1 (a) Clutch size

Clutch size is dependent on multiple variables:

Symbol	description
a	fraction of sons aborted
C	clutch size
E	total energy invested in clutch
$e\_daughter$	energy investment to produce one healthy daughter
e_son	energy investment to produce one healthy son
s	primary sex ratio

$$C\left(a\right) = \frac{E}{\left(1-a\right)\ e\_son + e\_daughter + a\ e\_abort}$$

2 (b) Secondary sex ratio

Secondary sex ratio with no sons aborted:

S(a=0) = 0/(0+1)

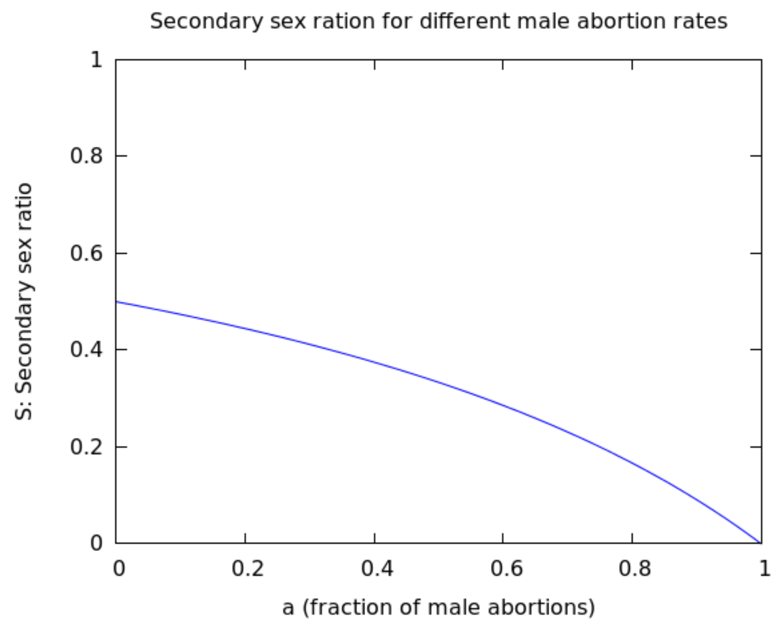
Secondary sex ratio with all aborted:

S(a=1) = 1/(1+1)

Secondary sex ratio in general:

$$S\left(a\right) = \frac{1-a}{2-a}$$

Plotted:



3 (c) Shaw-Mohler

The fitness of a mutant in a resident population is:

$$W\left(a, a\_star\right) = m\left(a\right)\ v\_male\left(a\_star\right) + f\left(a\right)\ v\_female\left(a\_star\right)$$

To calculate the fitness of a mutant in a resident population, we'll need:

- $m(a)$: number of surviving sons
- $f(a)$: number of surviving daughters
- $v\_male(a\_star)$: male reproductive value in resident population
- $v\_female(a\_star)$: female reproductive value in resident population

3.1 (c.1) Surviving sons

General formula of surviving sons:

$$m\left(a\right) = \left(1 - a\right)\ C\left(a\right)\ S\left(a\right)$$

Plug in $C(s, a)$:

$$m\left(a\right) = \frac{\left(1 - a\right)\ S\left(a\right)\ E}{\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort}$$

Set $s = 1/2$:

$$m\left(a\right) = \frac{\left(1 - a\right)\ S\left(a\right)\ E}{\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort}$$

Also plug in $S(a)$:

$$m\left(a\right) = \frac{\left(1 - a\right)^{2}\ E}{\left(2 - a\right)\ \left(\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort\right)}$$

3.2 (c.2) Surviving daughters

General formula of surviving daughters:

$$f\left(a\right) = C\left(a\right)\ \left(1 - S\left(a\right)\right)$$

Plug in $C(s,a)$:

$$f\left(a\right) = \frac{\left(1 - S\left(a\right)\right)\ E}{\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort}$$

Set $s = 1/2$:

$$f\left(a\right) = \frac{\left(1 - S\left(a\right)\right)\ E}{\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort}$$

Also plug in $S(a)$:

$$f\left(a\right) = \frac{\left(1 - \frac{1-a}{2-a}\right)\ E}{\left(1 - a\right)\ e\_son + e\_daughter + a\, e\_abort}$$

