Kin selection and the evolution of dispersal

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Symbol definitions

Symbol	Description
c	cost of dispersal, chance to die when dispersing
d	dispersal rate of mutant
$\hat{d}$	dispersal rate of resident
d^*	evolutionary stable singularity of dispersal rate
k	probability that an individual present in a patch after dispersal was born there
n	Patch size (=number of females, as haploid)

a)

The focal female stays in a patch:

$$N_{stay} = 1 - d$$

Her presence will be diluted by unrelated dispersers that enters the patch with rate:

$$N_{in} = d(1-c)$$

It might be imagined that a disperser lands in the same patch as it originated in. With an infinite amount of patches, this will never happen.

Therefore, the relatedness k equals:

$$k = \frac{N_{stay}}{N_{stay} + N_{in}} = \frac{1-d}{1-d+(1-c)d} = \frac{1-d}{1-d+d-cd} = \frac{1-d}{1-cd}$$

b)

Combining these equations:

$$\Delta w = \delta p \left(-c + Rk\right) = 0$$

$$k = \frac{1-d}{1-cd}$$

Solving this for d, calling the answer d^*:

$$-c + R\left(\frac{1-d}{1-cd}\right) = 0$$

$$d^* = \frac{c-R}{c^2-R}$$

c)

Solve

$$\frac{dw}{dd} = \left(r\frac{\partial w}{\partial d} + R\frac{\partial w}{\partial \bar{d}}\right)_{d=\bar{d}=d^*}$$

This can be done by using these Maxima equations (and are checked by hand as well):

```
W(d,d_bar,d_star)
   := ((1-d)/(1-d_bar+((1-c)*d_star)))
   + (((1-c)*d)/(1-(c*d_star)))
;
ChangeInFitnessA(w,d) := ''(diff(W(d,d_bar,d_star),d));
ChangeInFitnessB(w,d) := ''(diff(W(d,d_bar,d_star),d_bar));
ChangeInFitness(w,d)
   := (r * ''(diff(W(d,d_bar,d_star),d)))
   + (R * ''(diff(W(d,d_bar,d_star),d_bar)))
;
d_bar:d;
 d_star:d;
ChangeInFitness(w,d);
```

results in:

$$W(d,\bar{d},d^*) = \frac{(1-c)\ d}{1-c\,d} + \frac{1-d}{(1-c)\ d+d+1}$$

$$\frac{\partial w(d,\bar{d},d^*)}{\partial d} = \frac{1-c}{1-c\,d\_star} - \frac{1}{(1-c)\ d\_star - d\_bar + 1}$$

$$\frac{\partial w(d,\bar{d},d^*)}{\partial \bar{d}} = -\frac{1-d}{\left((1-c)\ d\_star + d\_bar + 1\right)^2}$$

$$\frac{dw}{dd} = \left(r\frac{\partial w}{\partial d} + R\frac{\partial w}{\partial \bar{d}}\right)_{d=\bar{d}=d^*} = \left(\frac{1-c}{1-c\,d} - \frac{1}{(1-c)\ d-\bar{d}+1}\right) r - \frac{(1-d)\ R}{\left((1-c)\ d-\bar{d}+1\right)^2}$$

When setting $r = 1$ (mother is totally related to her offspring, thanks to clonal reproduction), solve $\frac{dw}{dd} = 0$:

$$\frac{dw}{dd} = \left(\frac{1-c}{1-c\,d} - \frac{1}{(1-c)\;d - \bar{d} + 1}\right) r - \frac{(1-d)\;R}{\left((1-c)\;d - \bar{d} + 1\right)^2} = 0$$

$$\left(\frac{1-c}{1-c\,d} - \frac{1}{(1-c)\;d - \bar{d} + 1}\right) - \frac{(1-d)\;R}{\left((1-c)\;d - \bar{d} + 1\right)^2} = 0$$

$$d = \frac{R-c}{R-c^2}$$

d)

In this case, we can also assume $R = 1$.

