1 (c) Shaw-Mohler

The fitness of a mutant in a resident population is:

$$W\left(a, a\_star\right) = m\left(a\right)\ v\_male\left(a\_star\right) + f\left(a\right)\ v\_female\left(a\_star\right)$$

To calculate the fitness of a mutant in a resident population, we'll need:

- $m(a)$: number of surviving sons
- $f(a)$: number of surviving daughters
- $v\_male(a\_star)$: male reproductive value in resident population
- $v\_female(a\_star)$: female reproductive value in resident population

1.1 (c.5) Fitness

As a reminder, the fitness formula for a mutant in a resident population is:

$$W\left(a, a\_star\right) = m\left(a\right)\ v\_male\left(a\_star\right) + f\left(a\right)\ v\_female\left(a\_star\right)$$

Plugging in the four functions above:

$$W\left(a, a\_star\right) = \frac{0.5\ (0.5 - 0.5\,a)\ ((1 - a\_star)\ e\_son + 0.5\,e\_daughter + a\_star\ e\_abort)}{(0.5 - 0.5\,a\_star)\ ((1 - a)\ e\_son + 0.5\,e\_daughter + a\ e\_abort)}$$

The fitness of the resident population is:

$$W\left(a\_star, a\_star\right) = m\left(a\_star\right)\ v\_male\left(a\_star\right) + f\left(a\_star\right)\ v\_female\left(a\_star\right)$$

Plugging in the four functions (with $a = a\_star$) above, this simplifies to:

$$W\left(a\_star, a\_star\right) = 0.5$$

The relative fitness of the mutant on the resident population:

$$dW\left(a, a\_star\right) = W\left(a, a\_star\right) - dW\left(a\_star, a\_star\right)$$

Filling in both fitness functions:

$$dW\left(a, a\_star\right) = \frac{0.5\,\left(0.5-0.5\,a\right)\,\left(\left(1-a\_star\right)\,e\_son+0.5\,e\_daughter+a\_star\,e\_abort\right)}{\left(0.5-0.5\,a\_star\right)\,\left(\left(1-a\right)\,e\_son+0.5\,e\_daughter+a\,e\_abort\right)}-0.5$$

For an ESS, the fitness of a mutant must always be lower in a resident population. To calculate this, we need the partial derivative of the above fitness function to a.

$$-\frac{0.25\,\left(\left(1-a\_star\right)\,e\_son+0.5\,e\_daughter+a\_star\,e\_abort\right)}{\left(0.5-0.5\,a\_star\right)\,\left(\left(1-a\right)\,e\_son+0.5\,e\_daughter+a\,e\_abort\right)}-\frac{0.5\,\left(0.5-0.5\,a\right)\,\left(e\_abort-e\_son\right)\,\left(\left(1-a\_st\right.\right.}{\left(0.5-0.5\,a\_star\right)\,\left(\left(1-a\right)\,e\_so\right.}$$

Which is equal to:

$$-\frac{\left(\left(2\,a\_star-2\right)\,e\_daughter+\left(4\,a\_star-4\right)\,e\_abort\right.}{\left(\left(8\,a^2-16\,a+8\right)\,a\_star-8\,a^2+16\,a-8\right)\,e\_son^2+\left(\left(\left(8-8\,a\right)\,a\_star+8\,a-8\right)\,e\_daughter+\left(\left(16\,a-16\,a^2\right.\right.\right.}$$

Filling in $e_son = 10$ and $e_daughter = 10$:

$$-\frac{-4\,a\_star\,e\_abort^2+1}{\left(8\,a\_star^3-8\,a\_star^2\right)\,e\_abort^2+10\,\left(\left(16\,a\_star^2+a\_star\,\left(16\,a\_star-16\,a\_star^2\right)-16\,a\_star\right)\,e\_abort+1\right.}$$

Which is equal to:

$$\frac{e\_abort+5}{\left(2\,a\_star^2-2\,a\_star\right)\,e\_abort-20\,a\_star^2+50\,a\_star-30}$$

1.2 (c.1) Surviving sons

General formula of surviving sons:

$$m\left(s, a\right) = S\left(a\right)\,P\_son\,C\left(s, a\right)$$

Plug in $C(s, a)$:

$$m\left(s, a\right) = \frac{S\left(a\right)\,P\_son\,E}{\left(e\_daughter\,\left(1-s\right)+\left(1-a\right)\,e\_son+a\,e\_abort\right)\,s}$$

Set $s = 1/2$:

$$m\left(a\right) = \frac{2.0\,S\left(a\right)\,P\_son\,E}{\left(1-a\right)\,e\_son+0.5\,e\_daughter+a\,e\_abort}$$

Also plug in $S(a)$:

$$m(a) = \frac{2.0\ (0.5 - 0.5\,a)\ P\_son\,E}{(1-a)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

