STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*, but it also works with the Fourth Edition.

Pearson

CHAPTER 3

Arbitrage and Financial Decision Making

Chapter Synopsis

3.1 Valuing Decisions

When considering an investment opportunity, a financial manager must systematically compare the costs and benefits associated with the project in order to determine whether it is worthwhile. Determining the cash value today of the costs and benefits is one way to make such a comparison.

In a **competitive market,** a good can be bought and sold at the same price, so the market price can be used to determine the cash value today of the good. Because competitive markets exist for many assets, such as commodities and financial securities, they can be used to determine cash values and evaluate decisions in many situations. For example, if gold trades at $250/ounce in a competitive market, then 20 ounces of gold have a cash value today of $5000. A buyer wouldn't need to pay more, and a seller wouldn't need to accept less, so individual preferences are not relevant.

If a manager can observe competitive market prices, he may be able to use them to determine the current cash value of different costs and benefits so they can be compared. For example, if someone offers to trade the manager 20 ounces of gold for 10 ounces of platinum, which trades at $550 per ounce in a competitive market, he should reject the trade. The benefit (the cash value today of the gold, $5000) is smaller than the cost (the cash value today of the platinum, $5500).

By evaluating cost and benefits using competitive market prices, we can determine whether a decision will make the firm and its investors wealthier. This point is one of the central and most powerful ideas in finance, which we call the Valuation Principle:

> The value of an asset to the firm or its investors is determined by its competitive market price. The benefits and costs of a decision should be evaluated using these

> market prices, and when the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm.

The Valuation Principle provides the basis for decision making throughout this text.

3.2 Interest Rates and the Time Value of Money

Many financial problems require the valuation of cash flows occurring at different times. However, money received in the future is worth less than money received today because the money received today can be invested to grow to have a larger value in the future. Thus, money has **time value,** and it is only possible to compare cash flows occurring at different times by bringing them to the same point in time.

For example, suppose that there is an annual **risk-free rate**, r_f, of 7% at which you can borrow or lend without risk. If you have the opportunity to lend $100,000 dollars to receive $105,000 in one year, you should not accept this opportunity. The benefit (the cash value today of the $105,000 in 1 year = $105,000 / (1.07) = $98,131) is smaller than the cost (the cash value today of the $100,000, which is just $100,000). You would be better off investing the $100,000 at the risk-free rate and receiving $107,000 in one year.

3.3 Present Value and the NPV Decision Rule

When the value of a cost or benefit is computed in terms of cash today, it is referred to as the **present value** (PV). The **net present value** (NPV) of a project or investment is the difference between the present value of its benefits and the present value of its costs:

$$\text{Net Present Value (NPV)} = \text{PV(Benefits)} - \text{PV(Costs)}$$

Because the NPV represents a project in terms of cash today, it simplifies decision making and leads to the **net present value rule**:

> When making an investment decision, take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

Regardless of individual preferences for cash today versus cash in the future, everyone should always maximize NPV first. Investors can then borrow or lend to shift cash flows through time to achieve their preferred pattern of cash flows.

3.4 Arbitrage and the Law of One Price

In a competitive market, the price of a good cannot trade in two different markets at different prices. Such a price discrepancy represents an **arbitrage opportunity** because you can make a riskless profit without making an investment by buying in the low price market and selling in the high price market. Because an arbitrage opportunity has a positive NPV, whenever an arbitrage opportunity appears in financial markets, investors will quickly take advantage of it. The presence of such arbitrage activity leads to the **Law of One Price**:

> If equivalent goods or securities trade simultaneously in different competitive markets, then they will trade for the same price in both markets.

A competitive market in which there are no arbitrage opportunities can be referred to as a **normal market.**

3.5 No-Arbitrage and Security Prices

The Law of One Price has implications for valuing securities, such as a bond. (A bond is a security issued by governments and corporations to raise money from investors today in exchange for future payments.)

For example, suppose you can either (1) buy a riskless bond paying $1000 in one year, or (2) invest money in a riskless bank account that pays 5%. It would require a $1000 / (1.05) = $952.38 investment in the bank account to generate $952.38(1.05) = $1000 in one year. Thus, the price of the bond must be $952.38 or an arbitrage opportunity would exist:

- When the bond is priced below $952.38, the arbitrage strategy involves buying the bond and borrowing $952.38 from the bank. You will owe the bank $1000 in one year, but you can use the bond's payment to pay that back. Your profit today = $952.38 – P > 0.
- When the bond is priced above $952.38, the arbitrage strategy involves selling the bond and investing $952.38 of the proceeds in the bank account. [Note that if you do not own the bond you can **short sell** the bond by borrowing it from your broker and selling it with the promise to replace it in the future.] You will still receive $1000 (from the bank now instead of from the bond) in one year. Your profit today = P – $952.38 > 0.

Thus, the existence of investors trying to exploit such opportunities leads to the existence of the **no-arbitrage price,** $952.38.

When securities trade at no-arbitrage prices, then investing in securities is a zero NPV investment. Thus, in normal markets, trading securities neither creates nor destroys value. Instead, value is created by the real investment projects made by firms, such as developing new products, opening new stores, or creating more efficient production methods. It follows that the firm's investment decision can be separated from its financing choice. This concept is referred to as the **separation principle.**

The Law of One Price has implications for packages of securities as well. Consider two securities, A and B. Suppose a third security, C, has the same cash flows as A and B combined. Because security C is equivalent to the portfolio of A and B, by the Law of One Price, they must have the same price; otherwise, an obvious arbitrage opportunity would exist. This relationship is known as **Value Additivity**:

$$\text{Price}(C) = \text{Price}(A) + \text{Price}(B).$$

Value additivity has an important consequence for the value of an entire firm. Since the cash flows of the firm are equal to the total cash flows of all projects and investments within the firm, the value of the firm equals the sum of the values of all of its projects and other assets. Thus, to maximize the value of the firm, managers should make decisions that maximize the NPV of each project, which represents the project's contribution to the firm's total value.

Appendix: The Price of Risk and Arbitrage with Transaction Costs

Thus far we have considered only cash flows that have no risk. However, in many settings, cash flows are risky. Intuitively, investors will pay less to receive a risky cash flow in the future than they would to receive a certain cash flow because they don't like risk. The notion that investors prefer to have a safe income rather than a risky one of the same average amount is called **risk aversion.**

Because investors care about risk, we cannot use the risk-free interest rate to compute the present value of a risky future cash flow. The increase in the discount rate over the risk-free rate that investors use to value risky cash flows is called the **risk premium.** *When a cash flow is risky, to compute its present value you must discount the expected cash flow at a rate equal to the risk-free interest rate plus an appropriate risk premium.*

The risk of a security cannot be evaluated in isolation. Even when a security's returns are quite variable, if the returns vary in a way that offsets other risks investors are holding, the security may reduce rather than increase investors' risk. As a result, risk can only be assessed relative to the other risks that investors face, so *the risk of a security must be* *evaluated in relation to the fluctuations of other investments in the economy. A security's risk premium will be higher the more its returns tend to vary with the overall economy and the market index. If the security's returns vary in the opposite direction of the market index, it offers insurance and will have a lower or even a negative risk premium.*

In most markets, you must pay **transaction costs** to trade securities. First, you must pay your broker a commission on the trade. Second, because you will generally pay a slightly higher price when you buy a security (the ask price) than you receive when you sell (the bid price), you will also pay the bid-ask spread. *Thus, when there are transaction costs, arbitrage keeps prices of equivalent goods and securities close to each other. However, prices can deviate, but not by more than the transaction costs of the arbitrage.*

Selected Concepts and Key Terms

Competitive Market

A market in which goods can be bought and sold at the same price. Because competitive markets exist for most commodities and financial assets, we can use them to determine cash values and evaluate decisions in many situations.

Time Value of Money

The idea that it is only possible to compare cash flows occurring at different times by bringing them to the same point in time. When the expected rate of return on invested cash is positive, cash received in the future is worth less than cash received today because less cash can be invested today to equal the future amount. Thus, the **present value** of a future cash flow is less than the amount received in the future, and the **future value** of a cash flow invested in a previous period is worth more than the amount invested in the past.

Risk-Free Interest Rate

The interest rate at which you can borrow or lend without risk. $(1 + r_f)$ is the **interest rate factor** for risk-free cash flows; it defines the exchange rate across time.

Net Present Value (NPV)

The difference between the present value of an investment's benefits and the present value of its costs. The NPV of a project represents its value in terms of cash today.

NPV Decision Rule

Select all projects that have a positive NPV. When choosing among mutually exclusive alternatives, take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

Arbitrage

The practice of buying and selling equivalent goods in different markets to take advantage of a price difference. A situation in which it is possible to make a profit without taking any risk or making any investment is an **arbitrage opportunity.**

Normal Market

A competitive market in which there are no arbitrage opportunities. The term *efficient market* is also sometimes used to describe a market that, along with other properties, is without arbitrage opportunities.

Law of One Price

If equivalent goods or securities trade simultaneously in different competitive markets, then they will trade for the same price in both markets.

Separation Principle

The idea that a firm's investment decision can be separated from its financing choice. This follows from the idea that, in normal markets, trading securities neither creates nor destroys value. Instead, value is created by the real investment projects made by firms, such as developing new products, opening new stores, or creating more efficient production methods.

Concept Check Questions and Answers

3.1.1. **In order to compare the costs and benefits of a decision, what must we determine?**

In order to compare the costs and benefits, we need to evaluate them in the same terms—cash today.

3.1.2. **If crude oil trades in a competitive market, would an oil refiner that has a use for the oil value it differently than another investor?**

No, if crude oil trades in a competitive market, the value of crude oil depends only on the current market price. The personal opinion of an oil refiner or any investor does not alter the value of the decision today.

3.2.1. **How do you compare costs at different points in time?**

We can compare costs at different points in time by converting the costs in the future to dollars today using the interest rate.

3.2.2. **If interest rates rise, what happens to the value *today* of a promise of money in one year?**

When interest rates rise, the value of money today to be received in one year is lower. In other words, the higher the discount rate, the lower the value of money today.

3.3.1. **What is the NPV decision rule?**

The NPV decision rule states that when choosing among alternatives, we should take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

3.3.2. **Why doesn't the NPV decision rule depend on the investor's preferences?**

Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first. We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows.

3.4.1. **If the Law of One Price were violated, how could investors profit?**

If the Law of One Price were violated, investors can profit by arbitrage. They buy goods or securities at a lower price in one market and simultaneously resell the goods or securities at a higher price in a different market to take advantage of a price difference.

3.4.2. When investors exploit an arbitrage opportunity, how do their actions affect prices?

Investors exploit an arbitrage opportunity when taking advantage of price differences in two separate markets. In doing so, investors will buy in the market where it is cheap and simultaneously sell in the market where it is expensive. As more and more investors compete, the price will rise with increased buy orders in one market and fall with increased sell orders in the other. Arbitrage activities will continue until the prices in the two markets are equal.

3.5.1. If a firm makes an investment that has a positive NPV, how does the value of the firm change?

If a firm makes an investment that has a positive NPV, the value of the firm will increase by the NPV amount today.

3.5.2. What is the separation principle?

The separation principle states that security transactions in a normal market neither create nor destroy value on their own. Therefore, we can evaluate the NPV of an investment decision separately from any security transactions the firm is considering. That is, we can separate the firm's investment decision from its financing choice.

3.5.3. In addition to trading opportunities, what else do liquid markets provide?

Competitive markets depend upon liquidity because liquid markets allow market prices to be determined. When markets become illiquid, it may not be possible to trade at the posted price. As a consequence, we can no longer rely on market prices as a measure of value.

3.A.1 Why does the expected return of a risky security generally differ from the risk-free interest rate? What determines the size of its risk premium?

The expected return of a risky security generally differs from the risk-free interest rate because the expected return includes a risk premium. The higher the variability of returns, the higher the risk premium demanded by investors.

3.A.2. Explain why the risk of a security should not be evaluated in isolation.

The risk of a security must be evaluated in relation to the fluctuations of other investments in the economy. A security's risk premium will be higher the more its returns tend to vary with the overall economy and the market index.

3.A.3. In the presence of transactions costs, why might different investors disagree about the value of an investment opportunity?

In the presence of transaction costs, different investors might disagree about the value of an investment opportunity because investors with high transaction costs will value the investment opportunity less.

3.A.4. By how much could this value differ?

When there are transaction costs, arbitrage keeps prices of equivalent securities close to each other. Prices can deviate, but not by more than the transaction costs of the arbitrag

Examples with Step-by-Step Solutions

Solving Problems

Problems using the ideas in this chapter generally involve:

- Finding the NPV of an investment (or comparing different investment alternatives) by calculating the benefits minus the costs using either competitive market prices, the risk-free rate, or currency exchange rates; or
- Determining no arbitrage prices based on current market prices.

Below are examples of finding the net present value of an investment by calculating the benefits minus the costs using the risk-free rate, finding the net present value of an investment by calculating the benefits minus the costs using competitive market prices, and determining a no-arbitrage price based on current market prices.

Examples

1. **You are going to retire in one year, and your defined benefit retirement plan will pay you $3 million on the date you retire. If you work another year, you will get one more year's salary of $100,000 (paid one year from today). Your firm has offered to pay you an early retirement package of $2.9 million if you quit today. The risk-free interest rate is 5%, and there are no income tax effects. Ignoring the fact that one option involves having to show up at work one more year:**
   [A] Which option should you take if you compare them based on dollars today?
   [B] Which option should you take if you compare them based on dollars in one year?
   [C] What is the lowest retirement value today that would make you indifferent between the two options?

   Step 1. Determine the value of each option in today's dollars. Since there are no cash outflows, the NPV is just the present value of the cash inflows.

   Present value of retiring today = $2,900,000

   $$\text{Present value of retiring in one year} = \frac{\$3{,}000{,}000 + \$100{,}000}{1.05} = \$2{,}952{,}381$$

   Thus, you should keep working.

   Step 2. Determine the value of each option in next year's dollars.

   Future value of retiring today = $2,900,000(1.05) = $3,045,000

   Future value of retiring in one year = $3,000,000 + $200,000 = $3,200,000

   Thus, you should keep working.

   The two approaches, whether comparing the options based on present values or future values, will always provide the same answer.

   Step 3. Determine the lowest retirement package today that would make you indifferent between the two options.

   You would accept the present value of retiring in one year, $2,952,381. This is equivalent to having $3,200,000 in one year since $2,952,381(1.05) = $3,200,000. In other words, you can take the $2,952,381 payment and invest it at the risk-free rate and end up with $3,200,000.

2. **Suppose your employer offers you a choice between a $20,000 bonus and 30 ounces of gold. Whichever one you choose will be awarded today. Gold is trading today at $500 per ounce. Ignoring income tax implications:**
   [A] Which form of the bonuses should you choose?
   [B] What do you tell your broker, who advises you to take the gold because he predicts that the price of gold is going to double in value this year?

   Step 1. Determine the value of each option.

   Value of bonus = $20,000

   $$\text{Value of the gold} = 30\left(\frac{\$500}{1 \text{ ounce of gold}}\right) = \$15{,}000 < \$20{,}000 \text{ cash bonus.}$$

   So, you should take the cash bonus.

   Step 2. Address the concern that gold is a good investment.

   The reason you can compare the two options above is because gold trades in a competitive market and you can buy it and sell it for the same price. Thus, you would be better of taking the bonus and buying 30 ounces of gold for $15,000; you would still have $5,000 left over.

3. **A hedge fund has a portfolio consisting of 1 million shares of Microsoft, which trades at $30 per share, and 1 million shares of Intel, which trades at $20. A stockholder in the hedge fund, which has 500,000 shares outstanding, has offered to sell you 1,000 shares for $75 per share. Does this represent an arbitrage opportunity? If so, how can you exploit it?**

   Step 1. Determine the no-arbitrage price.

   Because the hedge fund is equivalent to a portfolio of Microsoft and Intel, by the Law of One Price, they must have the same price, so:

   $$\begin{aligned}\text{Value(hedge fund)} &= \text{Value(Microsoft stock)} + \text{Value(Intel Stock)} \\ &= \$30(1 \text{ million}) + \$20(1 \text{ million}) = \$50 \text{ million.}\end{aligned}$$

   So, the Value(hedge fund) per share $= \dfrac{\$50{,}000{,}000}{500{,}000} = \$100 > \$75 \Rightarrow$ an arbitrage opportunity.

   Step 2. Determine how you would exploit it.

   Since you can buy it at a lower price than the components are worth, you should buy the hedge fund shares and sell the components. You can short sell the stocks by borrowing them from your broker and selling them with the promise to replace them in the future.

   To take advantage of the situation, you should buy the 1,000 shares for $75(1,000) = $75,000.

   Next, short sell $\left(\dfrac{\$30(1{,}000{,}000)}{\$30(1{,}000{,}000) + \$20(1{,}000{,}000)}\right)(\$75{,}000) = \$45{,}000$ in Microsoft, and short sell $75,000 – $45,000 = $30,000 of Intel. This amounts to short selling $\dfrac{\$45{,}000}{\$30}$ = 1,500 shares of Microsoft and $\dfrac{\$30{,}000}{\$20}$ = 1,500 shares of Intel. The NPV of the transactions is $25,000.

Questions and Problems

1. Suppose a Treasury bill with a risk-free cash flow of $10,000 in one year trades for $9,615 today. If there are no arbitrage opportunities, what is the current risk-free interest rate?

2. You have an investment opportunity in Italy. It requires an investment of $1 million today and will produce a cash flow of 1 million euros in one year with no risk. Suppose the risk-free interest rate in the United States is 4%, the risk-free interest rate in Italy is 5%, and the current competitive exchange rate is €1.2 per $1. What is the NPV of this investment? Is it a good opportunity?

3. Suppose the risk-free interest rate is 5.5%.
   [A] Having $100,000 today is equivalent to having what amount in one year?
   [B] Having $100,000 in one year is equivalent to having what amount today?

4. Your firm has identified three potential investment projects. All of the projects would use the same a tract of land, so you can only select one of them. All of the projects would generate risk-free cash flows and the risk-free rate is 5%.

   - Project 1 costs $1 and pays $1 million in 1 year
   - Project 2 costs $10 million and pays $12 million in 1 year
   - Project 3 has a cash inflow of $10 million today but a cash outflow of $11 million in 1 year

   What should the firm do?

5. Your employer has notified you that in 1 year you will be lose your job that pays $200,000 per year when you reach mandatory retirement age. They have offered you the choice of leaving today and keeping your company car, a 2005 Mercedes SLK with a Kelley blue book market value of $40,000. If you stay, you don't keep the car. You believe that the most you could make elsewhere is $100,000 next year if you quit and forego your current salary. The risk-free rate is 6%. Ignoring taxes, and assuming that you are paid your salary at the end of the year, what should you do?

Solutions to Questions and Problems

1. The PV of the security's cash flow is ($10,000) / (1 + r), where r is the one-year risk-free interest rate. If there are no arbitrage opportunities, this PV equals the security's price of $9,615 today. Therefore,

$$\$9,615 = \frac{\$10,000}{(1+r)} \Rightarrow (1+r) = \frac{\$10,000}{\$9,615} = 1.04 \Rightarrow r = .04, \text{or } 4\%.$$

2. Cost = $1 million today

   Benefit = €1 million in one year

$$\text{PV} = \left(\frac{€1 \text{ million in one year}}{1.05}\right) = €0.9524 \text{ million today}$$

$$= €0.95 \text{ million today} \times \left(\frac{€1.2}{\$ \text{ today}}\right) = \$1.142857 \text{ million today}$$

$$\Rightarrow \text{NPV} = \$1.142857 - \$1 \text{ million } = \$142,857$$

   The NPV is positive, so it is a good investment opportunity.

3. [A] Having \$100,000 today is equivalent to having 100,000 × 1.055 = \$105,500 in 1 year.
   [B] Having \$100,000 in one year is equivalent to having 100,000/ 1.055 = \$94,787 today.

4. Projects 2 and 3 are equally as valuable—select either one.

$$NPV_1 = -\$1 + \frac{\$1{,}000{,}000}{1.05} = \$952{,}381$$

$$NPV_2 = -\$10{,}000{,}000 + \frac{\$12{,}000{,}000}{1.05} = \$1{,}428{,}571\,.$$

$$NPV_3 = \$10{,}000{,}000 + \frac{-\$9{,}000{,}000}{1.05} = \$1{,}428{,}571$$

5. NPV of staying = $\frac{\$200{,}000}{1.06} = \$186{,}679$

   NPV of leaving = $\frac{\$100{,}000}{1.06} + 40{,}000 = \$134{,}340$

   So, you should stay one more year.

