Case 2000 page 69 exercise

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Abstract

This article is created within the CAS program Maxima and shows how to the exercise of [1] at page 69.

1 Introduction

LaTeX is commonly used for writing publishable scientific articles[2]. Algebraic manipulations can be done by a CAS, for example Maxima, Maple or Mathematica. Maxima is the only free and open-source program, and it is the oldest free and open-source computer algebra system, with development started in 1967 (as Macsyma) or 1982 (as MAXIMA). This article is an example of writing a LaTeX article within Maxima

2 Exercise

symbol	description
I	Indentity matrix
L	Leslie matrix
M	Leslie matrix with λ subtracted at diagonal
x	population density vector
Z	Vector filled with zeroes
λ	population growth rate

Table 1: Definitions

(for definitions see table 1 on page 1).
The equation to solve, equation 3.21 is:

$$x\,L = x\,\lambda \tag{1}$$

The Leslie matrix, L, given is:

$$\begin{pmatrix} 1.0 & 1.0 \\ 0.8 & 0.8 \end{pmatrix}$$

The simplifies equation 3.21 to:

$$\begin{pmatrix} 1.0\,x & 1.0\,x \\ 0.8\,x & 0.8\,x \end{pmatrix} = x\,\lambda$$

Solving equation 3.21 can be done with equation 3.24:

$$det(L - \lambda * I) = Z$$

Where I is the identity matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

And Z is the vector of zeroes:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This simplifies equation 3.24 to:

$$det\left(\begin{pmatrix} 1.0 - \lambda & 1.0 \\ 0.8 & 0.8 - \lambda \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The determinant of that matrix (M), is:

$$(0.8 - \lambda)\ (1.0 - \lambda) - 0.8$$

Solving $M = 0$, the λs found are:

$$\left[\lambda = \frac{9}{5}, \lambda = 0\right]$$

There is one stable population structure, $\lambda = 0$, which is denotes an extinct population. Here I focus on the more interesting value, where $\lambda = 9/5$. This lambda is called the dominant eigenvalue, which equals the ultimate population growth.

Results in M:

$$\begin{pmatrix} -0.8 & 1.0 \\ 0.8 & -1.0 \end{pmatrix}$$

Solving 3.23: $M * x = Z$:

Create a population vector, x, with 1.0 as an initial value (it will be rescaled later):

$$\begin{pmatrix} 1.0 \\ x_2 \end{pmatrix}$$

Now $M * x$ simplifies to:

$$\begin{pmatrix} 1.0\,x_2 - 0.8 \\ 0.8 - 1.0\,x_2 \end{pmatrix}$$

Solving this, 3.23: $M * x = Z$, there are two equations that can be solved:

$$\begin{pmatrix} 1.0\,x_2 - 0.8 \\ 0.8 - 1.0\,x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Solving the upper, results in $x2 = 4/5$. This results in an x of:

$$\begin{pmatrix} 1.0 \\ \frac{4}{5} \end{pmatrix}$$

x must be rescaled so that its sum equals 1.0. x its current sum is 1.8, so dividing all elements by it, results in an x of:

$$\begin{pmatrix} 0.55555555555555 \\ 0.44444444444444 \end{pmatrix}$$

3 Conclusion

For this Leslie matrix:

$$\begin{pmatrix} 1.0 & 1.0 \\ 0.8 & 0.8 \end{pmatrix}$$

The dominant eigenvalue, λ, is:

9/5

The stable population size distribution is:

$$\begin{pmatrix} 0.55555555555555 \\ 0.44444444444444 \end{pmatrix}$$

4 Discussion

Writing LaTeX within Maxima can be done, but it is a bit cumbersome: Maxima does not know LaTeX syntax and just creates contextless strings, which might not be compilable by LaTeX. However, because the script does create a .tex file, this file can be inspected easily with a LaTeX tool like texmaker.

References

[1] Case, Ted J. 2000 An illustrated guide to theoretical ecology.