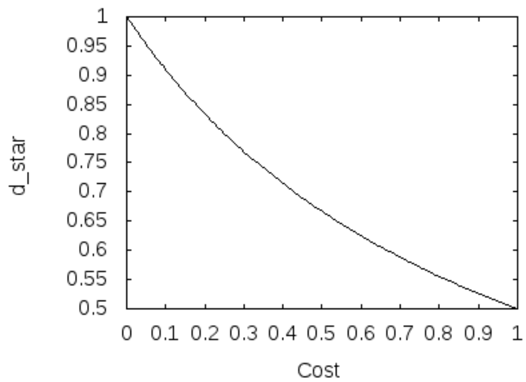
Plugging it in, using the Maxima equations:

```
W(d,d_bar,d_star)
   := ((1-d)/(1-d_bar+((1-c)*d_star)))
   + (((1-c)*d)/(1-(c*d_star)));
ChangeInFitnessA(w,d) := ''(diff(W(d,d_bar,d_star),d));
ChangeInFitnessB(w,d) := ''(diff(W(d,d_bar,d_star),d_bar));
ChangeInFitness(w,d)
   := (r * ''(ChangeInFitnessA(w,d)))
   + (R * ''(ChangeInFitnessB(w,d)));
d_bar:d;   d_star:d;
ChangeInFitness(w,d);
r:1;
ChangeInFitness(w,d);
OptimalDispersal(c,R)
   := ''(solve(ChangeInFitness(w,d)=0,d))[1];
OptimalDispersal(c,1);
wxplot2d(
   rhs(OptimalDispersal(c,1)),[c,0.0001,1],
   [xlabel,"Cost"],[ylabel,"d_star"],[color,black]);
```

This results in:

$$d^* = \frac{1-c}{1-c^2}$$

And the plot:



So, the less cost is associated with dispersal, the more offspring should disperse.

e)

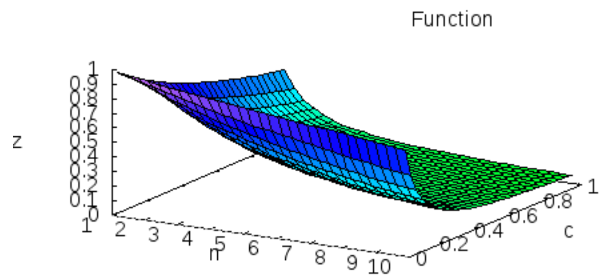
If the mother controls the dispersal, she will want to have only dispersers, except for exactly one.

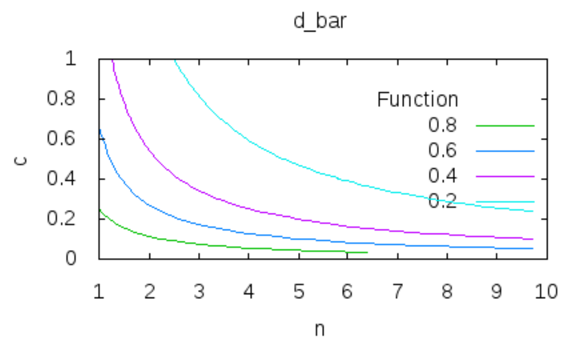
f)

Using the Maxima code

```
wxplot3d(
  d_bar(n,c),[n,1,10],[c,0.0,1.0],
  [title,"d_bar"],
  [xlabel,"n"],
  [ylabel,"c"],
  [zlabel,"d_star"]
);
wxcontour_plot(
  d_bar(n,c),[n,1,10],[c,0.0,1.0],
  [title,"d_bar"],
  [xlabel,"n"],
  [ylabel,"c"],
  [zlabel,"d_star"]
);
```

This results in:





This leads us to conclude that the bigger patches are, the less dispersal there should be. This can be explained as such: the bigger a remote patch, the more offspring you need to disperse to occupy it. The drawback is that per offspring a cost needs to be paid, rendering the residents stronger.

Acknowledgements

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