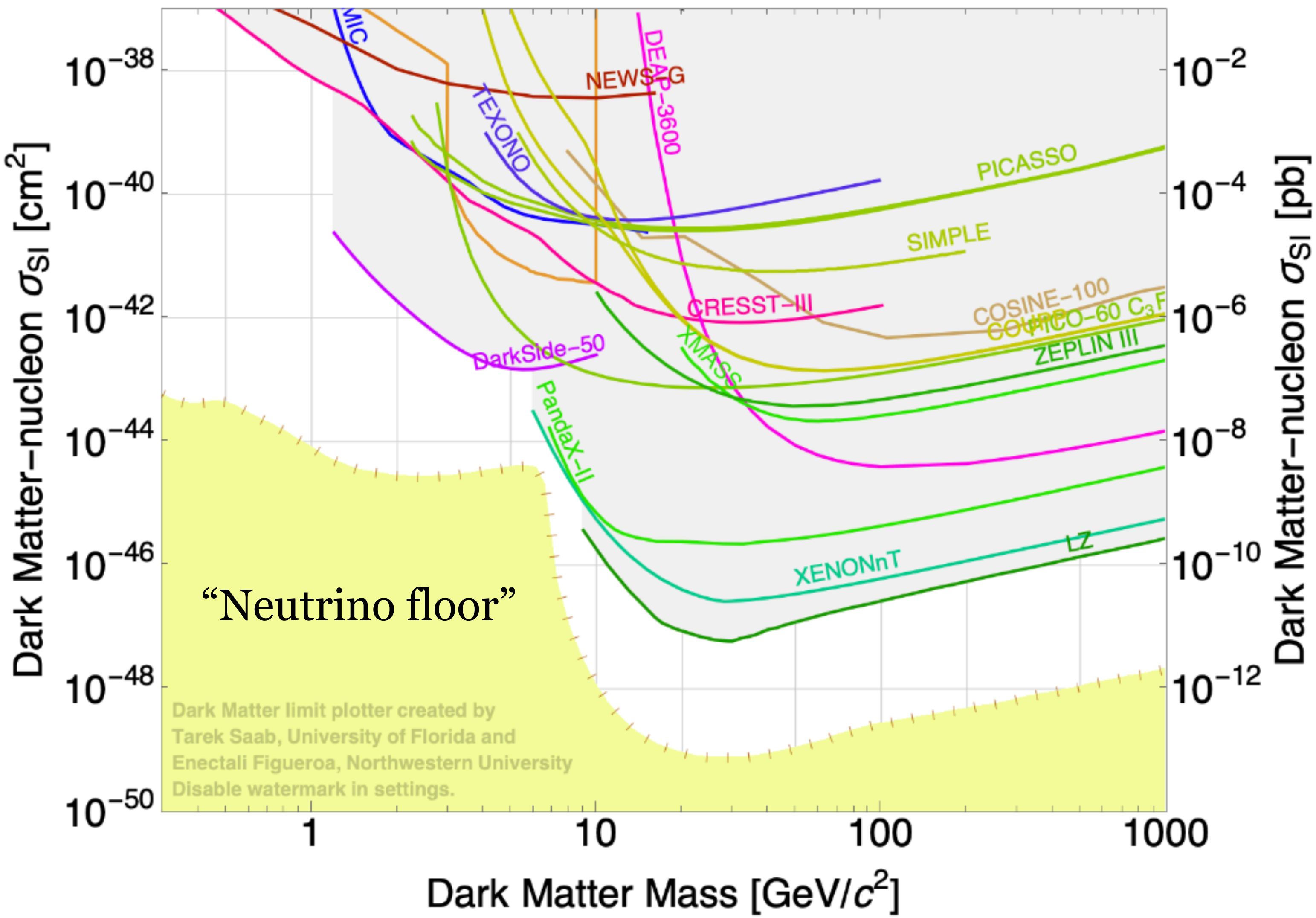
- Assume a dark matter "thermal history" similar to Standard Model particles : particles in thermal equilibrium in the early Universe when $T \gg m_{\rm DM}$.
- As the Universe expands, it cools down, dark matter is diluted through annihilations until it is too dilute to interact, at which point it's density "freezes out".

