Lab 1 Prelab

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1)

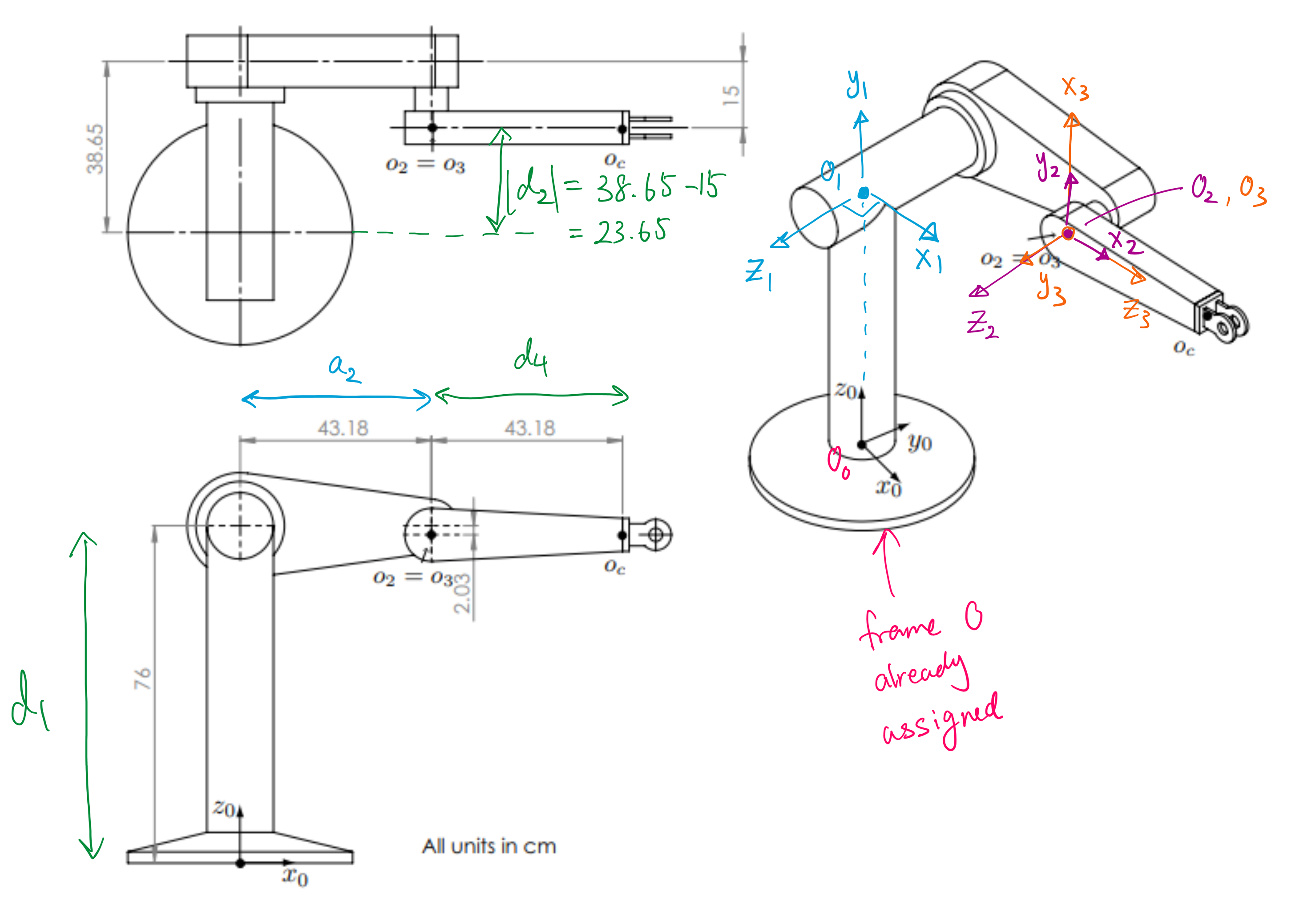
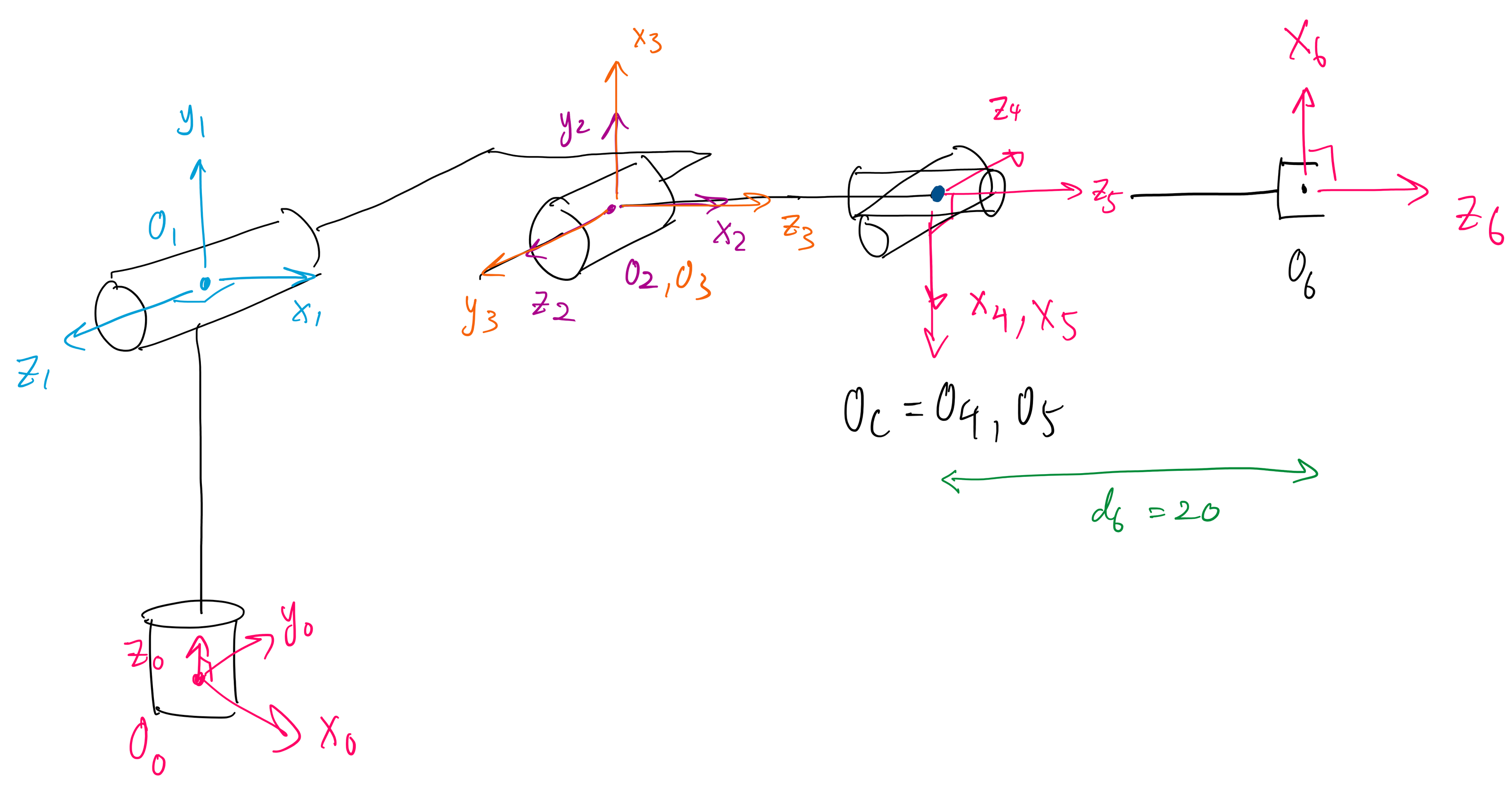


