

# STUDY GUIDE

to accompany

Berk/DeMarzo: *Corporate Finance*

This Study Guide was originally created for the Third Edition of *Corporate Finance*,  
but it also works with the Fourth Edition.



Pearson



# CHAPTER 6

## Valuing Bonds

### Chapter Synopsis

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#### 6.1 Bond Cash Flows, Prices, and Yields

A bond is a security sold at **face value** (FV), usually \$1,000, to investors by governments and corporations. Bonds generally obligate the borrower to make a promised future repayment of face value at the **maturity date** along with interest payments called **coupon payments** that are typically paid semiannually. The amount of each coupon payment is determined by the **coupon rate** of the bond. By convention, the coupon rate is expressed as an APR, so the amount of each coupon payment is:

$$CPN = \frac{\text{Coupon Rate} \times \text{Face value}}{\text{Number of Coupon Payments Per Year}}$$

For example, a “10-year, \$1,000 face value bond with a 10% semiannual coupon rate” will pay coupon payments of  $\$1000 \times 0.10/2 = \$50$  every six months and repay the face value, or **principal**, in 10 years. The terms of the bond are described as part of the bond certificate, which indicates the amounts and dates of all payments to be made.

A **zero-coupon bond**’s only payment is the face value of the bond on the maturity date—it does not make coupon payments. Treasury bills, which are U.S. government bonds with a maturity of up to one year, are zero-coupon bonds and can be valued easily using the present value of a cash flow equation. For example, a one-year, risk-free, zero-coupon bond with a \$1,000 face value with a required return of 3.5% is worth:

$$PV = \frac{\$1,000}{1.035} = \$966.18.$$

Zero-coupon bonds always trade at a **discount** (a price lower than the face value) and are sometimes referred to as **pure discount bonds**.

The **yield to maturity** (YTM) of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. For the zero-coupon

bond above, the YTM is the return an investor will earn from holding the bond to maturity and can be calculated as:

$$\$966.18 = \frac{\$1,000}{1 + \text{YTM}} = \$966.18 \Rightarrow \text{YTM} = \frac{\$1,000}{\$966.18} - 1 = 0.035 = 3.5\%.$$

In general, the YTM for a zero coupon bond can be calculated as:

$$P = \frac{FV}{(1 + \text{YTM})^n} \Rightarrow \text{YTM} = \left( \frac{FV}{P} \right)^{\frac{1}{n}} - 1.$$

The yield to maturity of an  $n$ -year, zero-coupon, risk-free bond is generally referred to as the **risk-free interest rate**, or the **spot rate**. The risk-free yield curve, which plots interest rate for risk-free bonds with different maturities, is often constructed using yields of zero coupon Treasury securities, which are generally considered to be risk free.

U.S. **Treasury notes**, which have original maturities from one to ten years, and **Treasury bonds**, which have original maturities of more than ten years, as well as most **corporate bonds** make semiannual coupon payments. The price of a coupon-paying bond with a required return of  $y$  can be calculated as:

$$P_0 = \frac{CPN}{1+y} + \frac{CPN}{(1+y)^2} + \frac{CPN}{(1+y)^3} + \dots + \frac{CPN + FV}{(1+y)^N} = CPN \times \frac{1}{y} \left( 1 - \frac{1}{(1+y)^N} \right) + \frac{FV}{(1+y)^N}.$$

Unlike zero-coupon bonds, the yield to maturity for coupon-paying bonds cannot be solved directly with a simple equation. Instead, the calculation requires iteration—guessing until the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond is found. Excel and financial calculators can be used to perform this iteration quickly.

## 6.2 Dynamic Behavior of Bond Prices

Coupon bonds may trade at:

- **par** (when their price is equal to their face value),
- a **discount** (when their price is less than their face value), or
- a **premium** (when their price is greater than their face value).

Bonds trading at a discount generate a return from both receiving the coupons and from receiving a face value that exceeds the price paid for the bond. As a result, the yield to maturity of discount bonds exceeds the coupon rate. Conversely, the YTM on bonds selling at a premium is lower than the coupon rate because the face value received is less than the price paid for the bond. A bond selling at par has a YTM equal to its coupon rate.