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CHAPTER 4

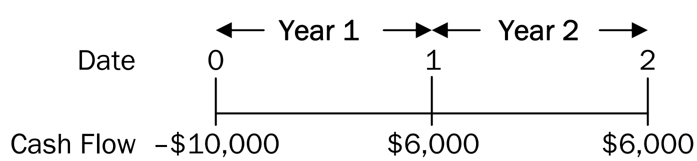
The Time Value of Money

Chapter Synopsis

Many financial problems require the valuation of cash flows occurring at different times. However, money received in the future is worth less than money received today because the money received today can be invested to grow to have a larger value in the future. Thus, money has time value, and it is only possible to compare cash flows occurring at different times by valuing them at the same point in time.

4.1 The Timeline

The first step in most problems is to put the cash flows involved on a timeline in which each point is a specific date, such as this representation of a $10,000 loan made by a bank to a borrower who promises to pay $6,000 to the bank in each of the next two years:



The space between date 0 and date 1 represents the time period between these dates. Date 0 is the beginning of the first year, and date 1 is the end of the first year. Similarly, date 1 is the beginning of the second year, and date 2 is the end of the second year. The signs of the cash flows are important: in the diagram, –$10,000 represents a cash outflow, and $6,000 represents a cash inflow.

4.2 The Three Rules of Time Travel

To correctly account for the time value of money, three general rules must be followed:

(1) It is only possible to compare or combine values at the same point in time;

(2) To move a cash flow forward in time, you must compound it; and

(3) To move a cash flow backward in time, you must discount it.

For example, valuation problems often require the determination of the **future value** (FV) of a series of cash flows at a given interest rate, such as in a retirement savings planning application. Invested cash that is earning a positive rate of interest grows at an increasing rate over time in a process called **compounding** in which interest earned in the later periods accrues on both the original value of the cash and the interest earned in the prior periods. The general expressions for the FV of a lump sum, *C,* invested at rate *r* for *n* periods is as follows.

Future Value of a Cash Flow

$$\mathrm{FV}_n = C(1+r)^n$$

Other problems seek to determine the **present value** (PV) of a series of future cash flows, such as in the valuation today of a bond that promises to make a series of payments in the future. The process of determining the present value of future cash flows is referred to as **discounting,** and the result is a discounted cash flow value. The general expression for the PV of a lump sum is as follows.

Present Value of a Cash Flow

$$\mathrm{PV} = \frac{C_n}{(1+r)^n}$$

4.3 Valuing a Stream of Cash Flows

Applications often involve accurately considering a stream of cash flows occurring at different points in time over N periods:

The PV of such a stream can be found by using:

Present Value of a Cash Flow Stream

$$\mathrm{PV}=\frac{C_1}{(1+r)^1}+\frac{C_2}{(1+r)^2}+\cdots+\frac{C_N}{(1+r)^N}=\sum_{n=0}^{N}\frac{C_n}{(1+r)^n}$$

While this equation can generally be used to calculate the present value of future cash flows, there are often certain types of cash flow streams, such as annuities and perpetuities discussed below, that make the calculation less tedious.

4.4 Calculating the Net Present Value

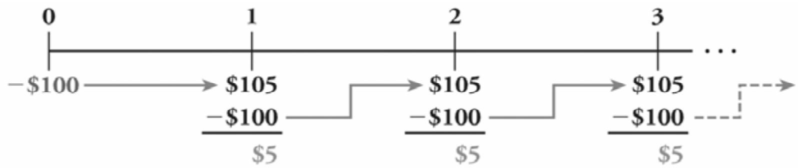
An investment decision can be represented on a timeline as a stream of cash flows. The Net Present Value (NPV) of the project is thus the present value of the stream of cash flows of the opportunity:

$$\mathrm{NPV} = \mathrm{PV(benefits)} - \mathrm{PV(costs)}.$$

4.5 Perpetuities and Annuities

A **perpetuity** is a stream of equal cash flows that occurs at regular intervals and lasts forever. For example, suppose you could invest $100 in a bank account paying 5% interest per year forever, and you want to create a perpetuity by taking $5 out each year. At the end of one year, you will have $105 in the bank, and you can withdraw the $5 interest and reinvest the

$100 for a second year. Again you will have $105 after one year, and you can withdraw $5 and reinvest $100 for another year as depicted in the diagram:

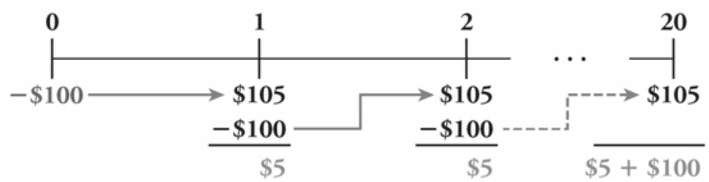


Thus, the PV of the $5 perpetuity with r = 5% must be $100 = $5 / .05 = C/r—the cost of replicating the cash flow stream. Thus, the present value of receiving C in perpetuity is:

Present Value of a Perpetuity

$$\text{PV}(C \text{ in perpetuity}) = \frac{C}{r}$$

An **annuity** is a stream of equal cash flows paid each period for N periods. Examples of annuities are home mortgage loans and corporate bonds. To determine the PV of an annuity suppose once again that you invest $100 in a bank account paying 5% interest. At the end of one year, you will have $105 in the bank. Using the same strategy as for a perpetuity, suppose you withdraw the $5 interest and reinvest the $100 for a second year. Once again you will have $105 after two years, and you can repeat the process, withdrawing $5 and reinvesting $100 every year for 20 years to close the account and withdraw the principal:



You have created a 20-year, $5 annuity. The value must be the NPV of the cash flows associated with creating it: the initial amount required to fund the annuity minus the PV of the return of the initial amount in N years, or $100 – $100 / $(1.05)^{20}$ = $62.31. In general, the PV of an N-year annuity paying C per period with the first payment one period from date 0 is as follows.

Present Value of an Annuity

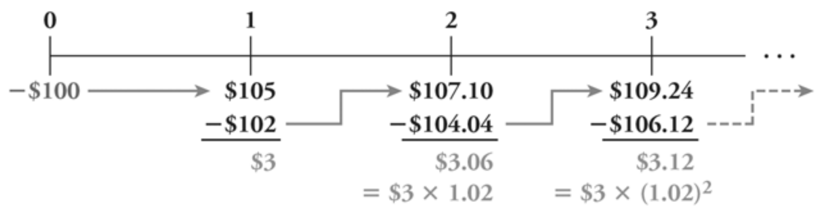
$$\text{PV(annuity of } C \text{ for } N \text{ periods)} = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)$$

Based on these equations, the future value of an annuity can now be calculated as:

Future Value of an Annuity

$$\text{FV(annuity of } C \text{ for } N \text{ periods)} = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)(1+r)^N = C \times \frac{1}{r}\left((1+r)^N - 1\right)$$

A **growing perpetuity** is a cash flow stream that occurs at regular intervals and grows at a constant rate forever. For example, suppose you invest $100 in a bank account that pays 5% interest. At the end of the first year, you will have $105 in the bank. If you withdraw $3, you will have $102 to reinvest. This amount will then grow to in the following year to $102(1.05) = $107.10 and you can withdraw $3.06 leaving the balance at $104.04.



You have created a 2% growing perpetuity with a first payment of $3 so the value must equal the cost to create it, $100 = $3/(.05 – .02)= $C/(r - g)$. In general, the PV of a perpetuity growing at g percent that pays C one period from date 0 is as follows.

Present Value of a Growing Perpetuity

$$\text{PV(perpetuity growing at } g) = \frac{C_1}{r-g}$$

Finally, a **growing annuity** is a stream of N growing cash flows paid at regular intervals, where $N < \infty$. Assuming that the first cash flow, C, is paid at the end of the first period, the present value of an N-period growing annuity is:

Present Value of a Growing Annuity

$$\text{PV(annuity growing at } g) = C \times \left(\frac{1}{r-g}\right)\left(1-\left(\frac{1+g}{1+r}\right)^N\right)$$

4.6 Solving Problems with a Spreadsheet or Calculator

Microsoft Excel has time value of money functions based on the variables defined above. In the program: N = NPER, r = RATE, PV = PV, C = PMT, and FV = FV. You must input four of these variables and then Excel finds the fifth (one can be zero). For example, if you invest $20,000 at 8% for 15 years, how much will you have in 15 years?

The Excel function is = FV(RATE,NPER,PMT,PV)=FV(0.08,15,0,-20000) and the resulting FV equals $63,443.

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	15	8.00%	−20,000	0		
Solve for FV					63,443	=FV(0.08,15,0,−20000)

You can also use a financial calculator to do the same calculations. The calculators work in much the same way as the annuity spreadsheet. You enter any four of the five variables, and the calculator calculates the fifth variable.

4.7 Non-Annual Cash Flows

When periodic cash flow streams occur in periods other than yearly, such as monthly or quarterly, the inputs to the valuation equations need to be adjusted. You apply the equations exactly as before, but set r equal to the interest rate per period and n equal to the number of periods.

4.8 Solving for the Cash Payments

Any of the equations above (the future value of a lump sum, the present value of a lump sum, the present value of a perpetuity, the present value of an annuity, the future value of an annuity, the present value of a growing annuity, and the present value of a growing perpetuity) can be solved for any of the variables as long as the remaining variables in the equation are known.

4.9 The Internal Rate of Return

In some situations, you know the present value and cash flows of an investment opportunity. The internal rate of return (IRR) is the interest rate that sets the net present value of the cash flows equal to zero.

For simple formulas, such as the present value of a single cash flow or the present value of a perpetuity, you enter the known variables into the equation and solve for r to find the IRR. For more complicated problems, such as the present value an annuity, you can use a spreadsheet or financial calculator to solve for r.

Excel has a built in function that allows you to solve for the IRR of a set of unequal cash flows. For example, the rate of return that makes the net present value of a stream of cash flows equal to zero, the internal rate of return (IRR), can be found using the present value of an annuity equation if the present value of the annuity payments (PV), the number of payments (*N*), and the periodic level cash flow (*C*) are all known.

Selected Concepts and Key Terms

Time Value of Money

The idea that it is only possible to compare cash flows occurring at different times by bringing them to the same point in time. When the expected rate of return on invested cash is positive, cash received in the future is worth less than cash received today because less cash can be invested today to equal the future amount. Thus, the present value of a future cash flow is less than the amount received in the future, and the future value of a cash flow invested in a previous period is worth more than the amount invested in the past.

Compounding

The process of moving cash forward in time over more than one time period. When cash is invested over multiple periods in the future, interest earned in the later periods grows at an increasing rate because it accrues on both the original value of the cash and the interest earned in the prior periods.

Discounting

The process of moving cash backwards in time. When interest rates are positive, the present value received is less than the value of the future cash flow. The process of calculating such present values is commonly referred to as discounting.

Perpetuity

A stream of cash flows that is received over equal, periodic intervals that lasts forever. A perpetuity can have level cash flows or it can have cash flows that grow at a constant rate, which is referred to as a **growing perpetuity.**

Annuity

A stream of cash flows that is received over equal, periodic intervals that ends at some future time period. An annuity generally has level cash flows, but it can also be a **growing annuity** and have cash flows that grow at a constant rate.

Internal Rate of Return

The rate of return that makes the net present value of a stream of cash flows equal to zero. The internal rate of return (IRR) is a popular measure used to evaluate the desirability of an investment based on its projected cash flows.

Concept Check Questions and Answers

4.1.1. What are the key elements of a timeline?

A timeline is a linear representation of the timing of the cash flows. Date 0 represents the present. Date 1 is one period (a month or a year) later; that is, it represents the end of the first period. The cash flow shown below date 1 is the payment you will receive at the end of the first period. You continue until all the cash flows and their timing are shown on the timeline.

4.1.2. How can you distinguish cash inflows from outflows on a timeline?

To differentiate between the two types of cash flows, we assign a positive sign to cash inflows and a negative sign to cash outflows.

4.2.1. Can you compare or combine cash flows at different times?

No, you cannot compare or combine cash flows at different times. A dollar today and dollar in one year are not equivalent.

4.2.2. What is compound interest?

Compound interest is the effect of earning "interest on interest." For compound interest, you can earn interest on the original investment and the interest reinvested from prior periods.

4.2.3. How do you move a cash flow backward and forward in time?

To move a cash flow forward in time, you must compound it. To move a cash flow back in time, you must discount it.

4.3.1. How do you calculate the present value of a cash flow stream?

The present value of cash flow stream is the sum of the present values of each cash flow.

4.3.2. How do you calculate the future value of a cash flow stream?

The future value of a cash flow stream is the sum of the future values of each cash flow.

4.4.1. How do you calculate the net present value of a cash flow stream?

The net present value of a cash flow stream is the present value of all the benefits minus the present value of all the costs. The benefits are the cash inflows and the costs are the cash outflows.

4.4.2. What benefit does a firm receive when it accepts a project with a positive NPV?

When a firm accepts a project with a positive NPV, the value of the firm will increase by the NPV today.

4.5.1. How do you calculate the present value of a

a. Perpetuity?

The present value of a perpetuity is the annual cash flow divided by the appropriate discount rate.

b. Annuity?

The present value of an annuity of C for n periods with interest rate r is:

$$\text{PV(annuity of } C \text{ for } N \text{ periods)} = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right).$$

c. Growing perpetuity?

The present value of a growing perpetuity is:

$$\text{PV(perpetuity growing at } g) = \frac{C_1}{r - g}.$$

d. Growing annuity?

$$\text{PV(annuity growing at } g) = C \times \left(\frac{1}{r-g}\right)\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)$$

4.5.2. **How are the formulas for the present value of a perpetuity, annuity, growing perpetuity, and growing annuity related?**

The formula for the present value of growing annuity is a general solution. From this formula, we can deduce the formulas for the present value of a perpetuity, annuity, and growing perpetuity.

4.6.1. **What tools can you use to simplify the calculation of present values?**

Spreadsheet programs such as Excel have a set of functions that performs the calculations that finance professionals do most often. Financial calculators also have a set of functions that simplify the calculations involved in many time value of money calculations.

4.6.2. **What is the process for using the annuity spreadsheet?**

In Excel, the functions are NPER, RATE, PV, PMT, and FV. You must determine what variable you are solving for and then enter other required information in the valuation formula. For an annuity, you could solve for the annuity payment by using the PMT formula with the following formula: =PMT(rate,nper,pv).

4.7.1. **Do the present and future value formulas depend upon the cash flows occurring at annual intervals?**

No, you can use the formulas for any periodic interval. When periodic cash flow streams occur in periods other than yearly, such as monthly or quarterly, the inputs to the valuation equations just need to be adjusted. You apply the equations exactly as before, but set r equal to the interest rate per period and n equal to the number of periods.

4.7.2. **When cash flows occur at a non-annual interval, what interest rate must you use? What number of periods must you use?**

You apply the equations exactly as before, but set r equal to the interest rate per period and n equal to the number of periods.

4.8.1. **How can we solve for the required annuity payment for a loan?**

Any of the equations above (the future value of a lump sum, the present value of a lump sum, the present value of a perpetuity, the present value of an annuity, the future value of an annuity, the present value of a growing annuity, and the present value of a growing perpetuity) can be solved for any of the variables as long as the remaining variables in the equation are known.

4.8.2. How can we determine the required amount to save each year to reach a savings goal?

You can solve for the cash flows that must be invested to have a specific future value using the future value of a lump sum and/or the future value of an annuity equations.

4.9.1. What is the internal rate of return?

The internal rate of return (IRR) is the interest rate that sets the net present value of the cash flows equal to zero.

4.9.2. In what two cases is the internal rate of return easy to calculate?

For simple formulas, such as the present value of a single cash flow or the present value of a perpetuity, you enter the known variables into the equation and solve for r to find the IRR

Examples with Step-by-Step Solutions

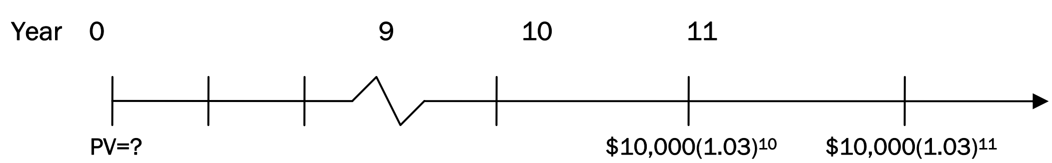
Solving Problems

Problems using the concepts in this chapter can be solved by first determining the timeline of the known and unknown cash flows and then determining which valuation equation is necessary. Some problems will involve using two or more of the equations and two or more steps. You just need to make sure that each equation you are solving has only one unknown variable: PV, FV, *C, r, N,* or *g*. The examples below demonstrate the general procedure for solving these types of problems.

Examples

1. **You would like to endow a scholarship. In-state tuition is currently $10,000, and the rate will grow by 3% per year. How much would it cost today to endow a scholarship that pays full tuition once every year forever starting 10 years from now? Assume a 5% annual percentage rate (APR) rate of return on your investment to fund the scholarship.**

   Step 1. Put the cash flows that are known and unknown on a timeline.

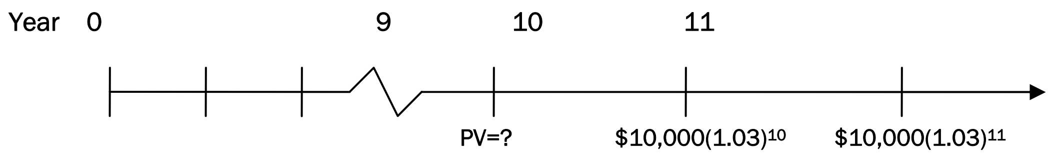


   Step 2. Determine the type or types of valuation problems involved.

   This problem involves the following additional steps 3 and 4:

   [3] finding the present value in year 9 of the known tuition payments in years 10 to infinity, and
   [4] finding the present value at time 0 of the time 9 value found in step 3.

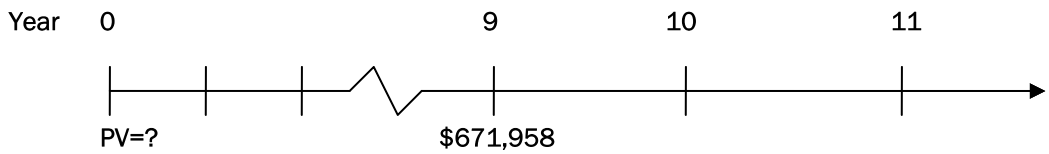
   Step 3. Find the present value in year 9 of the known tuition payments in years 10 to infinity.



Using the present value of a growing perpetuity with C = \$10,000(1.03)^{10}, r = .05, and g = .03:

$$PV_9 = \frac{C}{r-g}\left[\frac{10{,}000(1.03)^{10}}{.05-.03}\right] = \left[\frac{13{,}439}{.05-.03}\right] = \$671{,}958$$

Step 4. Find the present value of at time 0 of the time 9 value found in step 3.



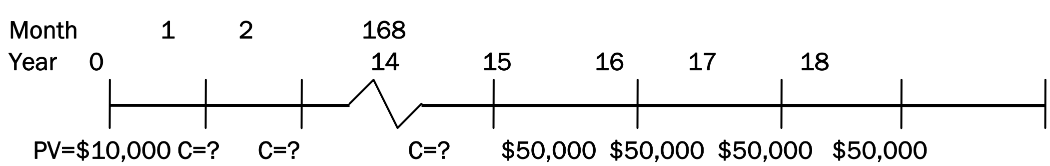
Using the present value of a lump sum equation with FV = \$671,958, r = .05, and N = 9:

$$\text{PV}_0 = \frac{\$671{,}958}{(1.05)^9} = \$433{,}150$$

You would need to make invest \$433,150 into the account today.

2. **You have determined that you will need \$50,000 per year for four years to send your daughter to college. You have already saved \$10,000 and placed the money in an account that you expect will yield a monthly compounded 12% APR (1% per month). Money for the first of the four payments will be removed from the account exactly 15 years from now and the last withdrawal will be made 18 years from now. You have decided to save more by making monthly payments into the same account yielding an expected 12% APR (1% per month) over the next 14 years beginning next month. You will take the money out of the 12% account and place it in a 6% APR account in 14 years and take the cash out as needed. How large must these monthly payments be?**

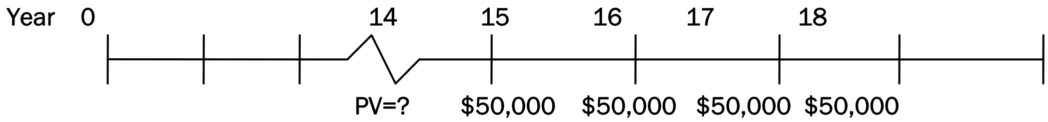
Step 1. Put the cash flows that are known and unknown on a timeline.



