ECE557 Lab 4 Report

PRA01 Group 5

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Introduction

In this lab, we again study the cart-pendulum system. The system dynamics and linearization is the same as in Lab 3. This time, we design an output feedback controller to make the cart position track a square wave signal while keeping the pendulum upright the entire time, as best as possible (i.e., the upright configuration). In the following sections, we discuss our experimental results: our best controllers, and their parameters, at attaining the objective of maintaining the upright configuration. We study two controllers: an output feedback controller, and the same thing but with integral action. Below are two block diagrams describing these systems.

Block Diagram

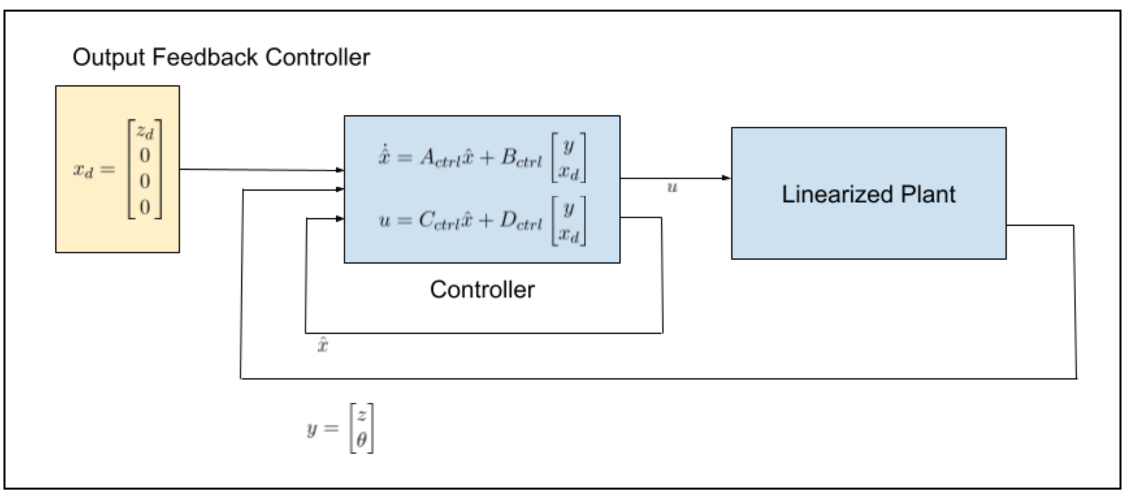

Figure 1. Output feedback controller system.

In Figure 1 (which shows the output feedback controller), the controller matrices are:

$$A_{ctrl} = A + BK - LC$$

$$B_{ctrl} = \begin{bmatrix} L & -BK \end{bmatrix}$$

$$C_{ctrl} = K$$

$$D_{ctrl} = \begin{bmatrix} 0_{1x2} & -K \end{bmatrix}$$

So the matrices that must be tuned are K and L.

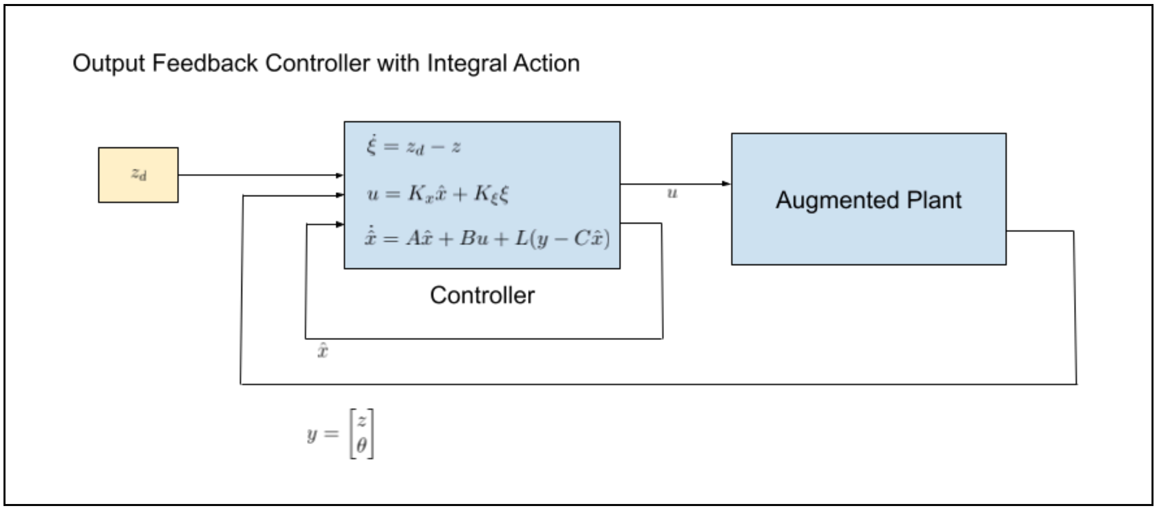

Figure 2. Output feedback controller system but with integral action.

Figure 2 shows the controller but with integral action, where $z_d - z$ is the tracking error. The augmented plant is:

$$\dot{X} = \bar{A}X + \bar{B}u + \begin{bmatrix} 0_{4x1} \\ 1 \end{bmatrix} z_d$$

$$\bar{A} = \begin{bmatrix} A & 0_{4x1} \\ -C_1 & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Where $X = (x, \xi)$. Again, L must be tuned. But now, $\bar{K} = \begin{bmatrix} K_x & K_\xi \end{bmatrix}$ needs to be tuned, where $\bar{K}$ is the controller for the $(\bar{A}, \bar{B})$ system.