Between coupon payments, the prices of all bonds rise at a rate equal to the semiannual yield to maturity. Also, as shown in Figure 6.1 below:

- As each coupon is paid, the price of a bond drops by the amount of the coupon.
- When the bond is trading at a premium, the price drop when a coupon is paid will be larger than the price increase between coupons, so the bond's premium will tend to decline as time passes.

- If bond is trading at a discount, the price increase between coupons will exceed the drop when a coupon is paid, so the bond's price will rise, and its discount will decline as time passes.
- When the bond matures, the price of the bond equals the bond's face value.

Bond prices are subject to the effects of both the passage of time and changes in interest rates. While, as shown in Figure 6.1 below, bond prices converge to the bond's face value due to the time effect, they also move up and down due to unpredictable changes in bond yields. A higher yield to maturity means a higher discount rate for a bond's remaining cash flows, reducing their present value and thus the bond's price. Therefore, there is an inverse relation between bond prices and yields: as the discount rate increases, a bond's price falls, and as the discount rate falls, a bond's price increases.

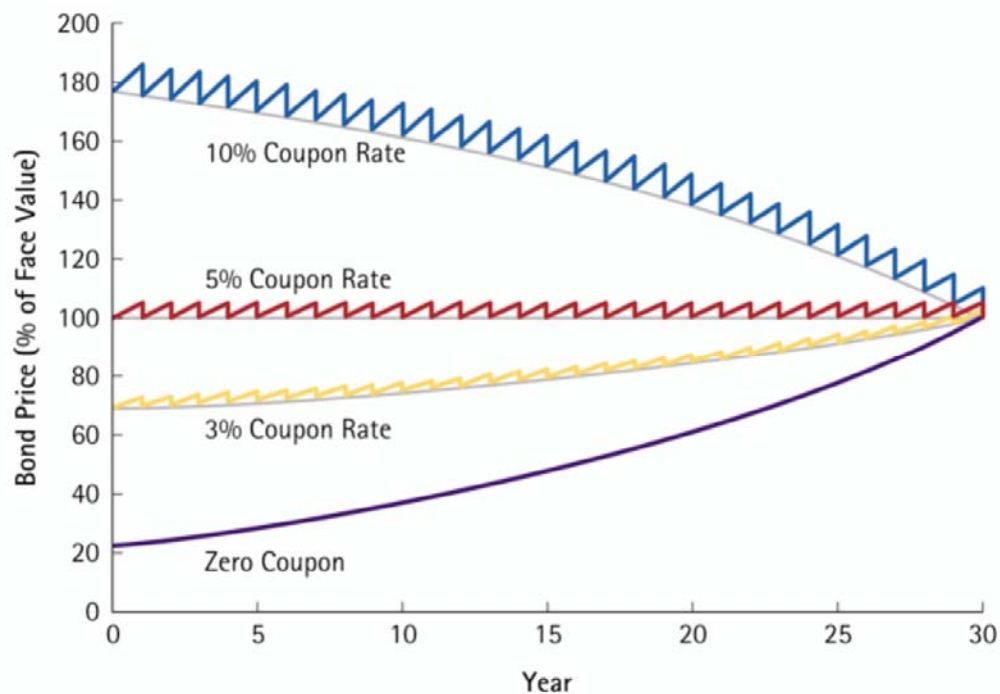


Figure 6.1

The graph illustrates the effects of the passage of time on bond prices per \$100 face value when the yield of a bond remains constant at 5%.

The sensitivity of a bond's price to changes in interest rates depends on the timing of its cash flows.

- Shorter maturity bonds are less sensitive to changes in interest rates because the present value of a cash flow that will be received in the near future is less dramatically affected by interest rates than a cash flow in the distant future.
- Bonds with higher coupon rates are less sensitive to interest rate changes than otherwise identical bonds with lower coupon rates because they pay a higher proportion of their cash flows sooner.

The sensitivity of a bond's price to changes in interest rate can be measured by its **duration**, which is discussed later in the text.