Step 2. Determine the type or types of valuation problems involved. This problem can be solved using the following additional steps 3–5:

[3] Find the present value in year 14 of the known \$50,000 payments in years 15–18,
[4] Find the future value of the \$10,000 you have today at time 14 years, and
[5] Find the unknown monthly annuity payment that has the future value at time 14 equal to the value found in step 3 minus the value found in step 4.

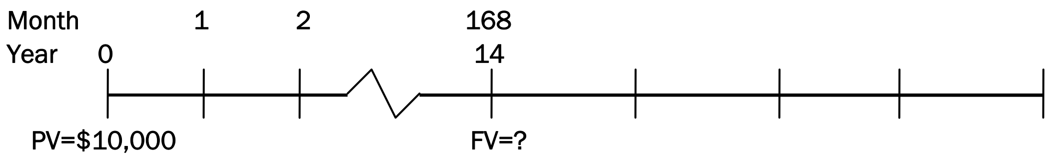
Step 3. Find the present value of the known \$50,000 payments. Since it is only possible to compare values at the same point in time, the first step in a problem like this is to find the value of the known cash flows at one point in time. The most straightforward time to value the cash flows is time 14 because then you can use the present value of an annuity equation, which assumes that the first cash flow occurs one period after it is being valued, time 15.



Using the present value of an annuity equation with N = 4, r = 0.06, and C = $50,000:

$$\text{PV} = \$50{,}000\left(\frac{1}{.06}\right)\left[1-\frac{1}{.06(1.06)^4}\right] = \$173{,}255$$

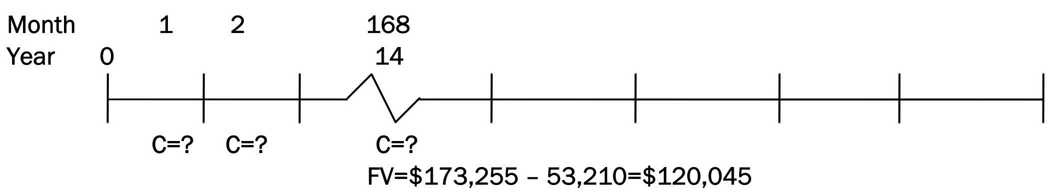
Step 4. Find the future value of the $10,000 you have today at time 14 years.



Using the future value of a lump sum equation with N = 12 x 14 = 168, r = 0.01, and PV = $10,000:

$$\text{FV} = 10{,}000(1.01)^{168} = \$53{,}210$$

Step 5. Find the unknown monthly annuity payment that has the future value at time 14 equal to the amount found in part a minus the amount found in part b.



Using the future value of an annuity equation with N = 12 x 14 = 168, r = 0.01, and FV = $120,045:

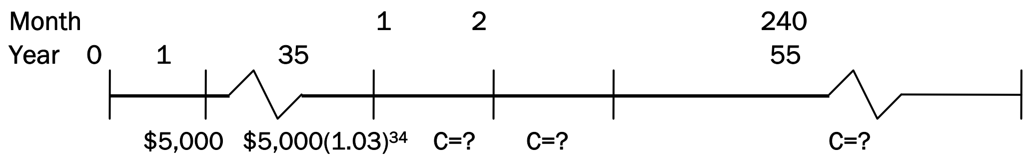
$$\text{FV} = 120{,}045 = C\left[\frac{1}{.01}-\frac{1}{.01(1.01)^{168}}\right](1.01)^{168} \Rightarrow C = \$278.$$

You would need to make monthly payments of $278 into the account.

3. **Some Republicans would like to give those contributing to Social Security the option of investing in their own personal accounts and in assets riskier than Treasury Bonds. Assume that the average worker will contribute $5,000 into his or her retirement account next year, and can choose option 1 and invest in T-bonds, which have a 4% expected return, or option 2 and invest in a stock index fund that has a 12% expected return. In both options at the date of retirement, the money will be placed in an account with an expected return of 3% APR (0.25% per month). Assume that the amount workers contribute will grow by 3% per year, and the average worker is 35 years from retirement age. If the money is withdrawn from the account beginning the month after retirement, and the average worker is expected to live for**

20 years after retirment, what size monthly payment would the average worker be able to withdraw in both of the options?

Step 1. Put the cash flows that are known and unknown on a timeline.



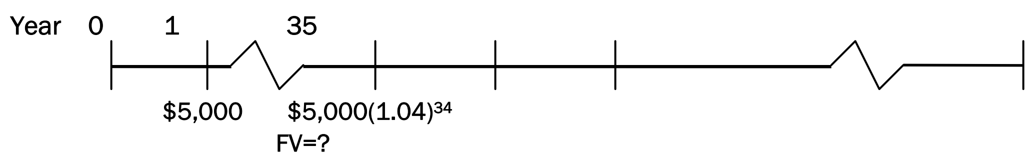
Step 2. Determine the type or types of valuation problems involved.

This problem involves the following additional steps 3–4:

[3] finding the future value of the known growing annuity of payments that will have accumulated in 35 years, and

[4] finding the unknown monthly annuity payment that has that present value.

Step 3. Find the future value of the growing annuity of payments.



The future value of a growing annuity can be found using the present value of a growing annuity equation and the future value of a lump sum equation as follows:

$$\text{The present value of the payments at time } 0 = PV_0 = \left(\frac{C}{r-g}\right)\left[1-\left(\frac{1+g}{1+r}\right)^N\right],$$

$$\text{so the future value at time } 35 = FV_{35} = PV_0(1+r)^{35}, \text{ or}$$

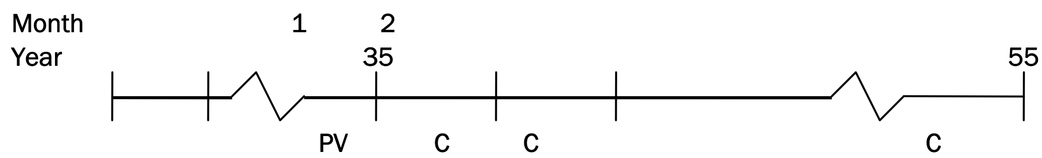
$$FV_{35} = \left(\frac{C}{r-g}\right)\left[1-\left(\frac{1+g}{1+r}\right)^N\right](1+r)^{35}.$$

For both of the options, C = \$5,000, g = 3%, and N = 35. For the T-bond option, r = 4%, and for the stock index fund option, r = 12%.

$$FV_{35}^{\text{T-bond}} = \left[\left(\frac{5{,}000}{.04-.03}\right)\left(1-\left(\frac{1.03}{1.04}\right)^{35}\right)\right](1.04)^{35} = \$566{,}113$$

$$FV_{35}^{\text{Stock}} = \left[\left(\frac{5{,}000}{.12-.03}\right)\left(1-\left(\frac{1.03}{1.12}\right)^{35}\right)\right](1.12)^{35} = \$2{,}776{,}986$$

Step 4. Find the 240 month annuity payment that has that present value.



Using the present value of an annuity equation with PV = FV_{35} from step 3, $N = 12 \times 20 = 240$,

$r = .03 / 12 = 0.0025$, and solving for C, you have:

$$FV_{35}^{\text{T-bond}} = PV = \$566{,}113 = C\left[\frac{1}{.0025} - \frac{1}{.0025(1.0025)^{240}}\right] \Rightarrow C = \$3{,}140$$

$$FV_{35}^{\text{Stock}} = PV = \$2{,}776{,}986 = C\left[\frac{1}{.0025} - \frac{1}{.0025(1.0025)^{240}}\right] \Rightarrow C = \$15{,}401$$

Under the 4% T-bond plan, workers could withdraw $3,140 per month, and under the 12% Stock Index plan, workers could withdraw $15,401 per month.

Questions and Problems

1. The historical average return on U.S. T-bills is 3.8% per year, while the average return for small company stocks is 16.9% per year. Assuming these rates occur annually in the future, how much more cash would you have in 20 years by investing $50,000 in small company stocks rather than T-bills?

2. Your daughter is currently 8 years old. You anticipate that she will be going to college in 10 years. You would like to have $100,000 in a savings account at that time. If the account promises to pay a fixed interest rate of 3% per year, how much money do you need to put into the account today to ensure that you will have $100,000 in ten years?

3. You have determined that you will need $3,000,000 when you retire in 40 years. You plan to set aside a series of payments each year in an account yielding 12% per year to reach this goal. You will put the first payment in the account one year from today, and the payments will grow with your income by 3% per year. Calculate your first annual payment into this account. Calculate the last payment.

4. Like in problem 3, you have determined that you will need $3,000,000 when you retire in 40 years, and you plan to set aside a series of payments each year in an account yielding 12% per year to reach this goal. You will put in the first payment in the account one year from today and the payments will grow with your income by 3% per year. Assuming that the money is placed in a 6% APR account throughout your retirement period, and you plan to withdraw $25,000 per month, approximately how many years will the money last you?

5. You are offering the employees in your small firm a so-called defined benefit pension plan. Beginning exactly 21 years from today you will pay out the first annual payment of a guaranteed 30-year stream of annual payments. The first payment will be $100,000 for 10 employees, or $1 million. The payment stream will grow by 3% per year each year to match expected inflation. You have already started investing in the pension account, which has a balance of $113,971.84 today. You expect that the account will always yield 13% APR, and you will always leave the money in the account, only withdrawing the money as needed by the plan. To supplement the plan, you will make 20 even, annual payments over the next 20 years, beginning one year from today. How big must the annual payment that you will contribute be?

Solutions to Questions and Problems

1. This problem requires using the FV of a lump sum equation:

$$FV_{20} = \$50{,}000(1.038)^{20} = \quad \$105{,}418.56$$
$$FV_{20} = \$50{,}000(1.169)^{20} = \quad \$1{,}135{,}691.11$$

2. This problem requires using the PV of a single cash flow equation:

$$PV_0 = \frac{FV_{10}}{(1+r)^{10}} = \frac{100{,}000}{(1.03)^{10}} = \$74{,}409.39$$

3. Set $3,000,000 equal to the present value a growing annuity equation and solve for the first payment, C_1.

$$FV_{40} = \left[\frac{C}{.12-.03}\right]\left[1-\left(\frac{1.03}{1.12}\right)^{40}\right](1.12)^{40} = 3{,}000{,}000$$

$$= C\left[11.1111\right]\left[.96494\right]93.05$$

$$= C(997.643) \Rightarrow C = \$3{,}007$$

Thus, the first payment is $3,007 and the last payment is ($3,007)$(1.03)^{39}$ = $9,523.

4. You need to solve the following present value of an annuity equation for N.

$$\$3{,}000{,}000 = \$25{,}000\left(\frac{1}{.005}\right)\left(1-\frac{1}{.005(1.005)^N}\right) \Rightarrow N = 184 \text{ months, or 15 years and 4 months.}$$

5. You must contribute $99,648:

$$PV_{20} = \left[\frac{1{,}000{,}000}{.13-.03}\right]\left[1-\left(\frac{1.03}{1.13}\right)^{30}\right] = \$9{,}379{,}469$$

$$PV_0 = \frac{\$9{,}379{,}469}{(1.13)^{20}} - \$113{,}971.84 = \$813{,}972 - 113{,}971.84 = \$700{,}000$$

$$\$700{,}000 = C\left[\frac{1}{.13} - \frac{1}{.13(1.13)^{20}}\right] \Rightarrow C = \$99{,}648$$

STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*, but it also works with the Fourth Edition.

Pearson

CHAPTER 5

Interest Rates

Chapter Synopsis

5.1 Interest Rate Quotes and Adjustments

Interest rates can compound more than once per year, such as monthly or semiannually. An **annual percentage rate** (APR) equals the periodic interest rate, *r,* times the number of compounding periods per year, *k.* Because it does not include the effect of compounding, an APR understates the amount of interest that will be received if interest compounds more than once per year.

To compute the actual amount of interest earned in one year, an APR can be converted to an **effective annual rate** (EAR), which includes the effect of compounding and provides a measure of the amount of interest that will actually be earned over a year:

Converting an APR to an EAR

$$\text{EAR} = \left(1 + \frac{\text{APR}}{k}\right)^k - 1$$

The more compounding periods, the greater the EAR. For example, suppose a bank offers a certificate of deposit with an interest rate of "6% APR with monthly compounding." In this case, you will earn 6% / 12 = 0.5% every month. To determine the value of $100 invested for one year, you can either compound over 12 months at the monthly rate of 0.5% or you can compound over one year at the EAR = $(1 + .06/12)^{12} - 1$ = 6.17%:

$$\text{FV}_1 = \$100(1.005)^{12} = \$100(1.0617) = \$106.17.$$

Many loans, such as home mortgages and car loans, have monthly payments and are quoted in terms of an APR with monthly compounding. These types of loans are typically **amortizing loans** in which each month's payment includes the interest that accrues that month along with some part of the loan's balance. Each monthly payment is the same, and the loan is fully repaid with the final payment. Since the loan balance declines over time, the interest portion of the payment declines over time while the principal repayment portion increases. The number of compounding periods is generally equal to the number of payments per year by convention.

For example, suppose you are offered a $30,000 car loan at "6.75% APR for 60 months." You can find the monthly payment using the PV of an annuity equation:

$$\$30{,}000 = C\frac{1}{.005625}\left[1-\frac{1}{(1.005625)^{60}}\right] \Rightarrow C = \$590.50$$

In the first month, interest equals $30,000(0.005625) = $168.75 and the loan's balance is reduced by $590.50 – 168.75 = $421.75 to $29,578.25. In the second month, interest equals $29,578.25(0.005625) = $166.38 and the loan's balance is reduced by $590.50 – 166.38 = $424.12 to $29,154.13. This process continues until the beginning of the 60th month when the loan balance will be $578.20, so interest equals $587.20(0.005625) = $3.30 and the loan's balance is reduced by $590.50 – 3.30 = $587.20 to $0. A tabular depiction of this process is called an **amortization table.**

5.2 Application: Discount Rates and Loans

To calculate a loan payment, you first equate the outstanding loan balance with the present value of the loan payments using the discount rate from the quoted interest rate of the loan and then solve for the loan payment. Many loans, such as mortgages and car loans, have monthly payments and are quoted in terms of an APR with monthly compounding. These types of loans are amortizing loans, which means that each month you pay interest on the loan plus some part of the loan balance. Each monthly payment is the same, and the loan is fully repaid with the final payment.

Typical terms for a new car loan might be "6.75% APR for 60 months." This quote means that the loan will be repaid with 60 equal monthly payments, computed using a 6.75% APR with monthly compounding. The payment, C, is set so that the present value of the cash flows, evaluated using the loan interest rate, equals the original principal amount of $30,000. So, using the annuity formula to compute the present value of the loan payments, the payment C must satisfy

$$C \times \frac{1}{0.005625}\left(1-\frac{1}{(1+0.005625)^{60}}\right) = 30{,}000$$

$$\text{and therefore, } C = \frac{30{,}000}{\frac{1}{0.005625}\left(1-\frac{1}{(1+0.005625)^{60}}\right)} = \$590.50$$

Alternatively, we can solve for the payment C using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	60	0.5625%	30,000		0	
Solve for PMT				−590.50		=PMT(0.005625,60,30000,0)

5.3 The Determinants of Interest Rates

Nominal interest rates, which indicate the actual rate at which interest will accrue, are typically stated in loan agreements and quoted in financial markets. If prices in the economy are also growing due to inflation, the nominal interest rate does not represent the increase in purchasing power that will result from investing at this rate. The rate of growth of purchasing power, after adjusting for inflation, is determined by the **real interest rate,** r_r. If *r* is the nominal interest rate and *i* is the rate of inflation, the real rate can be calculated as follows.

The Real Interest Rate

$$r_r = \frac{r - i}{1 + i}$$

Interest rates affect firms' incentives to raise capital and invest as well as individuals' propensities to save. For example, an increase in interest rates will generally decrease an investment's NPV and reduce the number of positive-NPV investments available to firms. The U.S. Federal Reserve as well as central banks in other countries use this idea to try and influence economic activity. Central banks can lower interest rates to stimulate investment if the economy is slowing and raise interest rates to reduce investment if the economy is perceived to be growing too fast ,which may lead to an increase in the inflation rate.

Interest rates generally depend on the horizon, or term, of the investment or loan. The relation between an investment's term and its interest rate is called the **term structure of interest rates,** and it can be plotted on a graph called the **yield curve.** Common equations used for computing present values, such as the annuity and perpetuity formulas, are based on discounting all of the cash flows at the same rate. In situations in which cash flows need to be discounted at different rates depending on when they occur, the following equation can be used:

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{1+r_2} \cdots \frac{C_N}{1+r_N} = \sum_{n=0}^{N} \frac{C_n}{\left(1+r_n\right)^n}$$

The Federal Reserve determines short-term interest rates through its influence on the **federal funds rate**, which is the rate at which banks can borrow cash reserves over one night. All other interest rates on the yield curve are set in the market and are adjusted until the supply of lending matches the demand for borrowing at each loan term. Expectations of future interest rate changes have a major effect on investors' willingness to lend or borrow for longer terms and, therefore, on the shape of the yield curve. An increasing yield curve, with long-term rates higher than short-term rates, generally indicates that interest rates are expected to rise in the future. A decreasing (inverted) yield curve, with long-term rates lower than short-term rates, generally signals an expected decline in future interest rates. Because interest rates tend to drop in response to an economic slowdown, an inverted yield curve is often interpreted as a negative economic forecast.

5.4 Risk and Taxes

U.S. Treasury securities are widely regarded as risk-free because there is virtually no chance the U.S. government will fail to pay the interest or default on these bonds; thus, the rate on Treasury securities is often referred to as the **risk-free rate.** All other borrowers are generally assumed to have some risk of default. For these loans, the stated interest rate is the maximum amount that investors will receive. Investors may receive less if the company is unable to fully repay the loan. To compensate for the risk that they will receive less if the firm defaults, investors demand a higher interest rate than the rate on U.S. Treasuries. The difference between the interest rate of the loan and the Treasury rate is called the **credit spread.**

If the cash flows from an investment are taxed, the net cash flow that the investor will receive will be reduced by the amount of the taxes paid. In general, if the interest rate is *r* and the tax rate is τ, then for each $1 invested you will earn interest equal to *r* and owe tax of $\tau \times r$ on the interest. Thus, the equivalent **after-tax interest rate** is $r(1-\tau)$. For example, if an investment pays 8% interest for one year, and you invest $100 at the start of the year, you will earn 8% × $100 = $8 in interest at year-end. If you must pay taxes at 40% on this interest, you will owe 40% × $8 = $3.20. Thus you will receive only $8 − $3.20 = $4.80 after paying taxes. This amount is equivalent to earning 4.80% interest and not paying any taxes, so the after-tax interest rate is $r(1-\tau) = 8\%(1-.40) = 4.80\%$.

5.5 The Opportunity Cost of Capital

The discount rate used to evaluate cash flows is the **cost of capital**, or **opportunity cost of capital,** which is the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted. The cost of capital is the return the investor forgoes when making a new investment. For a risk-free project, it will typically correspond to the interest rate on U.S. Treasury securities with a similar term. For risky projects, it will include a **risk premium.**

Selected Concepts and Key Terms

Amortizing Loan

A loan in which each month you pay interest on the loan plus some part of the loan principal, or amount borrowed. Each monthly payment is the same, and the loan is fully repaid with the final payment. Since the loan balance declines over time, the interest portion of the payment declines over time while the principal repayment portion increases.

Annual Percentage Rate (APR)

The periodic interest rate, *r,* times the number of compounding periods per year, *k.* Because it does not include the effect of compounding, the APR quote is less than the actual amount of interest that will be received if $k > 1$.

Opportunity Cost of Capital

The best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted. The cost of capital is the return the investor forgoes when the making a new investment. For a risk-free project, it will typically correspond to the interest rate on U.S. Treasury securities with a similar term. For risky projects, it will include a risk premium.

Credit Spread

The difference between the interest rate of the loan and the risk-free Treasury security rate.

Effective Annual Rate (EAR)

The amount of interest that will be earned over a year. The more compounding periods, the greater the EAR for a given APR.

Nominal Interest Rate

The actual rate at which money will grow. Nominal rates are typically stated in loan agreements and quoted in financial markets. If prices in the economy are also growing due to inflation, the nominal interest rate does not represent the increase in purchasing power that will result from investing at the nominal rate.

Real Interest Rate

The rate of growth of purchasing power after adjusting for inflation.

Term Structure

The relation between an investment's term and its interest rate is called the term structure of interest rates, and it can be plotted on a graph called the yield curve.

Concept Check Questions and Answers

5.1.1. What is the difference between an EAR and an APR quote?

An annual percentage rate is the rate that interest earns in one year before the effect of compounding. An effective annual rate is the rate that the amount of interest actually earns at the end of one year. Because the APR does not include the effect of compounding, it is typically less than the EAR.

