Lab 4 Prelab

2025-11-17 6:50 PM

2. repulsive:

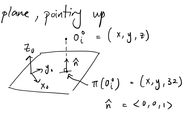
$$U_{i,rep}(O_i^o) = \begin{cases} \frac{1}{2}\eta_i\left(\frac{1}{\|O_i^o - \pi(o_i^o)\|} - \frac{1}{\rho_0}\right)^2, & \|O_i^o - \pi(o_i^o)\| \le \rho_0 \\ 0, & \|O_i^o - \pi(o_i^o)\| > \rho_0 \end{cases}$$

$$F_{i,rep}(O_i^o) = -\nabla U_{i,rep}(O_i^o) = \begin{cases} \eta_i\left(\frac{1}{\|O_i^o - \pi(O_i^o)\|} - \frac{1}{\rho_0}\right)\frac{(O_i^o - \pi(O_i^o))}{\|O_i^o - \pi(O_i^o)\|^3}, & \|O_i^o - \pi(O_i^o)\| \le \rho_0 \\ 0, & \|O_i^o - \pi(O_i^o)\| > \rho_0 \end{cases}$$

i) repulsion upward from Workspace plane which is parallel to x_0-y_0 plane, with z_0 value of 32 mm

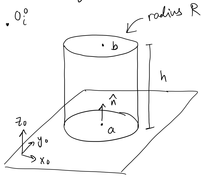
→ let $\hat{n}$ be unit normal vector of the plane, pointing up

$$O_i^o - \pi(O_i^o) = \|O_i^o - \pi(O_i^o)\|\,\hat{n}$$



$$\Rightarrow \text{so, } F_{i,rep}(O_i^o) = \begin{cases} \eta_i\left(\frac{1}{z-32} - \frac{1}{\rho_0}\right)\frac{1}{(z-32)^2}\begin{bmatrix}0\\0\\1\end{bmatrix}, & z-32 \le \rho_0 \text{ and } z-32 \ne 0 \\ 0, & z-32 > \rho_0 \end{cases}$$

2) repulsion from cylinder of finite length with bottom on x_0-y_0 plane, and height is h



$$b^o = a^o + h\hat{n}$$

$$a^o = \begin{bmatrix}c_x\\c_y\\0\end{bmatrix},\ b^o = \begin{bmatrix}c_x\\c_y\\h\end{bmatrix},\ \hat{n} = \begin{bmatrix}0\\0\\1\end{bmatrix},\ O_i^s = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

* robot will be either above cylinder or around it (should not go under the bottom or within the cylinder)

i) robot is around cylinder: $z \le h$

$$\|O_i^o - \pi(O_i^o)\| = \left\|\begin{bmatrix}x\\y\end{bmatrix} - \begin{bmatrix}c_x\\c_y\end{bmatrix}\right\| - R$$

$$= \sqrt{(x-c_x)^2 + (y-c_y^2)^2} - R$$

$$= d - R$$

$$O_i^o - \pi(O_i^o) = \begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{d-R} - \frac{1}{\rho_0}\right)\frac{1}{(d-R)^3}\begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}, & d-R \le \rho_0,\ d-R \ne 0 \\ 0, & d-R > \rho_0 \end{cases}$$

ii) robot is right above cylinder: $z > h$

$$\|O_i^o - \pi(O_i^o)\| = z - h$$

$$O_i^s - \pi(O_i^s) = (z-h)\hat{n}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{z-h} - \frac{1}{\rho_0}\right)\frac{1}{(z-h)^2}\begin{bmatrix}0\\0\\1\end{bmatrix}, & z-h \le \rho_0,\ z-h \ne 0 \\ 0, & z-h > \rho_0 \end{cases}$$

iii) $z > h$ but robot is not right above cylinder

$$\|O_i^o - \pi(O_i^o)\| = \sqrt{d_\perp^2 + d_{//}^2}$$

$$d_\perp = z - h,\quad d_{//} = d - R$$

$$O_i^o - \pi(O_i^o) = d_\perp \hat{r} + d_{//}\hat{n}$$

$$\hat{r} = \frac{1}{d_\perp}\begin{bmatrix}x-c_x\\y-c_y\\0\end{bmatrix}$$

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{\sqrt{d_\perp^2 + d_{//}^2}} - \frac{1}{\rho_0}\right)\frac{1}{(d_\perp^2 + d_{//}^2)^{\frac{3}{2}}}\begin{bmatrix}x-c_x\\y-c_y\\d_{//}\end{bmatrix}, & \sqrt{d_\perp^2 + d_{//}^2} \le \rho_0,\ \sqrt{d_\perp^2 + d_{//}^2} \ne 0 \\ 0, & \sqrt{d_\perp^2 + d_{//}^2} > \rho_0 \end{cases}$$

MATLAB Testing

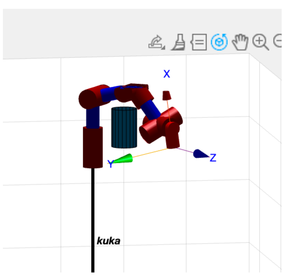
(1) testing τ:

```
%% test forces
setupobstacle_lab4prep;
q = [pi/10, pi/12, pi/6, pi/2, pi/2, -pi/6];
tau = rep(q, kuka_forces, prepobs{1});


%% test motion planning
p1 = [620 375 50];
```

Command Window

```
F_rep =

   1.0e-06 *

  -0.1269    0.0059    0.0073    0.0168    0.0168    0.0189
   0.0043    0.0096    0.0092    0.0096    0.0096    0.0018
   0.1005    0.1037    0.0615         0         0         0


ans =

    0.1795    0.9540    0.2353   -0.0344   -0.0344    0.0000
```

(2) Motion plane, around cylinder:




```
%% test motion planning
p1 = [620 375 50];
p2 = [620 -375 50];
R = [0 0 1; 0 -1 0; 1 0 0];
H1 = [R p1'; zeros(1,3) 1];
H2 = [R p2'; zeros(1,3) 1];
q1 = inverse_kuka(H1, kuka);
q2 = inverse_kuka(H2, kuka);
qref = motionplan(q1, q2, 0, 10, kuka_forces, prepobs, 0.01);

% plot trajectory
hold on
plotobstacle(prepobs);
t = linspace(0,10,300);
q = ppval(qref,t)';
plot(kuka,q);
hold off
```