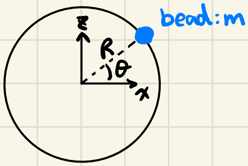
AE352 Quiz 2



a) Generalized Coordinates

$q_1 = R \qquad q_2 = \theta$

$$x = R\cos\theta \longrightarrow \boxed{x = q_1 \cos q_2}$$
$$z = R\sin\theta \longrightarrow \boxed{z = q_1 \sin q_2}$$

b) Lagrangian

Kinetic

$T = \frac{1}{2}mv^2 \qquad v = R\dot{\theta} = q_1\dot{q}_2$

$$T = \frac{1}{2}m(q_1\dot{q}_2)^2$$

Potential

$$U = mgq_1 \sin q_2$$

$$L = T - U$$
$$L = \frac{1}{2}m(q_1\dot{q}_2)^2 - mgq_1\sin q_2$$
$$L = \frac{1}{2}m(R\dot{\theta})^2 - mgR\sin\theta$$

c) EOM

O'Alembert: $\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$

For $q_1 = R$, because q_1 is constant, δq_1 & $\delta \dot{q}_1 = 0$

For $q_2 = \theta$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q_\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}}\left(\frac{1}{2}mR^2\dot{\theta}^2\right) = mR^2\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mgR\cos\theta$$

$Q_\theta = -kR\dot{\theta}$ from friction $F = -kv = -kR\dot{\theta}$

$$mR^2\ddot{\theta} + mgR\cos\theta = -kR\dot{\theta}$$

$$\boxed{mR\ddot{\theta} + mg\cos\theta = -k\dot{\theta}}$$

d) there are 2 equilibrium points

one at the very top of the ring $\overset{x}{(0},\overset{y}{0},\overset{z}{R})$ is unstable equilibrium

another at the very bottom of the ring $(0, 0, -R)$ is stable equilibrium