5.1.2. Why can't the APR be used as a discount rate?

Because the APR does not reflect the true amount you will earn one year, the APR itself cannot be used as a discount rate.

5.2.1. How can you compute the outstanding balance on a loan?

The outstanding balance can be computed by constructing an amortization table or by finding the present value of the remaining payments.

5.2.2. What is an amortizing loan?

It is a loan in which each month you pay interest on the loan plus some part of the loan principal, or amount borrowed. Each monthly payment is the same, and the loan is fully repaid with the final payment. Since the loan balance declines over time, the interest portion of the payment declines over time while the principal repayment portion increases.

5.3.1. What is the difference between a nominal and real interest rate?

The nominal interest rate is the rate quoted by banks and other financial institutions, whereas the real interest rate is the rate of growth of purchasing power, after adjusting for inflation. The real interest rate is approximately equal to the nominal rate less the rate of inflation.

5.3.2. How do investors' expectations of future short-term interest rates affect the shape of the current yield curve?

The shape of the yield curve tends to vary with investors' expectations of future economic growth and interest rates. It tends to be inverted prior to recessions and to be steep coming out of a recession.

5.4.1. Why do corporations pay higher interest rates on their loans than the U.S. government?

Corporations pay higher interest rates on their loans than the U.S. government does because all corporations have some risk of default, while there is virtually no chance the U.S. government will fail to pay the interest or default on the loans.

5.4.2. How do taxes affect the interest earned on an investment? What about the interest paid on a loan?

The interest the investor earned on an investment is taxable and will be reduced by the amount of the tax payments. In some cases, since the interest on loans is tax deductible, the cost of paying interest on the loan is offset by the benefit of the tax deduction.

5.5.1. **What is the opportunity cost of capital?**

The opportunity cost of capital is the best available return offered in the market on an investment of comparable risk and term to the cash flow being discounted.

5.5.2. **Why do different interest rates exist, even in a competitive market?**

The interest rates we observe in the market will vary based on quoting conventions, the term of investment, and risk. The actual return kept by an investor will also depend on how the interest is taxed.

Examples with Step-by-Step Solutions

Solving Problems

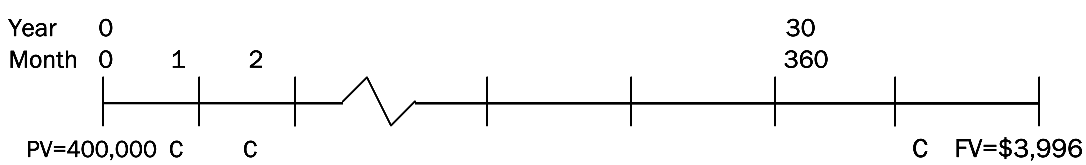
Problems using the concepts in this chapter often involve solving problems using the valuation equations in Chapter 4. It is helpful to represent the cash flows involved on a timeline, and it is important to use the correct periodic interest rate. For instance, in example 1 below, the loan involves monthly payments at a 6% APR with monthly compounding, so the correct rate to use is the monthly rate = 6% / 12 = 0.5%. Other problems may involve finding real cash flows. To do this, it is generally necessary to calculate and use real interest rates using the relation between nominal rates, real rates and inflation. Example 2 below provides such an example. Finally, problems may involve understanding the mechanics of an amortizing loan, as in example 3 below.

Examples

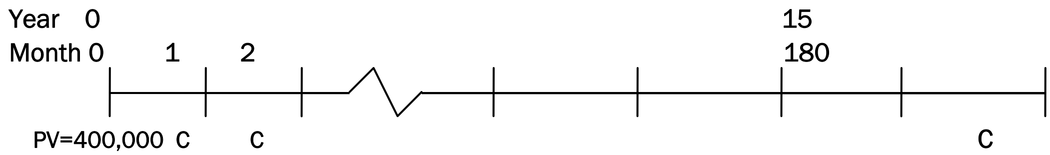
1. **You want to buy a vacation house in Hood River, Oregon, by borrowing $400,000.**
   - **[A] If you obtain a 30-year loan at 6% APR with monthly compounding, what is your monthly payment? How much goes to interest and how much to principal over the loan's life?**
   - **[B] If you obtain a 15-year loan at 6% APR with monthly compounding, what is your monthly payment? How much goes to interest and how much to principal over the loan's life?**

Step 1: Put the known and unknown cash flows on a timeline.

The 30-year Loan.



The 15-year Loan.



Step 2: Since this problem involves the present value of an annuity in which C is unknown, set the PV of annuity equation equal to \$400,000 with r = 0.05 and N = 360 months for part [A] and 180 months for part [B]. Solve for C to get each loan's payment.

$$\text{[A]}\ \$400{,}000 = C\left[\frac{1}{.005} - \frac{1}{.005(1.005)^{360}}\right] \Rightarrow\ C = \$2{,}398.20$$

$$\text{[B]}\ \$400{,}000 = C\left[\frac{1}{.005} - \frac{1}{.005(1.005)^{180}}\right] \Rightarrow C = \$3{,}375.43$$

Step 3. Once the payment is calculated, you can calculate the total interest paid as the total payments made minus the \$400,000 principal that was paid.

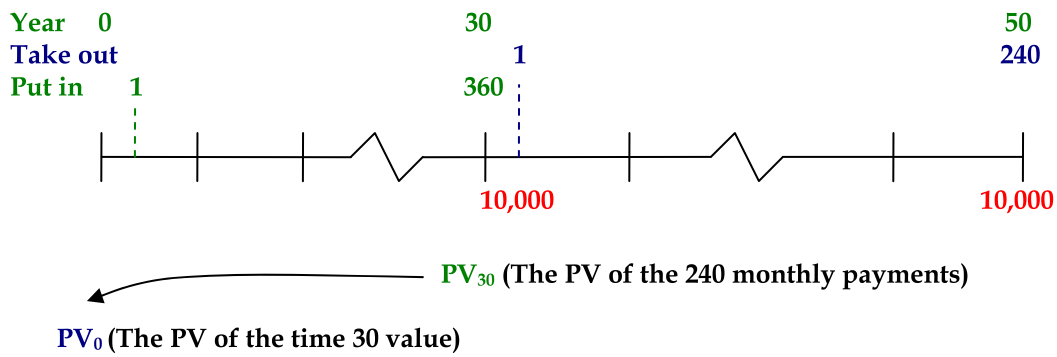
[A] Total = 360(2,398.20) = 863,353
−Principal = 400,000
= Interest = 463,353

[B] Total = 180(3,375.48) = 607,577
−Principal = 400,000
= Interest = 207,577

Thus, the payment and interest paid is \$2,398 and \$463,353 for the 30-year loan and \$3,375 and \$207,577 for the 15-year loan.

2. You plan on drawing on retirement income exactly 30 years from today. You have liquidated \$1 million worth of investments and are going to move to Spain, taking whatever cash is left after funding your retirement account. You want the retirement account to pay exactly \$10,000 per month in today's (real) dollars for 20 years. You are going to place enough into an account yielding a nominal 12% APR, or 1% per month. At retirement, you will place the accumulated money into an account yielding a nominal 6% APR, which compounds monthly at 0.5%, and remove cash from this account according to the payment schedule. You expect that inflation will be 0.25% per month. What payment to the 12% account is required to fund the retirement plan?

Step 1. Put the cash flows that are known and unknown on a timeline.



Step 2. Determine the type or types of valuation problems involved.

This problem can be solved using the following additional steps 3–6:

[3] find the real monthly interest rates in the first 30 years and last 20 years,
[4] find the present value in real terms in year 30 of the $10,000 payments in months 361–600,
[5] find the present value of the year 30 value found in step 4,
[6] subtract the value in step 5 from the $1 million.

Step 3. The real rates are:

$$r_{\text{real}}^{\text{Years 30-50}} = \frac{r-i}{1+i} = \frac{.005-.0025}{1+.025} \Rightarrow r_{\text{real}} \approx 0.0025 = 0.25\% \text{ per month.}$$

$$r_{\text{real}}^{\text{Years 1-30}} = \frac{r-i}{1+i} = \frac{.01-.0025}{1+.025} \Rightarrow r_{\text{real}} \approx 0.75\% \text{ per month.}$$

Step 4. Using the present value of an annuity equation:

$$PV_{30} = 10{,}000\left[\frac{1}{.0025} - \frac{1}{.0025(1.0025)^{240}}\right] = \$1{,}803{,}109 \text{ in real dollars.}$$

Step 5. Using the PV of a single cash flow equation:

$$PV = \frac{1{,}803{,}109}{(1.0075)^{360}} = \$122{,}406.$$

Step 6. Make a conclusion.

You can take $\$1{,}000{,}000 - \$122{,}406 = \$877{,}594$ with you and put the rest in the account.

3. The Boston Beer Company is shopping for a new bottling machine. The machine has a manufacturer's suggested retail price of $350,000.
   [A] Dealer A offers to sell them the machine for $290,000 with a 6% APR monthly amortizing 10-year loan. Dealer B will charge the full $350,000 but offers them 0% APR monthly payment loan with financing over 10 years. Which of these two options is a better deal?
   [B] If they decide to buy the machine from dealer A, how much of the first two payments goes to paying down the $290,000 principal? How much is interest?
   [C] If they decide to buy the machine from dealer A and sell it in three years, how much must they sell it for in order to pay back the remaining balance of the loan? (Ignore tax effects.)

Step 1. To answer part [A], you need to determine which payment option has the lowest present value.

Since both options have the same term and monthly payments, this is the same as finding which option has the lowest payment.

Using the PV of an annuity equation:

$$\$290{,}000 = C\left[\frac{1}{.005} - \frac{1}{.005(1.005)^{120}}\right] \Rightarrow C = \$3{,}219.59 \text{ is the payment for dealer A.}$$

$$\frac{\$350{,}000}{120} = \$2{,}916.67 \text{ is the payment for dealer B.}$$

Also, note that the present value of dealer B's payments at 6% APR is $262,714 which is less than the present value of dealer A's payments at 6% APR, $290,000.

Thus, you should select dealer B.

Step 2. To answer part [B] of the problem, you can construct the first two months of an amortization table with a payment of $3,219.59, an original balance of $290,000, and a periodic (monthly) interest rate of 6% / 12 = 0.50%:

Month	Principal	Interest=0.005 x Principal	Payment	Ending Balance
1	290,000.00	1,450.00	3,219.59	288,230.41
2	288,230.41	1,441.15	3,219.59	286,451.97

Now, calculate the principal repaid = 290,000 – 286,451.97 = $3,548.03.

After two months, the total principal repaid is $3,548.03 and the total interest paid is 2(3,219.59) – $3,548.03 = 1,450.00 + 1,441.15 = $2,891.15.

Step 3. To answer part [C] you could construct an amortization table, but without the aid of a spreadsheet this would be too time-consuming. Thus, you can solve for the present value of the remaining payments, which must be the remaining balance of the loan.

Using the present value of an annuity equation with C = $3,219.59, r = 6%/12 = 0.5%, and N = 120 – 36 = 84:

$$\text{PV} = \frac{3{,}219.59}{0.005}\left(1 - \frac{1}{(1.005)^{84}}\right) = \$220{,}390.73.$$

Thus, they must sell it for at least $220,390.73 or else they will have to pay off some of the balance with cash from a different source.

Questions and Problems

1. You won $1 in million the Lottery. The prize is paid out in equal, semi-annual payments over 50 years with the first payment immediately. GenexCapital.com has offered to buy the ticket for $250,000 in cash today. In the contract, they claim to be using an 8% APR with semi-annual compounding. Are they? (Ignore taxes)

2. You have a $50,000 balance on your credit card, and you have set your Wells Fargo checking account bill pay for monthly payments of $1,000. The interest rate is 18% APR with monthly compounding. How many years until you have paid it off? How long would it take if your balance was $70,000?

3. You are considering paying for a 2006 Mercedes SLK 350 with an MSRP of $50,000 using a 5-year loan. Based on the MSRP, the dealer's finance manager has quoted you a zero down, 4.8% APR (compounded monthly) loan with a payment of $966.64 and your first payment is due one month from today.
   [A] Is the rate you would be paying really 4.8% APR?
   [B] For every $500 that you get the dealer to lower the price of the car at a 4.8% APR, how much does your monthly payment decrease?
   [C] Based on a price of $45,000, how much would your down payment need to be to make your payments equal $700 per month at 4.8% APR?

4. You have decided to refinance your mortgage. You plan to borrow whatever is outstanding on your current mortgage. The current monthly payment is $5,200, and there are exactly 27 years left on the loan. You have just made your 36th monthly payment and the mortgage interest rate is 6% APR. How much do you owe on the mortgage today?

5. You have just sold your house for $2,000,000. Your mortgage was originally a 30-year mortgage with monthly payments, and an initial balance of $400,000. The mortgage is exactly 10 years old, and you have just made a monthly payment. If the fixed interest rate on the mortgage is 3.6% (APR), how much will you have from the sale once you pay off the mortgage?

Solutions to Questions and Problems

1. If they are paying 8% APR (4% per six months), then the PV of the annuity payments at this rate must be $250,000.

$$PV = \$10{,}000\left[\frac{1}{.04} - \frac{1}{.04(1.04)^{99}}\right] + 10{,}000 = \$254{,}852 > \$250{,}00.$$

Since they are paying less than $250,000, they are using a bit higher rate.

The actual rate is: $\$10{,}000\left[\frac{1}{r} - \frac{1}{r(1+r)^{99}}\right] + 10{,}000 = \$250{,}000 \Rightarrow \text{APR} \approx 8.175\%$.

2. This is a present value of an annuity problem in which you must solve for *N*.

$$\$1{,}000\left[\frac{1}{.015} - \frac{1}{.015(1.015)^{N}}\right] = \$50{,}000$$

$\Rightarrow$ T = 93.11 months, or about 7 years and 10 months.

For the $70,000 balance, note that if you paid $1,000 in perpetuity:

$\left(\frac{\$1{,}000}{.015}\right) = \$66{,}667,$ so you could never pay it off, since $\$70{,}000 > \$66{,}667.$

3. [A] The actual payment at 4.8% APR would be:

$$50{,}000 = C\left[\frac{1}{.004} - \frac{1}{.004(1.004)^{60}}\right] \Rightarrow C = \$938.99.$$

So the actual rate is higher. The implied rate in the payment can be found as follows:

$$50{,}000 = 966.64\left[\frac{1}{r} - \frac{1}{r(1+r)^{60}}\right] \Rightarrow r = 0.5\%, \text{ or } 6\% \text{ APR.}$$

[B] $\$500 = C\left(\frac{1}{.004} - \frac{1}{.004(1.004)^{60}}\right) \Rightarrow C = \$9.39.$

So, for every $500 reduction, the payment would decrease by $9.39.

[C] $\$45{,}000 = 700\left(\frac{1}{.004} - \frac{1}{.004(1.004)^{60}}\right) + \text{Down Payment}$

$\Rightarrow \text{Down Payment} = \$45{,}000 - \$37{,}274 = \$7{,}726$

4. To find out what is owed, compute the PV of the remaining payments:

$$PV = \frac{5{,}200}{0.005}\left(1 - \frac{1}{(1.005)^{324}}\right) = \$833{,}352.89.$$

5. First compute the original loan payment:

$$C = \frac{400{,}000 \times 0.003}{\left(1 - \frac{1}{(1.003)^{360}}\right)} = \$1{,}818.58.$$

Now compute the PV of continuing to make these payments.

Using the formula for the PV of an annuity:

$$PV = 1{,}818.58\left(\frac{1}{0.003} - \frac{1}{(1.003)^{240}}\right) = \$310{,}809.15.$$

So you would get to keep $2,000,000 – $310,809.15 = $1,689,190.85.

STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*, but it also works with the Fourth Edition.

Pearson

CHAPTER 6

Valuing Bonds

Chapter Synopsis

6.1 Bond Cash Flows, Prices, and Yields

A bond is a security sold at **face value** (FV), usually $1,000, to investors by governments and corporations. Bonds generally obligate the borrower to make a promised future repayment of face value at the **maturity date** along with interest payments called **coupon payments** that are typically paid semiannually. The amount of each coupon payment is determined by the **coupon rate** of the bond. By convention, the coupon rate is expressed as an APR, so the amount of each coupon payment is:

$$\text{CPN} = \frac{\text{Coupon Rate} \times \text{Face value}}{\text{Number of Coupon Payments Per Year}}$$

For example, a "10-year, $1,000 face value bond with a 10% semiannual coupon rate" will pay coupon payments of $1000 × 0.10/2 = $50 every six months and repay the face value, or **principal**, in 10 years. The terms of the bond are described as part of the bond certificate, which indicates the amounts and dates of all payments to be made.

A **zero-coupon bond**'s only payment is the face value of the bond on the maturity date—it does not make coupon payments. Treasury bills, which are U.S. government bonds with a maturity of up to one year, are zero-coupon bonds and can be valued easily using the present value of a cash flow equation. For example, a one-year, risk-free, zero-coupon bond with a $1,000 face value with a required return of 3.5% is worth:

$$\text{PV} = \frac{\$1,000}{1.035} = \$966.18.$$

Zero-coupon bonds always trade at a **discount** (a price lower than the face value) and are sometimes referred to as **pure discount bonds.**

The **yield to maturity** (YTM) of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. For the zero-coupon bond above, the YTM is the return an investor will earn from holding the bond to maturity and can be calculated as:

$$\$966.18 = \frac{\$1{,}000}{1 + \text{YTM}} = \$966.18 \Rightarrow \text{YTM} = \frac{\$1{,}000}{\$966.18} - 1 = 0.035 = 3.5\%.$$

In general, the YTM for a zero coupon bond can be calculated as:

$$\text{P} = \frac{\text{FV}}{(1 + \text{YTM})^{n}} \Rightarrow \text{YTM} = \left(\frac{\text{FV}}{\text{P}}\right)^{\frac{1}{n}} - 1.$$

The yield to maturity of an *n*-year, zero-coupon, risk-free bond is generally referred to as the **risk-free interest rate**, or the **spot rate.** The risk-free yield curve, which plots interest rate for risk-free bonds with different maturities, is often constructed using yields of zero coupon Treasury securities, which are generally considered to be risk free.

U.S. **Treasury notes**, which have original maturities from one to ten years, and **Treasury bonds**, which have original maturities of more than ten years, as well as most **corporate bonds** make semiannual coupon payments. The price of a coupon-paying bond with a required return of y can be calculated as:

$$P_0 = \frac{\text{CPN}}{1 + y} + \frac{\text{CPN}}{(1 + y)^2} + \frac{\text{CPN}}{(1 + y)^3} + \cdots + \frac{\text{CPN} + \text{FV}}{(1 + y)^N} = \text{CPN} \times \frac{1}{y}\left(1 - \frac{1}{(1 + y)^N}\right) + \frac{\text{FV}}{(1 + y)^N}.$$

Unlike zero-coupon bonds, the yield to maturity for coupon-paying bonds cannot be solved directly with a simple equation. Instead, the calculation requires iteration—guessing until the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond is found. Excel and financial calculators can be used to perform this iteration quickly.

6.2 Dynamic Behavior of Bond Prices

Coupon bonds may trade at:

- **par** (when their price is equal to their face value),
- a **discount** (when their price is less than their face value), or
- a **premium** (when their price is greater than their face value).

Bonds trading at a discount generate a return from both receiving the coupons and from receiving a face value that exceeds the price paid for the bond. As a result, the yield to maturity of discount bonds exceeds the coupon rate. Conversely, the YTM on bonds selling at a premium is lower than the coupon rate because the face value received is less than the price paid for the bond. A bond selling at par has a YTM equal to its coupon rate.

Between coupon payments, the prices of all bonds rise at a rate equal to the semiannual yield to maturity. Also, as shown in Figure 6.1 below:

- As each coupon is paid, the price of a bond drops by the amount of the coupon.
- When the bond is trading at a premium, the price drop when a coupon is paid will be larger than the price increase between coupons, so the bond's premium will tend to decline as time passes.

- If bond is trading at a discount, the price increase between coupons will exceed the drop when a coupon is paid, so the bond's price will rise, and its discount will decline as time passes.
- When the bond matures, the price of the bond equals the bond's face value.

Bond prices are subject to the effects of both the passage of time and changes in interest rates. While, as shown in Figure 6.1 below, bond prices converge to the bond's face value due to the time effect, they also move up and down due to unpredictable changes in bond yields. A higher yield to maturity means a higher discount rate for a bond's remaining cash flows, reducing their present value and thus the bond's price. Therefore, there is an inverse relation between bond prices and yields: as the discount rate increases, a bond's price falls, and as the discount rate falls, a bond's price increases.

