Date
Page

Random sample $(x_1, x_2, x_3, \_\_ x_n)$ from a normal population with parameters θ_1 (mean) and θ_2 (variance).

Maximum likelihood estimates for θ_1, θ_2 are as follows :-

Likelihood function for normal distribution

$$L(\theta_1, \theta_2 \mid x_1 x_2 \_\_ x_n) = \Pi_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(\frac{-(x_i-\theta_1)^2}{2\theta_2}\right)}$$

log likelihood function -

$$l(\theta_1 \theta_2 \mid x_1 x_2 - - x_n) = \sum_{i=1}^{n} \left(\frac{-1}{2} \log(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right)$$

→ we have to maximize this log likelihood function so we differentiate it wrt θ_1, θ_2 and equate it to 0, then solve for θ_1, θ_2.

wrto θ_1

$$\frac{\partial l}{\partial \theta_1} = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$\Rightarrow \frac{\partial l}{\partial \theta_1} = \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\theta_1 = 0$$

$$\boxed{\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

wrt θ_2

$$\frac{\partial l}{\partial \theta_2} = \sum_{i=1}^{n}\left(\frac{-1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2}\right) = 0$$

$$\Rightarrow -n + \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2} = 0$$

$$\Rightarrow \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2} = n$$

$$\Rightarrow \boxed{\theta_2 = \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{n}}$$

Random sample $(X_1, X_2, - X_n)$ from a binomial distribution with parameter m and θ, θ is unknown,

log - likelihood function for binomial distribution,

$$l(\theta \mid x_1, x_2 - x_n) = \sum_{i=1}^{n}\left(\log\binom{m}{x_i} + x_i\log(\theta) + (m-x_i)\log(1-\theta)\right)$$

differentiate wrt θ

$$\frac{\partial l}{\partial \theta} = \sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(xi(1-\theta) - (m-xi)\theta\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (xi - m\theta) = 0$$

$$\Rightarrow \sum_{i=1}^{n} xi - nm\theta = 0$$

$$\Rightarrow \boxed{\theta = \frac{1}{nm} \sum_{i=1}^{n} xi}$$