1 (c) Shaw-Mohler

The fitness of a mutant in a resident population is:

$$W\left(a, a\_star\right) = m\left(a\right)\ v\_male\left(a\_star\right) + f\left(a\right)\ v\_female\left(a\_star\right)$$

To calculate the fitness of a mutant in a resident population, we'll need:

- $m(a)$: number of surviving sons
- $f(a)$: number of surviving daughters
- $v\_male(a\_star)$: male reproductive value in resident population
- $v\_female(a\_star)$: female reproductive value in resident population

1.1 (c.5) Fitness

As a reminder, the fitness formula for a mutant in a resident population is:

$$W\left(a, a\_star\right) = m\left(a\right)\ v\_male\left(a\_star\right) + f\left(a\right)\ v\_female\left(a\_star\right)$$

Plugging in the four functions above:

$$W\left(a, a\_star\right) = \frac{0.5\ \left(0.5 - 0.5\, a\right)\ \left(\left(1 - a\_star\right)\ e\_son + 0.5\, e\_daughter + a\_star\ e\_abort\right)}{\left(0.5 - 0.5\, a\_star\right)\ \left(\left(1 - a\right)\ e\_son + 0.5\, e\_daughter + a\ e\_abort\right)}$$

The fitness of the resident population is:

$$W\left(a\_star, a\_star\right) = m\left(a\_star\right)\ v\_male\left(a\_star\right) + f\left(a\_star\right)\ v\_female\left(a\_star\right)$$

Plugging in the four functions (with $a = a\_star$) above, this simplifies to:

$$W\left(a\_star, a\_star\right) = 0.5$$

The relative fitness of the mutant on the resident population:

1.2 (c.1) Surviving sons

General formula of surviving sons:

$$m\left(s,a\right) = S\left(a\right)\ P\_son\,C\left(s,a\right)$$

Plug in $C(s,a)$:

$$m\left(s,a\right) = \frac{S\left(a\right)\ P\_son\,E}{\left(e\_daughter\ \left(1 - s\right) + \left(1 - a\right)\ e\_son + a\,e\_abort\right)\ s}$$

Set $s = 1/2$:

$$m\left(a\right) = \frac{2.0\,S\left(a\right)\ P\_son\,E}{\left(1 - a\right)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

Also plug in $S(a)$:

$$m\left(a\right) = \frac{2.0\ \left(0.5 - 0.5\,a\right)\ P\_son\,E}{\left(1 - a\right)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

1.3 (c.2) Surviving daughters

General formula of surviving daughters:

$$f\left(s,a\right) = \left(1 - S\left(a\right)\right)\ P\_daughter\,C\left(s,a\right)$$

Plug in $C(s,a)$:

$$f\left(s,a\right) = \frac{\left(1 - S\left(a\right)\right)\ P\_daughter\,E}{\left(e\_daughter\ \left(1 - s\right) + \left(1 - a\right)\ e\_son + a\,e\_abort\right)\ s}$$

Set $s = 1/2$:

$$f\left(a\right) = \frac{2.0\ \left(1 - S\left(a\right)\right)\ P\_daughter\,E}{\left(1 - a\right)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

Also plug in $S(a)$:

$$f\left(a\right) = \frac{2.0\ \left(0.5\,a + 0.5\right)\ P\_daughter\,E}{\left(1 - a\right)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

1.4 (c.3) Male reproductive value

General formula for male reproductive value:

$$v\_male\,(a\_star) = \frac{\alpha}{m\,(a\_star)}$$

Filling in $alpha = 1/2$:

$$v\_male\,(a\_star) = \frac{0.5}{m\,(a\_star)}$$

Filling in $m(a\_star)$:

$$v\_male\,(a\_star) = \frac{0.25\ ((1 - a\_star)\ e\_son + 0.5\,e\_daughter + a\_star\ e\_abort)}{(0.5 - 0.5\,a\_star)\ P\_son\,E}$$

1.5 (c.4) Female reproductive value

General formula for female reproductive value:

$$v\_female\,(a\_star) = \frac{\alpha}{f\,(a\_star)}$$

Filling in $alpha = 1/2$:

$$v\_female\,(a\_star) = \frac{0.5}{f\,(a\_star)}$$

Filling in $f(a\_star)$:

$$v\_female\,(a\_star) = \frac{0.25\ ((1 - a\_star)\ e\_son + 0.5\,e\_daughter + a\_star\ e\_abort)}{(0.5\,a\_star + 0.5)\ P\_daughter\,E}$$

2 (b) Secondary sex ratio

Secondary sex ratio with no sons aborted:

S(a=0) = 1/2

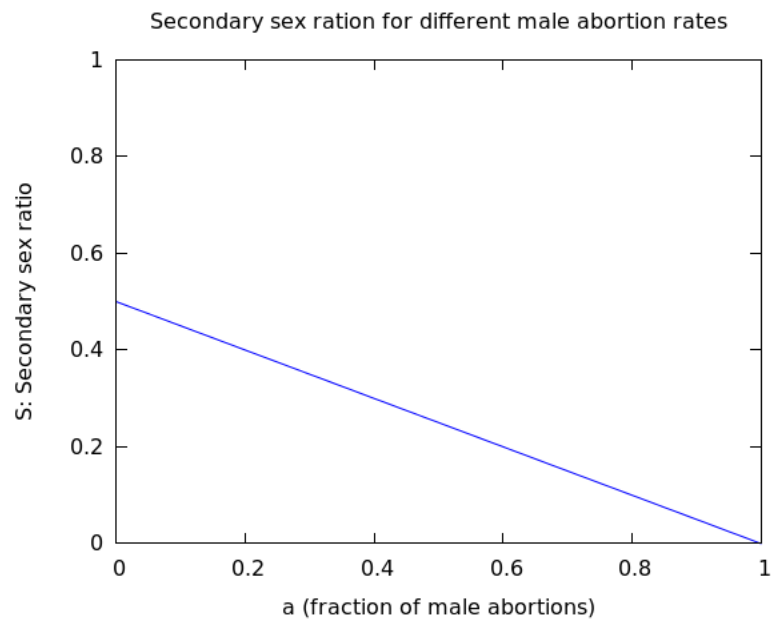
Secondary sex ratio with all aborted:

S(a=1) = 0

Secondary sex ratio in general:

$$S(a) = 0.5 - 0.5\,a$$

Plotted:



3 (a) Clutch size

Clutch size is dependent on multiple variables:

Symbol	description
a	fraction of sons aborted
C	clutch size
E	total energy invested in clutch
$e\_daughter$	energy investment to produce one healthy daughter
e_son	energy investment to produce one healthy son
s	primary sex ratio

Clutch size general formula without abortion:

$$C(s) = \frac{E}{e\_son\, s + e\_daughter\ (1 - s)}$$

Clutch size general formula with all sons aborted:

$$C(s) = \frac{E}{e\_abort\, s + e\_daughter\, (1 - s)}$$

Clutch general formula, where a denotes the fraction of sons that are aborted:

$$C(s, a) = \frac{E}{(e\_daughter\, (1 - s) + (1 - a)\, e\_son + a\, e\_abort)\, s}$$

Assuming $s = 0.5$, this results in:

$$C(0.5, a) = \frac{2.0\, E}{(1 - a)\, e\_son + 0.5\, e\_daughter + a\, e\_abort}$$