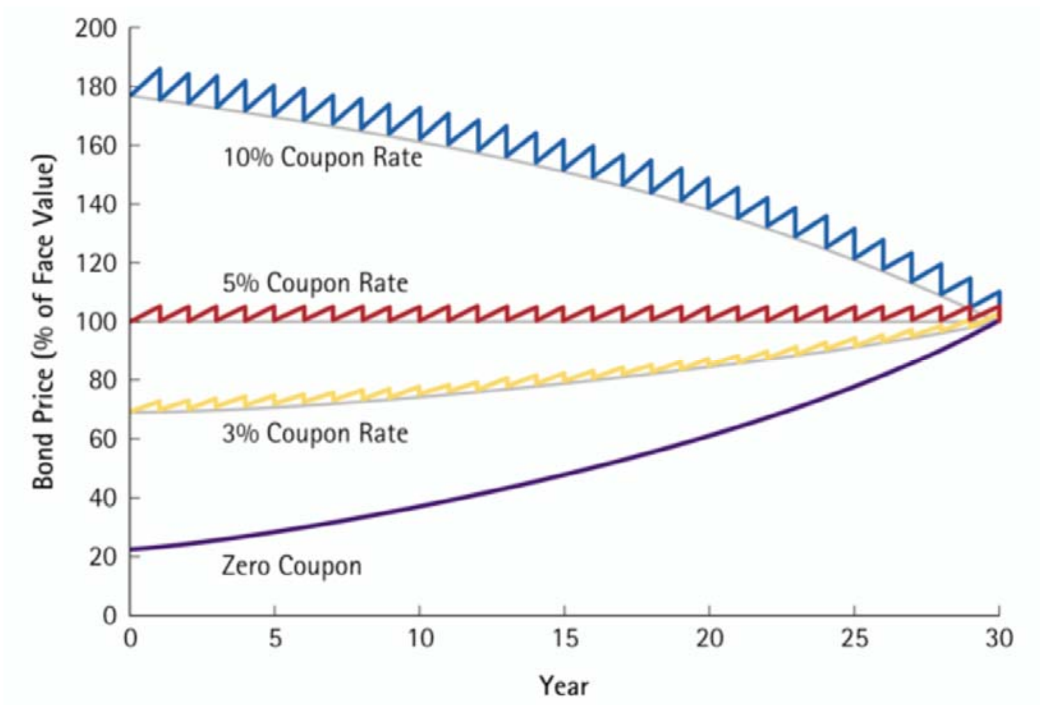


Figure 6.1

The graph illustrates the effects of the passage of time on bond prices per $100 face value when the yield of a bond remains constant at 5%.

The sensitivity of a bond's price to changes in interest rates depends on the timing of its cash flows.

- Shorter maturity bonds are less sensitive to changes in interest rates because the present value of a cash flow that will be received in the near future is less dramatically affected by interest rates than a cash flow in the distant future.
- Bonds with higher coupon rates are less sensitive to interest rate changes than otherwise identical bonds with lower coupon rates because they pay a higher proportion of their cash flows sooner.

The sensitivity of a bond's price to changes in interest rate can be measured by its **duration,** which is discussed later in the text.

6.3 The Yield Curve and Bond Arbitrage

It is possible to replicate the cash flows of a risk-free coupon bond using zero-coupon bonds. For example, a three-year, $1,000 bond can be replicated with 1-year, 2-year, and 3-year zero coupon bonds. By the Law of One Price, the three-year coupon bond must trade for the price it costs to replicate the payoffs using the zero-coupon bonds. If the price of the coupon bond were higher, you could earn an arbitrage profit by selling the coupon bond and buying the zero-coupon bond portfolio. If the price of the coupon bond were lower, you could earn an arbitrage profit by buying the coupon bond and short selling the zero-coupon bonds. The no-arbitrage price of a risk-free coupon bond can also be found by discounting its cash flows using the risk-free zero-coupon yields using:

$$P_0 = \frac{CPN}{1 + YTM_1} + \frac{CPN}{\left(1 + YTM_2\right)^2} + \frac{CPN}{\left(1 + YTM_3\right)^3} + \cdots + \frac{CPN + FV}{\left(1 + YTM_N\right)^N}.$$

where YTM_n is the yield to maturity of a zero-coupon bond that matures at the same time as the *n*th coupon payment. Thus, the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.

6.4 Corporate Bonds

Corporate bonds have **credit risk,** which is the risk that the borrower will default and not pay all specified payments. As a result, investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond. Because the YTM for a bond is calculated using the promised cash flows, the yields of bonds with credit risk will be higher than that of otherwise identical default-free bonds. However, the YTM of a bond with default risk is always higher than the expected return of investing in the bond because it is calculated using the promised cash flows rather than the expected cash flows.

Bond rating agencies, such as Standard & Poor's and Moody's, evaluate the creditworthiness of bonds and publish bond ratings. The ratings encourage widespread investor participation and bond market liquidity. In descending order of credit quality, ratings by the two firms are as follows.

Standard & Poor's	Moody's	
AAA	Aaa	Investment grade
AA	Aa	
A	A	
BBB	Baa	
BB	Ba	Junk bonds
B	B	
CCC	Caa	

Bonds in the top four categories are often referred to as **investment-grade bonds** and have very low default risk. Bonds in the bottom categories are often called **junk bonds** or **high-yield bonds** because their likelihood of default is relatively high.

6.5 Sovereign Bonds

Sovereign bonds are bonds issued by national governments. While U.S. Treasury bonds are generally considered to be default free, bonds issued by many other countries may have significant default risk. For example, in 2012, Greece defaulted and wrote-off over $100 billion, or about 50%, of its outstanding debt, in the largest sovereign debt restructuring in world history.

Unlike a corporation, a country facing difficulty meeting its financial obligations typically has the option to print additional currency to avoid default. However, this likely will lead to high inflation and a sharp devaluation of the currency. Debt holders, thus carefully consider inflation expectations when determining the yield they are willing to accept on the sovereign bonds.

Selected Concepts and Key Terms

Corporate Bonds

Bonds issued by corporations. They typically have $1,000 face values and pay semiannual coupon payments.

Coupon Bonds

The promised interest payments of a bond. The bond certificate typically specifies that the coupons will be paid semiannually until the maturity date of the bond.

Coupons, Coupon rate

The interest payments on a bond that are usually paid semiannually. The amount of each coupon payment is determined by the **coupon rate** of the bond. By convention, the coupon rate is expressed as an APR, so the amount of each coupon payment equals [coupon rate × face value] ÷2.

Credit Risk, Credit Spread

Bonds that are not risk free, such as corporate bonds, have **credit risk,** which is the risk that the borrower will default and not make all specified payments. As a result, investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond. Because the YTM for a bond is calculated using the promised cash flows, the yield of bonds reflect a **credit spread** and thus will be higher than that of otherwise identical default-free bonds.

Discount Bond

A bond with a price lower than the face value. For example, zero-coupon bonds are **pure discount bonds.**

High-Yield Bonds, Junk Bonds

Bonds rated below BBB by Standard & Poor's or below Baa by Moody's that have relatively high default risk and relatively high yields.

Investment-Grade Bonds

Bonds rated BBB and above by Standard & Poor's or Baa and above by Moody's that have low default risk.

Maturity Date

The date that a bond repays its face value.

On-the-Run Bond

The most recently issued Treasury bonds for a given maturity.

Premium

The term used for bonds selling at a price greater than their face value.

Face Value (FV)

The amount that a bond pays at its maturity data, typically $1,000. Also referred to as the principal or par value.

Spot Interest Rates

The yield to maturity of an *n*-year, zero-coupon, risk-free bond.

Treasury Bills, Notes, and Bonds

Securities issued by the U.S. Treasury. Treasury bills have original maturities less than one year and are zero-coupon bonds that are sold at a discount. Treasury notes, which have original maturities from one to ten years, and Treasury bonds, which have original maturities of more than ten years, typically make semiannual coupon payments.

Yield to Maturity (YTM)

The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. The YTM is the return an investor will earn from holding the bond to maturity

Zero-Coupon Bond

A bond in which the only payment is the face value of the bond on the maturity date—it does not make coupon payments. Treasury bills, which are U.S. government bonds with a maturity of up to one year, are zero-coupon bonds.

Concept Check Questions and Answers

6.1.1. **What is the relationship between a bond's price and its yield to maturity?**

The yield to maturity of a bond (or the IRR of a bond) is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. Thus, the bond price is negatively related to its yield to maturity. When interest rate and bond's yield to maturity rise, the bond price will fall (and vice versa).

6.1.2. **The risk-free interest rate for a maturity of *n*-years can be determined from the yield of what type of bond?**

The risk-free interest rate for a maturity of *n*-years can be determined from the yield of a default free zero-coupon bond with the same maturity. Because a default-free, zero-coupon bond that matures on date *n* provides a risk-free return over the same period, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.

6.2.1. **If a bond's yield to maturity does not change, how does its cash price change between coupon payments?**

Between coupon payments, the prices of all bonds rise at a rate equal to the yield to maturity as the remaining cash flows of the bonds become closer. But as each coupon is paid, the price of a bond drops by the amount of the coupon.

6.2.2. **What risk does an investor in a default-free bond face if he or she plans to sell the bond prior to maturity?**

An investor in a default-free bond will face the interest rate risk if she plans to sell the bond prior to maturity. If she chooses to sell and the bond's yield to maturity has decreased, then she will receive a high price and earn a high return. If the yield to maturity has increased and the bond price is low at the time of sale, she will earn a low return.

6.2.3. **How does a bond's coupon rate affect its duration—the bond price's sensitivity to interest rate changes?**

The higher the coupon rate, all else equal, the lower the duration.

6.3.1. **How do you calculate the price of a coupon bond from the prices of zero-coupon bonds?**

Because we can replicate a coupon-paying bond using a portfolio of zero-coupon bonds, the price of a coupon-paying bond can be determined based on the zero-coupon yield curve using the Law of One Price. In other words, the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.

6.3.2. **How do you calculate the price of a coupon bond from the yields of zero-coupon bonds?**

Since zero-coupon bond yields represent competitive market interest rate for a risk-free investment with a term equal to the term of the zero-coupon bond, the price of a coupon bond must equal the present value of its coupon payments and face value discounted at these the zero-coupon bond yields.

6.3.3. **Explain why two coupon bonds with the same maturity may each have a different yield to maturity.**

The coupon bonds with the same maturity can have different yields depending on their coupon rates. The yield to maturity of a coupon bond is a weighted average of the yields on the zero-coupon bonds. As the coupon increases, earlier cash flows become relatively more important than later cash flows in the calculation of the present value.

6.4.1. **There are two reasons the yield of a defaultable bond exceeds the yield of an otherwise identical default-free bond. What are they?**

Because the yield must be higher to compensate for the risk of not receiving the required cash flows and to compensate for the fact that the expected cash flow is lower than the required cash flows

6.4.2. **What is a bond rating?**

A bond rating is a classification provided by several companies that assess the creditworthiness of bonds and make this information available to investors. By consulting these ratings, investors can assess the creditworthiness of a particular bond issue. The ratings therefore encourage widespread investor participation and relatively liquid markets. The two best-known bond-rating companies are Standard & Poor's and Moody's.

6.5.1. **Why do sovereign debt yields differ across countries?**

Investors consider inflation expectations and default risk when determining the yield they are willing to accept on the sovereign bonds, thus the yields differ due to differing inflation expectations and default risk.

6.5.2. What options does a country have if it decides it cannot meet its debt obligations?

To avoid default, a country has the option to print additional currency to pay its debts, but, doing so is likely to lead to high inflation and a sharp devaluation of the currency.

Examples with Step-by-Step Solutions

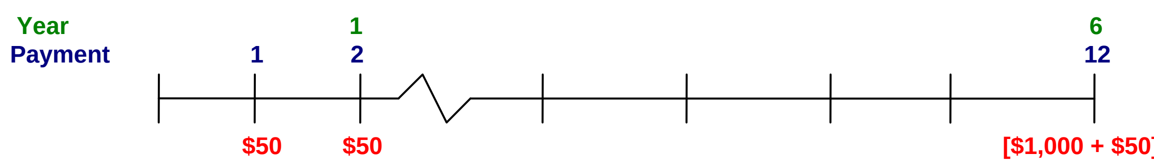
Solving Problems

Problems using the concepts in this chapter generally involve determining the value of a bond. This requires understanding the cash flows associated with bonds and bond terminology, such coupon rate, face value, maturity, and yield to maturity. The valuation of a bond generally involves utilizing the present value of an annuity equation and the present value of a cash flow equation. The yield to maturity can be found by setting the price equal to the present value of the bond's cash flows and solving for the discount rate that equates these two values.

Examples

1. You have an opportunity to buy several B-rated bonds for $641. B-rated bonds currently yield 12% APR, and these bonds have a coupon rate of 10%. The bonds have a face value of $1,000, mature in exactly six years, and the next semiannual coupon payment will occur in exactly 6 months.
   [A] What is the value of one bond?
   [B] Is the bond's yield to maturity at a price of $641 equal to, above, or below 12% APR?
   [C] If you bought some of these bonds and held them until maturity, what would your annual return be?

Step 1. Determine the bond's cash flows. The bond pays semi-annual coupon payments of 0.10($1,000)/2=$50 and pays $1,000 in 6 years.



Step 2. Calculate the value of the bond.

Since similar bonds yield 12% APR, the semi-annual rate of 12%/2=6% should be used to value this bond.

The next coupon is 6 months from today and the face value is repaid exactly 6 years from today. So, using the PV of an annuity equation and the PV of a single cash flow equation:

$$P_0 = 50\left[\frac{1}{.06} - \frac{1}{.06(1.06)^{12}}\right] + \frac{1{,}000}{(1.06)^{12}} = \$419.19 + 496.97 = \$916.16.$$

Thus, you should buy as many bonds as you can at a price of \$641 because they are selling below the market value. Even if you don't want to hold them until maturity, you can expect to sell them for \$916.16.

Step 3. Determine if the bond's yield to maturity at a price of \$644 is equal to, above, or below 12% APR.

This part does not require any calculations. Since the YTM would be 12% if the value that was found in step 2 was \$641, it does not equal 12%. Since the price found in step 2 is greater than \$641, the only way to lower the value from \$916.16 is to raise the discount rate above 12%, so the YTM must be higher than 12%.

Step 4. Your return is the yield to maturity.

The YTM is the rate that makes the bond's value equal to \$641, so

$$P_0 = \$841 = 50\left[\frac{1}{\frac{YTM}{2}} - \frac{1}{\frac{YTM}{2}(1+\frac{YTM}{2})^{12}}\right] + \frac{1{,}000}{(1+\frac{YTM}{2})^{12}}.$$

Without a financial calculator or spreadsheet, this problem requires iteration—guessing until the equation is correct. From step 3, we know that the YTM must be higher than 12% APR, or 6% per 6 months. So try 7%:

$$P_0 = \$841 = 50\left[\frac{1}{\frac{YTM}{2}} - \frac{1}{\frac{YTM}{2}(1+\frac{YTM}{2})^{12}}\right] + \frac{1{,}000}{(1+\frac{YTM}{2})^{12}} \Rightarrow \frac{YTM}{2}=0.07 \Rightarrow YTM=14\%.$$

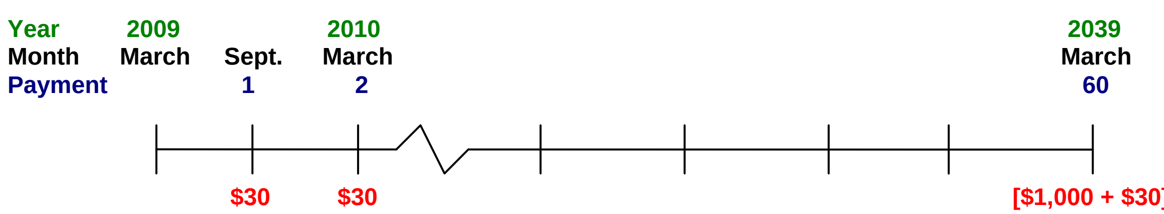
So your return would be the annual YTM, 14% APR.

2. It is March 16, 2009. Assume that a BBB-rated, 6% semiannual coupon, \$1,000 face value bond matures on March 15, 2039. The 30-year Treasury-bond yield is 6.5%.

[A] If BBB-rated currently have a 4.5% credit spread, how much should you pay for the bond?

[B] What would the value of the bond be if yields do not change in four months? What would the clean price be at this time?

Step 1. Put the cash flows on a time line. The bond pays semi-annual coupon payments of 0.06(\$1,000)/2=\$30 and pays \$1,000 in 30 years.



Step 2. Determine the discount rate.

The risk-free rate on bonds with the same term is 6.5%, and the credit spread is 4.5%, so the market rate on this bond is 6.5% + 4.5% = 11%. So, the semi-annual rate of 11%/2=5.5% should be used to value this bond.

Step 3. Determine the value today.

Since the bond paid a coupon yesterday, the next coupon is 6 months from today and the face value is repaid exactly 30 years from today. So using the PV of an annuity equation and the PV of a single cash flow equation:

$$\text{Value} = \frac{\text{Coupon}}{2}\left[\frac{1}{\left(\frac{r}{2}\right)} - \frac{1}{\left(\frac{r}{2}\right)\left(1+\frac{r}{2}\right)^{2M}}\right] + \frac{\text{Face Value}}{(1+\frac{r}{2})^{2M}}$$

$$= 30\left[\frac{1}{.055} - \frac{1}{.055(1.055)^{60}}\right] + \frac{1{,}000}{(1.055)^{60}} = 523.49 + 40.26 = \$563.75.$$

Step 4. Determine the cash value in four months.

The value in four months is the value on the date that the next coupon is paid discounted back two months. Since the EAR = $(1 + .055)^2 - 1$ = 11.30%, the two-month discount rate is $(1.113)^{2/12} - 1$ = 1.60%.

So

$$\text{Value} = \frac{\left\{\frac{\text{Coupon}}{2}\left[\frac{1}{\left(\frac{r}{2}\right)} - \frac{1}{\left(\frac{r}{2}\right)\left(1+\frac{r}{2}\right)^{2M}}\right] + 30 + \frac{\text{Face Value}}{(1+\frac{r}{2})^{2M}}\right\}}{1 + r_{2\text{ months}}}$$

$$= \frac{\left\{30\left[\frac{1}{.055} - \frac{1}{.055(1.055)^{59}}\right] + 30 + \frac{1{,}000}{(1.055)^{59}}\right\}}{1.018} = \$584.24.$$

To verify that this is correct, calculate the holding period return that this implies:

Return = $\frac{584.24 - 563.75}{563.75}$ = 0.036, which is the four-month return implied in a 11.3%:

EAR = $(1.113)^{\frac{4}{12}} - 1$ = 0.036.

Step 5. Determine the clean price.

This is the value (or cash price or dirty price) that was found in step 4, less the accrued interest. The accrued interest is (4/6) × \$30 = \$20, so the clean price is \$564.24.

3. The following table summarizes prices of zero-coupon U.S. Treasury securities per $100 of face value.

Maturity in Years	Price
1.	96.04
2.	93.35
3.	66.36
4.	79.21

[A] Plot the zero-coupon yield curve based on these bonds.
[B] Describe the shape of the yield curve.
[C] What is the value of a four-year 7% annual coupon Treasury bond with a face value of $1,000 that pays its first coupon in one year?

Step 1. First, the yield to maturity of each bond must be calculated.

Using the equation $P_0 = \frac{\$100}{(1+IRR)^N}$ and solving for IRR:

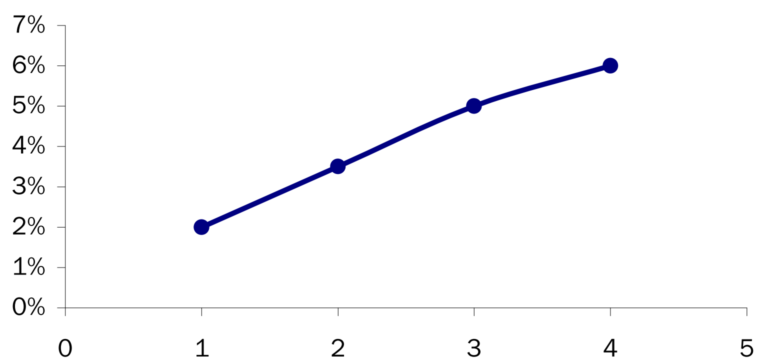
$$98.04 = \frac{100}{(1+IRR)^1} \Rightarrow IRR = \frac{100}{98.04} - 1 = 2.0\%$$

$$93.35 = \frac{100}{(1+IRR)^2} \Rightarrow IRR = \left(\frac{100}{93.35}\right)^{1/2} - 1 = 3.5\%$$

$$86.38 = \frac{100}{(1+IRR)^3} \Rightarrow IRR = \left(\frac{100}{86.38}\right)^{1/3} - 1 = 5.0\%$$

$$79.21 = \frac{100}{(1+IRR)^3} \Rightarrow IRR = \left(\frac{100}{79.21}\right)^{1/4} - 1 = 6.0\%$$

Step 2. Now, the zero-coupon Treasury yield curve can be drawn.