- Relic density depends on interaction cross section (it increases the smaller the cross section)
- **WIMP miracle:** right dark matter cosmic abundance if a cross section characteristic of the weak interaction and $m_{\rm DM} \sim 100\,\mathrm{GeV}$



See D. Baumann, https://cmb.wintherscoming.no/pdfs/baumann.pdf

WIMPs: constraints



Source: https://cmb.wintherscoming.no/pdfs/baumann.pdf

Vacuum realignment

See e.g. D. Marsh, arXiv:1510.07633

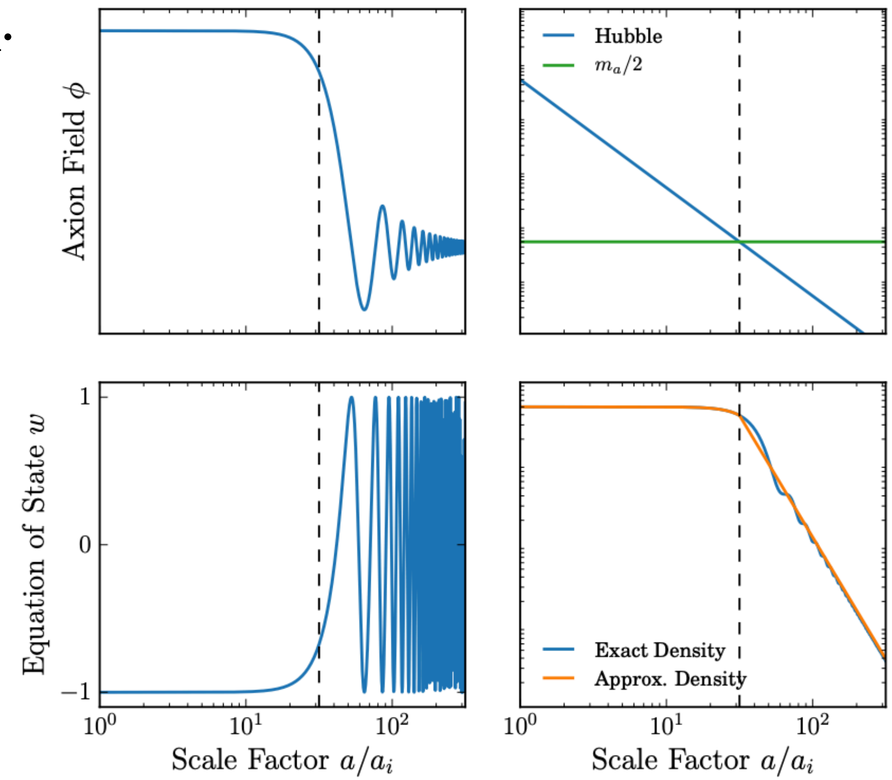
If ultralight bosons are dark matter they cannot be produced thermally. Alternative production mechanism: *vacuum realignment*

In a background FLRW metric, Klein-Gordon eq. reads (neglecting spatial inhomogeneities):

$$\ddot{\phi} + 3H(t)\dot{\phi} + m_\phi^2 \phi = 0$$

- For $H \gg m_\phi$: $\phi(t) \approx \phi(t=0)$
- For $H \ll m_\phi$:
  $\phi(t) \propto a(t)^{-3/2} \cos\left(m_\phi t + \theta_0\right) \implies \rho_\phi \propto a(t)^{-3}$

Behaves as CDM at matter-radiation equality as long $m_\phi \gtrsim H(a_{\text{eq}}) \sim 10^{-28}$ eV and can reproduce dark matter abundance with the right initial conditions.