[2] Gaudeul, A. 2006 Do Open Source Developers Respond to Competition?: The (La)TeX Case Study. Available at SSRN: http://ssrn.com/abstract=908946 or http://dx.doi.org/10.2139/ssrn.908946

A Script file

```
#!/bin/bash
maxima_input_file="case_2000_69.txt"
tex_output_file="case_2000_69_output.tex"

if [ -e $tex_output_file ]
then
  rm $tex_output_file
fi

maxima -b $maxima_input_file
pdflatex $tex_output_file
pdflatex $tex_output_file
```

B Maxima file

```
/* Maxima batch file */

/* Load libraries */
load("stringproc")$

/* Input filename */
bash_filename:"case_2000_69.sh"$
maxima_filename:"case_2000_69.txt"$ /* this file */

/* Output filenames */
tex_filename:"case_2000_69_output.tex"$

/* Write results to TeX file */
stream: openw(tex_filename)$
printf(stream,"\\documentclass{article}~%")$
printf(stream,"~%")$
printf(stream,"\\usepackage{listings}~%")$
printf(stream,"\\usepackage{graphicx}~%")$
printf(stream,"~%")$
printf(stream,"\\title{Case 2000 page 69 exercise}~%")$
printf(stream,"\\author{Richel Bilderbeek}~%")$
printf(stream,"\\date{\\today}~%")$
printf(stream,"~%")$
printf(stream,"\\begin{document}~%")$
printf(stream,"~%")$
printf(stream,"\\maketitle~%")$
printf(stream,"~%")$
```

```
printf(stream,"\\begin{abstract}~%")$
printf(stream,"This article is created within the CAS
   program Maxima~%")$
printf(stream,"and shows how to the exercise of \\cite{
   case2000} at page 69.~%")$
printf(stream,"\\end{abstract}~%")$
printf(stream,"~%")$
printf(stream,"\\section{Introduction}~%")$
printf(stream,"~%")$
printf(stream,"\\LaTeX~~is commonly used for writing
   publishable scientific articles\\cite{gaudeul2006}.~%"
   )$
printf(stream,"Algebraic manipulations can be done by a
   CAS, for example Maxima, Maple or Mathematica.~%")$
printf(stream,"Maxima is the only free and open-source
   program, and it is the oldest free and open-source
   computer algebra system, with development started in
   1967 (as Macsyma) or 1982 (as MAXIMA).~%")$
printf(stream,"This article is an example of writing a \\
   LaTeX~~ article within Maxima~%")$
printf(stream,"~%")$
printf(stream,"\\section{Exercise}~%")$
printf(stream,"~%")$
printf(stream,"\\begin{table}[here]~%")$
printf(stream,"  \\centering~%")$
printf(stream,"  \\begin{tabular}{ | r | l | }~%")$
printf(stream,"    \\hline~%")$
printf(stream,"    symbol & description \\\\~%")$
printf(stream,"    \\hline~%")$
printf(stream,"    $I$ & Indentity matrix \\\\~%")$
printf(stream,"    $L$ & Leslie matrix \\\\~%")$
printf(stream,"    $M$ & Leslie matrix with $\\lambda$
   subtracted at diagonal \\\\~%")$
printf(stream,"    $x$ & population density vector \\\\~%
   ")$
printf(stream,"    $Z$ & Vector filled with zeroes \\\\~%
   ")$
printf(stream,"    $\\lambda$ & population growth rate
   \\\\~%")$
printf(stream,"    \\hline~%")$
printf(stream,"  \\end{tabular}~%")$
printf(stream,"  \\caption{Definitions}~%")$
printf(stream,"  \\label{table:table_definition}~%")$
printf(stream,"\\end{table}~%")$
printf(stream,"~%")$
```

```
printf(stream,"(for definitions see table \\ref{table:
    table_definition} on page \\pageref{table:
    table_definition}).~%")$
printf(stream,"~%")$
printf(stream,"The equation to solve, equation 3.21 is:~%
    ")$

Eq3_21(L,x,lambda) := L * x = lambda * x;

printf(stream,"\\begin{equation}~%")$