### 6.3 The Yield Curve and Bond Arbitrage

It is possible to replicate the cash flows of a risk-free coupon bond using zero-coupon bonds. For example, a three-year, \$1,000 bond can be replicated with 1-year, 2-year, and 3-year zero coupon bonds. By the Law of One Price, the three-year coupon bond must trade for the price it costs to replicate the payoffs using the zero-coupon bonds. If the price of the coupon bond were higher, you could earn an arbitrage profit by selling the coupon bond and buying the zero-coupon bond portfolio. If the price of the coupon bond were lower, you could earn an arbitrage profit by buying the coupon bond and short selling the zero-coupon bonds. The no-arbitrage price of a risk-free coupon bond can also be found by discounting its cash flows using the risk-free zero-coupon yields using:

$$P_0 = \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \frac{CPN}{(1 + YTM_3)^3} + \dots + \frac{CPN + FV}{(1 + YTM_N)^N}.$$

where  $YTM_n$  is the yield to maturity of a zero-coupon bond that matures at the same time as the  $n$ th coupon payment. Thus, the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.

### 6.4 Corporate Bonds

Corporate bonds have **credit risk**, which is the risk that the borrower will default and not pay all specified payments. As a result, investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond. Because the YTM for a bond is calculated using the promised cash flows, the yields of bonds with credit risk will be higher than that of otherwise identical default-free bonds. However, the YTM of a bond with default risk is always higher than the expected return of investing in the bond because it is calculated using the promised cash flows rather than the expected cash flows.

Bond rating agencies, such as Standard & Poor's and Moody's, evaluate the creditworthiness of bonds and publish bond ratings. The ratings encourage widespread investor participation and bond market liquidity. In descending order of credit quality, ratings by the two firms are as follows.

Standard & Poor's	Moody's	
AAA	Aaa	} Investment grade
AA	Aa	
A	A	
BBB	Baa	
BB	Ba	} Junk bonds
B	B	
CCC	Caa	

Bonds in the top four categories are often referred to as **investment-grade bonds** and have very low default risk. Bonds in the bottom categories are often called **junk bonds** or **high-yield bonds** because their likelihood of default is relatively high.

### 6.5 Sovereign Bonds

Sovereign bonds are bonds issued by national governments. While U.S. Treasury bonds are generally considered to be default free, bonds issued by many other countries may have significant de-

fault risk. For example, in 2012, Greece defaulted and wrote-off over \$100 billion, or about 50%, of its outstanding debt, in the largest sovereign debt restructuring in world history.

Unlike a corporation, a country facing difficulty meeting its financial obligations typically has the option to print additional currency to avoid default. However, this likely will lead to high inflation and a sharp devaluation of the currency. Debt holders, thus carefully consider inflation expectations when determining the yield they are willing to accept on the sovereign bonds.

## Selected Concepts and Key Terms

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### Corporate Bonds

Bonds issued by corporations. They typically have \$1,000 face values and pay semiannual coupon payments.

### Coupon Bonds

The promised interest payments of a bond. The bond certificate typically specifies that the coupons will be paid semiannually until the maturity date of the bond.

### Coupons, Coupon rate

The interest payments on a bond that are usually paid semiannually. The amount of each coupon payment is determined by the **coupon rate** of the bond. By convention, the coupon rate is expressed as an APR, so the amount of each coupon payment equals  $[\text{coupon rate} \times \text{face value}] \div 2$ .

### Credit Risk, Credit Spread

Bonds that are not risk free, such as corporate bonds, have **credit risk**, which is the risk that the borrower will default and not make all specified payments. As a result, investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond. Because the YTM for a bond is calculated using the promised cash flows, the yield of bonds reflect a **credit spread** and thus will be higher than that of otherwise identical default-free bonds.

### Discount Bond

A bond with a price lower than the face value. For example, zero-coupon bonds are **pure discount bonds**.

### High-Yield Bonds, Junk Bonds

Bonds rated below BBB by Standard & Poor's or below Baa by Moody's that have relatively high default risk and relatively high yields.

### Investment-Grade Bonds

Bonds rated BBB and above by Standard & Poor's or Baa and above by Moody's that have low default risk.

**Maturity Date**

The date that a bond repays its face value.

**On-the-Run Bond**

The most recently issued Treasury bonds for a given maturity.

**Premium**

The term used for bonds selling at a price greater than their face value.

