Multivariate Selection exercise

Richèl Bilderbeek

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a) Show that the model defined above corresponds to what Lande calls the 'psychophysical' model of mating preferences

Starting with equation [9] from Lande:

$$\frac{S}{\sigma^2} = \beta = \frac{\frac{1}{\alpha}\bar{y} - \left[1 + \frac{\epsilon}{\alpha}\right]\bar{z} + \theta}{\omega^2}$$

Filling in $\epsilon = 0$ and $\alpha = \frac{1}{\omega^2}$ yields the equation on the document:

$$\beta = \frac{\omega^2\bar{y} - \bar{z} + \theta}{\omega^2} = \frac{\omega^2\bar{y} - \bar{z} + \theta}{\omega^2} = \frac{\omega^2\bar{y}}{\omega^2} - \frac{\bar{z} + \theta}{\omega^2} = \bar{y} - \frac{\bar{z} + \theta}{\omega^2} = \beta_z$$

b) Give a biological interpretation for the parameters ϑ and ω , and explain why there is a factor 1/2 in front of the G-matrix in equation (1).

ϑ: male trait value with highest viability, the optimal trait value without sexual selection. For example, the most optimal tail length.

ω: range of male phenotypes around the optimum for high viability, the strength of the effect on viability when a trait deviates from the optimum. If ω is infinite, all trait values result in the same (optimal) viability, if ω is close to zero, only males close the ϑ have a high viability.

$\frac{1}{2}$: the effect of allele change for the male trait happens within males only, that is, in half of the sexes. Also, the alleles for the female preference are only actively changed within females

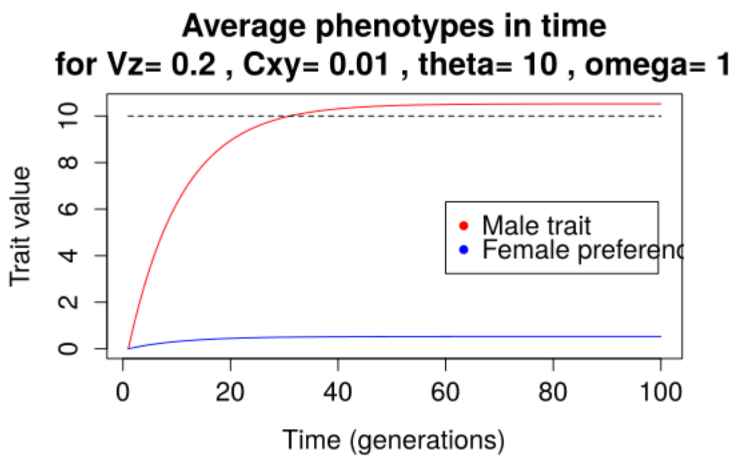
c) Which part of the selection gradient β_z captures the effect of sexual selection on the male secondary sexual character; which part describes the effect of natural selection?

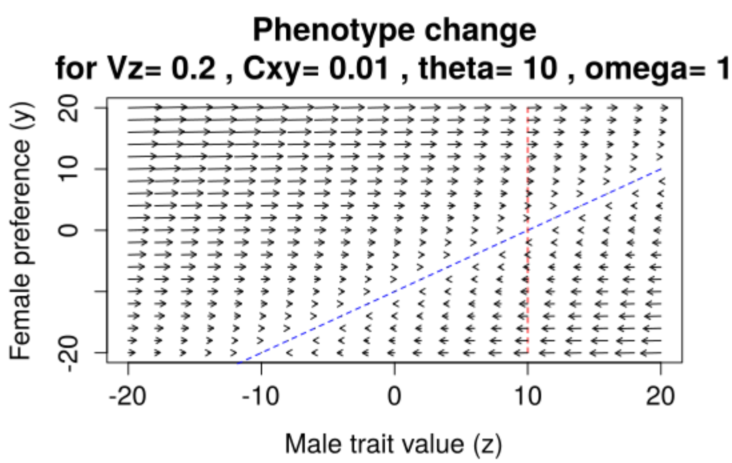
$$\beta_z = \bar{y} - (\bar{z} - \theta)/\omega^2$$

The effect of sexual selection is (simply) $\bar{y}$, the effect of natural (in this case: viability) selection is $-(\bar{z} - \theta)/\omega^2$.

