Lab 2 Prelab

Tuesday, October 14, 2025 8:36 AM

1)

d_1	400 mm
a_1	25 mm
a_2	315 mm
a_3	35 mm
d_4	365 mm
a_6	296.23 mm
d_6	161.44 mm

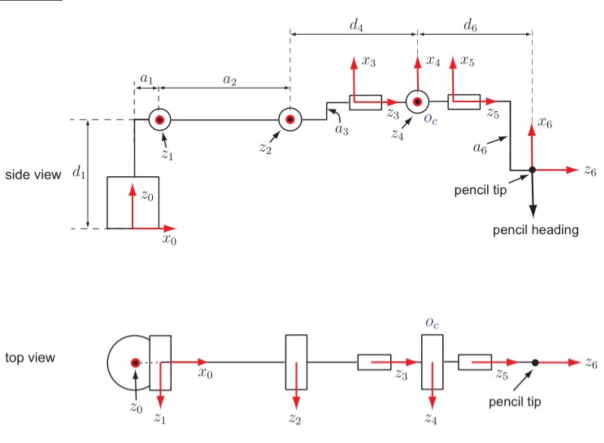


Figure 2: Schematic diagrams illustrating the KUKA kinematic parameters

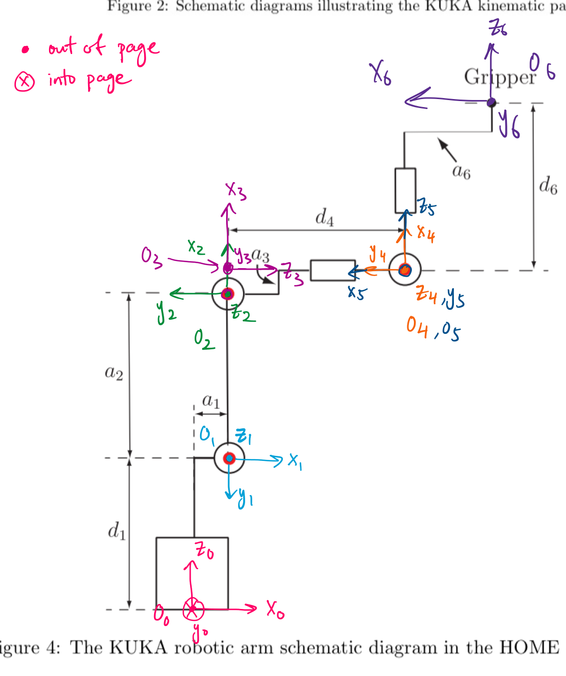


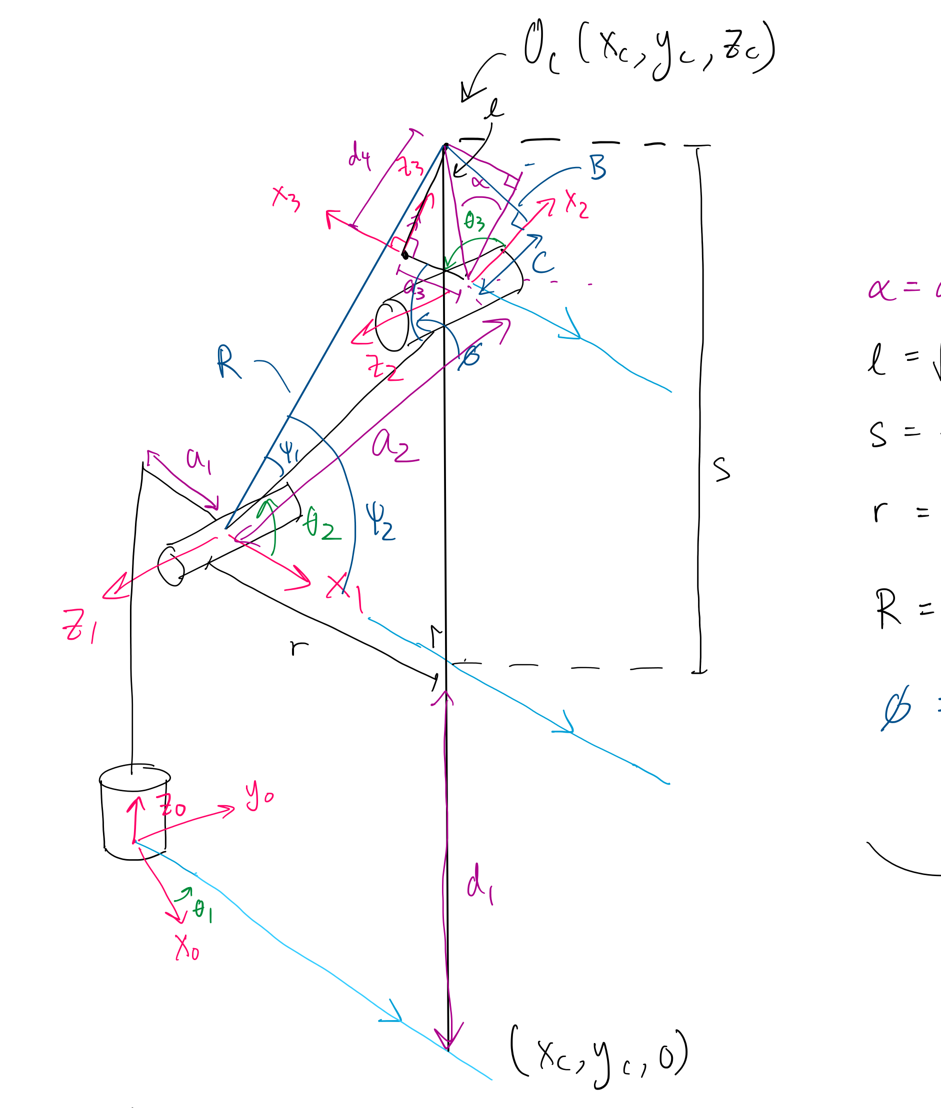
Figure 4: The KUKA robotic arm schematic diagram in the HOME configuration