1.3 (c.2) Surviving daughters

General formula of surviving daughters:

$$f(s,a) = (1 - S(a))\ P\_daughter\,C(s,a)$$

Plug in $C(s,a)$:

$$f(s,a) = \frac{(1 - S(a))\ P\_daughter\,E}{(e\_daughter\ (1-s) + (1-a)\ e\_son + a\,e\_abort)\ s}$$

Set $s = 1/2$:

$$f(a) = \frac{2.0\ (1 - S(a))\ P\_daughter\,E}{(1-a)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

Also plug in $S(a)$:

$$f(a) = \frac{2.0\ (0.5\,a + 0.5)\ P\_daughter\,E}{(1-a)\ e\_son + 0.5\,e\_daughter + a\,e\_abort}$$

1.4 (c.3) Male reproductive value

General formula for male reproductive value:

$$v\_male\,(a\_star) = \frac{\alpha}{m\,(a\_star)}$$

Filling in $alpha = 1/2$:

$$v\_male\,(a\_star) = \frac{0.5}{m\,(a\_star)}$$

Filling in $m(a\_star)$:

$$v\_male\,(a\_star) = \frac{0.25\ ((1 - a\_star)\ e\_son + 0.5\,e\_daughter + a\_star\ e\_abort)}{(0.5 - 0.5\,a\_star)\ P\_son\,E}$$

1.5 (c.4) Female reproductive value

General formula for female reproductive value:

$$v\_female\left(a\_star\right) = \frac{\alpha}{f\left(a\_star\right)}$$

Filling in $alpha = 1/2$:

$$v\_female\left(a\_star\right) = \frac{0.5}{f\left(a\_star\right)}$$

Filling in $f(a\_star)$:

$$v\_female\left(a\_star\right) = \frac{0.25\ \left(\left(1 - a\_star\right)\ e\_son + 0.5\, e\_daughter + a\_star\ e\_abort\right)}{\left(0.5\, a\_star + 0.5\right)\ P\_daughter\ E}$$

2 (b) Secondary sex ratio

Secondary sex ratio with no sons aborted:

S(a=0) = 1/2

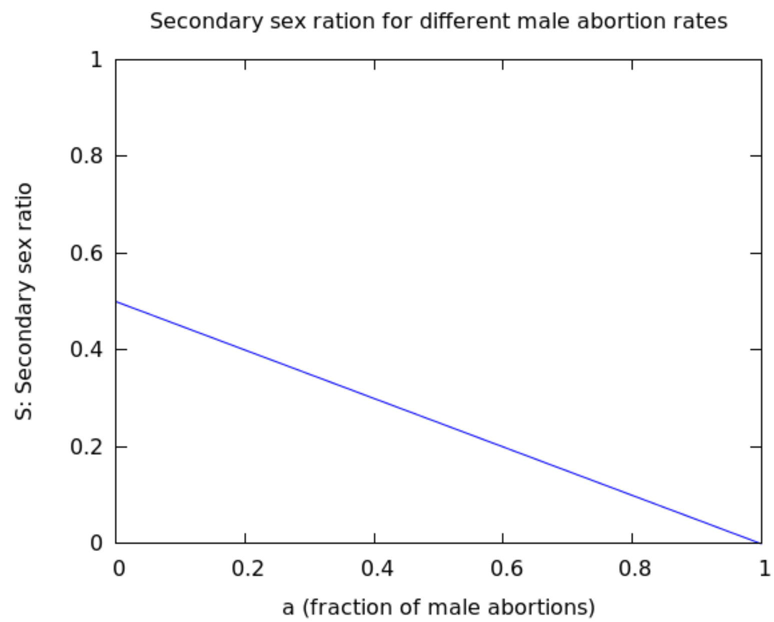
Secondary sex ratio with all aborted:

S(a=1) = 0

Secondary sex ratio in general:

$$S\left(a\right) = 0.5 - 0.5\, a$$

Plotted:



3 (a) Clutch size

Clutch size is dependent on multiple variables:

Symbol	description
a	fraction of sons aborted
C	clutch size
E	total energy invested in clutch
$e\_daughter$	energy investment to produce one healthy daughter
e_son	energy investment to produce one healthy son
s	primary sex ratio

Clutch size general formula without abortion:

$$C(s) = \frac{E}{e\_son\, s + e\_daughter\ (1-s)}$$

Clutch size general formula with all sons aborted:

$$C(s) = \frac{E}{e\_abort\, s + e\_daughter\ (1-s)}$$

Clutch general formula, where a denotes the fraction of sons that are aborted:

$$C(s, a) = \frac{E}{(e\_daughter\ (1 - s) + (1 - a)\ e\_son + a\, e\_abort)\ s}$$

Assuming $s = 0.5$, this results in:

$$C(0.5, a) = \frac{2.0\, E}{(1 - a)\ e\_son + 0.5\, e\_daughter + a\, e\_abort}$$