printf(stream,tex1(Eq3_21(L,x,lambda)))$
printf(stream,"\\end{equation}~%")$

printf(stream,"The Leslie matrix, L,  given is:~%")$
printf(stream,"~%")$

L:matrix([1.0,1.0],[0.8,0.8]);

printf(stream,tex(L,false))$
printf(stream,"~%")$
printf(stream,"The simplifies equation 3.21 to:~%")$
printf(stream,"~%")$
printf(stream,tex(Eq3_21(L,x,lambda),false))$
printf(stream,"~%")$
printf(stream,"Solving equation 3.21 can be done with
    equation 3.24:~%")$
printf(stream,"~%")$
printf(stream,"$$det(L - \\lambda*I) = Z$$~%")$
printf(stream,"~%")$

I:ident(2);
Z:zeromatrix(2,1);

printf(stream,"Where I is the identity matrix:~%")$
printf(stream,tex(I,false))$
printf(stream,"And Z is the vector of zeroes:~%")$
printf(stream,tex(Z,false))$
printf(stream,"~%")$

Eq3_24(L,lambda,I,Z) := det(L - lambda * I) = Z;

printf(stream,"This simplifies equation 3.24 to:~%")$
printf(stream,"~%")$
printf(stream,tex(Eq3_24(L,lambda,I,Z),false))$
printf(stream,"~%")$
```

```
/*
printf(stream,"Here I define $M$ as:~%")$

Eq3_24_b(L,lambda,I) := M = L - lambda * I;
printf(stream,tex(Eq3_24_b(L,lambda,I),false))$

*/
/* Great that M is in Eq3_24_b, but I cannot get it out,
   so I do it manually*/
M:copymatrix(L);
M[1][1] : M[1][1] - lambda;
M[2][2] : M[2][2] - lambda;

/*

printf(stream,tex(M,false))$
printf(stream,"~%")$
printf(stream,"Solving for which $\\lambda$ equation 3.24
    holds:~%")$
printf(stream,"~%")$
printf(stream,"$$det(L - \\lambda*I) = Z$$~%")$
printf(stream,"$$det(M) = Z$$~%")$
printf(stream,"~%")$
printf(stream,"'det' denotes taking the determinant, L is
    the Leslie matrix, I is an identity matrix~%")$
printf(stream,"and Z is a vector of all zeroes. M already
    has been calculated, so the equation becomes:~%")$
printf(stream,"~%")$
printf(stream,"$$det(M) = Z$$~%")$
*/

printf(stream,"~%")$
printf(stream,"The determinant of that matrix ($M$), is
   :~%")$
printf(stream,tex(determinant(M),false))$
printf(stream,"~%")$

stable_lambdas : solve(determinant(M) = 0,lambda);

printf(stream,"~%")$
printf(stream,"Solving $M=0$, the $\\lambda$s found are
   :~%")$
printf(stream,"~%")$
printf(stream,tex(stable_lambdas,false))$
printf(stream,"~%")$
```

```
printf(stream,"There is one stable population structure,
    $\\lambda=0$, which is~%")$
printf(stream,"denotes an extinct population.~%")$

lambda:rhs(stable_lambdas[1]);

printf(stream,"Here I focus on the more interesting value
    ,~%")$
printf(stream,"where $\\lambda=")$
printf(stream,string(lambda))$ /* Don't forget the string
     function */
printf(stream,"$.~%")$
printf(stream,"This lambda is called the dominant
    eigenvalue, which equals the ultimate population
    growth.~%")$
printf(stream,"~%")$

M:''(M); /* Filling it in */

printf(stream,"Results in M:~%")$
printf(stream,tex(M,false))$
printf(stream,"~%")$

printf(stream,"Solving 3.23: $M * x = Z$:~%")$
printf(stream,"~%")$
printf(stream,"Create a population vector, $x$, with $1.0
    $ as an initial value~%")$
printf(stream,"(it will be rescaled later):~%")$
printf(stream,"~%")$

x:transpose(matrix([1.0,x2]));
EqSolve(M,x,Z):= M.x = Z;

printf(stream,tex(x,false))$
printf(stream,"Now $M * x$ simplifies to:~%")$
printf(stream,tex(lhs(EqSolve(M,x,Z)),false))$
printf(stream,"Solving this, 3.23: $M * x = Z$, there are
     two equations that can be solved:~%")$