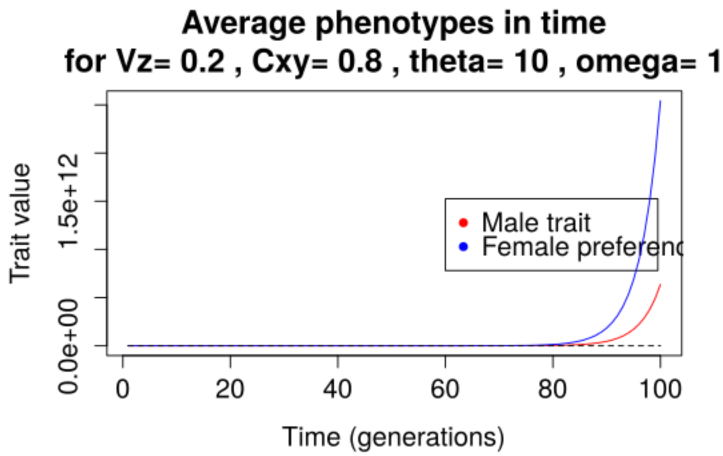
d) Can you recover the two scenarios depicted in figure 1 of Lande's paper? What do you conclude about the evolution of secondary sexual characters in each of the two scenarios?

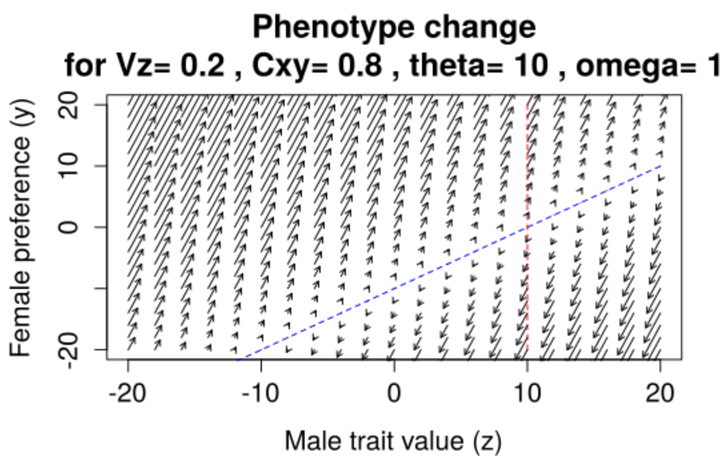
The first figure of Lande showed selection reaching an equilibrium:





Increasing the covariance:





Sexual selection can take male trait expression to extremes, if the covariance in the female is larger than the genetic additive variance in males. In other words: the males have to be able to keep up with the female preferences. Stability is calculated by Lande (equation 12), and it is stable if the next equation is true:

$$-2 < C_{xy} - \frac{[\alpha + \epsilon] V_z}{2\alpha\omega^2} < 0$$

Again, filling in $\epsilon = 0$ and $\alpha = \frac{1}{\omega^2}$ yields the following:

$$-2 < C_{xy} - \frac{V_z}{2\omega^2} < 0$$

(e) Is there a reason to expect that the additive genetic covariance C_{xy} is large enough to destabilize the evolutionary dynamics of trait and preference? Would you expect C_{xy} to be positive or negative in a population with sexual selection?

I would expect $V_z < C_{xy}$, because (1) the male trait is selected on directly (2) selection decreases variance.

I expect C_{xy} to be positive, as I expect it is to be beneficial for a mother to create (1) sons with the preferred trait (2) daughters with preference for the current population its trait.

f) Extend the model by assuming that the mating preference is subject to weak stabilizing natural selection. How does direct selection acting on the preference affect the outcome of evolution? Can costly preferences be maintained in a population at equilibrium?

