- The error in Bayesian phylogenetic reconstruction
- when speciation co-occurs
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- 8 Abstract

Keywords: computational biology, evolution, phylogenetics, Bayesian analysis, tree prior [RJCB: Have you already looked up for a target journal?]
[GL: Honestly I have literally no idea how to select a good journal

for this kind of article.] [RJCB: May I suggest we aim for Molecular

Phylogenetics and Evolution, the same journal as the raket paper?

1 Introduction

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• There are many contemporary tools that provide the possibility to infer a phylogeny from genetic data (DNA, RNA, proteins). A popular Bayesian

- phylogenetic tool is called BEAST (Drummond & Rambaut 2007) and its cousin BEAST2 (Bouckaert *et al.* 2014).
- BEAST is very flexible, providing the user with the option to set up all possible phylogenetic priors (e.g. site/clock/speciation model).
- However, currently available priors can be not suitable to analyze some specific datasets. With this work we aim to test whether or not the implementation of a new prior model is beneficial to study a specific kind of diversification process.
- BEAST2 gives us the possibility to introduce new tree priors to infer
 phylogenies based on different assumptions on how the speciation process
 takes place.
- One of such speciation processes is the multiple birth hypothesis, a new model (described below) and thus currently absent in BEAST.
- The Multiple birth hypothesis can be useful to explain a phenomenon 31 that has always puzzled evolutionary biologists: what are the drivers of 32 the diversification processes for those phylogenies that show an impressive 33 amount of speciation events in relatively short times? The (constant-rate) 34 birth-death (BD) model embodies the common assumption that only a single speciation event can occur at any given time. The multiple-birthdeath (MBD) model relaxes this assumption, allowing events in which large-scale environmental changes lead to a great number of species in relatively short time intervals. Such a hypothesis may be a better fit to describe the burst in systems like cichlid fish diversification in the African Great Lakes: Malawi, Tanganyika and Victoria (Janzen et al. 2016, Janzen et al. 2017). 42

- However, it may be that current BD tree priors are good enough at detecting such events, with a (preferred) lower level of complexity. If this is the case one should always be more keen to adopt the simplest model.
- Here we present our study with the aim of exploring when using a more complex MBD tree prior is warranted.

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• We hypothesize that the error made today, using BD tree priors, increases with an increased number or stronger effect of multiple birth events. This is straightforward: without multiple birth events or such event having no effect, the MBD model falls back to a BD model. We expect larger errors when we deviate more from the BD model's assumptions. Additionally, we hypothesize MBD having a stronger effect if the normal speciation process is less pronounced. The more speciations are caused by the BD process, there are relatively less multiple-birth events. To put this hypothesis, H1, into an explicit equation, we expect the error made be correlated to the number of species created by the multiple-birth process over the total number of species created:

$$\langle e \rangle = f(\frac{n_{taxa}^{MBD}}{n_{taxa}^{BD} + n_{taxa}^{MBD}})$$
 (1)

- Where $\langle e \rangle$ denotes the expected error, f is a monotonously increasing function of unknown shape, n_{taxa}^{MBD} is the number of taxa created in multiple-birth events and n_{taxa}^{BD} is the number of taxa created by the standard BD specition process.
 - We have the hypothesis, H7, that there is no difference in inference errors
 if the MBD process generates many species in many modest MB events
 or few intense MB events.

H1: we expect the error made be correlated to the number of species created by the multiple-birth process over the total number of species created

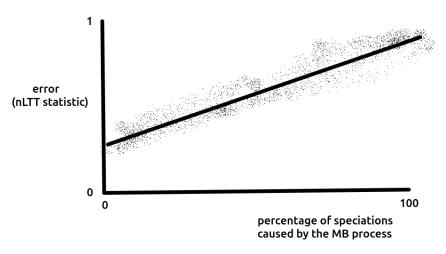
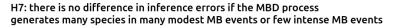


Figure 1: Hypothesis 1: we expect the error made be correlated to the number of species created by the multiple-birth process over the total number of species created



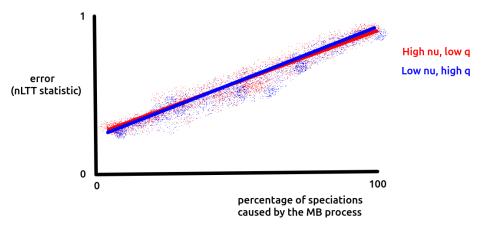


Figure 2: Hypothesis 7: there is no difference in inference errors if the MBD process generates many species in many modest MB events or few intense MB events

• We have the hypothesis, H2, that the effect of extinction rates is neutral, as extinctions will hit lineages created by both speciation processes equally.