**Face Value (FV)**

The amount that a bond pays at its maturity data, typically \$1,000. Also referred to as the **principal** or **par value**.

**Spot Interest Rates**

The yield to maturity of an  $n$ -year, zero-coupon, risk-free bond.

**Treasury Bills, Notes, and Bonds**

Securities issued by the U.S. Treasury. Treasury bills have original maturities less than one year and are zero-coupon bonds that are sold at a discount. Treasury notes, which have original maturities from one to ten years, and Treasury bonds, which have original maturities of more than ten years, typically make semiannual coupon payments.

**Yield to Maturity (YTM)**

The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. The YTM is the return an investor will earn from holding the bond to maturity

**Zero-Coupon Bond**

A bond in which the only payment is the face value of the bond on the maturity date—it does not make coupon payments. Treasury bills, which are U.S. government bonds with a maturity of up to one year, are zero-coupon bonds.

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**Concept Check Questions and Answers**

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**6.1.1. What is the relationship between a bond's price and its yield to maturity?**

The yield to maturity of a bond (or the IRR of a bond) is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond. Thus, the bond price is negatively related to its yield to maturity. When interest rate and bond's yield to maturity rise, the bond price will fall (and vice versa).

**6.1.2. The risk-free interest rate for a maturity of  $n$ -years can be determined from the yield of what type of bond?**

The risk-free interest rate for a maturity of  $n$ -years can be determined from the yield of a default free zero-coupon bond with the same maturity. Because a default-free, zero-coupon bond that matures on date  $n$  provides a risk-free return over the same period, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.

**6.2.1. If a bond's yield to maturity does not change, how does its cash price change between coupon payments?**

Between coupon payments, the prices of all bonds rise at a rate equal to the yield to maturity as the remaining cash flows of the bonds become closer. But as each coupon is paid, the price of a bond drops by the amount of the coupon.

**6.2.2. What risk does an investor in a default-free bond face if he or she plans to sell the bond prior to maturity?**

An investor in a default-free bond will face the interest rate risk if she plans to sell the bond prior to maturity. If she chooses to sell and the bond's yield to maturity has decreased, then she will receive a high price and earn a high return. If the yield to maturity has increased and the bond price is low at the time of sale, she will earn a low return.

**6.2.3. How does a bond's coupon rate affect its duration—the bond price's sensitivity to interest rate changes?**

The higher the coupon rate, all else equal, the lower the duration.

**6.3.1. How do you calculate the price of a coupon bond from the prices of zero-coupon bonds?**

Because we can replicate a coupon-paying bond using a portfolio of zero-coupon bonds, the price of a coupon-paying bond can be determined based on the zero-coupon yield curve using the Law of One Price. In other words, the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.

**6.3.2. How do you calculate the price of a coupon bond from the yields of zero-coupon bonds?**

Since zero-coupon bond yields represent competitive market interest rate for a risk-free investment with a term equal to the term of the zero-coupon bond, the price of a coupon bond must equal the present value of its coupon payments and face value discounted at these the zero-coupon bond yields.

**6.3.3. Explain why two coupon bonds with the same maturity may each have a different yield to maturity.**

The coupon bonds with the same maturity can have different yields depending on their coupon rates. The yield to maturity of a coupon bond is a weighted average of the yields on the zero-coupon bonds. As the coupon increases, earlier cash flows become relatively more important than later cash flows in the calculation of the present value.

**6.4.1. There are two reasons the yield of a defaultable bond exceeds the yield of an otherwise identical default-free bond. What are they?**

Because the yield must be higher to compensate for the risk of not receiving the required cash flows and to compensate for the fact that the expected cash flow is lower than the required cash flows

**6.4.2. What is a bond rating?**

A bond rating is a classification provided by several companies that assess the creditworthiness of bonds and make this information available to investors. By consulting these ratings, investors can assess the creditworthiness of a particular bond issue. The ratings therefore encourage widespread investor participation and relatively liquid markets. The two best-known bond-rating companies are Standard & Poor's and Moody's.

**6.5.1. Why do sovereign debt yields differ across countries?**



Investors consider inflation expectations and default risk when determining the yield they are willing to accept on the sovereign bonds, thus the yields differ due to differing inflation expectations and default risk.

