Sex ratio exercise

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a) Clutch size

Clutch size for no abortion:

$$C(a=0) = \frac{E}{\frac{1}{2}e_s + \frac{1}{2}e_d}$$

Clutch size for full abortion:

$$C(a=1) = \frac{E}{\frac{1}{2}e_a + \frac{1}{2}e_d}$$

Clutch size in general:

$$C(a) = \frac{E}{\frac{1}{2}(a.e_a + (1-a)e_s) + \frac{1}{2}e_d}$$

$$C(a) = \frac{2E}{a.e_a + (1-a)e_s + e_d}$$

b) Secondary sex ratio

Secondary sex ratio for no abortion:

$$S(a=0) = \frac{sons}{sons + daughters} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

Secondary sex ratio for full abortion:

$$S(a=1) = \frac{sons}{sons + daughters} = \frac{0}{0 + \frac{1}{2}} = 0$$

Secondary sex ratio in general:

$$S(a) = \frac{sons}{sons + daughters} = \frac{\frac{1}{2} - \frac{1}{2}a}{\frac{1}{2} - \frac{1}{2}a + \frac{1}{2}} = \frac{1-a}{2-a} \tag{1}$$

c) Calculate a mutant its fitness

Shaw-Mohler equation, with x replaced by a:

$$W(a,a^*) = \frac{1}{2}\left[\frac{m(a)}{m(a^*)} + \frac{f(a)}{f(a^*)}\right] \tag{2}$$

To solve this, we need $m(a)$ and $f(a)$:

$$m(a) = C(a)s(x)P_s(a) = C(a)\frac{1}{2}(1-a) = C(a)\left[\frac{1}{2} - \frac{1}{2}a\right]$$

$$f(a) = C(a)s(x)P_d(a) = C(a)\frac{1}{2}.1 = \frac{1}{2}C(a)$$

Filling this in:

$$W(a,a^*) = \frac{1}{2}\left[\frac{C(a)[\frac{1}{2} - \frac{1}{2}a]}{C(a^*)[\frac{1}{2} - \frac{1}{2}a^*]} + \frac{\frac{1}{2}C(a)}{\frac{1}{2}C(a^*)}\right] = \frac{1}{2}.\frac{C(a)}{C(a^*)}\left[\frac{1-a}{1-a^*} + 1\right] \tag{3}$$

d) For $e_a < e_d$, Is the equal allocation principle still valid?

This equation should work, for the equal allocation principle to be valid:

$$\frac{s^*}{1-s^*} = \frac{e_d}{e_s}$$

Too bad, this does not take into account the cost of abortion. So, using equation 1 to determine the abortion strategy:

$$\frac{s^*}{1-s^*} = \frac{\frac{1-a^*}{2-a^*}}{1 - \frac{1-a^*}{2-a^*}} = 1 - a^*$$

In section (g) we calculate a^*, so we can plug it in:

$$\frac{s^*}{1-s^*} = 1 - a^* = 1 - \left[\frac{-e_s + e_d + 2.e_a}{e_a - e_s}\right] = \frac{e_d + e_a}{e_s - e_a} \tag{4}$$

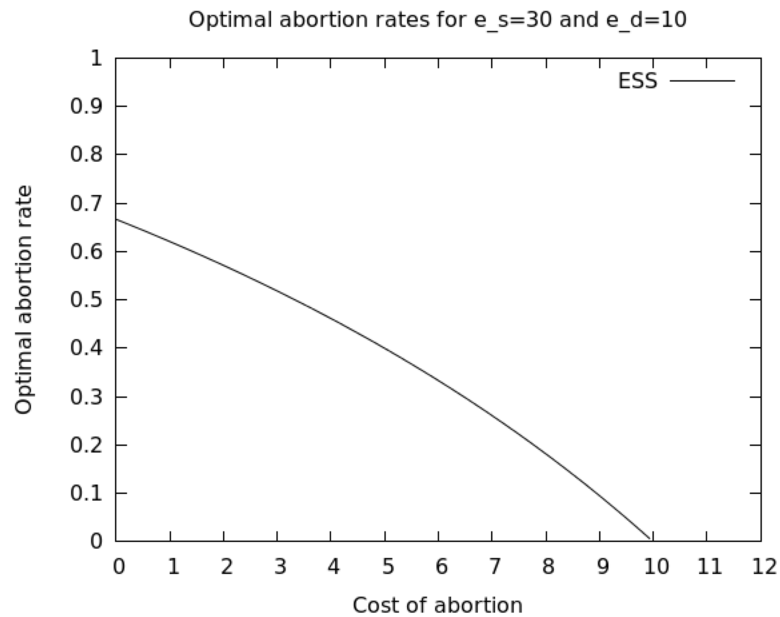
Now we can see that abortion is an investment at the expense of sons (see the denominator) at the gain of daughters (numerator). Taking into account abortion, the equal allocation principle is still unscathed.

e) For $e_a = e_d$, is the equal allocation principle still valid?

From equation 4 we can see that abortion is invested in the creation of daughters. If this investment (the expense of a son) equals its return (a daughter), nothing is gained by aborting sons.

f) Plot of a graph

Using equation 6 (which is derived below).



g) Derive a^*

First equation 3 needs to be reworked before taking the partial differential of $W(a, a^*)$ to a:

$$W(a, a^*) = \frac{1}{2} \cdot \frac{C(a)}{C(a^*)} \left[\frac{1-a}{1-a^*} + 1 \right] = \left[\frac{E}{1 - a^* C(a^*)} \right] \left[\frac{2 - a - a^*}{a.e_a + (1-a)e_s + e_d} \right]$$

Because the first term is indepent of a, it does not take part in the partial differential:

$$k = \left[\frac{E}{1 - a^* C(a^*)} \right]$$

Extracting k and differentiating with the quotient rule:

$$\left[\frac{f}{g} \right]' = \frac{f'g - f.g'}{(g)^2} = \frac{dW}{da} = \frac{e_s - e_d - 2.e_a + a^*.e_a - a^*.e_s}{(g)^2}$$

To find the maximum $k\frac{dW}{da} = 0$ must be solved, of which either k or $\frac{dW}{da}$is zero:

$$k = \left[\frac{E}{1 - a^* C(a^*)} \right] = 0$$

$$a^* = \frac{-e_s - e_d}{e_a - e_s} \tag{5}$$

$$\frac{dW}{da} = 0$$

$$e_s - e_d - 2.e_a + a^*.e_a - a^*.e_s = 0$$

$$a^* = \frac{-e_s + e_d + 2.e_a}{e_a - e_s} \tag{6}$$

Note that equation 5 is mathematically valid, but only works when some costs are negative.

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