Kin selection and the evolution of dispersal

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Symbol definitions

Symbol	Description
c	cost of dispersal, chance to die when dispersing
d	dispersal rate of mutant
$\hat{d}$	dispersal rate of resident
d^*	evolutionary stable singularity of dispersal rate
n	Patch size (=number of females, as haploid)

a)

From T&F96, equation 9:

$$k = \frac{1 - \hat{d}}{1 - \hat{d}.c}$$

b)

From T&F96, equation 11:

$$d^* = \frac{\sqrt{1 + 4N(N-1)c^2} + 1 - 2Nc}{\sqrt{1 + 4N(N-1)c^2} + 1 - 2Nc^2}$$

c)

Solve

$$\frac{dw}{dd} = \left(r\frac{\partial w}{\partial d} + R\frac{\partial w}{\partial \bar{d}} \right)_{d=\bar{d}=d^*}$$

Using the Maxima equations:

```
W(d,d_bar,d_star)
   := ((1-d)/(1+d_bar+((1-c)*d_star)))
   + (((1-c)*d)/(1-(c*d)))
;
ChangeInFitness(w,d)
   := (r * ''(diff(W(d,d_bar,d_star),d)))
   + (R * ''(diff(W(d,d_bar,d_star),d_bar)))
;
d_bar:d;
d_star:d;
ChangeInFitness(w,d);
```

results in:

$$W(d,\bar{d},d^*) = \frac{(1-c)\ d}{1-c\,d} + \frac{1-d}{(1-c)\ d+d+1}$$

$$\frac{dw}{dd} = \left(r\frac{\partial w}{\partial d} + R\frac{\partial w}{\partial \bar{d}}\right)_{d=\bar{d}=d^*} = \left(\frac{1-c}{1-c\,d} + \frac{(1-c)\ c\,d}{(1-c\,d)^2} - \frac{1}{(1-c)\ d+d+1} - \frac{(2-c)\ (1-d)}{((1-c)\ d+d+1)^2}\right) R + \left(\frac{1-c}{1-c\,d} + \frac{(1-c)\ c\,d}{(1-c\,d)^2} - \frac{1}{(1-c)\ d+d+1} - \frac{(2-c)\ (1-d)}{((1-c)\ d+d+1)^2}\right) r$$

I guess this simplifies to this:

$$\frac{dw}{dd} = \left(r\frac{\partial w}{\partial d} + R\frac{\partial w}{\partial \bar{d}}\right)_{d=\bar{d}=d^*} = R + r$$

d)

Relatedness between disperser and other inividuals in the patch:

?

Plugging it in, using the Maxima equations:

```
d_star(N)
   :=   (sqrt(1+4*N*(N-1)*(c^2))+1-(2*N*c))
      / (sqrt(1+4*N*(N-1)*(c^2))+1-(2*N*(c^2)))
;
d_star(1);
tex1(d_star(1));
```

This results in:

$$d^* = \frac{2-2\,c}{2-2\,c^2}$$

e)

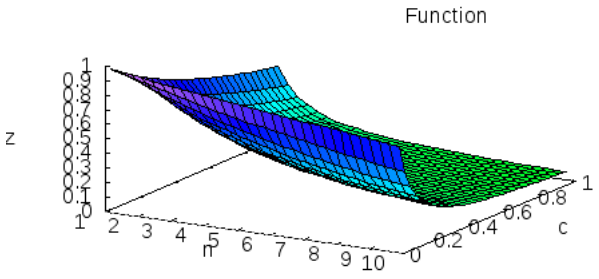
Didn't she already???

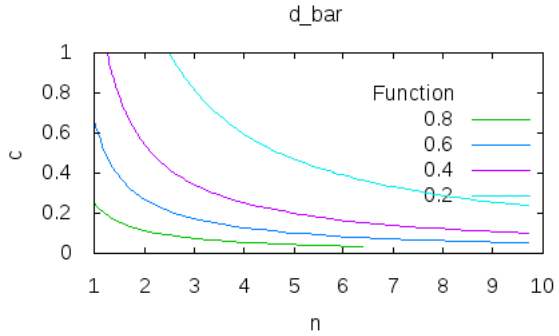
f)

Using the Maxima code

```
d_bar(n,c) := (1 + (2*n*c) - sqrt(1 + (4*n*(n-1)*(c^2))))/(2*n*c*(1+c));
wxplot3d(
  d_bar(n,c),[n,1,10],[c,0.0,1.0],
  [title,"d_bar"],
  [xlabel,"n"],
  [ylabel,"c"],
  [zlabel,"d_star"]
);
wxcontour_plot(
  d_bar(n,c),[n,1,10],[c,0.0,1.0],
  [title,"d_bar"],
  [xlabel,"n"],
  [ylabel,"c"],
  [zlabel,"d_star"]
);
```

This results in:





References

T&F96: Taylor & Frank, 1996, How to make a kin selection model