### 6.5.2. What options does a country have if it decides it cannot meet its debt obligations?

To avoid default, a country has the option to print additional currency to pay its debts, but, doing so is likely to lead to high inflation and a sharp devaluation of the currency.

## Examples with Step-by-Step Solutions

### Solving Problems

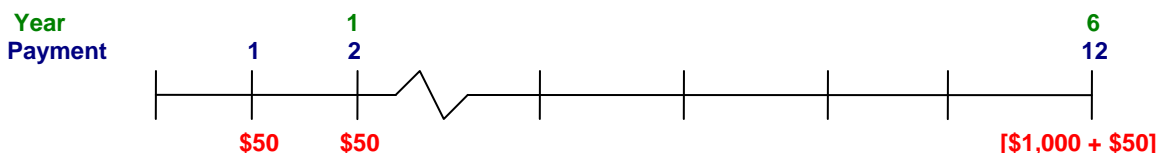
Problems using the concepts in this chapter generally involve determining the value of a bond. This requires understanding the cash flows associated with bonds and bond terminology, such as coupon rate, face value, maturity, and yield to maturity. The valuation of a bond generally involves utilizing the present value of an annuity equation and the present value of a cash flow equation. The yield to maturity can be found by setting the price equal to the present value of the bond's cash flows and solving for the discount rate that equates these two values.

### Examples

1. You have an opportunity to buy several B-rated bonds for \$641. B-rated bonds currently yield 12% APR, and these bonds have a coupon rate of 10%. The bonds have a face value of \$1,000, mature in exactly six years, and the next semiannual coupon payment will occur in exactly 6 months.

- [A] What is the value of one bond?
- [B] Is the bond's yield to maturity at a price of \$641 equal to, above, or below 12% APR?
- [C] If you bought some of these bonds and held them until maturity, what would your annual return be?

**Step 1.** Determine the bond's cash flows. The bond pays semi-annual coupon payments of  $0.10(\$1,000)/2 = \$50$  and pays \$1,000 in 6 years.



**Step 2.** Calculate the value of the bond.

Since similar bonds yield 12% APR, the semi-annual rate of  $12\%/2 = 6\%$  should be used to value this bond.

The next coupon is 6 months from today and the face value is repaid exactly 6 years from today. So, using the PV of an annuity equation and the PV of a single cash flow equation:

$$P_0 = 50 \left[ \frac{1}{.06} - \frac{1}{.06(1.06)^{12}} \right] + \frac{1,000}{(1.06)^{12}} = \$419.19 + 496.97 = \$916.16.$$

Thus, you should buy as many bonds as you can at a price of \$641 because they are selling below the market value. Even if you don't want to hold them until maturity, you can expect to sell them for \$916.16.

**Step 3.** Determine if the bond's yield to maturity at a price of \$644 is equal to, above, or below 12% APR.

This part does not require any calculations. Since the YTM would be 12% if the value that was found in step 2 was \$641, it does not equal 12%. Since the price found in step 2 is greater than \$641, the only way to lower the value from \$916.16 is to raise the discount rate above 12%, so the YTM must be higher than 12%.

**Step 4.** Your return is the yield to maturity.

The YTM is the rate that makes the bond's value equal to \$641, so

$$P_0 = \$841 = 50 \left[ \frac{1}{\frac{YTM}{2}} - \frac{1}{\frac{YTM}{2} \left(1 + \frac{YTM}{2}\right)^{12}} \right] + \frac{1,000}{\left(1 + \frac{YTM}{2}\right)^{12}}.$$