Figure 1: Schematics of the PUMA560 robot

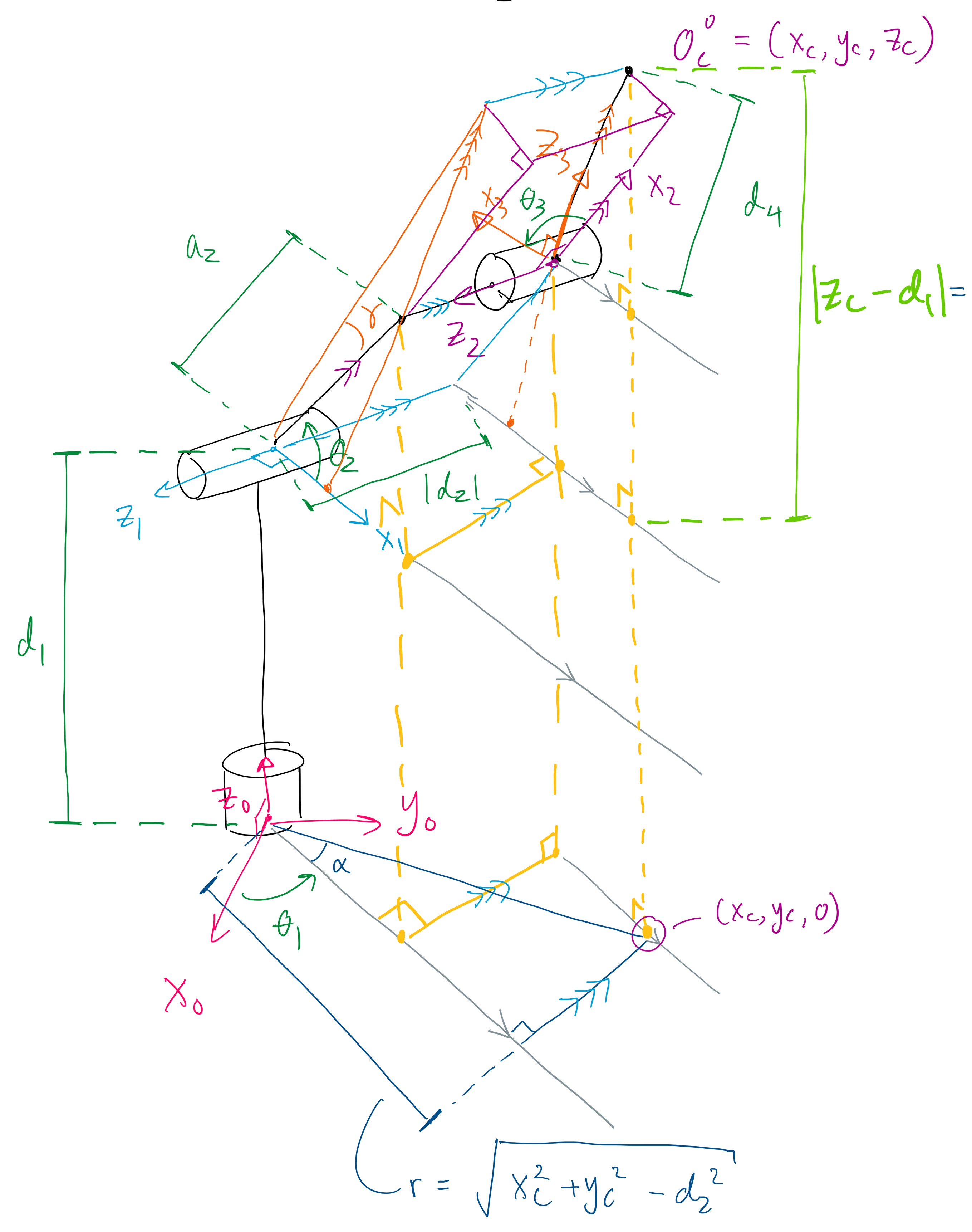
2)