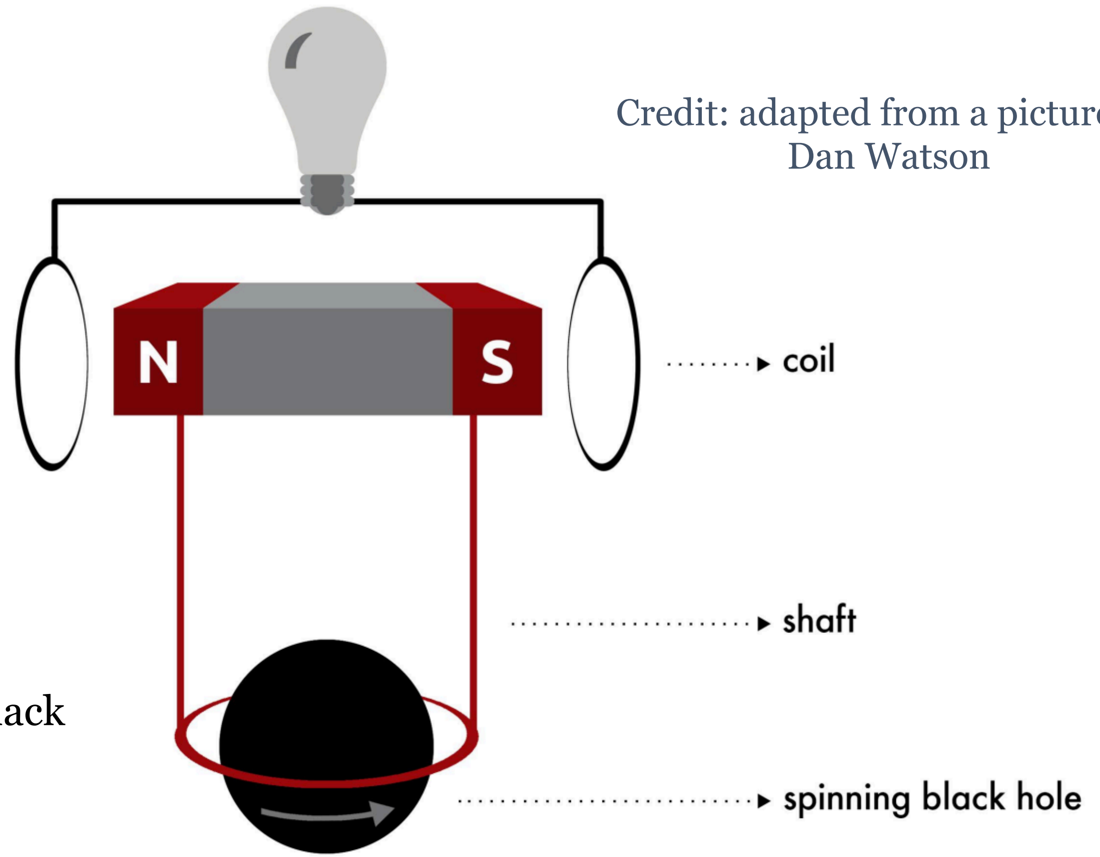
From D. Marsh, arXiv:1510.07633

“Black-hole generator”

Review: RB, Cardoso & Pani “Superradiance” Lect. Notes Phys. 971 (2020), 2nd ed.

Credit: adapted from a picture by Dan Watson

Rods turn magnet over generating electric current



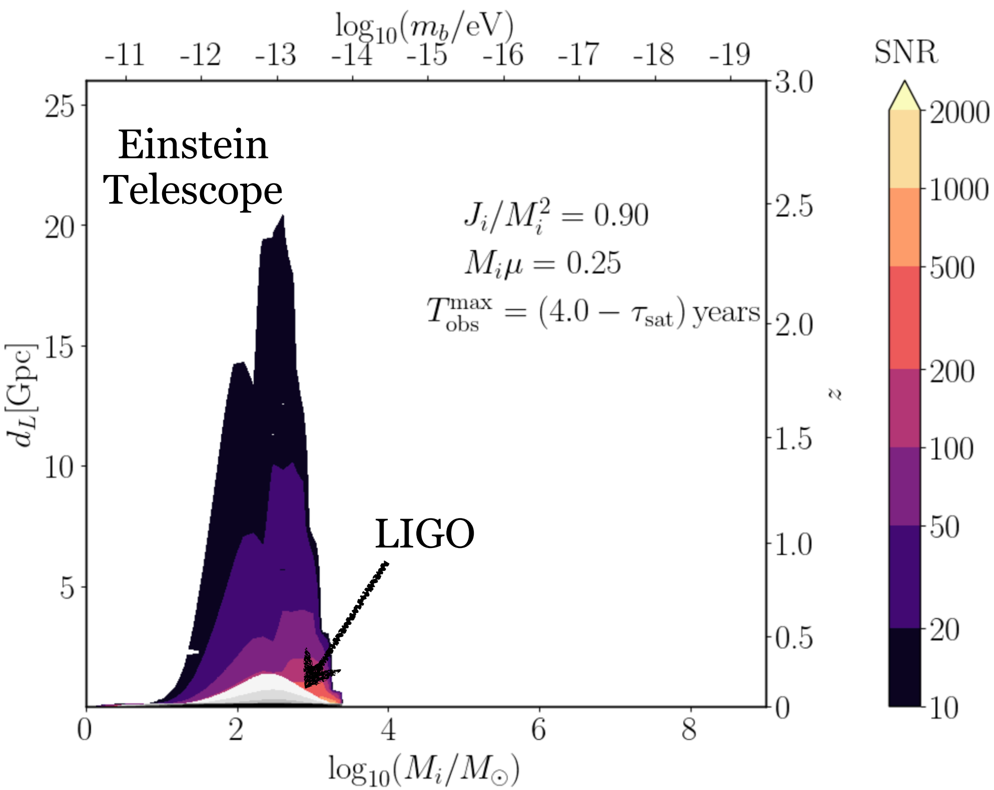
Ends of shaft placed in the ergoregion of the spinning black hole.