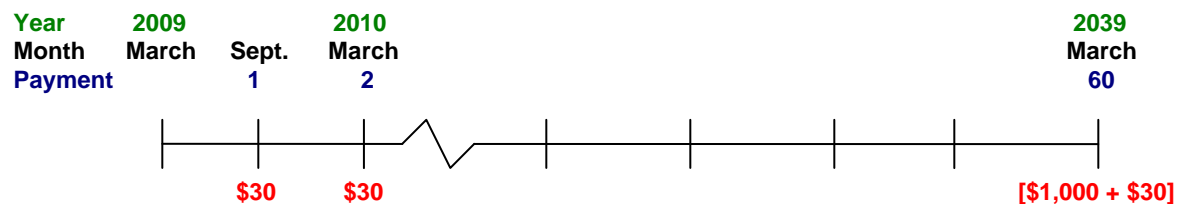
Without a financial calculator or spreadsheet, this problem requires iteration—guessing until the equation is correct. From step 3, we know that the YTM must be higher than 12% APR, or 6% per 6 months. So try 7%:

$$P_0 = \$841 = 50 \left[ \frac{1}{\frac{YTM}{2}} - \frac{1}{\frac{YTM}{2} \left(1 + \frac{YTM}{2}\right)^{12}} \right] + \frac{1,000}{\left(1 + \frac{YTM}{2}\right)^{12}} \Rightarrow \frac{YTM}{2} = 0.07 \Rightarrow YTM = 14\%.$$

So your return would be the annual YTM, 14% APR.

2. It is March 16, 2009. Assume that a BBB-rated, 6% semiannual coupon, \$1,000 face value bond matures on March 15, 2039. The 30-year Treasury-bond yield is 6.5%.
  - [A] If BBB-rated currently have a 4.5% credit spread, how much should you pay for the bond?
  - [B] What would the value of the bond be if yields do not change in four months? What would the clean price be at this time?

**Step 1.** Put the cash flows on a time line. The bond pays semi-annual coupon payments of  $0.06(\$1,000)/2 = \$30$  and pays \$1,000 in 30 years.



**Step 2.** Determine the discount rate.

The risk-free rate on bonds with the same term is 6.5%, and the credit spread is 4.5%, so the market rate on this bond is  $6.5\% + 4.5\% = 11\%$ . So, the semi-annual rate of  $11\%/2 = 5.5\%$  should be used to value this bond.

**Step 3.** Determine the value today.

Since the bond paid a coupon yesterday, the next coupon is 6 months from today and the face value is repaid exactly 30 years from today. So using the PV of an annuity equation and the PV of a single cash flow equation:

$$\begin{aligned}\text{Value} &= \frac{\text{Coupon}}{2} \left[ \frac{1}{\left(\frac{r}{2}\right)} - \frac{1}{\left(\frac{r}{2}\right)\left(1 + \frac{r}{2}\right)^{2M}} \right] + \frac{\text{Face Value}}{\left(1 + \frac{r}{2}\right)^{2M}} \\ &= 30 \left[ \frac{1}{.055} - \frac{1}{.055(1.055)^{60}} \right] + \frac{1,000}{(1.055)^{60}} = 523.49 + 40.26 = \$563.75.\end{aligned}$$

**Step 4.** Determine the cash value in four months.

The value in four months is the value on the date that the next coupon is paid discounted back two months. Since the EAR =  $(1 + .055)^2 - 1 = 11.30\%$ , the two-month discount rate is  $(1.113)^{2/12} - 1 = 1.60\%$ .

So

$$\begin{aligned}\text{Value} &= \frac{\left\{ \frac{\text{Coupon}}{2} \left[ \frac{1}{\left(\frac{r}{2}\right)} - \frac{1}{\left(\frac{r}{2}\right)\left(1 + \frac{r}{2}\right)^{2M}} \right] + 30 + \frac{\text{Face Value}}{\left(1 + \frac{r}{2}\right)^{2M}} \right\}}{1 + r_{2 \text{ months}}} \\ &= \frac{\left\{ 30 \left[ \frac{1}{.055} - \frac{1}{.055(1.055)^{59}} \right] + 30 + \frac{1,000}{(1.055)^{59}} \right\}}{1.018} = \$584.24.\end{aligned}$$

To verify that this is correct, calculate the holding period return that this implies:

$$\text{Return} = \frac{584.24 - 563.75}{563.75} = 0.036, \text{ which is the four-month return implied in a } 11.3\%:$$

$$\text{EAR} = (1.113)^{\frac{4}{12}} - 1 = 0.036.$$

**Step 5.** Determine the clean price.

This is the value (or cash price or dirty price) that was found in step 4, less the accrued interest. The accrued interest is  $(4/6) \times \$30 = \$20$ , so the clean price is  $\$564.24$ .