printf(stream,tex(EqSolve(M,x,Z),false))$

q:''(M.x)[1,1];
x2:rhs(solve(q)[1]);

printf(stream,"Solving the upper, results in $x2=")$
printf(stream,string(x2))$
printf(stream,"$.~%")$
```

```
printf(stream,"This results in an $x$ of:~%")$

x;
x:''(x);

printf(stream,tex(x,false))$
printf(stream,"$x$ must be rescaled so that its sum
    equals $1.0$.~%")$

sz: matrix_size(x);
my_sum: sum(sum(x[i,j],i,1,sz[1]),j,1,sz[2]);

printf(stream,"$x$ its current sum is $")$
printf(stream,string(my_sum))$
printf(stream,"$, so dividing all elements by it, results
     in an $x$ of:~%")$

x:x/my_sum;
x;

printf(stream,tex(x,false))$

/* Conclusion */

printf(stream,"\\section{Conclusion}~%")$
printf(stream,"~%")$
printf(stream,"For this Leslie matrix:~%")$
printf(stream,tex(L,false))$
printf(stream,"~%")$
printf(stream,"The dominant eigenvalue, $\\lambda$, is:~%
    ")$
printf(stream,"\\\\\\\\~%")$
printf(stream,string(lambda))$ /* Don't forget the string
     function */
printf(stream,"\\\\~%")$
printf(stream,"~%")$
printf(stream,"The stable population size distribution is
    :~%")$
printf(stream,"~%")$
printf(stream,tex(x,false))$
printf(stream,"~%")$

/* Discussion */

printf(stream,"\\section{Discussion}~%")$
printf(stream,"~%")$
```

```
printf(stream,"Writing \\LaTeX~~within Maxima can be done
    , but it is a bit cumbersome:~%")$
printf(stream,"Maxima does not know \\LaTeX~~syntax and
    just creates contextless strings,~%")$
printf(stream,"which might not be compilable by \\LaTeX
    .~%")$
printf(stream,"However, because the script does create a
    .tex file,~%")$
printf(stream,"this file can be inspected easily with a
    \\LaTeX~~tool like texmaker.~%")$
printf(stream,"~%")$

/* Bibliography */

printf(stream,"\\begin{thebibliography}{9}~%")$
printf(stream,"~%")$
printf(stream,"\\bibitem{case2000}~%")$
printf(stream,"  Case, Ted J.~%")$
printf(stream,"  2000~%")$
printf(stream,"  An illustrated guide to theoretical
    ecology.~%")$
printf(stream,"~%")$
printf(stream,"\\bibitem{gaudeul2006}~%")$
printf(stream,"  Gaudeul, A.~%")$
printf(stream,"  2006~%")$
printf(stream,"  Do Open Source Developers Respond to
    Competition?: The (La)TeX Case Study.~%")$
printf(stream,"  Available at SSRN: http://ssrn.com/
    abstract=908946 or http://dx.doi.org/10.2139/ssrn
    .908946~%")$
printf(stream,"~%")$
printf(stream,"\\end{thebibliography}~%")$
printf(stream,"~%")$

/* Appendix */

printf(stream,"\\appendix~%")$
printf(stream,"~%")$
printf(stream,"\\section{Script file}~%")$
printf(stream,"~%")$
printf(stream,"\\lstinputlisting[language=C++,
    showstringspaces=false,breaklines=true,frame=single]{"
    )$
printf(stream,bash_filename)$
printf(stream,"}~%")$
printf(stream,"~%")$
```

```
printf(stream,"\\section{Maxima file}~%")$
printf(stream,"~%")$
printf(stream,"\\lstinputlisting[language=C++,
    showstringspaces=false,breaklines=true,frame=single]{"
    )$
printf(stream,maxima_filename)$
printf(stream,"}~%")$
printf(stream,"~%")$
printf(stream,"\\section{\\LaTeX~~file}~%")$
printf(stream,"~%")$
printf(stream,"\\lstinputlisting[language=tex,
    showstringspaces=false,breaklines=true,frame=single]{"
    )$
printf(stream,tex_filename)$
printf(stream,"}~%")$
printf(stream,"~%")$
printf(stream,"\\end{document}~%")$
close(stream)$
```