The yield curve is upward sloping. A yield curve with a positive slope is sometimes referred to as a normal yield curve.

Step 3. By the Law of One Price, the 4-year coupon bond must trade for the price it costs to replicate the payoffs using the zero-coupon bonds. The no-arbitrage price can be found by discounting its cash flows using the risk-free zero-coupon yields.

The annual coupon payment is 0.07($1,000) = $70, so

$$P_0 = \frac{70}{1.02} + \frac{70}{(1.035)^2} + \frac{70}{(1.05)^3} + \frac{70 + 1{,}000}{(1.06)^4}$$

$$= 68.63 + 65.35 + 60.47 + 847.54 = \$1{,}041.99.$$

Questions and Problems

1. Several major companies like Citigroup, Disney, and AT&T have issued Century Bonds. These bonds pay regular semiannual coupons, but do not mature until 100 years after they are issued. Some critics have stated that they are extremely risky because you can't predict what will happen to the companies in 100 years. Assume that such bonds were just issued with a $1,000 par value and an 6% semiannual coupon rate.
   [A] If current market rates are 6%, what is the present value of the principal repayment at maturity?
   [B] What is the total value today of the final 40 years (years 61–100) of payments, including coupons and principal?

2. Below is a quote from finance.yahoo.com for a Northrop Grumman bond. Assume it is March 2, 2006 and the $1,000 face value bond just paid a coupon payment yesterday.

Price	61.95
Coupon (%)	7.750
Maturity Date	1-March-2016
Debt Rating	BBB
Coupon Payment Frequency	Semiannual
First Coupon Date	1-Sept-1996
Type	Corporate
Industry	Industrial

   [A] What is the value of the bond if your required return is 6% APR?
   [B] If the bond were a zero-coupon bond and its only payment was the return of face value at maturity, what would the yield-to-maturity be at the price quoted?
   [C] Assume the bond was originally issued for a price of $1,000. In one sentence, explain something specifically could have happened in the economy or to the firm that could have made the bond sell for its current price.
   [D] The day before the bond matures and pays its last coupon payment, what will its value be?

3. Suppose that Ford has a B-rated bond with exactly 30 years until maturity, a face value of $1,000, and a semiannual coupon rate of 6%. The yield to maturity on B-rated bonds today is 10%.
   [A] What was the price of this bond today?
   [B] Assuming the yield to maturity remains constant, what is the price of the bond immediately before and after it makes its next coupon payment?

4. Suppose a ten-year, $1000 bond with a 9% coupon rate and semiannual coupons is trading for a price of $1,156.
   [A] What is the bond's yield to maturity (expressed as an APR with semiannual compounding)?
   [B] If the bond's yield to maturity changes to 12% APR, what will the bond's price be?

5. The following table summarizes the yields to maturity on several one-year, zero-coupon bonds:

Bond	% Yield
Treasury	4.1
AA corporate	4.6
BBB corporate	6.2
CCC corporate	10.5

   [A] What is the value of a one-year, $1,000 face value, zero-coupon corporate bond with a CCC rating?
   [B] What is the credit spread on AA-rated corporate bonds?
   [C] What is the credit spread on B-rated corporate bonds?

Solutions to Questions and Problems

1. [A] $\frac{1{,}000}{(1.04)^{200}}=\$0.39$

   [B] $$\frac{40\left[\frac{1}{.04}-\frac{1}{.04(1.04)^{80}}\right]}{(1.04)^{120}}+0.39=\frac{956.61}{110.66}=\$8.64+.39=\$9.03$$

2. [A] $$P_0=38.75\left[\frac{1}{.03}-\frac{1}{.03(1.03)^{20}}\right]+\frac{1{,}000}{(1.03)^{20}}=576.50+553.68=\$1{,}130.18$$

   [B] $$819.50=\frac{1{,}000}{(1+r)^{20}}\Rightarrow(1+r)^{20}=1.22\Rightarrow1+r=(1.22)^{\frac{1}{20}}=1.01\Rightarrow r=1\%$$

   So the YTM APR = 2(1%) = 2%.

   [C] Either the firm's default risk increased leading to a higher credit spread or rates in the economy increased leading to an increase in the risk-free interest rates.

   [D] $1,000 + 36.75 = $1,036.75.

3. [A] $$P=\$30\left[\frac{1}{.05}-\frac{1}{1.05(1.05)^{60}}\right]+\frac{1{,}000}{(1.05)^{60}}=567.88+53.54=\$621.42$$

   [B] Before the next coupon payment, the price of the bond is

   $$P=\$30\left[\frac{1}{.05}-\frac{1}{1.05(1.05)^{59}}\right]+\frac{1{,}000}{(1.05)^{59}}+30=566.27+56.21+30=\$652.48$$

   After the next coupon payment, the price of the bond will be

$$P=\$30\left[\frac{1}{.05}-\frac{1}{1.05(1.05)^{59}}\right]+\frac{1{,}000}{(1.05)^{59}}=566.27+56.21=\$622.48$$

4. [A] $P_0 = 45\left[\frac{1}{\frac{.12}{2}} - \frac{1}{\frac{.12}{2}(1+\frac{.12}{2})^{20}}\right]+\frac{1{,}000}{(1+\frac{.12}{2})^{20}} \Rightarrow P = \$827.95$

   [B] $P_0 = \$45\left[\frac{1}{\frac{.12}{2}} - \frac{1}{\frac{.12}{2}(1+\frac{.12}{2})^{20}}\right]+\frac{1{,}000}{(1+\frac{.12}{2})^{20}} \Rightarrow P = \$827.95$

5. [A] The price of this bond will be $P=\frac{1{,}000}{1.105}=\$904.98$

   [B] The credit spread on AA-rated corporate bonds is 0.046 – 0.041 = 0.7%

   [C] The credit spread on BBB-rated corporate bonds is 0.062 – 0.041 = 2.1%

STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*, but it also works with the Fourth Edition.

Pearson

CHAPTER 7

Investment Decision Rules

Chapter Synopsis

7.1 NPV and Stand-Alone Projects

The net present value (NPV) of a project is the difference between the present value of its benefits and the present value of its costs. Since a project's NPV represents its value in terms of cash today, the NPV investment rule, which states that all positive NPV projects should be accepted, is consistent with maximizing the value of the firm.

The internal rate of return (IRR) is the rate of return that makes the net present value of a stream of cash flows equal to zero. Thus, accepting projects with an IRR above the required return, or cost of capital, is generally equivalent to accepting projects with a positive NPV. The difference between the cost of capital and the IRR can be thought of as the maximum amount of estimation error in the cost of capital estimate that can exist without altering the original decision.

7.2 The Internal Rate of Return Rule

The IRR investment rule advises taking investment opportunities in which the IRR exceeds the opportunity cost of capital. The IRR rule will give the same answer as the NPV rule in many, but not all, applications. The following are cases in which the IRR rule may fail to reliably provide the correct decision.

- When the initial cash flow is positive and all later cash flows are negative, the IRR rule will provide the opposite decision provided by the NPV rule.
- In some cases, such as when there is no required investment for a project, the IRR does not exist.
- There may be multiple IRRs when the sign of the project's cash flows changes more than once, so the IRR rule cannot be relied upon.

While these are limitations to the usefulness of the IRR rule, the IRR itself remains a useful tool. Not only does the IRR measure the sensitivity of the NPV to estimation error in the cost of capital, but it also measures the average return of the investment.

7.3 The Payback Rule

The **payback investment rule** is based on the idea that an opportunity that pays back its initial investment quickly is a good idea. To apply the payback rule, you first calculate the amount of time it takes to pay back the initial investment, called the **payback period.** If the payback period is less than a pre-specified length of time, you accept the project. The payback rule is not a reliable method of determining if projects will increase the value of the firm since it does not consider the timing of a project's cash flows or cost of capital.

7.4 Choosing Between Projects

Sometimes a firm must choose among **mutually exclusive projects** in which only one of two or more projects being considered can be selected. In this case, the NPV rule advises picking the project with the highest NPV and provides the best answer.

Picking one project over another simply because it has a larger IRR can lead to errors.

- Because the IRR measures only the return of the investment opportunity, it does not depend on the scale of the investment opportunity. Hence the IRR rule cannot be used to compare projects of different scales because larger scale projects may be more valuable.
- Investment opportunities with the same NPV can have different IRRs because the IRR depends on the timing of the cash flows even when a change in timing does not affect the NPV. By altering the timing of the cash flows, it is possible to change the ranking of the IRRs of two mutually exclusive projects without changing either project's NPV.

The **incremental IRR investment rule** applies the IRR rule to the difference between the cash flows of the two mutually exclusive alternatives. For example, assume you are comparing two mutually exclusive opportunities, A and B, and the IRRs of both opportunities exceed the cost of capital. If you subtract the cash flows of opportunity B from the cash flows of opportunity A, then you should take opportunity A if the incremental IRR exceeds the cost of capital. Otherwise, you should take opportunity B. Although the incremental IRR rule often provides a reliable method for choosing among mutually exclusive projects, it can be difficult to apply correctly, and it is much simpler to just use the NPV rule.

7.5 Project Selection with Resource Constraints

Sometimes there is a fixed supply of capital, or other resources, so that all possible opportunities cannot be undertaken. The **profitability index** can be used to identify the optimal combination of projects to undertake in such situations, where:

$$\text{Profitability Index} = \frac{\text{NPV}}{\text{Resource Consumed}}$$

Projects should be selected in order of profitability index ranking starting with the project with the highest index and moving down the ranking until the resource is consumed. While this procedure generally leads to the most valuable combination of projects, the only guaranteed way to find the best combination of projects is to search through all of them. Linear programming techniques have been developed to solve this kind of problem.

Selected Concepts and Key Terms

Net Present Value (NPV) Investment Rule

Select all projects that have a positive net present value (NPV), where NPV is the difference between the present value of an investment's benefits and the present value of its costs. A project's NPV represents its value in terms of cash today. Choosing this alternative is equivalent to receiving its NPV in cash today, so positive NPV projects should be accepted. When choosing among mutually exclusive alternatives, the alternative with the highest NPV should be selected.

NPV profile

A graph of a project's NPV over a range of discount rates.

Internal Rate of Return (IRR) Investment Rule

Take investment opportunities in which the IRR exceeds the opportunity cost of capital. The **internal rate of return** (IRR) is the rate of return that makes the net present value of a stream of cash flows equal to zero. The IRR investment rule will give the same answer as the NPV rule in many, but not all, applications.

Mutually Exclusive Projects

A situation where only one of two or more projects being considered can be selected. In this case, the NPV rule provides the best answer: Pick the project with the highest NPV. Picking one project over another simply because it has a larger IRR can lead to errors.

Incremental IRR

The **incremental IRR investment rule** applies the IRR rule to the difference between the cash flows of the two mutually exclusive alternatives. Although the incremental IRR rule often provides a reliable method for choosing among mutually exclusive projects, it can be difficult to apply correctly, and it is much simpler to just use the NPV rule.

Profitability Index

The NPV of a project divided by the amount of a resource (such as capital) consumed. When there is a limited resource (such as capital), projects should be selected in order of profitability index ranking starting with the project with the highest index and moving down the ranking until the resource is consumed.

Payback Investment Rule

If the payback period is less than a pre-specified length of time, you accept the project. The **payback period** is the amount of time it takes to pay back the initial investment. The payback rule is not a reliable method of determining if projects will increase the value of the firm since it does not consider the timing of a project's cash flows or cost of capital.

Concept Check Questions and Answers

7.1.1. Explain the NPV rule for stand-alone projects.

The NPV rule for stand-alone projects states that when choosing among alternatives, we should take the project with the highest positive NPV.

7.1.2. What does the difference between the cost of capital and the IRR indicate?

In general, the difference between the cost of capital and the IRR is the maximum amount of estimation error in the cost of capital estimate that can exist without altering the original decision.

7.2.1. Under what conditions do the IRR rule and the NPV rule coincide for a stand-alone project?

The IRR rule is only guaranteed to work for a stand-alone project if all of the project's negative cash flows precede its positive cash flows. If this is not the case, the IRR rule can lead to incorrect decisions.

7.2.2. If the IRR rule and the NPV rule lead to different decisions for a stand-alone project, which should you follow? Why?

When investment rules conflict, you should follow the NPV rule because following the alternative rules means you are not taking a positive NPV project, and thus, you are not maximizing wealth. In these cases, the alternative rules lead to bad decisions.

7.3.1. Can the payback rule reject projects that have positive NPV? Can it accept projects that have negative NPV?

Yes, because the payback rule does not take into consideration the required rate of return and the exact timing of the cash flows.

7.3.2. If the payback rule does not give the same answer as the NPV rule, which rule should you follow? Why?

The NPV rule because it correctly accounts for the required rate of return and the exact timing of the cash flows while the payback rule does not.

7.4.1. For mutually exclusive projects, explain why picking one project over another because it has a larger IRR can lead to mistakes.

For mutually exclusive projects, picking one project over another because it has a larger IRR can lead to mistakes. Problems arise when projects have differences in scale (require different initial investments) and when they have different cash flow patterns.

7.4.2. What is the incremental IRR rule and what are its shortcomings?

The incremental IRR rule applies to the difference between the cash flows of two mutually exclusive projects. Suppose you compare two mutually exclusive projects, A and B, and the IRR of both projects exceeds the cost of capital. If you subtract the cash flows of project B from the cash flows of project A, then you should choose project A if the incremental IRR exceeds the cost of capital. Otherwise, choose project B.

7.5.1. Explain why ranking projects according to their NPV might not be optimal when you evaluate projects with different resource requirements.

When there is a fixed supply of the resource so that you cannot undertake all the mutually exclusive projects, choosing the highest NPV project may not lead to the best decision. The project that has the highest NPV may use up the entire resource. Therefore, it would be a mistake to take it. A combination of other projects may produce a combined NPV that exceeds the NPV of the best single project.

7.5.2. How can the profitability index be used to identify attractive projects when there are resource constraints?

Practitioners often use the profitability index to identify the optimal combination of projects to undertake because the profitability index measures the value created in terms of NPV per unit of resources consumed. After computing the profitability index, practitioners rank projects from the highest index down until the resource is used up.

Examples with Step-by-Step Solutions

Solving Problems

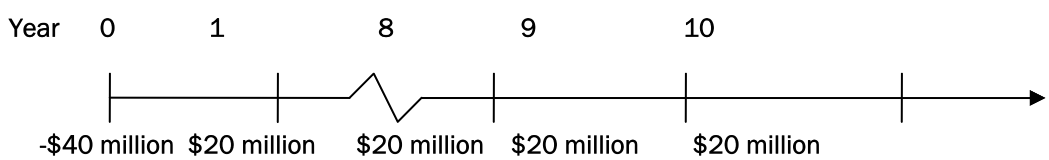
Problems using the concepts in this chapter generally involve determining the NPV or IRR for a simple project. The ability to evaluate mutually exclusive projects using the NPV rule may be asked as well. Finally, there may be applications involving selecting projects in the presence of a limited amount of capital (or some other resource) using the profitability index. The examples below demonstrate these three types of problems.

Examples

1. **Microsoft is considering moving 1,000 employees from a help-desk call center in Seattle to Bombay. The total after-tax cost of a Seattle worker is $50,000 per year and the total after-tax cost of a Bombay worker is $30,000 per year. The move would require paying an upfront severance package worth $40,000 after taxes per former Seattle employee. Assume for this analysis that the cost savings would last forever and that Microsoft's cost of capital is 20%.**
   [A] Should the project be accepted based on the NPV rule?
   [B] What is the IRR of the project?
   [C] Can the IRR be relied on in this application?

Step 1. Put the cash flows on a time line.

The time 0 cost is $40,000(1,000) = $40 million. The annual savings is $50,000(1,000) = $50 million, and the new annual cost is$30,000(1,000) = $30 million, so the annual net incremental cash flow is $20 million.



Step 2. Determine the NPV. Since the cash flows after time 0 are a perpetuity:

$$\text{NPV} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n} = \sum_{n=0}^{\infty} \frac{C_n}{(1.2)^n} = C_0 + \frac{C}{r} = -40{,}000{,}000 + \frac{20{,}000{,}000}{.2} = \$60 \text{ million}$$

Since the NPV > 0, the project should be accepted.

Step 3. Determine the IRR by setting the NPV equal to zero and solving for the rate.

$$\text{NPV} = 0 \Rightarrow \sum_{n=0}^{\infty} \frac{C_n}{(1+\text{IRR})^n} = -40{,}000{,}000 + \frac{20{,}000{,}000}{\text{IRR}} = 0 \Rightarrow \text{IRR} = \frac{20{,}000{,}000}{40{,}000{,}000} = 50\%$$

Since the IRR > 20%, the IRR rule says to accept the project as well.

Step 4. Determine if the IRR rule can be relied on.

The IRR rule can be relied on here because the cash flow at time 0 is negative and all future cash flows are positive. Also, the decision being made involves a stand-alone project, not mutually exclusive projects, in which case the IRR could not be relied in.

2. **Pulte Homes purchased 100 acres in suburban Los Angeles. They are considering the following development options:**

	NPV in millions	**Acres used**
Housing development A	$30	100
Housing development B	$24.5	70
Drug store	$3	3
Strip mall	$3.5	7
Golf course	$8	20

Which project(s) should the firm choose?

Step 1. Since the amount of land is a limited resource, calculate the profitability indices for each project relative to how much land they use.

$$\text{Profitabiliy Index} = \frac{\text{NPV}}{\text{Land Used}}$$

	Profitability Index	**Acres used**
Housing development A	.30	100
Housing development B	.35	70
Drug store	1.00	3
Strip mall	.50	7
Golf course	.40	20

Step 2. Rank the projects based on how much land they use:

	Rank	**Acres used**
Drug store	1	3
Strip mall	2	7
Golf course	3	20
Housing development B	4	70
Housing development A	5	100

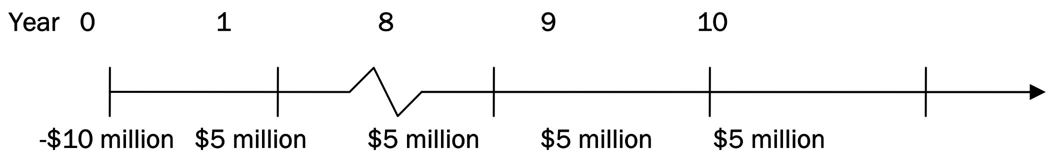
Step 3. Select the projects in descending order of profitability index until all the land is used.

Select the drug store, strip mall, golf course, and housing development A.

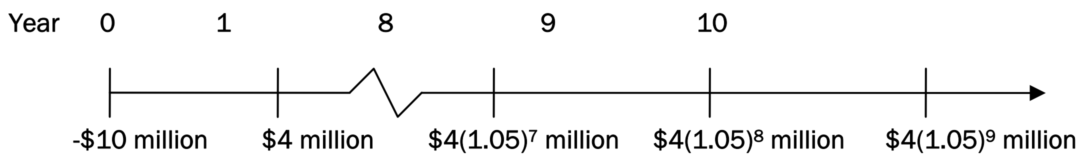
3. **You are deciding between two mutually exclusive investment opportunities. They both require the same initial investment of $10 million. Project X generates $5 million per year (starting at the end of the first year) in perpetuity. Project Y generates $4 million at the end of the first year and will grow at 5% per year for every year after that. The cost of capital is 10%.**
   - [A] **Which investment has the higher IRR?**
   - [B] **What project should be chosen?**
   - [C] **In this case, when does picking the higher IRR give the correct answer as to which investment is the best opportunity?**

Step 1. Put the cash flows of each project on a time line.

Project X



Project Y



Step 2. Calculate the IRR of each project.

Project X is a perpetuity and project Y is a growing perpetuity, so set the NPV of those valuation equations equal to zero.

$$NPV_A = 0 = -10{,}000{,}000 + \left(\frac{5{,}000{,}000}{IRR}\right) \Rightarrow IRR = \frac{5{,}000{,}000}{10{,}000{,}000} = 50\%$$

$$NPV_B = 0 = -10{,}000{,}000 + \left(\frac{4{,}000{,}000}{IRR\text{-}.05}\right) \Rightarrow IRR = \frac{4{,}000{,}000}{10{,}000{,}000} + .05 = 45\%$$

Step 3. Since the projects are mutually exclusive, the NPV must be calculated to determine which investment is better.

$$NPV_A = -10{,}000{,}000 + \left(\frac{5{,}000{,}000}{.10}\right) \Rightarrow NPV = \$40 \text{ million}$$

$$NPV_B = -10{,}000{,}000 + \left(\frac{4{,}000{,}000}{.10\text{-}.05}\right) \Rightarrow NPV = \$70 \text{ million}$$

Since the NPV of Project Y is much higher, it is the best project. In this case relying on the IRR rule would lead to the wrong conclusion since Project X has a higher IRR.