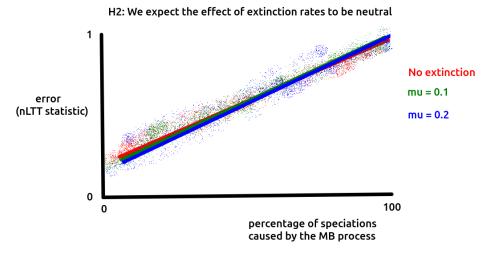


Figure 3: Hypothesis 2: the effect of extinction rates is neutral

• Due to the proportionality of the term within f, we have the hypothesis, H3, that the timing of a multiple birth event (be it close to the crown age or close to the present) to have no effect. Compared to a late multiple birth event, an early multiple birth event may have a longer-lasting effect (as the next speciation event will be later), but it will create less new species, as there are still fewer taxa.

74 2 Methods

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• Current phylogenetic tools assume that only a single speciation event can occur at any given time. While this assumption is useful to construct a wide variety of successful models (e.g Maddison *et al.* 2007, Valente *et al.* 2015, Etienne *et al.* 2012, Etienne *et al.* 2014), they disallow for

the timing of a multiple birth event has no effect

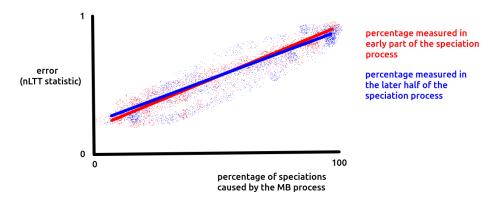


Figure 4: Hypothesis 3: the timing of a multiple birth event has no effect.

environmental changes that trigger speciations in multiple clades at a same point in time.

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- The (constant-rate) birth-death (BD) model embodies the common assumption that only a single speciation event can occur at any given time. The multiple-birth-death (MBD) model relaxes this assumption, allowing events in which large-scale environmental changes lead to a great number of species in relatively short time intervals. Such hypothesis can be useful to describe, for example, systems like cichlid fish diversification in the African Great Lakes: Malawi, Tanganyika and Victoria (Janzen et al. 2016, Janzen et al. 2017).
- In the MBD model, parameters λ and μ correspond, respectively, to the common per-species speciation and extinction rates present also in the standard BD model. Additionally, MBD relies on two additional parameters. Parameter ν is the rate at which an environmental change is triggered. When such event is triggered, all species present in the phylogeny at that moment have a probability q to speciate at that time, which is independent on λ. Polytomies are not allowed in such process as each

- species can speciate only once at the time.
- It is also possible to write down a likelihood function for such processes
 as in Laudanno 2018.

$_{\circ}$ 2.2 Simulations

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- To prove our hypothesis we simulate two twin datasets. All the simulations
 are produced in continuous time, using the Doob-Gillespie algorithm.
- We start simulating $N_S = 1000$ MBD trees, with either 50, 100 and 200 taxa. We have the hypothesis H5, that the number of taxa does not have an effect on the error being made, as there is no diversity dependency in any of the processes. We have the hypothesis H6, that for a higher number of taxa the variance in the error decreases, as more information is present in the simulated phylogenies.

H5: the number of taxa does not have an effect on the error being made

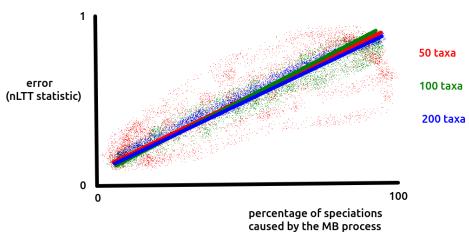


Figure 5: Hypothesis 5: the number of taxa does not have an effect on the error being made

• From each MBD tree, a DNA sequence alignment is simulated. For each

H6: for a higher number of taxa the variance in the error decreases

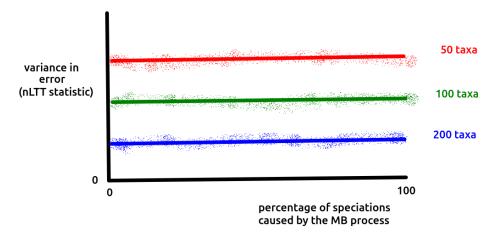


Figure 6: Hypothesis 6: for a higher number of taxa the variance in the error decreases

sequence alignment we then perform a Bayesian analysis to recover a posterior distribution of trees, each composed of N_P phylogenies. Such analysis is performed using the 'pirouette' package (Bilderbeek 2018) to call the BEAST2 tool suite from R. We let the Bayesian analysis assume a BD prior in both cases, to investigate the extent of the error we make under this assumption.

