Lab 3 Prelab

2025-11-03 11:31 AM

3. write expressions for $\| o_i(q) - b \|$ and $o_i(q) - b$ appearing in eq. (7.5):

$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(o_i(q))} \nabla \rho(o_i(q)),$$

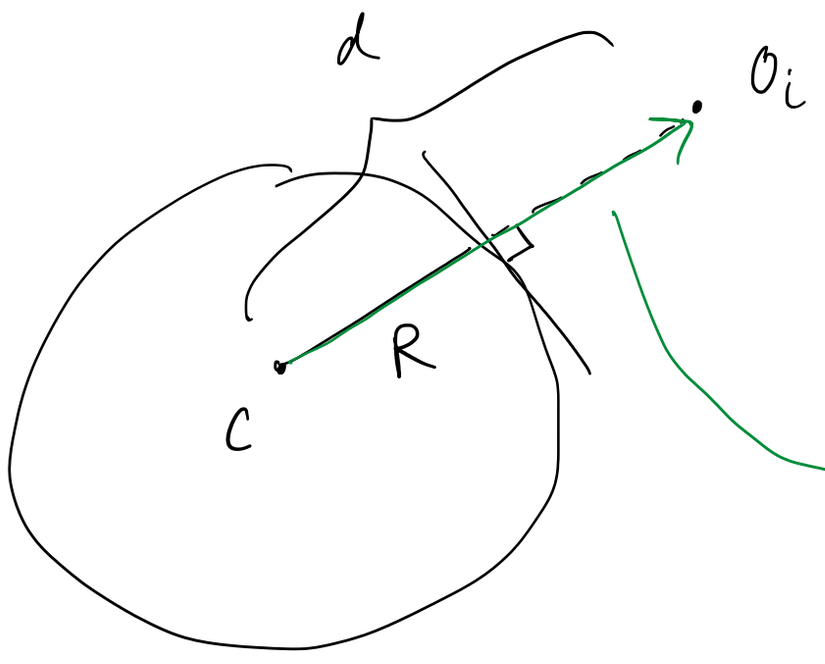
$\rho(o_i(q))$ is distance from o_i to nearest obstacle,

$$\rho(o_i(q)) = \min_{x \in \partial O} \| o_i(q) - x \|$$

$$= \| o_i(q) - b \|, \quad b \text{ is point on obstacle closest to } o_i$$

$$\nabla \rho(x) \Big|_{x = o_i(q)} = \frac{o_i(q) - b}{\| o_i(q) - b \|}$$

Case 1 : obstacle is sphere of radius R centred at $c = (c_x, c_y, c_z)$



$$d = \| o_i - c \|$$

$$\| o_i - b \| = d - R$$

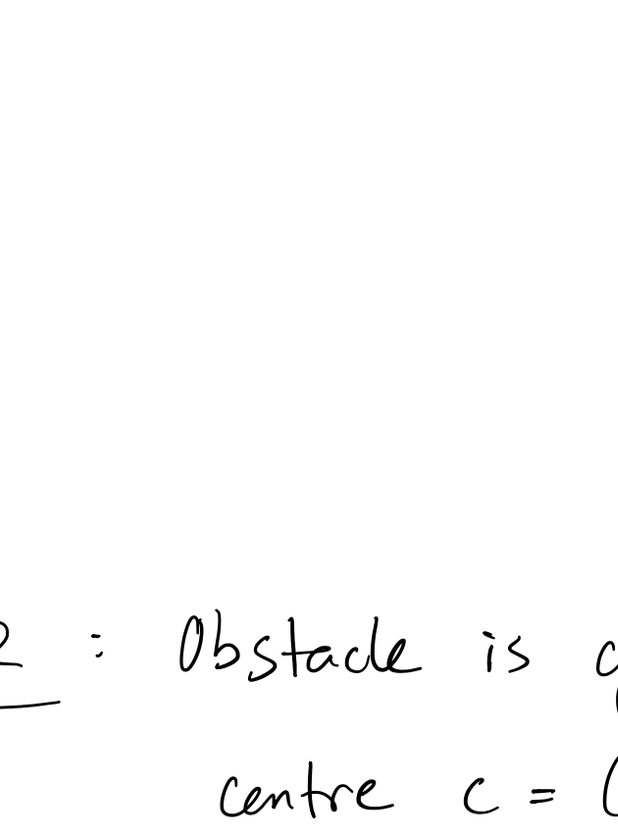
$$o_i - b = \left(\frac{o_i - c}{\| o_i - c \|} \right) \| o_i - b \|$$

unit direction vector

$$= (o_i - c) \frac{d - R}{d}$$

$$\boxed{o_i - b = (o_i - c)\left(1 - \frac{R}{d}\right)}$$

Case 2 : Obstacle is cylinder of ∞ height w/ centre $c = (c_x, c_y)$, axis parallel to z_0, radius R



$$\| o_i - b \| = \left\| \begin{bmatrix} o_{ix} - c_x \\ o_{iy} - c_y \end{bmatrix} \right\| - R$$

d

$$o_i - b = \left(\frac{o_i - c}{\| o_i - c \|} \right) \| o_i - b \|$$

$$\boxed{o_i - b = \begin{bmatrix} o_{ix} - c_x \\ o_{iy} - c_y \end{bmatrix} \left(1 - \frac{R}{d} \right)}$$