3.3 (c.3) Male reproductive value

General formula for male reproductive value:

$$v\_male\left(a\_star\right) = \frac{\alpha}{m\left(a\_star\right)}$$

Filling in $alpha = 1/2$:

$$v\_male\,(a\_star) = \frac{0.5}{m\,(a\_star)}$$

Filling in $m(a\_star)$:

$$v\_male\,(a\_star) = \frac{0.5\ (2 - a\_star)\ ((1 - a\_star)\ e\_son + e\_daughter + a\_star\ e\_abort)}{(1 - a\_star)^2\ E}$$

3.4 (c.4) Female reproductive value

General formula for female reproductive value:

$$v\_female\,(a\_star) = \frac{\alpha}{f\,(a\_star)}$$

Filling in $alpha = 1/2$:

$$v\_female\,(a\_star) = \frac{0.5}{f\,(a\_star)}$$

Filling in $f(a\_star)$:

$$v\_female\,(a\_star) = \frac{0.5\ ((1 - a\_star)\ e\_son + e\_daughter + a\_star\ e\_abort)}{\left(1 - \frac{1 - a\_star}{2 - a\_star}\right)\ E}$$

3.5 (c.5) Fitness

As a reminder, the fitness formula for a mutant in a resident population is:

$$W\,(a, a\_star) = m\,(a)\ v\_male\,(a\_star) + f\,(a)\ v\_female\,(a\_star)$$

Plugging in the four functions above:

$$W\,(a, a\_star) = \frac{0.5\ (1 - a)^2\ (2 - a\_star)\ ((1 - a\_star)\ e\_son + e\_daughter + a\_star\ e\_abort)}{(2 - a)\ (1 - a\_star)^2\ ((1 - a)\ e\_son + e\_daughter + a\ e\_abort)}$$

The fitness of the resident population is:

$$W\,(a\_star, a\_star) = m\,(a\_star)\ v\_male\,(a\_star) + f\,(a\_star)\ v\_female\,(a\_star)$$

Plugging in the four functions (with $a = a\_star$) above, this simplifies to:

$$W\left(a\_star, a\_star\right) = 0.5$$

The relative fitness of the mutant on the resident population:

$$dW\left(a, a\_star\right) = W\left(a, a\_star\right) - dW\left(a\_star, a\_star\right)$$

Filling in both fitness functions:

$$dW\left(a, a\_star\right) = \frac{0.5\,\left(1-a\right)^2\,\left(2-a\_star\right)\,\left(\left(1-a\_star\right)\,e\_son + e\_daughter + a\_star\,e\_abort\right)}{\left(2-a\right)\,\left(1-a\_star\right)^2\,\left(\left(1-a\right)\,e\_son + e\_daughter + a\,e\_abort\right)} - 0.5$$

For an ESS, the fitness of a mutant must always be lower in a resident population. To calculate this, we need the partial derivative of the above fitness function to a.

$$-\frac{1.0\,\left(1-a\right)\,\left(2-a\_star\right)\,\left(\left(1-a\_star\right)\,e\_son + e\_daughter + a\_star\,e\_abort\right)}{\left(2-a\right)\,\left(1-a\_star\right)^2\,\left(\left(1-a\right)\,e\_son + e\_daughter + a\,e\_abort\right)} + \frac{0.5\,\left(1-a\right)^2\,\left(2-a\_star\right)\,\left(\left(1-\right.\right.}{\left(2-a\right)^2\,\left(1-a\_star\right)^2\,\left(\left(\right.\right.}$$

Which is equal to:

$$-\frac{}{\left(\left(2\,a^4-12\,a^3+26\,a^2-24\,a+8\right)\,a\_star^2+\left(-4\,a^4+24\,a^3-52\,a^2+48\,a-16\right)\,a\_star+2\,a^4-12\,a^3+26\,a^2\right.}$$

Filling in $e_son = 10$ and $e_daughter = 10$:

$$-\frac{}{10\,\left(5\,\left(a\_star\,\left(8\,a\_star^4-40\,a\_star^3+64\,a\_star^2-32\,a\_star\right)-4\,a\_star^4+a\_star^2\,\left(-4\,a\_star^4+20\,a\_star\right.\right.\right.}$$

Which is equal to:

$$-\frac{2\,a\_star-5}{a\_star^3-7\,a\_star^2+14\,a\_star-8}$$