- For each tree generated under the MBD model we aim to generate a "twin" tree under the BD model. With the word "twin" we denote a tree generated starting from the respective MBD tree, in order to perform a fair comparison with it. This operation has to be done, because we want to compare two trees that are generated by different processes. To do so we infer the parameters λ_{BD} and μ_{BD} from the MBD maximizing the likelihood under a BD model. To perform this operation we use the function "bd_ML" from the package "DDD" (Etienne et al. 2012).
- We then exploit such parameters to generate a BD tree using the func-

tion "tess.sim.taxa.age" from the package "TESS" (Hhna 2013). We simulate the tree in such a way the new tree has the same number of tips and the same crown age as the MBD tree. We furthermore require that the BD tree conserve the topology of the MBD tree. We have hypothesis H4 that, compared to the MBD trees, the error will be less in the BD twin tree. The difference between the errors made in MBD and twin BD trees indicates the impact the MBD process has on the error we make in inference using a contemporary BD prior.

H4: compared to the MBD trees, the error will be less in the BD twin tree

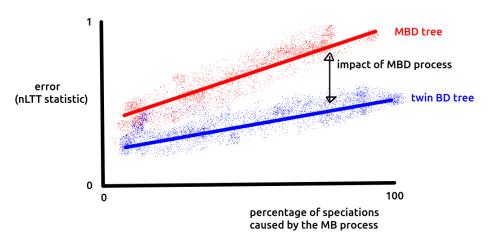


Figure 7: Hypothesis 4: compared to the MBD trees, the error will be less in the BD twin tree

We want the MBD and twin BD trees to contain the same amount of information, i.e. the same number of DNA mutations and the same number of taxa at the present:

$$m_{MBD} = m_{BD} \tag{2}$$

The expected number of mutations m of a phylogeny with crown age -T

(with T > 0) in fact is given by [RJCB: So one of use likes '-T', the other likes 'T'. How to resolve this?

$$m = L \cdot \rho \cdot \int_0^T n(t) \ dt \tag{3}$$

where L is the number of DNA nucleotides, ρ is the per-site per-species mutation rate and n(t) the number of species at each time.

The parameter we'll tune is ρ ... [RJCB: elaborate here :-)]

Since we cannot know $n_{BD}(t)$ before running simulations we need to replace it with a proxy. For this reason we will use the average number of species in time according to the BD model. It's well known that this is equal to [GL: insert proper citation]

$$< n_{BD} > (t) = n_0 \cdot e^{(\mu_{BD} - \lambda_{BD})t}$$
 (4)

where $n_0 = n_{BD}(-T) = n_{MBD}(-T)$ is the initial number of species at the crown age. From 2, 3 and 4 follows:

$$m_{MBD} = L \cdot \rho \cdot \int_0^T \langle n_{BD} \rangle (t) dt = L \cdot \rho \cdot n_0 \cdot \left[\frac{e^{(\mu_{BD} - \lambda_{BD})T} - 1}{\mu_{BD} - \lambda_{BD}} \right] (5)$$

If we set $\mu_{BD} = \mu_{MBD}$ and reverse this relation we can extrapolate the value of λ_{BD} to use to generate BD trees.

• We explained how we set the parameters for each twin BD tree. Using this rules we generate a BD dataset. We repeat the analysis, producing alignments for each tree and subsequently using BEAST to produce a posterior for each of them.

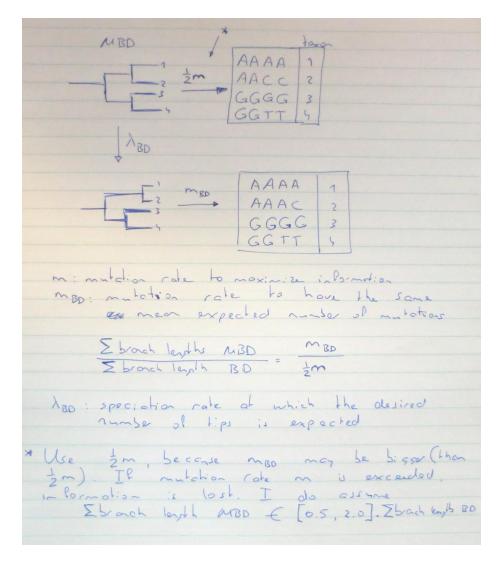


Figure 8: How to create twin trees and alignments. From a focal MBD tree, a twin tree is produced as such: (1) estimate the λ_{BD} to get the same expected number of tips, (2) simulate a BD tree with that amount of tips (discard trees with different number of tips), (3) estimate a mutation rate to get an alignment with the same expected number of mutations, (4) simulate alignments with that amount of mutations (discard those that don't, the picture shows an alignment that should be discarded)