C LaTeX file

```
\documentclass{article}

\usepackage{listings}
\usepackage{graphicx}

\title{Case 2000 page 69 exercise}
\author{Richel Bilderbeek}
\date{\today}

\begin{document}

\maketitle

\begin{abstract}
This article is created within the CAS program Maxima
and shows how to the exercise of \cite{case2000} at page
    69.
\end{abstract}

\section{Introduction}

\LaTeX~is commonly used for writing publishable
    scientific articles\cite{gaudeul2006}.
```

```
Algebraic manipulations can be done by a CAS, for example
    Maxima, Maple or Mathematica.
Maxima is the only free and open-source program, and it
   is the oldest free and open-source computer algebra
   system, with development started in 1967 (as Macsyma)
   or 1982 (as MAXIMA).
This article is an example of writing a \LaTeX~ article
   within Maxima

\section{Exercise}

\begin{table}[here]
  \centering
  \begin{tabular}{ | r | l | }
    \hline
    symbol & description \\
    \hline
    $I$ & Indentity matrix \\
    $L$ & Leslie matrix \\
    $M$ & Leslie matrix with $\lambda$ subtracted at
       diagonal \\
    $x$ & population density vector \\
    $Z$ & Vector filled with zeroes \\
    $\lambda$ & population growth rate \\
    \hline
  \end{tabular}
  \caption{Definitions}
  \label{table:table_definition}
\end{table}

(for definitions see table \ref{table:table_definition}
   on page \pageref{table:table_definition}).

The equation to solve, equation 3.21 is:
\begin{equation}
x\,L=x\,\lambda\end{equation}
The Leslie matrix, L,  given is:

$$\pmatrix{1.0&1.0\cr 0.8&0.8\cr }$$

The simplifies equation 3.21 to:

$$\pmatrix{1.0\,x&1.0\,x\cr 0.8\,x&0.8\,x\cr }=x\,\lambda
   $$

Solving equation 3.21 can be done with equation 3.24:
```

```
$$det(L - \lambda*I) = Z$$

Where I is the identity matrix:
$$\pmatrix{1&0\cr 0&1\cr }$$
And Z is the vector of zeroes:
$$\pmatrix{0\cr 0\cr }$$

This simplifies equation 3.24 to:

$${\it det}\left(\pmatrix{1.0-\lambda&1.0\cr 0.8&0.8-\
    lambda\cr }
 \right)=\pmatrix{0\cr 0\cr }$$


The determinant of that matrix ($M$), is:
$$\left(0.8-\lambda\right)\,\left(1.0-\lambda\right)
    -0.8$$


Solving $M=0$, the $\lambda$s found are:

$$\left[ \lambda={{9}\over{5}} , \lambda=0 \right] $$

There is one stable population structure, $\lambda=0$,
    which is
denotes an extinct population.
Here I focus on the more interesting value,
where $\lambda=9/5$.
This lambda is called the dominant eigenvalue, which
    equals the ultimate population growth.

Results in M:
$$\pmatrix{-0.8&1.0\cr 0.8&-1.0\cr }$$

Solving 3.23: $M * x = Z$:

Create a population vector, $x$, with $1.0$ as an initial
     value
(it will be rescaled later):

$$\pmatrix{1.0\cr {\it x_2}\cr }$$
Now $M * x$ simplifies to:
$$\pmatrix{1.0\,,{\it x_2}-0.8\cr 0.8-1.0\,,{\it x_2}\cr
    }$$
```

```
Solving this, 3.23: $M * x = Z$, there are two equations
    that can be solved:
$$\pmatrix{1.0\,,{\it x_2}-0.8\cr 0.8-1.0\,,{\it x_2}\cr
    }=\pmatrix{0
 \cr 0\cr }$$
Solving the upper, results in $x2=4/5$.
This results in an $x$ of:
$$\pmatrix{1.0\cr {{4}\over{5}}\cr }$$
$x$ must be rescaled so that its sum equals $1.0$.
$x$ its current sum is $1.8$, so dividing all elements by
     it, results in an $x$ of:
$$\pmatrix{0.55555555555555\cr 0.44444444444444\cr }$$
\section{Conclusion}

For this Leslie matrix:
$$\pmatrix{1.0&1.0\cr 0.8&0.8\cr }$$

The dominant eigenvalue, $\lambda$, is:
\\\\
9/5\\

The stable population size distribution is:

$$\pmatrix{0.55555555555555\cr 0.44444444444444\cr }$$

\section{Discussion}

Writing \LaTeX~within Maxima can be done, but it is a bit
     cumbersome:
Maxima does not know \LaTeX~syntax and just creates
    contextless strings,
which might not be compilable by \LaTeX.
However, because the script does create a .tex file,
this file can be inspected easily with a \LaTeX~tool like
     texmaker.

\begin{thebibliography}{9}

\bibitem{case2000}
  Case, Ted J.
  2000
  An illustrated guide to theoretical ecology.

\bibitem{gaudeul2006}
  Gaudeul, A.
  2006
```

```
  Do Open Source Developers Respond to Competition?: The
      (La)TeX Case Study.
  Available at SSRN: http://ssrn.com/abstract=908946 or
      http://dx.doi.org/10.2139/ssrn.908946

\end{thebibliography}

\appendix

\section{Script file}

\lstinputlisting[language=C++,showstringspaces=false,
    breaklines=true,frame=single]{case_2000_69.sh}

\section{Maxima file}

\lstinputlisting[language=C++,showstringspaces=false,
    breaklines=true,frame=single]{case_2000_69.txt}

\section{\LaTeX~file}

\lstinputlisting[language=tex,showstringspaces=false,
    breaklines=true,frame=single]{case_2000_69_output.tex}

\end{document}
```