3. The following table summarizes prices of zero-coupon U.S. Treasury securities per \$100 of face value.

Maturity in Years	Price
1.	96.04
2.	93.35
3.	66.36
4.	79.21

- [A] Plot the zero-coupon yield curve based on these bonds.  
 [B] Describe the shape of the yield curve.  
 [C] What is the value of a four-year 7% annual coupon Treasury bond with a face value of \$1,000 that pays its first coupon in one year?

**Step 1.** First, the yield to maturity of each bond must be calculated.

Using the equation  $P_0 = \frac{\$100}{(1+IRR)^N}$  and solving for IRR:

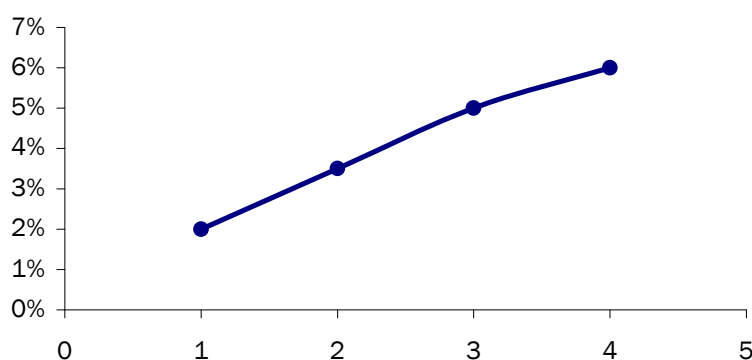
$$96.04 = \frac{100}{(1+IRR)^1} \Rightarrow IRR = \frac{100}{96.04} - 1 = 2.0\%$$

$$93.35 = \frac{100}{(1+IRR)^2} \Rightarrow IRR = \left( \frac{100}{93.35} \right)^{1/2} - 1 = 3.5\%$$

$$66.36 = \frac{100}{(1+IRR)^3} \Rightarrow IRR = \left( \frac{100}{66.36} \right)^{1/3} - 1 = 5.0\%$$

$$79.21 = \frac{100}{(1+IRR)^4} \Rightarrow IRR = \left( \frac{100}{79.21} \right)^{1/4} - 1 = 6.0\%$$

**Step 2.** Now, the zero-coupon Treasury yield curve can be drawn.



The yield curve is upward sloping. A yield curve with a positive slope is sometimes referred to as a normal yield curve.

**Step 3.** By the Law of One Price, the 4-year coupon bond must trade for the price it costs to replicate the payoffs using the zero-coupon bonds. The no-arbitrage price can be found by discounting its cash flows using the risk-free zero-coupon yields.

The annual coupon payment is  $0.07(\$1,000) = \$70$ , so

$$P_0 = \frac{70}{1.02} + \frac{70}{(1.035)^2} + \frac{70}{(1.05)^3} + \frac{70 + 1,000}{(1.06)^4}$$

$$= 68.63 + 65.35 + 60.47 + 847.54 = \$1,041.99.$$

## Questions and Problems

- Several major companies like Citigroup, Disney, and AT&T have issued Century Bonds. These bonds pay regular semiannual coupons, but do not mature until 100 years after they are issued. Some critics have stated that they are extremely risky because you can't predict what will happen to the companies in 100 years. Assume that such bonds were just issued with a \$1,000 par value and an 6% semiannual coupon rate.
  - If current market rates are 6%, what is the present value of the principal repayment at maturity?
  - What is the total value today of the final 40 years (years 61–100) of payments, including coupons and principal?
- Below is a quote from finance.yahoo.com for a Northrop Grumman bond. Assume it is March 2, 2006 and the \$1,000 face value bond just paid a coupon payment yesterday.

Price	61.95
Coupon (%)	7.750
Maturity Date	1-March-2016
Debt Rating	BBB
Coupon Payment Frequency	Semiannual
First Coupon Date	1-Sept-1996
Type	Corporate
Industry	Industrial

- What is the value of the bond if your required return is 6% APR?
- If the bond were a zero-coupon bond and its only payment was the return of face value at maturity, what would the yield-to-maturity be at the price quoted?
- Assume the bond was originally issued for a price of \$1,000. In one sentence, explain something specifically could have happened in the economy or to the firm that could have made the bond sell for its current price.
- The day before the bond matures and pays its last coupon payment, what will its value be?