Rotational energy can be extracted from spinning black holes.

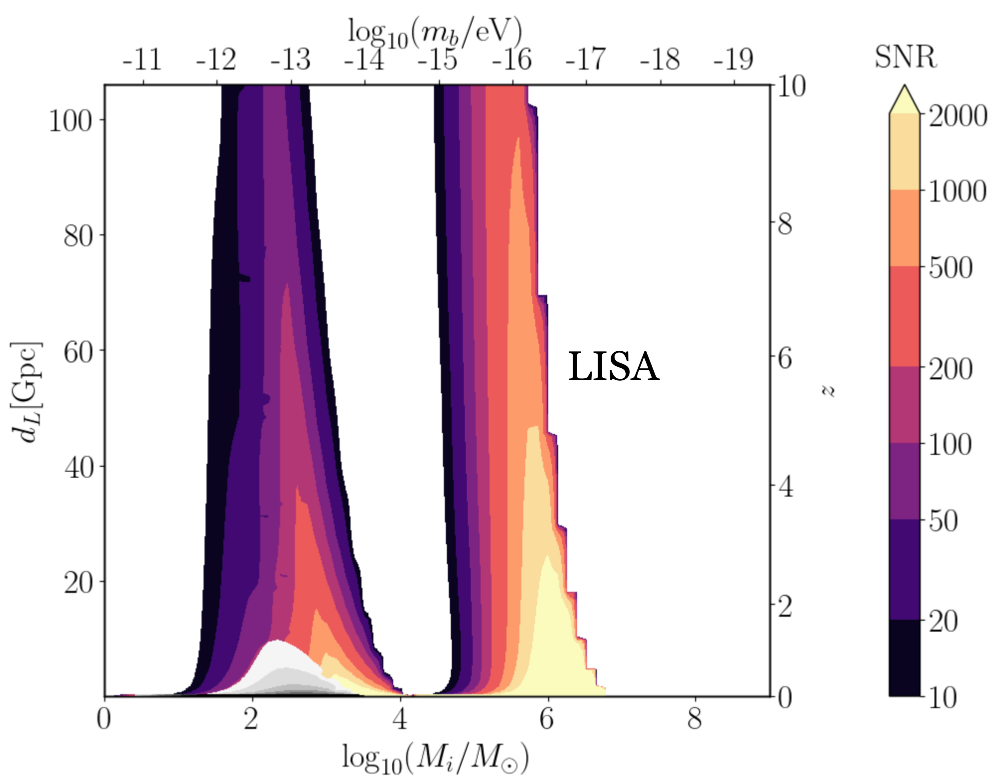
Follow-up searches

- Next-generation GW detectors could be used to search for boson clouds formed around **merger black hole remnants**, for which we know the natal mass and spin, time of formation and location (with some error) [Arvanitaki+, '16, Isi+ '19, Jones+ '23].
- Such follow-up searches requires $\tau_{\rm sat} \lesssim T_{\rm obs}$. Imposing this constraint, only a subset of potentially detectable sources could be followed up. In particular only vector clouds can be followed up in the LISA band, because the instability timescale is too long for scalar clouds around supermassive BHs.