Questions and Problems

1. You own a gold mining company and are considering opening a new mine. The mine is expected to generate $10 million for the next 21 years. After 21 years, the gold is expected to be depleted, but the site can be sold for an expected $20 million. If the cost of capital is 8%, what is the most you should invest to open the mining operation at time 0?

2. You are considering opening a new hotel. The hotel will cost $150 million upfront and will be built immediately. It is expected to produce profits of $20 million every year forever. Calculate the NPV of this investment opportunity if your cost of capital is 10%. Should you make the investment? Calculate the IRR and use it to determine the maximum deviation allowable in the cost of capital estimate to leave the decision unchanged.

3. The Professional Golf Association (PGA) is considering developing a new PGA-branded golf ball. Development will take 3 years at a cost of $250,000 per year. Once in production, the ball is expected to make $250,000 per year for 5 years at which time new technology will make it obsolete. The cost of capital is 10%. Calculate the NPV of this investment opportunity. Should the PGA make the investment?

4. You are considering making a movie. The movie is expected to cost $100 million upfront and take a year to make. After that, it is expected to make $85 million in the first year it is released and $5 million for the following 20 years. What is the payback period of this investment? If you require a payback period of two years, will you make the movie? Does the NPV rule agree with the payback rule if the cost of capital is 10%?

5. Your corporation has $1 million to spend on capital investments this year and is evaluating four investments. The following table summarizes NPV and cost of these investments.

	NPV	Cost
1	$400,000	$400,000
2	$300,000	$200,000
3	$750,000	$400,000
4	$150,000	$700,000

Which project(s) should the firm choose?

Solutions to Questions and Problems

1. Using X as the initial investment:

$$\text{NPV} = X + \frac{10}{.08}\left(1 - \frac{1}{(1.08)^{21}}\right) + \frac{20}{(1.08)^{21}} = X + 100 + 4 = 0 \Rightarrow X = -\$104\text{ million.}$$

Thus, the most you should invest is $104 million.

2. $\text{NPV} = -150 + \dfrac{20}{.10} = -150 + 200 = \$50\text{ million,}$ so you should accept the project.

The IRR can be found by setting the NPV = 0:

$$\text{NPV} = -150 + \frac{20}{\text{IRR}} = 0 \Rightarrow \text{ IRR} = \frac{20}{150} = 13.3\%.$$

So the cost of capital can be underestimated by 3.3% without changing the decision.

3. $$\text{NPV} = \frac{-250{,}000}{.10}\left(1 - \frac{1}{(1.10)^3}\right) + \left(\frac{1}{(1.10)^3}\right)\frac{250{,}000}{.10}\left(1 - \frac{1}{(1.10)^5}\right)$$
$$= -621{,}713 + 712{,}019 = 90{,}036 > 0.$$

NPV > 0, so the company should take the project.

4. It will take 4 years to pay back the initial investment, so the payback period is 4 years. You will not make the movie.

$$\text{NPV} = -100 + \frac{85}{(1.10)^2} + \frac{5}{.10}\left(1 - \frac{1}{(1.10)^{20}}\right)\frac{1}{(1+r)^2}$$
$$= -100 + 70.2 + 42.6 = \$12.8 \text{ million} > 0.$$

So the NPV does not agree with the payback rule in this case.

5.

	Profitability Index	Cost
1	1.000	$400,000
2	1.500	$200,000
3	1.725	$400,000
4	0.250	$700,000

Select the projects in descending order of profitability index until all the money is used. They should select 3, 2, and 1.

STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*, but it also works with the Fourth Edition.

Pearson

CHAPTER 8

Fundamentals of Capital Budgeting

Chapter Synopsis

8.1 Forecasting Earnings

A firm's **capital budget** lists all of the projects that a firm plans to undertake during the next period. The selection of projects that should be included in the capital budget is called the **capital budgeting** decision. To evaluate a project, the project's future free cash flows must first be estimated. Some aspects of a project will affect the firm's revenues, while others will affect its costs.

The first step is generally to generate revenue and cost estimates and forecast expected incremental income statements for the project. For example, in the HomeNet project example in this chapter, the following income statements were forecasted:

		Year 0	1	2	3	4	5
Incremental Earnings Forecast ($000s)							
1	Sales	—	26,000	26,000	26,000	26,000	—
2	Cost of Goods Sold	—	(11,000)	(11,000)	(11,000)	(11,000)	—
3	**Gross Profit**	—	15,000	15,000	15,000	15,000	—
4	Selling, General, and Administrative	—	(2,800)	(2,800)	(2,800)	(2,800)	—
5	Research and Development	(15,000)	—	—	—	—	
6	Depreciation	—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	**EBIT**	(15,000)	10,700	10,700	10,700	10,700	(1,500)
8	Income Tax at 40%	6,000	(4,280)	(4,280)	(4,280)	(4,280)	600
9	**Unlevered Net Income**	**(9,000)**	**6,420**	**6,420**	**6,420**	**6,420**	**(900)**

- **Capital Expenditures and Depreciation.** Investments in plant, property, and equipment are not directly listed as expenses when calculating earnings. Instead, the firm deducts a fraction of the cost of these items each year as depreciation. Several different methods are used to compute depreciation. The simplest method is straight-line depreciation, in which the asset's cost is divided equally over its life.
- **Interest Expenses.** When evaluating a capital budgeting decision, interest expense is generally not included in the income statement. The effects of using debt financing, such

as incurring interest expense, is accounted for in the appropriate discount rate used to evaluate this project, the weighted average cost of capital, which is discussed in detail later in the text. Thus, the net income computed in Spreadsheet 8.1 is referred to as the **unlevered net income** of the project, indicating that it does not include any interest expenses associated with using debt financing.

- **Taxes.** The correct tax rate to use is the firm's marginal corporate tax rate, which is the tax rate it will pay on an incremental dollar of pre-tax income.

Project externalities are indirect effects of the project that may increase or decrease the cash flow of other business activities of the firm.

- The **opportunity cost** of using a resource is the value it could have provided in its best alternative use. Because this value is lost when the resource is used by another project, the opportunity cost should be included as an incremental cost of the project.
- A **sunk cost** is any cost that has been paid (such as past research and development expenses) or will be paid regardless of the decision whether to proceed with the project. Therefore, it is not incremental with respect to the current decision and should not be included in the analysis.
- When sales of a new product displace sales of an existing product, the situation is often referred to as **cannibalization.**
- **Overhead expenses** are associated with activities that are not directly attributable to a single business activity but instead affect many different areas of the corporation. To the extent that these overhead costs are fixed and will be incurred in any case, they are not incremental to the project and should not be included.

8.2 Determining Free Cash Flow and NPV

The incremental effect of a project on the firm's available cash is the project's **free cash flow** (FCF). It can be calculated as:

$$\text{Free Cash Flow (FCF)} = \text{EBIT} \times (1-\tau) + \text{Depreciation} - \text{Capital Expenditures} - \Delta\text{NWC}$$

- Since depreciation is not a cash expense (it is a method used for accounting and tax purposes to allocate the original purchase cost of the asset over its life), it should be added back to the unlevered net income. Depreciation does have an effect on FCF—it reduces taxes be $\text{Depreciation} \times (1-\tau)$, the **depreciation tax shield.**
- **Capital expenditures** are cash payments made to acquire fixed assets.
- **Net working capital** is the difference between current assets and current liabilities. The main components of net working capital are cash, inventory, accounts receivable, and accounts payable.

$$\begin{aligned}\text{Net Working Capital (NWC)} &= \text{Current Assets} - \text{Current Liabilities}\\ &\approx \text{Cash} + \text{Inventory} + \text{Accounts Receivable} - \text{Accounts Payable}\end{aligned}$$

Most projects will require the firm to invest in net working capital—often at a project's inception (time 0). The annual investment required investment is as follows.

$$\Delta NWC_t = NWC_t - NWC_{t\text{-}1}$$

While it is generally assumed that cash flows occur at annual intervals beginning in one year, in reality, cash flows will typically be spread throughout the year. Cash flows can also be forecasted on a quarterly, monthly, or even continuous basis when greater accuracy is required.

Because depreciation contributes positively to the firm's cash flow through the depreciation tax shield, the most accelerated method of depreciation that is allowable for tax purposes increases the value of a project. In the United States, the most accelerated depreciation method allowed by the IRS is Modified Accelerated Cost Recovery System (MACRS) depreciation. With MACRS depreciation, assets are categorized according to their asset class, and a corresponding MACRS depreciation tables assigns a fraction of the purchase price that the firm can depreciate each year.

For the HomeNet project considered in the chapter, FCFs were forecasted as follows:

	Year	0	1	2	3	4	5
Incremental Earnings Forecast ($000s)							
1	Sales	—	23,500	23,500	23,500	23,500	—
2	Cost of Goods Sold	—	(9,500)	(9,500)	(9,500)	(9,500)	—
3	**Gross Profit**	—	14,000	14,000	14,000	14,000	—
4	Selling, General, and Administrative	—	(3,000)	(3,000)	(3,000)	(3,000)	—
5	Research and Development	(15,000)	—	—	—	—	—
6	Depreciation	—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	**EBIT**	(15,000)	9,500	9,500	9,500	9,500	(1,500)
8	Income Tax at 40%	6,000	(3,800)	(3,800)	(3,800)	(3,800)	600
9	**Unlevered Net Income**	**(9,000)**	**5,700**	**5,700**	**5,700**	**5,700**	**(900)**
Free Cash Flow ($000s)							
10	Plus: Depreciation	—	1,500	1,500	1,500	1,500	1,500
11	Less: Capital Expenditures	(7,500)	—	—	—	—	—
12	Less: Increases in NWC	—	(2,100)	—	—	—	2,100
13	**Free Cash Flow**	**(16,500)**	**5,100**	**7,200**	**7,200**	**7,200**	**2,700**

Once the FCFs over the life of a project have been determined, the NPV can be calculated as:

$$NPV = FCF_0 + \frac{FCF_1}{(1+r)^1} + \frac{FCF_2}{(1+r)^2} + \cdots + \frac{FCF_T}{(1+r)^T} = \sum_{t=0}^{T} \frac{FCF_t}{(1+r)^t}.$$

For the HomeNet example in the chapter, the NPV was found to be positive; it was calculated in the following spreadsheet.

	Year	0	1	2	3	4	5
Net Working Capital Forecast ($000s)							
1	Cash Requirements	—	—	—	—	—	—
2	Inventory	—	—	—	—	—	—
3	Receivables (15% of Sales)	—	3,525	3,525	3,525	3,525	—
4	Payables (15% of COGS)	—	(1,425)	(1,425)	(1,425)	(1,425)	—
5	**Net Working Capital**	**—**	**2,100**	**2,100**	**2,100**	**2,100**	**—**

8.3 Choosing Among Alternatives

In many situations, you must compare mutually exclusive alternatives, each of which has consequences for the firm's cash flows. In such cases, you can make the best decision by first computing the free cash flow associated with each alternative and then choosing the alternative with the highest NPV.

8.4 Further Adjustments to Free Cash Flow

A number of complications can arise when estimating a project's free cash flow, such as non-cash charges, alternative depreciation methods, liquidation or continuation values, and tax loss carryforwards.

Other non-cash items that appear as part of incremental earnings should not be included in the project's free cash flow. The firm should include only actual cash revenues or expenses.

For example, the firm adds back any amortization of intangible assets (such as patents) to unlevered net income when calculating free cash flow.

Because depreciation contributes positively to the firm's cash flow through the depreciation tax shield, it is in the firm's best interest to use the most accelerated method of depreciation that is allowable for tax purposes. By doing so, the firm will accelerate its tax savings and increase its present value. In the United States, the most accelerated depreciation method allowed by the IRS is MACRS (Modified Accelerated Cost Recovery System) depreciation. With MACRS depreciation, the firm first categorizes assets according to their recovery period. Based on the recovery period, MACRS depreciation tables assign a fraction of the purchase price that the firm can recover each year.

Assets that are no longer needed often have a resale value or some salvage value if the parts are sold for scrap. When an asset is liquidated, any gain on sale is taxed. The gain on sale is the difference between the sale price and the book value of the asset. The book value is equal to the asset's original cost less the amount it has already been depreciated for tax purposes. You must adjust the project's free cash flow to account for the after-tax cash flow that would result from an asset sale as:

After-Tax Cash Flow from Asset Sale = Sale Price – (t_c x Gain on Sale).

Sometimes the firm explicitly forecasts free cash flow over a shorter horizon than the full horizon of the project or investment. This is necessarily true for investments with an indefinite life, such as an expansion of the firm. In this case, we estimate the value of the remaining free cash flow beyond the forecast horizon by including an additional, one-time cash flow at the end of the forecast horizon called the **terminal** or **continuation value** of the project. This amount represents the market value (as of the last forecast period) of the free cash flow from the project at all future dates. For example, when analyzing investments with long lives, it is common to explicitly calculate free cash flow over a short horizon, and then assume that cash flows grow at some constant rate beyond the forecast horizon.

Since 1998, companies can utilize **tax loss carrybacks** from the last two years to offset taxable income in the current year. They can also utilize **tax loss carryforwards** and use losses in the current year to reduce taxable income for up to 20 years in the future.

8.5 Analyzing the Project

Sensitivity analysis shows how the NPV varies when changing one variable. **Scenario** analysis considers the effect on the NPV of changing multiple project variables together. As part of a project analysis it is useful to perform **break-even analysis** by studying how far a variable can be changed until the project's NPV is 0.

Selected Concepts and Key Terms

Capital Budgeting, Capital Budget

The capital budget lists all of the projects that a firm plans to undertake during the next period. The selection of projects that should be included in the capital budget is called the capital budgeting decision.

Unlevered Net Income

When evaluating a capital budgeting decision, interest expense is generally not included in the income statement, and the effects of using debt financing, such as incurring interest expense, is accounted for in the appropriate discount rate used to evaluate this project, the weighted average cost of capital, discussed in detail later. Thus, the net income does not include interest expense, and it is referred to as the unlevered net income.

Marginal Corporate Tax Rate

The tax rate a firm will pay on the next incremental dollar of pre-tax income.

Cannibalization

When sales of a new product displace sales of an existing product.

Opportunity Cost

The value an asset could provide in its best alternative use. Because this value is lost when the resource is used by another project, it should be included as an incremental cost of the project.

Sunk Cost

Any unrecoverable cost for which the firm is already liable. Sunk costs have been or will be paid regardless of the decision whether to proceed with the project, and therefore they should not be included in the analysis of the project.

Free Cash Flow

The periodic incremental effect of a project on the firm's available cash. It can generally be calculated as operating cash flow minus capital spending minus the increase in net working capital.

Trade Credit

The difference between accounts receivable and accounts payable; it is the net amount of the firm's capital that is used as a result of credit transactions.

Depreciation Tax Shield

The tax savings that results from the ability to deduct depreciation. It generally equals depreciation expense × tax rate.

Modified Accelerated Cost Recovery System (MACRS) Depreciation

The most accelerated depreciation method allowed by the Internal Revenue Service. Assets are categorized according to their asset class and a corresponding MACRS depreciation tables assigns a fraction of the purchase price that the firm can depreciate each year.

Terminal Value, Continuation Value

The present value (as of the last forecast period) of the free cash flow from the project at all future dates after the last forecast period.

Break-Even Analysis

Studying how far a variable can be changed until the project's NPV is 0.

Sensitivity Analysis

An analysis of how the NPV of a project varies when changing one variable.

Scenario Analysis

An analysis of how the NPV of a project varies when changing multiple project variables together.

Concept Check Questions and Answers

8.1.1. How do we forecast unlevered net income?

Interest and other financing-related expenses are excluded from the forecasted income statements to determine a project's unlevered net income.

8.1.2. Should we include sunk costs in the cash flows of a project? Why or why not?

We should not include sunk costs in the cash flows of a project because sunk costs must be paid regardless of whether or not the firm decides to proceed with the project. Sunk costs are not incremental with respect to the current decision.

8.1.3. Explain why you must include the opportunity cost of using a resource as an incremental cost of a project.

We must include the opportunity cost of using a resource as an incremental cost of a project because that resource can be used in the next-best alternative way. It is a mistake to assume that the resource is free.

8.2.1. What adjustments must you make to a project's unlevered net income to determine its free cash flows?

You must add depreciation back (because it is a non-cash expense) and subtract capital spending and the change in working capital.

8.2.2. What is the depreciation tax shield?

The depreciation tax shield is the reduction in tax expense from the ability to deduct depreciation expense before determining taxable income.

8.3.1. How do you choose between mutually exclusive capital budgeting decisions?

You can make the best decision by first computing the free cash flows and NPVs of each alternative and then choosing the alternative with the highest NPV.

8.3.2. When choosing between alternatives, what cash flows can be ignored?

Components of free cash flow that are the same in each alternative can be ignored.

8.4.1. Explain why it is advantageous for a firm to use the most accelerated depreciation schedule possible for tax purposes.

Because depreciation contributes positively to the firm's cash flow through the depreciation tax shield, it is in the firm's best interest to use the most accelerated method of depreciation that is allowable for tax purposes. By doing so, the firm will accelerate its tax savings and increase its present value.

8.4.2. What is the continuation or terminal value of a project?

The continuation or terminal value of a project is the estimated value of the remaining free cash flow beyond the forecast horizon of the project. This amount represents the market value (as of the last forecast period) of the free cash flow from the project at all future dates.

8.5.1. What is sensitivity analysis?

Sensitivity analysis breaks the NPV calculation into its component assumptions and shows how the NPV varies as the underlying assumptions change. In this way, sensitivity analysis allows us to explore the impact of errors in NPV estimates for the project.

8.5.2. How does scenario analysis differ from sensitivity analysis?

Sensitivity analysis changes one parameter at a time. Scenario analysis changes the effect on NPV of changing multiple project parameters simultaneously.

Examples with Step-by-Step Solutions

Solving Problems

Problems using the concepts in this chapter generally involve finding the NPV of potential projects given a cost of capital. The NPV is the present value of a project's free cash flow from time 0 to the end of the project. This requires forecasting income statements over the life of the project, calculating free cash flow = EBIT(1 – t) + depreciation – capital expenditures – the increase in net working capital each year, and calculating the NPV. You may also need to determine a project's IRR, which is the discount rate that makes the NPV of the project's FCFs equal to $0.

Examples

1. Your firm owns a Volkswagen dealership, and you are considering entering into a 5-year agreement to also sell Audi A4s. The cars would cost $26,000, and you believe that you can sell 50 Audis per year at an average price of $30,000. You would have to hire 2 new sales people that you would pay $30,000 per year each plus 5% of the revenue they each generate. Audi would require that you invest $200,000 (depreciable straight line over 5 years) in Audi-related signs, equipment, and furniture to place in your dealership. You would also be required to invest in 20 cars to keep in inventory over the life of the project. After 5 years, you can recover any investment in working capital, and the unneeded equipment would have a market value of $50,000. Your firm requires a 12% return on all new investments, and the tax rate is 40%. Should you accept the project? Show your work and justify your answer.

Step 1. Determine how you should make the decision.

The project's NPV will indicate whether the project will add value to your firm, so the NPV should be calculated.

Step 2. Determine the income statements for years 1 through 5 of the project.

Since there is straight line depreciation on the $200,000 investment in capital assets, annual depreciation is $200,000 / 5 = $40,000.

Each year will be the same.

Sales	1,500,000
Cost of goods sold	1,300,000
Gross profit	200,000
Selling, general & admin. costs	
Salary	60,000
Commission	85,000
EBITDA	65,000
Depreciation	40,000
EBIT	25,000
–Tax @ 40%	10,000
Net Income	$15,000

Step 3. Determine the Free Cash Flows for years 0 through 5.

Operating cash flow equals net income + depreciation each year, which is $15,000 + $40,000 = $55,000.

Capital spending is $200,000 at time 0.

At the end of year 5, the equipment can be sold for $50,000, resulting in an after-tax cash flow = sale price – $\tau \times$(sale price – book value) = $50,000 – 0.40($50,000 – 0) = $30,000.

The time 0 investment in working capital is 20 cars at $26,000, or $520,000.

At the end of the project, the investment in working capital could be recovered. The year 5 income statement's cost of goods sold would be overstated by $520,000, since you effectively bought 20 of the cars you sold in year 5 at time 0, so the decrease in working capital is a $520,000 cash inflow.