3. Suppose that Ford has a B-rated bond with exactly 30 years until maturity, a face value of \$1,000, and a semiannual coupon rate of 6%. The yield to maturity on B-rated bonds today is 10%.
- [A] What was the price of this bond today?
- [B] Assuming the yield to maturity remains constant, what is the price of the bond immediately before and after it makes its next coupon payment?
4. Suppose a ten-year, \$1000 bond with a 9% coupon rate and semiannual coupons is trading for a price of \$1,156.
- [A] What is the bond's yield to maturity (expressed as an APR with semiannual compounding)?
- [B] If the bond's yield to maturity changes to 12% APR, what will the bond's price be?
5. The following table summarizes the yields to maturity on several one-year, zero-coupon bonds:

Bond	% Yield
Treasury	4.1
AA corporate	4.6
BBB corporate	6.2
CCC corporate	10.5

- [A] What is the value of a one-year, \$1,000 face value, zero-coupon corporate bond with a CCC rating?
- [B] What is the credit spread on AA-rated corporate bonds?
- [C] What is the credit spread on B-rated corporate bonds?

### Solutions to Questions and Problems

1. [A]  $\frac{1,000}{(1.04)^{200}} = \$0.39$

[B]  $\frac{40 \left[ \frac{1}{.04} - \frac{1}{.04(1.04)^{80}} \right]}{(1.04)^{120}} + 0.39 = \frac{956.61}{110.66} = \$8.64 + .39 = \$9.03$

2. [A]  $P_0 = 38.75 \left[ \frac{1}{.03} - \frac{1}{.03(1.03)^{20}} \right] + \frac{1,000}{(1.03)^{20}} = 576.50 + 553.68 = \$1,130.18$

[B]  $819.50 = \frac{1,000}{(1+r)^{20}} \Rightarrow (1+r)^{20} = 1.22 \Rightarrow 1+r = (1.22)^{\frac{1}{20}} = 1.01 \Rightarrow r = 1\%$

So the YTM APR = 2(1%) = 2%.

- [C] Either the firm's default risk increased leading to a higher credit spread or rates in the economy increased leading to an increase in the risk-free interest rates.

[D]  $\$1,000 + 36.75 = \$1,036.75$ .

3. [A]  $P = \$30 \left[ \frac{1}{.05} - \frac{1}{1.05(1.05)^{60}} \right] + \frac{1,000}{(1.05)^{60}} = 567.88 + 53.54 = \$621.42$

- [B] Before the next coupon payment, the price of the bond is

$$P = \$30 \left[ \frac{1}{.05} - \frac{1}{1.05(1.05)^{59}} \right] + \frac{1,000}{(1.05)^{59}} + 30 = 566.27 + 56.21 + 30 = \$652.48$$

After the next coupon payment, the price of the bond will be

$$P = \$30 \left[ \frac{1}{.05} - \frac{1}{1.05(1.05)^{59}} \right] + \frac{1,000}{(1.05)^{59}} = 566.27 + 56.21 = \$622.48$$

4. [A]  $P_0 = 45 \left[ \frac{1}{\frac{.12}{2}} - \frac{1}{\frac{.12}{2} \left(1 + \frac{.12}{2}\right)^{20}} \right] + \frac{1,000}{\left(1 + \frac{.12}{2}\right)^{20}} \Rightarrow P = \$827.95$

[B]  $P_0 = \$45 \left[ \frac{1}{\frac{.12}{2}} - \frac{1}{\frac{.12}{2} \left(1 + \frac{.12}{2}\right)^{20}} \right] + \frac{1,000}{\left(1 + \frac{.12}{2}\right)^{20}} \Rightarrow P = \$827.95$

5. [A] The price of this bond will be  $P = \frac{1,000}{1.105} = \$904.98$

[B] The credit spread on AA-rated corporate bonds is  $0.046 - 0.041 = 0.7\%$

[C] The credit spread on BBB-rated corporate bonds is  $0.062 - 0.041 = 2.1\%$