Lab 4 Prelab

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2. repulsive:

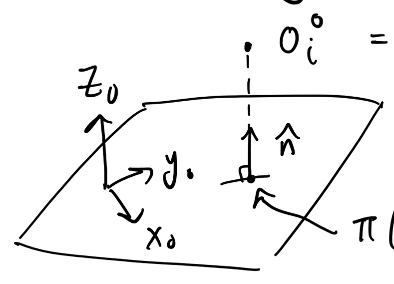
$$U_{i,rep}(O_i^o) = \begin{cases} \frac{1}{2}\eta_i\left(\frac{1}{\|O_i^o - \pi(\sigma_i^o)\|} - \frac{1}{\rho_o}\right)^2, & \|O_i^o - \pi(\sigma_i^o)\| \le \rho_o \\ 0, & \|O_i^o - \pi(\sigma_i^o)\| > \rho_o \end{cases}$$

$$F_{i,rep}(O_i^o) = -\nabla U_{i,rep}(O_i^o) = \begin{cases} \eta_i\left(\frac{1}{\|O_i^o - \pi(O_i^o)\|} - \frac{1}{\rho_o}\right)\frac{(O_i^o - \pi(O_i^o))}{\|O_i^o - \pi(O_i^o)\|^3}, & \|O_i^o - \pi(O_i^o)\| \le \rho_o \\ 0, & \|O_i^o - \pi(O_i^o)\| > \rho_o \end{cases}$$

1) repulsion upward from Workspace plane which is parallel to $x_o - y_o$ plane, with z_o value of 32 mm

→ let $\hat{n}$ be unit normal vector of the plane, pointing up

$$O_i^o - \pi(O_i^o) = \|O_i^o - \pi(O_i^o)\|\,\hat{n}$$



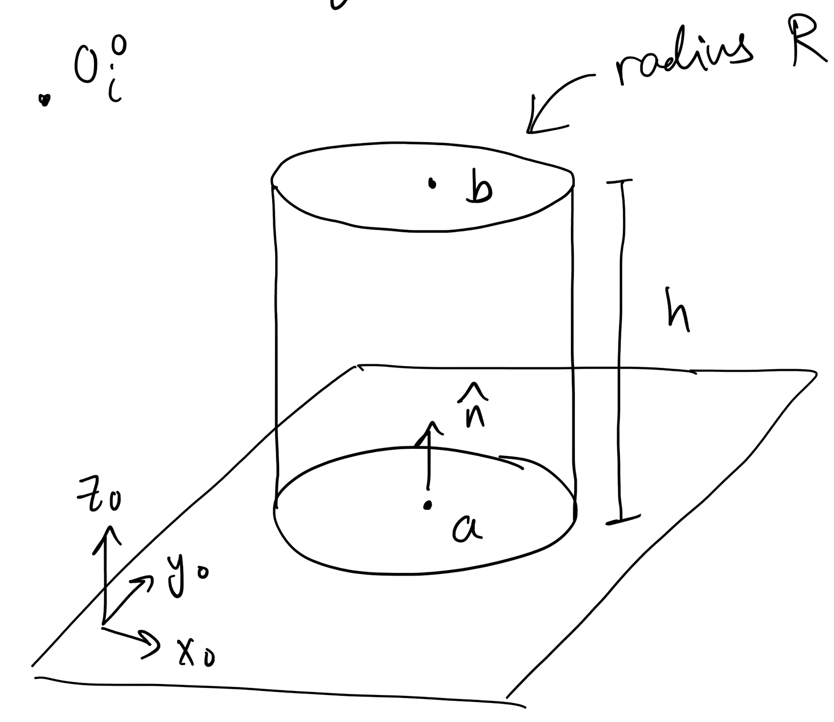
$O_i^o = (x, y, z)$

$\pi(O_i^o) = (x, y, 32)$

$\hat{n} = \langle 0, 0, 1 \rangle$

$$\Rightarrow \text{so, } F_{i,rep}(O_i^o) = \begin{cases} \eta_i\left(\frac{1}{z-32} - \frac{1}{\rho_o}\right)\frac{1}{(z-32)^2}\begin{bmatrix}0\\0\\1\end{bmatrix}, & z-32 \le \rho_o \text{ and } z-32 \ne 0 \\ 0, & z-32 > \rho_o \end{cases}$$

2) repulsion from cylinder of finite length with bottom on $x_o - y_o$ plane, and height is h



$$b^o = a^o + h\hat{n}$$

$$a^o = \begin{bmatrix}c_x\\c_y\\0\end{bmatrix}, \quad b^o = \begin{bmatrix}c_x\\c_y\\h\end{bmatrix}, \quad \hat{n} = \begin{bmatrix}0\\0\\1\end{bmatrix}, \quad O_i^o = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

(*) robot will be either above cylinder or around it (should not go under the bottom or within the cylinder)

i) robot is around cylinder: $z \le h$

$$\|O_i^o - \pi(O_i^o)\| = \left\|\begin{bmatrix}x\\y\end{bmatrix} - \begin{bmatrix}c_x\\c_y\end{bmatrix}\right\| - R$$

$$= \sqrt{(x-c_x)^2 + (y-c_y^2)^2} - R$$

$$= d - R$$

$$O_i^o - \pi(O_i^o) = \begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{d-R} - \frac{1}{\rho_o}\right)\frac{1}{(d-R)^3}\begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}, & d-R \le \rho_o,\ d-R \ne 0 \\ 0, & d-R > \rho_o \end{cases}$$

ii) robot is right above cylinder: $z > h$

$$\|O_i^o - \pi(O_i^o)\| = z - h$$

$$O_i^o - \pi(O_i^o) = (z-h)\,\hat{n}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{z-h} - \frac{1}{\rho_o}\right)\frac{1}{(z-h)^2}\begin{bmatrix}0\\0\\1\end{bmatrix}, & z-h \le \rho_o,\ z-h \ne 0 \\ 0, & z-h > \rho_o \end{cases}$$

iii) $z > h$ but robot is not right above cylinder

$$\|O_i^o - \pi(O_i^o)\| = \sqrt{d_\perp^2 + d_\parallel^2}$$

$$d_\perp = z - h, \quad d_\parallel = d - R$$

$$O_i^o - \pi(O_i^o) = d_\perp \hat{r} + d_\parallel \hat{n}$$

$$\hat{r} = \frac{1}{d_\perp}\begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{\sqrt{d_\perp^2 + d_\parallel^2}} - \frac{1}{\rho_o}\right)\frac{1}{(d_\perp^2 + d_\parallel^2)^{3/2}}\begin{bmatrix}x-c_x\\y-c_y\\d_\parallel\end{bmatrix}, & \sqrt{d_\perp^2 + d_\parallel^2} \le \rho_o,\ \sqrt{d_\perp^2 + d_\parallel^2} \ne 0 \\ 0, & \sqrt{d_\perp^2 + d_\parallel^2} > \rho_o \end{cases}$$