DH Table

link	a_i	α_i	d_i	θ_i
1	25	$\pi/2$	400	θ_1^*
2	315	0	0	θ_2^*
3	35	$\pi/2$	0	θ_3^*
4	0	$-\pi/2$	365	θ_4^*
5	0	$\pi/2$	0	θ_5^*
6	-296.23	0	161.44	θ_6^*

2) Find Inverse Kinematics

① $$O_c^0(\theta_1,\theta_2,\theta_3) = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d^0 \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$$



$$\alpha = \operatorname{atan2}(a_3, d_4)$$
$$\ell = \sqrt{d_4^2 + a_3^2}$$
$$s = z_c - d_1$$
$$r = \sqrt{x_c^2 + y_c^2} - a_1$$
$$R = \sqrt{r^2 + s^2} = \sqrt{\left(\sqrt{x_c^2+y_c^2} - a_1\right)^2 + (z_c - d_1)^2}$$
$$\phi = \pi - \alpha - \left(\theta_3 - \tfrac{\pi}{2}\right)$$
$$= \frac{3\pi}{2} - (\alpha + \theta_3)$$

θ_1

$$\boxed{\theta_1 = \operatorname{atan2}(y_c, x_c)}$$

θ_3

$$\Rightarrow \quad R^2 = \ell^2 + a_2^2 - 2\ell a_2 \cos\phi$$
$$s^2 + r^2 = \ell^2 + a_2^2 - 2\ell a_2 \cos\left(\tfrac{3\pi}{2} - (\alpha+\theta_3)\right)$$

$$s^2 + r^2 - \ell^2 - a_2^2 = 2\ell a_2 \sin(\alpha + \theta_3)$$
$$\Rightarrow \quad \sin(\alpha+\theta_3) = \frac{s^2+r^2-\ell^2-a_2^2}{2\ell a_2} = D$$

$$\boxed{\theta_3 = \sin^{-1}(D) - \alpha}$$

θ_2

$$\boxed{\theta_2 = \psi_2 - \psi_1}$$
$$\psi_2 = \operatorname{atan2}(s, r)$$
$$\sin\left(\alpha + \left(\theta_3 - \tfrac{\pi}{2}\right)\right) = \frac{B}{\ell} \Rightarrow B = \ell \sin\left(\theta_3 + \alpha - \tfrac{\pi}{2}\right)$$
$$B = -\ell\cos(\theta_3 + \alpha)$$
$$\cos\left(\alpha + \left(\theta_3 - \tfrac{\pi}{2}\right)\right) = \frac{C}{\ell} \Rightarrow C = \ell \sin(\theta_3 + \alpha)$$
$$\psi_1 = \operatorname{atan2}(B, a_2 + C)$$

② i) $$H_3^0(\theta_1,\theta_2,\theta_3) = H_1^0(\theta_1)\, H_2^1(\theta_2)\, H_3^2(\theta_3)$$

$$H_i^{i-1}(\theta_i) = \left[\begin{array}{ccc|c} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$H_3^0 = \begin{bmatrix} R_3^0 & O_3^0 \\ 0 & 1 \end{bmatrix}$$

ii) $$R_6^3(\theta_4,\theta_5,\theta_6) = \underbrace{(R_3^0)^T R_d}_{\text{compute this, } R_d \text{ given to us}}$$

$$R_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

iii) compute euler angles:

$$\left.\begin{aligned} \theta &= \operatorname{atan2}\left(+\sqrt{1-r_{33}^2},\ r_{33}\right) \\ \phi &= \operatorname{atan2}(r_{23}, r_{13}) \\ \psi &= \operatorname{atan2}(r_{32}, -r_{31}) \end{aligned}\right\} \quad s_\theta > 0$$

set: $\theta_4 = \theta,\ \theta_5 = \phi,\ \theta_6 = \psi$

3) MATLAB

Editor – /Users/richardwu/Documents/University/4th-year/ECE470/Labs/Lab2/

mykuka.m | forward_kuka.m | inverse_kuka.m | inverse.m

```matlab
function myrobot = mykuka(DH)
    for i=1:6
        L(i) = Link(DH(i,:));
    end
    myrobot = SerialLink(L, 'name', 'kuka');
end
```