2.3 Measuring the inference error

- So far we have simulated two datasets of trees under the two models: $\{T_i^{BD}\}_{i=1}^{N_S}$ and $\{T_i^{MBD}\}_{i=1}^{N_S}.$ We used them to generate a dataset of align-ments for each model: $\{X_i^{BD}\}_{i=1}^{N_S}$ and $\{X_i^{MBD}\}_{i=1}^{N_S}$. From each dataset we produced a posterior distribution from a BD prior: $P_i(\theta|X_i^{BD},BD)$ and $P_i(\theta|X_i^{MBD}, BD)$. [GL: 1) We might want to rename the models, e.g. BD = (0) and MBD = (1). These names with capital letters are too big and ugly; | [RJCB: I would strongly prefer MBD and BD, as I feel replacing the big ugly capital letters by short pretty numbers hurts readability even more
 - To compare the results for the two models we measure the inference error using the nLTT statistic between known/true tree and posterior/inferred trees (Janzen 2015). To obtain such statistics the procedure is the following:
 - From each tree $T_{i,j}^M$ (with $j=1,...,N_S$) belonging to the posterior $P_i(\theta|X_i^M,BD)$ and relative to the model M, we extrapolate the lineage-through-time (LTT), in other words we measure the number of species as a function of time $n_{i,j}(t)$. To allow a comparison we normalize dividing by the maximum number of species of each tree, i.e. the number of tips at the present $N_{i,j}(t) = \frac{n_{i,j}(t)}{n_{i,j}^{max}}$. We then define the nLTT measure as $nLTT_{i,j} = \int_0^T |N_{i,j}(t) N_{T_i}| dt$
 - [GL: I am running out of letters:() [RJCB: Haha! I suggest to use the same equation and symbols as equation 1 in the nLTT article of Janzen, Hoehna and Etienne, 2015:]

$$\Delta nLTT = \int_0^1 |nLTT_1(t) - nLTT_2t| dt$$

2.4 Model selection

We simulate alignments using the simplest nucleotide substitution model (JC69), the simplest clock model (strict). It is thus imminent to assume these models in our Bayesian inference. Nevertheless, the phylogeny the alignment was based on, could have followed either an MBD or BD tree model, where we in both cases assume a BD tree model. This will have an unknown effect on our inference: it may theoretically be that an MBD model generates (a tree that generates) an alignment in which a different site and/or clock model is favored.

We investigate this by measuring if the generative model (with the simplest nucleotide substitution and simplest clock model) is indeed selected to be the best fitting model. To be precise, we look at the model with the highest marginal likelihood (also called evidence MacKay & Mac Kay 2003), f(D|M), which is the probability of the data D given model M. In the context of this research, D consists of the DNA alignment, and M is the combination of site, clock and tree models.

To estimate the marginal likelihood, we use an algorithm named nested sampling Skilling *et al.* 2006. Nested sampling is attractive to use in a phylogentic context, as it gives a good estimation, requires little tuning Russel *et al.* 2018. Nested sampling is available as a BEAST2 package and can be used by babette Bilderbeek & Etienne 2018.

The nested sampling algorithm stops its run when the marginal likelihood estimation error reaches below a certain tolerance. Similar to Russel *et al.* 2018, we use a (relative) error tolerance ϵ of 10^{-13} , 1 particle to explore the parameter space and 100 active points. To achieve the latter, we use the MCMC chain length L_c of 1M (as also used in the parameter estimates), and a sub-chain length L_{sc} of 10K.

The models we use in our model comparison are the four combinations of two site models and two clock models. We use the JC69 site model, which is the (generative and) simplest model and GTR, the site model with most degrees of freedom. For the clock models, we use the strict clock model, which is the (generative and) simplest clock model, and the RLN clock model. [RJCB: Could also just be all site models and clock models = 8 models]

From these four marginal likelihood estimates, we calculate the weight of the generative model and plot this in figure 2. We do this for both the alignments derived from the MBD tree and the BD twin tree. We expect that the generative model has the heighest weight in both the MBD and BD alignments. We expect this weight to be higher in the BD alignments.

217 3 Results

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	Description	Value(s)
L_c	MCMC chain length	10^{6}
L_{sc}	MCMC sub-chain length	10^{4}
ϵ	relative error tolerance in marginal likelihood es-	10^{-13}
	timation	

Table 1: Overview of the simulation parameters.

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