

THE COULOMB COHESION OF WET GRANULAR MATERIALS

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Summary. *We propose a model for the Coulomb cohesion of wet polydisperse granular media in the pendular state. A fairly good agreement is found between the theoretical estimate and the data from DEM simulations. A detailed analysis of the microstructure and stress transmission suggests that the gap liquid bridges play a minor role compared to contact bridges in sustaining tensile stresses.*

1 INTRODUCTION

Capillary cohesion is at the origin of tensile strength and it enhances the shear strength of wet granular materials in the pendular state, i.e. the state where the liquid phase is discontinuous. By definition, the Coulomb cohesion c is the yield shear stress at zero confining pressure. This limit is often obtained by extrapolating the yield loci at nonzero compressive pressures. Assuming a linear behavior, we have $c = \mu\sigma_t$, where μ and σ_t are the internal coefficient of friction and the tensile strength, respectively. We propose an expression σ^{th} for the tensile strength, from which a theoretical cohesion $c^{th} = \mu\sigma^{th}$ is then defined. Our three-dimensional simulations by the discrete element method (DEM) allow to control the approximations made in each step of derivation.

2 CAPILLARY FORCE LAW

The capillary attraction force between two particles is a consequence of the liquid surface tension and the pressure difference between liquid and gas phases. Recently, Soulié et al. showed that the capillary force can be expressed in the following form [1]:

$$f_n^c = \begin{cases} -\pi\gamma_s \sqrt{R_1 R_2} \{\exp(A\delta_n^* + B) + C\} & \text{for } \delta_n^* > 0 \\ -\pi\gamma_s \sqrt{R_1 R_2} \{\exp(B) + C\} & \text{for } \delta_n^* \leq 0 \end{cases}, \quad (1)$$

where R_1 and R_2 are the sphere radii ($R_1 \leq R_2$), γ_s is the liquid surface tension, $\delta_n^* = \delta_n/R_2$. The parameters A , B and C are functions of the liquid volume V_b of the bond and the contact angle θ . The liquid bond is stable as long as the gap is below a de-bonding distance δ_n^{max} given by $\delta_n^{max} = (1 + 0.5\theta)V_b^{1/3}$ [2].

The largest absolute value f_0 of the capillary force occurs at $\delta_n = 0$. It is remarkable that f_0 is proportional to $\sqrt{R_1 R_2}$ and only very weakly dependent on the liquid volume. Hence, we may write

$$f_0 = \kappa \sqrt{R_1 R_2}, \quad (2)$$

where κ is a function only of the surface tensions and the contact angle. For example, with glass beads and water bridges, we have $\kappa \simeq 0.4$ N/m.

3 DEM SIMULATIONS

We implemented a DEM algorithm for spherical particles. The equations of motion are integrated according to the velocity Verlet scheme. The normal force between two particles is the sum of a repulsive force as a linear function of the overlap and an attraction force obeying Eq. 1. Inelastic dissipation is accounted for by viscous damping and the Coulomb friction. The material is sheared quasistatically in a direct shear cell in the presence of gravity and confining stresses. We attribute a capillary bond to each pair of particles within the de-bonding distance and the sample is consolidated under the action of a vertical confining pressure. Since all particle pairs within the de-bonding distance are considered, the wet coordination number z (number of liquid bonds per particle) obtained by this procedure has the highest possible value.

4 THEORETICAL ESTIMATION

Let us consider the general expression of the stress tensor σ in a granular material. This is an average quantity with a well-established expression involving contact forces f^k and inter-center vectors ℓ^k :

$$\sigma_{ij} = \frac{1}{V} \sum_{k \in V} f_i^k \ell_j^k, \quad (3)$$

where i and j refer to the components and V is the control volume. The set of contact points in a wet material is extended to include gap capillary bridges. From Eq. 3, we get the following expression for the stress p in the direction of extension:

$$p = n_w \langle f_n \ell \rangle, \quad (4)$$

where n_w is the number of bonds per unit volume, f_n is the normal force and ℓ is the distance between particle centers. The symbol $\langle \cdot \cdot \rangle$ designs averaging over all bonds in the control volume V .