Editor – /Users/richardwu/Documents/University/4th-year/ECE470/Labs/Lab2/forward_kuka.m

mykuka.m | forward_kuka.m | inverse_kuka.m | prelab.m | untitled3 * | untitled4 *

```matlab
function [H] = forward_kuka(joint,myrobot)
    % get DH parameters
    d = myrobot.d;
    alpha = myrobot.alpha;
    a = myrobot.a;

    % compute H_i-1_i using DH parameters
    A = zeros(6,4,4);
    for i=1:6
        theta = joint(i);
        A(i,:,:) = [cos(theta) -sin(theta)*cos(alpha(i)) sin(theta)*sin(alpha(i)) a(i)*cos(theta);
                sin(theta) cos(theta)*cos(alpha(i)) -cos(theta)*sin(alpha(i)) a(i)*sin(theta);
                0          sin(alpha(i))        cos(alpha(i))             d(i);
                0 0 0 1];
    end

    % get H_0_6
    H = squeeze(A(1,:,:)) * squeeze(A(2,:,:)) * squeeze(A(3,:,:)) * squeeze(A(4,:,:)) * squeeze(A(5,:,:)) * squeeze(A(6,:,:));
end
```

mykuka.m | forward_kuka.m | inverse_kuka.m | prelab.m | untitled3 *

```matlab
function [q] = inverse_kuka(H,myrobot)
    q = zeros(1,6);

    % get desired position of end effector
    o_d = H(1:3,4);

    % get R_d
    R_d = H(1:3, 1:3);

    % get needed DH parameters
    d = myrobot.d;
    a = myrobot.a;

    % get position of wrist
    o_c = o_d - R_d*[a(6); 0; d(6)];
    x_c = o_c(1);
    y_c = o_c(2);
    z_c = o_c(3);

    % theta1:
    q(1,1) = atan2(y_c,x_c);

    % intermediate calculations
    alpha = atan2(a(3),d(4));
    l = sqrt((d(4))^2 + a(3)^2);
    s = z_c - d(1);
    r = sqrt(x_c^2 + y_c^2) - a(1);
    D = (s^2 + r^2 - l^2 - a(2)^2) / (2*l*a(2));

    % theta3:
    q(1,3) = asin(D) - alpha;

    % more intermediate calculations
    psi2 = atan2(s, r);
    B = real(-l*cos(q(1,3) + alpha));
    C = real(l*sin(q(1,3) + alpha));
    psi1 = atan2(B, a(2)+C);

    % theta2:
    q(1,2) = psi2 - psi1;

    % get R_6_3
    joint = [q(1,1); q(1,2); q(1,3); 0; 0; 0];
    H_3_0 = forward_kuka(joint,myrobot);
    R_3_0 = H_3_0(1:3,1:3);
    R_6_3 = transpose(R_3_0)*R_d;

    % for some reason, expected theta4 in 5th pos and theta5 in 4th pos...
    % theta4:
    q(1,5) = atan2(real(sqrt(1-(R_6_3(3,3))^2)),real(R_6_3(3,3)));
    % theta5:
    q(1,4) = atan2(real(R_6_3(2,3)),real(R_6_3(1,3)));
    % theta6:
    q(1,6) = atan2(real(R_6_3(3,2)),real(-R_6_3(3,1)));
end
```

4) Test MATLAB

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mykuka.m | forward_kuka.m | inverse_kuka.m | prelab.m | untitl

```matlab
%% DH table
DH = [0  400.0    25.0  pi/2;
      0      0   315.0     0;
      0      0    35.0  pi/2;
      0  365.0       0 -pi/2;
      0      0       0  pi/2;
      0 161.44  -296.23    0;];

%% setup kuka robot
kuka = mykuka(DH);

%% test
q = [pi/5 pi/3 -pi/4 pi/4 pi/3 pi/4];
H = forward_kuka(q,kuka)
q_check = inverse_kuka(H,kuka)
```

Command Window

```
>> prelab

H =

    0.1173   -0.3109    0.9432  368.9562
   -0.8419   -0.5349   -0.0717  420.4832
    0.5268   -0.7856   -0.3245  120.8570
         0         0         0    1.0000


q_check =

    0.6283    1.0472   -0.7854    0.7854    1.0472    0.7854

>>
```