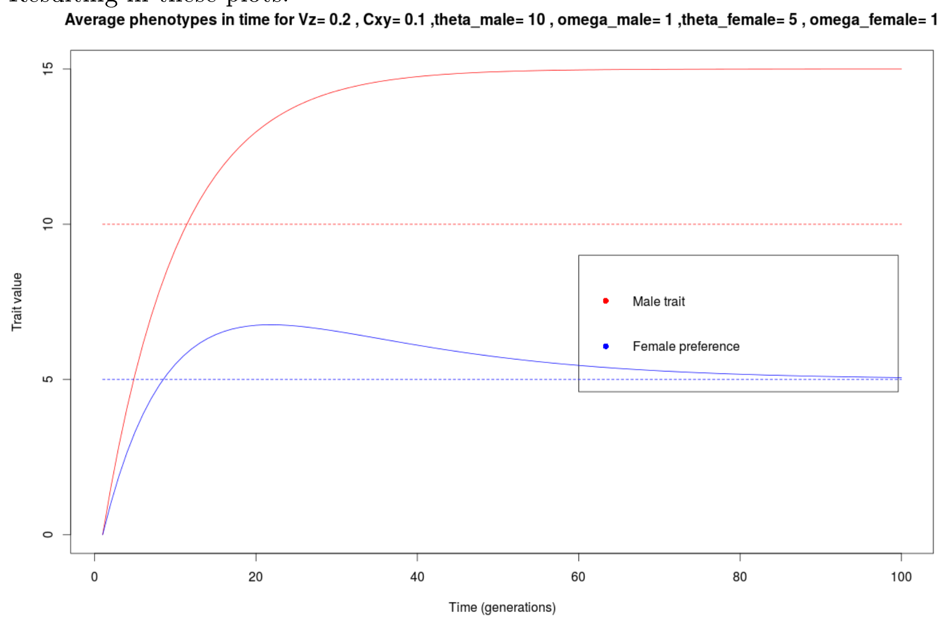
I extended the model to this:

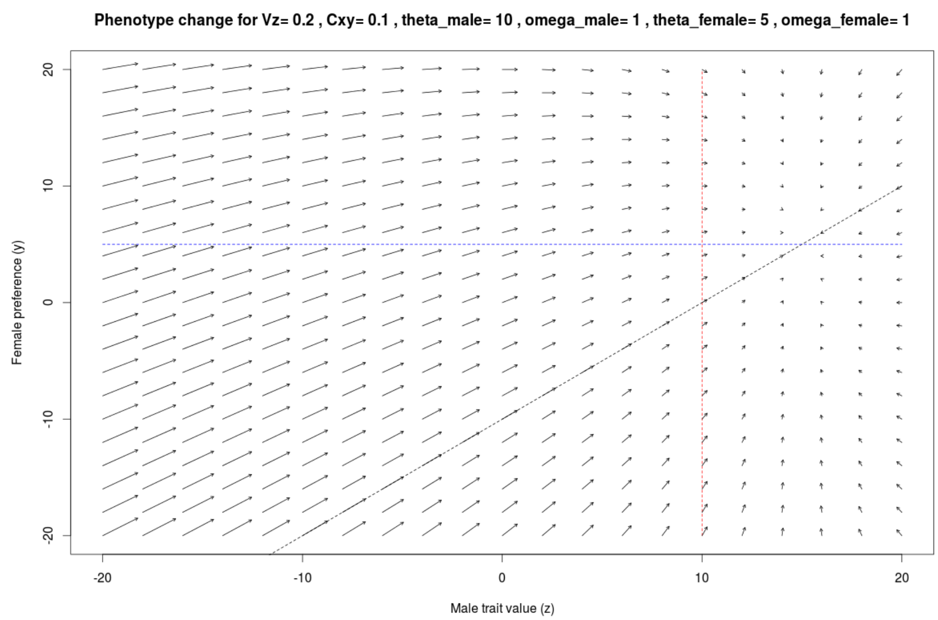
$$\beta_z = \bar{y} - \frac{\bar{z} + \theta_{\male}}{\omega^2_{\male}}$$

$$\beta_y = -\frac{\bar{y} + \theta_{\female}}{\omega^2_{\female}}$$

$$\begin{pmatrix} \Delta\bar{z} \\ \Delta\bar{y} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} V_z & C_{xy} \\ C_{xy} & V_y \end{pmatrix} \cdot \begin{pmatrix} \beta_z \\ \beta_y \end{pmatrix}$$

Resulting in these plots:





It shows that the females will evolve to the preference with maximum viability. Additionaly, a female being choosy, does not give her extra costs beyond this cost of being choosy itself: she does not have to make a trade-off between viability and preference.