Our simulations show that the fraction of liquid bonds in the range $\delta_n > 0$ (gap liquid bridges) is always below 15%. This means that most tensile stresses occur at contact bonds and, as a result, the capillary failure threshold f_0 is far more important for the failure of a wet material than the de-bonding distance δ_n^{max} . We may thus get a good estimation of the tensile strength for a wet particle assembly by replacing f_n in Eq. 4 by the capillary force threshold f_0 and by assuming, for simplicity, that the material is isotropic (a correction due to anisotropy can then easily be made):

$$\sigma^{th} = \frac{n_w}{2} \langle f_0 \ell \rangle. \quad (5)$$

The bond density n_w is equal to half the average number of bonds per particle divided by the free volume, i.e. the mean volume of a Voronoi cell surrounding the particle. The latter is simply the average particle volume $(1/6)\pi\langle d^3 \rangle$ divided by the solid fraction ϕ . Introducing these expressions in Eq. 5 and using Eq. 2, we get

$$\sigma^{th} = \frac{3}{2\pi} \kappa \phi z \frac{\langle (R_1 + R_2) \sqrt{R_1 R_2} \rangle}{\langle d^3 \rangle} = \frac{3}{4\pi} s \frac{\kappa \phi z}{\langle d \rangle}, \quad (6)$$

where z is the average number of bonds per particle, and we have

$$s = \frac{\langle d^{1/2} \rangle \langle d \rangle \langle d^{3/2} \rangle}{\langle d^3 \rangle}. \quad (7)$$

In derivation of the expression of s , it is assumed that the particle radii R_1 and R_2 are not correlated. It is easy to see that for a uniform size distribution, s varies from 8/15 to 1 as the smallest particle size increases from 0 to the mean particle size. Eq. 6 is similar to the expression proposed first by Rumpf for monodisperse materials without the s prefactor, and recently derived from the stress tensor by Gröger et al. [4]. Our equation 6 accounts for polydispersity and the correlation between the capillary force threshold f_0 and the particle size d .

Our simulations show that the internal angle of friction μ is practically independent of water content. Hence, from Eq. 6 we get the following expression for the Coulomb cohesion $c^{th} = \mu \sigma^{th}$:

$$c^{th} = \frac{3}{4\pi} \mu \kappa s \frac{\phi z}{\langle d \rangle}. \quad (8)$$

We found that the Coulomb cohesion calculated from the simulations of polydisperse materials is in good agreement with the theoretical estimate by Eq. 8. It is worth noting that an important discrepancy would arise if the polydispersity were ignored; i.e. with $s = 1$. On the other hand, Eq. 8 suggests that the Coulomb cohesion increases nonlinearly with water content as the average bond connectivity z . The simulations are in good agreement with this prediction.

REFERENCES

- [1] Soulié F., Cherblanc, F., El Youssoufi, M.S. and Saix, C. (2005) : Influence of liquid bridges on the mechanical behaviour of polydisperse granular materials, *Int. J. for Numerical and Analytical Methods in Geomechanics*, in press, <http://www3.interscience.wiley.com/cgi-bin/jissue/11043584>.
- [2] Mikami, T., Kamiya, H. and Horio M. (1998) : Numerical Simulation of Cohesive Powder Behavior in Fluidized Bed, *Chemical Engineering Science*, Vol. 53, issue 10, pp. 1927-1940.
- [3] El Youssoufi, M.S., Delenne, J.-Y. and Radjaï, F. (2005) : Self-stresses and crack formation by particle swelling in cohesive granular media, *Phys. Rev. E*, Vol. 71, p. 051307.
- [4] Gröger, T., Tüzün, U. and Heyes, D.M. (2003) : Modelling and measuring of cohesion in wet granular materials, *Powder Technology*, Vol. 133, pp. 203-215.