link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76	θ_1^*
2	43.18	0	-23.65	θ_2^*
3	0	$\pi/2$	0	θ_3^*
4	0	$-\pi/2$	43.18	θ_4^*
5	0	$\pi/2$	0	θ_5^*
6	0	0	20	θ_6^*

3) 1

$$O_c^0(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$



θ_1

$$\theta_1 + \alpha = \text{atan2}(y_c, x_c)$$

$$\alpha = \sin^{-1}\left(\frac{|d_2|}{\sqrt{x_c^2 + y_c^2}}\right)$$

$$\boxed{\theta_1 = \text{atan2}(y_c, x_c) - \sin^{-1}\left(\frac{|d_2|}{\sqrt{x_c^2 + y_c^2}}\right)}$$

θ_2, θ_3

$$r^2 + s^2 = a_2^2 + d_4^2 - 2a_2 d_4 \cos\Big(\underbrace{\pi - (\theta_3 - \tfrac{\pi}{2})}_{\frac{3\pi}{2} - \theta_3}\Big)$$

$$r^2 + s^2 = a_2^2 + d_4^2 + 2a_2 d_4 \sin\theta_3$$

$$\sin\theta_3 = \frac{r^2 + s^2 - a_2^2 - d_4^2}{2a_2 d_4} = D$$

$$\boxed{\theta_3 = \text{atan2}\left(D, \pm\sqrt{1 - D^2}\right)}$$

↖ elbow up/down

$$\gamma = \text{ata2}\left(d_4 \sin(\theta_3 - \tfrac{\pi}{2}),\ a_2 + d_4 \cos(\theta_3 - \tfrac{\pi}{2})\right)$$

$$\boxed{\theta_2 = \text{atan2}(s, r) - \gamma}$$

2 i) $H_3^0(\theta_1, \theta_2, \theta_3) = H_1^0(\theta_1)\, H_2^1(\theta_2)\, H_3^2(\theta_3)$

$$H_i^{i-1}(\theta_i) = \left[\begin{array}{ccc|c} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$H_3^0 = \begin{bmatrix} R_3^0 & O_3^0 \\ 0 & 1 \end{bmatrix}$$

ii) $R_6^3(\theta_4, \theta_5, \theta_6) = \underbrace{(R_3^0)^T R_d}_{\text{compute this, } R_d \text{ given to us}}$

$$R_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

iii) compute euler angles:

$$\left.\begin{aligned} \theta &= \text{atan2}\left(+\sqrt{1 - r_{33}^2},\ r_{33}\right) \\ \phi &= \text{atan2}(r_{23}, r_{13}) \\ \psi &= \text{atan2}(r_{32}, -r_{31}) \end{aligned}\right\} s_\theta > 0$$

OR:

$$\left.\begin{aligned} \theta &= \text{atan2}\left(-\sqrt{1 - r_{33}^2},\ r_{33}\right) \\ \phi &= \text{atan2}(-r_{23}, -r_{13}) \\ \psi &= \text{atan2}(-r_{32}, r_{31}) \end{aligned}\right\} s_\theta < 0$$

set: $\theta_4 = \theta$, $\theta_5 = \phi$, $\theta_6 = \psi$