Scalar field



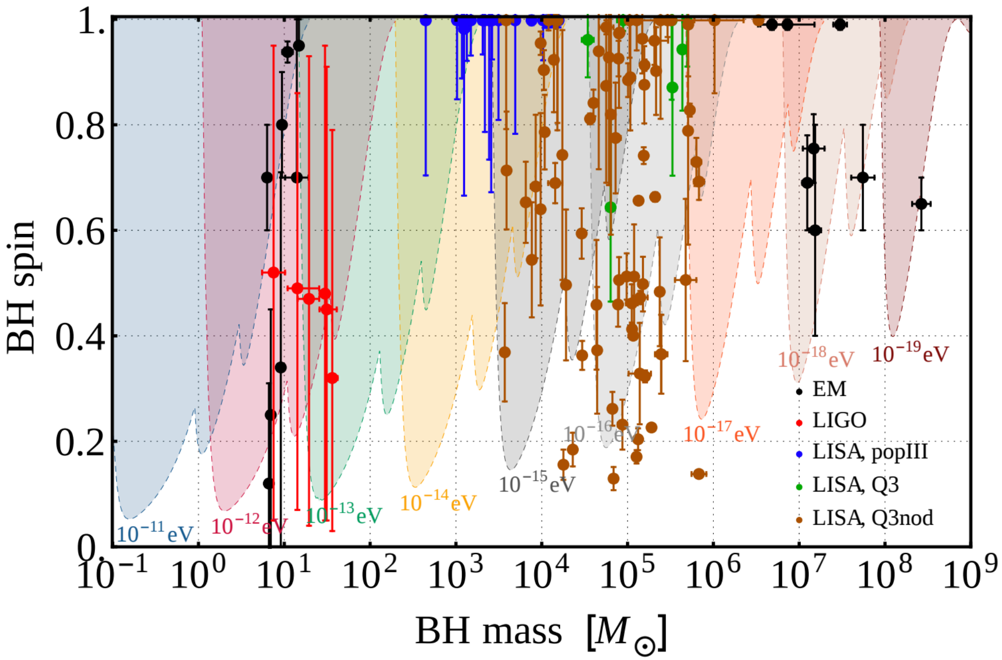
Vector field



Other signatures (not directly related to GWs)

- “Gaps” in the BH mass-spin astrophysical distribution:

Arvanitaki *et al* ’09; Arvanitaki & Dubovsky ’10; Arvanitaki, Baryakthar & Huan ’15; Pani *et al* ‘12; Baryakthar, Lasenby & Teo ’17; RB *et al* ’17; Cardoso et al ’18; RB, Grillo & Pani ’20; Stott ’20; K. Ng *et al* ’21,...



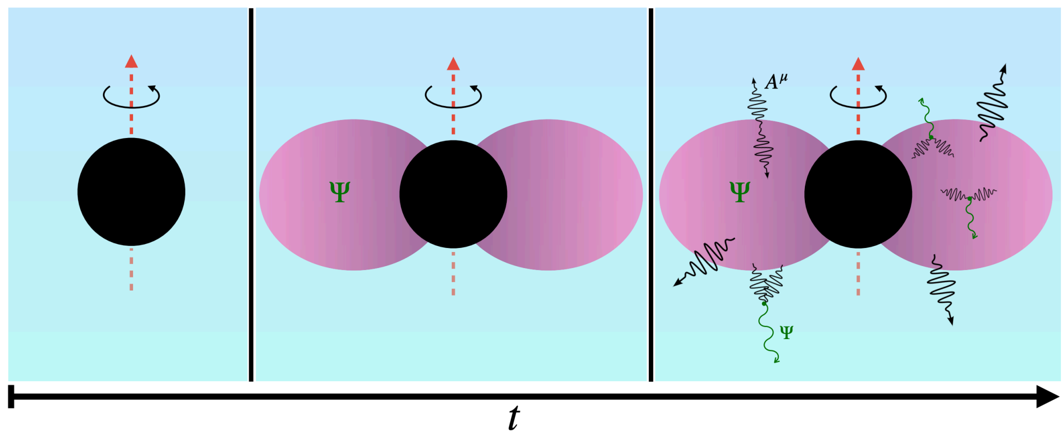
From: RB *et al*, PRD96 (2017) 6, 064050

- Non-gravitational interactions of the ultralight boson e.g. axionic couplings to photons:

$$(\Box - \mu^2)\Phi = \frac{k_a}{2}\tilde{F}^{\mu\nu}F_{\mu\nu}$$

$$\nabla_\nu F^{\mu\nu} = j^\mu - 2k_a \tilde{F}^{\mu\nu}\nabla^\nu \Phi$$

Yoshino & Kodama ’12, ’15; Rosa & Kephart ’18; Ikeda, RB & Cardoso ’19; Baryakthar+ ’20; Omiya+ ’22 Caputo+ ’21; East ’22, Spieksma+ ’23...



From: Spieksma *et al*, arXiv:2306.16447

Sensitivity Curves

Generated using gwplotter (http://gwplotter.com/)