	0	1	2	3	4	5
Operating cash flow	–	55,000	55,000	55,000	55,000	55,000
– Capital expenditures	200,000	0	0	0	0	–30,000
– Increases in working capital	520,000	0	0	0	0	–520,000
Free Cash Flow	–820,000	55,000	55,000	55,000	55,000	605,000

Step 4. Calculate the project's NPV.

$$\text{NPV} = -720{,}000 + \frac{55{,}000}{(1.12)} + \frac{55{,}000}{(1.12)^2} + \frac{55{,}000}{(1.12)^3} + \frac{55{,}000}{(1.12)^4}$$
$$+ \frac{55{,}000 + 30{,}000 + 520{,}000}{(1.12)^5} = -\$209{,}653 < 0$$

Since the NPV is less than zero, the project cannot be justified given the forecasts. In other words, the project costs $820,000 and is only worth about $510,348 (820,000 – 209,653). This implies that the IRR is less than 12% and can be calculated to be about 3%.

2. **Calaveras Vineyards, a highly profitable wine producer, is considering the purchase of 10,000 French oak barrels at a cost of $900 each, or $9 million for all of them. The barrels would be considered a capital expense and would be depreciated straight line over 5 years. After 4 years, the barrels will be useless for making fine wine, but they expect to be able to sell them for $3 million to Gallo. The increase in the quality of its zinfandel line of wines due to the use of the new French oak barrels is expected to increase revenue by $8 million in**

years 3 and 4. The barrels would have no influence on COGS, SG&A, other operating expenses, or working capital. The tax rate is 40%, and the required return is 15%.

[A] According to the NPV rule, is the purchase of the barrels a good idea?

[B] According to the IRR rule, is the purchase of the barrels a good idea?

Step 1. In order to calculate the NPV and IRR, the FCFs over the life of the project (years 0 through 4) need to be calculated. Thus, the each year's income statement must be determined.

Step 2. Determine the income statements from years 1 through 4 for the project.

	1	2	3	4
Sales	0	0	8,000,000	8,000,000
– Cost of goods sold	0	0	0	0
– Selling, general & admin. costs	0	0	0	0
EBITDA	0	0	8,000,000	8,000,000
– Depreciation	1,800,000	1,800,000	1,800,000	1,800,000
EBIT	–1,800,000	–1,800,000	6,200,000	6,200,000
– Tax	–720,000	–720,000	2,480,000	2,480,000
Net income	–1,080,000	–1,080,000	3,720,000	3,720,000

Step 3. Determine operating cash flow in years 1 through 4.

	0	1	2	3	4
Net income	–1,080,000	–1,080,000		3,720,000	3,720,000
+ Depreciation	1,800,000	1,800,000		1,800,000	1,800,000
= Operating cash flow	720,000	720,000		5,520,000	5,520,000

Step 4. Determine the Free Cash Flows for years 0 through 4.

	0	1	2	3	4
Operating cash flow	0	720,000	720,000	5,520,000	5,520,000
– Capital expenditures	9,000,000	0	0	0	2,520,000
– Increase in working capital	0	0	0	0	0
Free Cash Flow	–9,000,000	720,000	720,000	5,520,000	8,040,000

Since the book value of the barrels is $1.8 million after 5 years, the after-tax cash flow from selling the barrels in year 5 is = sale price – $\tau \times$(sale price – book value) = $3 million – 0.40($3 million – 1.8 million) = $2.52 million.

Step 5. Calculate the project's NPV.

$$NPV = -9{,}000{,}000 + \frac{720{,}000}{(1.15)} + \frac{720{,}000}{(1.15)^2} + \frac{5{,}520{,}000}{(1.15)^3} + \frac{8{,}040{,}000}{(1.15)^4} = \$396{,}896 > 0$$

Type equation here.

Since the NPV greater than zero, the project can be justified given the forecasts.

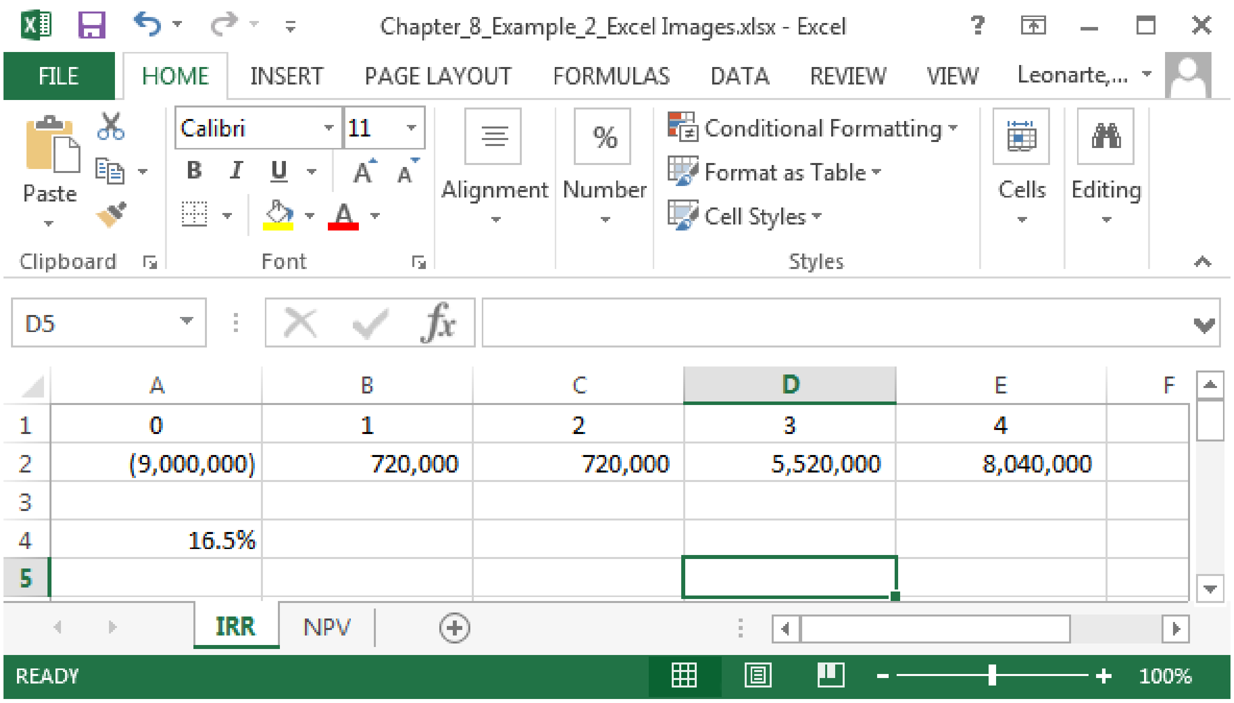
Step 6. Calculate the project's IRR.

$$NPV = -9{,}000{,}000 + \frac{720{,}000}{(1+IRR)} + \frac{720{,}000}{(1+IRR)^2} + \frac{5{,}520{,}000}{(1+IRR)^3} + \frac{8{,}040{,}000}{(1+IRR)^4} = 0$$

$$\rightarrow IRR = 16.5\%$$

The IRR is more than 15%, so the project can be justified based on the forecasts.

Note that the determination of IRR requires iteration, i.e., trying various IRRs until the NPV=$0. This process is best performed using the IRR function in a spreadsheet as shown below.



3. Your firm has a vacant warehouse in Louisiana that has a market value of $15 million and a book value of $0. You are considering entering into a 10-year contract to become the exclusive Coca-Cola bottler for your region. You would need to purchase $10 million worth of equipment, which you would depreciate straight line over 5 years; after 10 years, the equipment would have a market value of $2 million. You would also need to invest $2 million in working capital, which you can recover at the end of the project. You would sell $5 million worth of Coke products each year. Total costs (excluding taxes and depreciation) would be 60% of sales per year. You anticipate that the value of the warehouse will increase by about 2% per year and be worth $18 million in 10 years. The tax rate is 40% and the firm's cost of capital is 12%. Does the project satisfy your investment criteria? Show your work and justify your answer.

Step 1. Determine how you should make the decision.

The project's NPV will indicate whether the project will add value to your firm, so the NPV should be calculated.

Step 2. Determine the income statements from years 1 through 10 for the project along with the operating cash flows.

	1-5	6-10
Sales	5,000,000	5,000,000
Cost of goods sold + SG&A	3,000,000	3,000,000
EBITDA	2,000,000	2,000,000
Depreciation	2,000,000	0
EBIT	0	2,000,000
– Tax @ 40%	0	800,000
Net Income	0	1,200,000
+ Depreciation	2,000,000	0
= Operating cash flow	2,000,000	1,200,000

Note that depreciation =\$10,000,000/5 = \$2,000,000 for years 1–5 and \$0 in years 6–10.

Step 3. Calculate the Free Cash Flows.

Free Cash Flows for years 0 through 10 are:

	0	1-5	6-9	10
Operating cash flow	0	2,000,000	1,200,000	1,200,000
– Capital expenditures	19,000,000	0	0	–12,000,000
– Increase in working capital	2,000,000	0	0	–2,000,000
Free Cash Flow	–21,000,000	2,000,000	1,200,000	15,200,000

Time 0 capital spending includes the \$10 million of equipment that would need to be purchased. There is also an opportunity cost associated with using the warehouse. The after-tax cash flow = sale price – τ × (sale price – book value) = \$15 million – 0.40(\$15 million – 0) = \$9 million. Thus, capital spending is \$19 million.

NWC will increase by \$2 million at time 0, and it will decrease by \$2 million in year 10.

Step 4. Calculate the NPV.

$$\text{NPV} = -21,000,000 + 2,000,000\left[\frac{1}{.12} - \frac{1}{.12(1.12)^5}\right] + \frac{1,200,000\left[\frac{1}{.12} - \frac{1}{.12(1.12)^4}\right]}{1.12^5} + \frac{15,200,000}{1.12^{10}}$$

$$= -\$6,828,286 < 0$$

Since the NPV is above zero, the project cannot be justified given the forecasts, and it should be rejected.

Questions and Problems

1. Your large, highly profitable golf course management firm owns 200 acres in Surprise, Arizona. The land is surrounded by a housing development and is zoned exclusively for a golf course. The non-depreciable land has increased in value over the year since you bought it from \$10 million to \$35 million, and someone has offered to buy it for this price. Your original plan was to develop the land over the next 3 years by spending \$20 million per year in pre-tax development costs. You would also have to spend \$30 million in capital equipment today, and this would be depreciated straight line over 3 years. Based on the performance of the other courses you own, you expect annual revenue from the course to be \$40 million when it opens 3 years from today and that all operating costs (excluding tax and depreciation) will amount to 50% of revenue. From then on, you expect that the free cash flow the course generates will grow by 3% forever. The tax rate is 40%, and you require a 15% return.
2. You are considering the purchase of super-automatic espresso machines to replace the existing manually operated machines in your chain of 1,000 coffee shops around the country. Each machine has a cost of \$10,000, and you would have to buy 1,000. You could then sell the 1,000 existing La Marzocco machines in the stores now for \$2,000 each. The old machines have zero book values. The new machines would have no effect on revenues, but you could save an estimated \$3,000 per store per year in labor and training costs since operating a super-automatic espresso machine is easier. The new machines would have a 10-year depreciable life and be worthless after 10 years. You require a 15% return on all investments and are taxed at 40%. Does the project satisfy your investment criteria? Show your work and justify your answer.
3. You are considering the purchase of 1,000 Coke machines in the greater Chicago area. The machines cost \$2,500 each and are depreciable straight-line over 5 years. Sales are expected to be 3,000 bottles per machine in the first year at a selling price of \$1 per bottle. Sales revenue is expected to be constant every year thereafter. The cost of each bottle is \$0.30. Operating expenses include stocking and maintenance and are expected to amount to \$1,000 per year per machine. You would have to stock each machine with 200 bottles at the beginning of the project. After 5 years, you plan to sell the machines for \$1,000 each and recover any investment in working capital. The tax rate is 40%. The firm uses all equity financing, and stockholders require a 15% return. Determine whether the project is a good idea. Does the project have an IRR above or below 15%?
4. Your firm manufactures custom-labeled, purified bottled water. Your plant generates \$10 million in annual sales and runs at full capacity. You currently have four full-time employees who are paid \$50,000 each per year and are responsible for removing the bottles from the manufacturing line and packaging them in boxes for delivery. You are considering replacing these four employees with a packaging machine that will do their same jobs. The machine costs \$900,000 and would be depreciable straight line over 4 years. The purchase price includes a full warranty that guarantees to keep the machine in working order for 6 years. After 6 years, you would sell the machine back for \$100,000. You require a 15% return on investments and the tax rate is 40%. Should you buy the machine?
5. 3com is considering producing a new handheld, wireless internet device. Management spent \$3 million last year on test marketing and has developed a set of forecasts. Total cash costs (COGS, SG&A, etc…) of the device will be \$30 each, and they will sell them all for \$100 each. They can produce 50,000 each year for the next five years, and they expect to sell them all each year. They would have to construct a manufacturing plant, which would cost \$10 million to be constructed immediately and be depreciable over 10 years using straight-line depreciation. They would have to invest \$2 million in inventory beginning today, and this amount

would not change over the life of the project. In 5 years, they will quit, dispose of the plant for \$1 million, and recover working capital. The tax rate is 40%, the firm uses stock financing, and stockholders require a 15% return. Should 3com accept the project? Show any needed calculations and justify the answer.

Solutions to Questions and Problems

1. The project's NPV will indicate whether the project is worth more than the value of just selling the land, so it should be calculated.

   Determine the income statements from years 1 through 4 for the project along with the operating cash flows.

	1-3	4
Sales	0	40,000,000
Cost of goods sold + SG&A	20,000,000	20,000,000
EBITDA	–20,000,000	20,000,000
Depreciation	10,000,000	0
EBIT	–30,000,000	20,000,000
– Tax @ 40%	–12,000,000	8,000,000
Net Income	–18,000,000	12,000,000
+ Depreciation	10,000,000	0
= Operating cash flow	-8,000,000	12,000,000

Depreciation =\$30,000,000 / 3 = \$10,000,000 for years 1–3 and \$0 in year 4 and on.

Calculate the Free Cash Flows for years 0–4:

	0	1-3	4
Operating cash flow	--	–8,000,000	12,000,000
– Capital expenditures	30,000,000	0	0
– Increases in working capital	0	0	0
Free Cash Flow	–30,000,000	–8,000,000	12,000,000

Calculate the NPV.

$$\text{NPV} = -30 + \frac{-8}{1.15} + \frac{-8}{1.15^2} + \frac{-8}{1.15^3} + \frac{\left(\frac{12}{.15-.03}\right)}{1.15^3} = \$17.5 \text{ million}$$

You would generate 35 – (35 – 10).4 = \$25 million > \$18.5 million by selling the land, so you should sell.

2 . The project's NPV or IRR will indicate whether the project will add value to your firm, so at least the NPV should be calculated.

Determine the income statements from years 1 through 10 for the project.

Each year will be the same.

Sales	0
Cost of goods sold	0
Gross profit	0
Selling, general & admin. costs	–3,000,000
EBITDA	3,000,000
Depreciation	1,000,000
EBIT	2,000,000
–Tax @ 40%	800,000
Net Income	$1,200,000

Since there is straight line depreciation on the $10 million investment in capital assets, annual depreciation is $10 millon/10 = $1 million.

The Free Cash Flows for years 0 through 10 are:

Year	0	1-10
Operating cash flow	--	2,200,000
Capital expenditures	–8,800,000	0
Increases in working capital	0	0
Free Cash Flow	–8,800,000	2,200,000

Operating cash flow equals net income + depreciation each year, which is $1,200,000 + $100,000 = $2,200,000.

Capital spending on the new machines is 1,000($10,000) = $10 million at time 0. You can also sell the old machines for $2,000 each resulting in an after-tax cash flow = sale price – $\tau \times$ (sale price – book value) = $2,000 – 0.40($2,000 – 0) = $1,200 each or $1,200(1,000) = $1.2 million for all of them. Thus, capital spending is $8.8 million.

Calculate the project's NPV.

$$\text{NPV} = -8{,}800{,}000 + 2{,}200{,}000\left[\frac{1}{.15} - \frac{1}{.15(1.15)^{10}}\right] = 2{,}241{,}291 > 0$$

Since the NPV is above zero, the project can be justified given the forecasts. This implies that the IRR is above 15%, and can be calculated to be about 21%.

3. If the NPV is greater than 0, the project is acceptable. Once the NPV is calculated, it can be determined if the IRR is above or below 15%.

Determine the income statements from years 1 through 5 for the project (each year is the same) and calculate operating cash flow each year.

Sales	3,000,000
Cost of goods sold	900,000
Gross profit	2,100,000
Selling, general & admin. Costs	1,000,000
EBITDA	1,100,000
Depreciation	500,000
EBIT	600,000

– Tax @ 40%	240,000
Net Income	360,000
+ Depreciation	500,000
= Operating cash flow	860,000

Sales = 3,000(1,000)$1 = $3 million.

Cost of goods sold is 0.30($1)/$1 = 30% of sales each year.

SG&A is $1,000 per machine, or 1,000($1,000) = $1,000,000 for all of them.

Depreciation = $2,500(1,000)/5 = $500,000 each year.

Free Cash Flows for years 0 through 5.

Year	0	1-4	5
Operating cash flow	--	860,000	860,000
– Capital expenditures	2,500,000	0	–600,000
– Increases in working capital	60,000	0	–60,000
Free Cash Flow	–2,560,000	860,000	1,520,000

The after-tax cash flow = sale price – $\tau \times$(sale price – book value) = $1 million – 0.40($1 million – 0) = $600,000.

Now, the NPV can be calculated.

$$NPV = -2{,}560{,}000 + 860{,}000\left[\frac{1}{.15} - \frac{1}{.15(1.15)^4}\right] + \frac{1{,}520{,}000}{1.15^5} = 650{,}990 > 0$$

Since the NPV is above zero, the project can be justified given the forecasts. This implies that the IRR is above 15%, and it can be calculated to be about 24%.

4. The project's NPV will indicate whether the project will add value to your firm, so the NPV should be calculated.

Determine the income statements in years 1 through 6 for the project along with the operating cash flows.

	1-4	5-6
Sales	0	0
Cost of goods sold + SG&A	–200,000	–200,000
EBITDA	–200,000	200,000
Depreciation	225,000	0
EBIT	–25,000	200,000
– Tax @ 40%	–10,000	80,000
Net Income	–15,000	120,000
+ Depreciation	225,000	0
= Operating cash flow	210,000	120,000

Depreciation =$900,000 / 4 = $225,000 for years 1–4 and $0 in years 5–6.

Free Cash Flows for years 0 through 6 are:

	0	1-4	5	6
Operating cash flow	--	210,000	120,000	120,000
– Capital expenditures	900,000	0	0	–60,000
– Increases in working capital	0	0	0	0
Free Cash Flow	–900,000	210,000	120,000	180,000

The after-tax cash flow in year 6 from selling the machine = sale price – $\tau\times$(sale price – book value) = \$100,000 – 0.40(\$100,000 – 0) = \$60,000.

Calculate the NPV.

$$\text{NPV}=-900{,}000+\frac{210{,}000}{(1.15)}+\frac{210{,}000}{(1.15)^2}+\frac{210{,}000}{(1.15)^3}+\frac{210{,}000}{(1.15)^4}+\frac{120{,}000}{(1.15)^5}+\frac{120{,}000+60{,}000}{(1.15)^6}$$

$=-\$162{,}974$, so the firm is worth more with the employees given these assumptions.

5. If the NPV is greater than 0, the project is acceptable.

   Determine the income statements and operating cash flow in years 1 through 5.

Sales	5,000,000
Cost of goods sold	1,500,000
Gross profit	3,500,000
Selling, general & admin. Costs	0
EBITDA	3,500,000
Depreciation	1,000,000
EBIT	2,500,000
– Tax @ 40%	1,000,000
Net Income	1,500,000
+ Depreciation	1,000,000
= Operating cash flow	2,500,000

Free Cash Flows for years 0 through 5 are:

Year	0	1–4	5
Operating cash flow	--	2,500,000	2,500,000
– Capital expenditures	10,000,000	0	–2,600,000
– Increases in working capital	2,000,000	0	–2,000,000
Free Cash Flow	–12,000,000	2,500,000	8,100,000

The after-tax cash flow = sale price – τ × (sale price – book value) = \$1 million – 0.40(\$1 million – \$5 million) = \$2.6 million.

Now, the NPV can be calculated.

$$\text{NPV} = -12{,}000{,}000 + 2{,}500{,}000\left[\frac{1}{.15}-\frac{1}{.15(1.15)^4}\right] + \frac{7{,}100{,}000}{1.15^5} = -\$1{,}332{,}599 < 0$$

Since the NPV is below zero, the project cannot be justified given the forecasts. This implies that the IRR is below 15%, and it can be calculated to be about 11%.