<u>Part 1</u>

In this part, we design the output feedback controller (Figure 1) in simulation. We pick K such that the eigenvalues of $A + BK$ are at -5 (or more accurately, slightly perturbed from -5) and L such that the eigenvalues of $A - LC$ are at -10. The matrices are:

$$K = \begin{bmatrix} 14.5279 & 17.7041 & -47.0339 & -8.3938 \end{bmatrix}$$

$$L = \begin{bmatrix} 11.1793 & -0.0002 \\ 1.3968 & 1.8921 \\ -26.6118 & 20.0053 \\ -297.6048 & 135.4683 \end{bmatrix}$$

Figure 3 below shows plots of the simulation of the state and control input with the state and output feedback controllers.

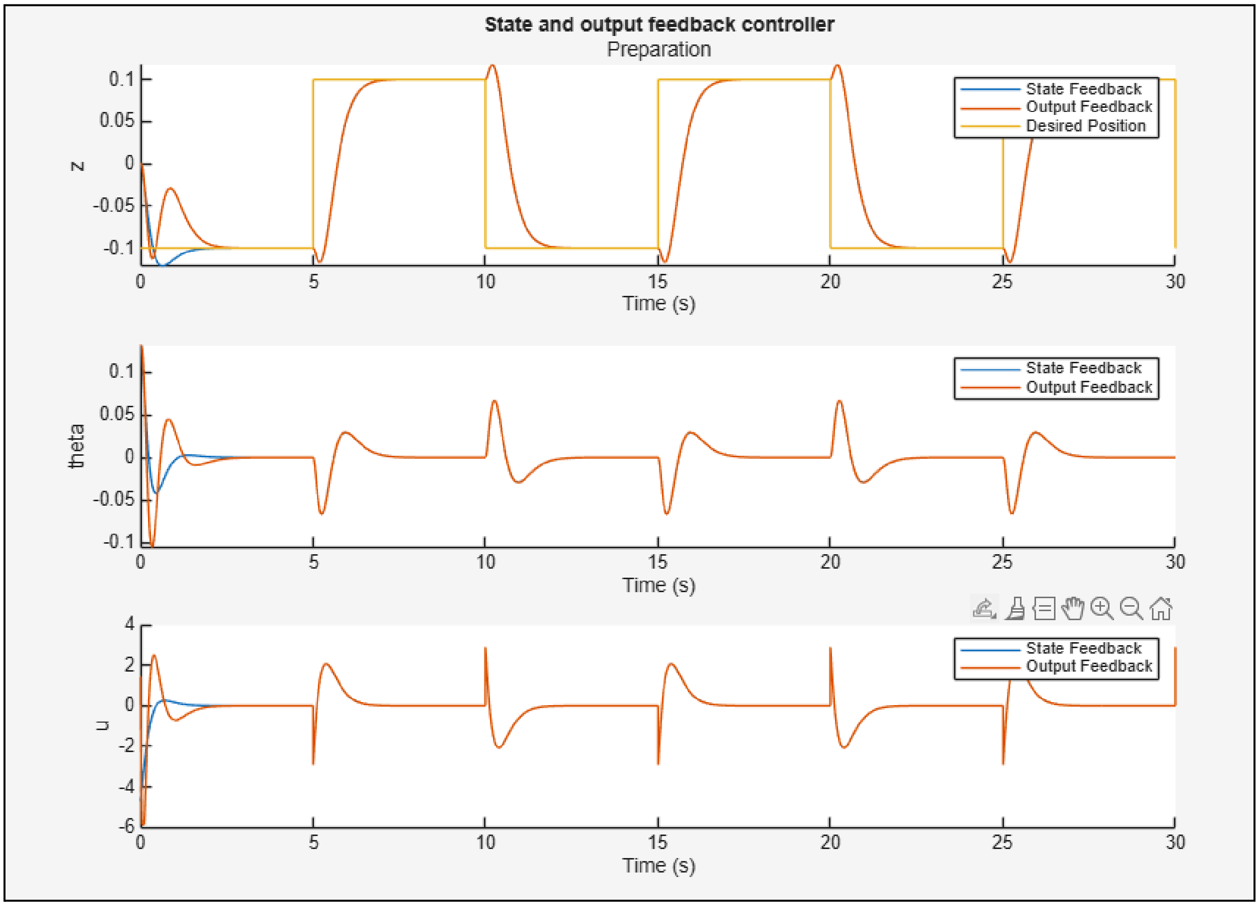

Figure 3. Plots of z, θ, u for $\mathrm{eig}(A + BK)$ at -5 and $\mathrm{eig}(A - LC)$ at -10.

We see that the state and output feedback controllers result in almost identical behaviour. The only differences exist in the transient component. Both controllers exhibit overshoot for z, θ and u. However, the output feedback controller has greater overshoot and also brief oscillation for z and θ.

We now move the eigenvalues of $A - LC$ to -40. The resulting L matrix is:

$$L = \begin{bmatrix} 71.1845 & 0 \\ 972.4779 & 1.8896 \\ -26.6971 & 80.0001 \\ -1900.4 & 1635.4 \end{bmatrix}$$

And the behaviour is plotted in Figure 4 below.

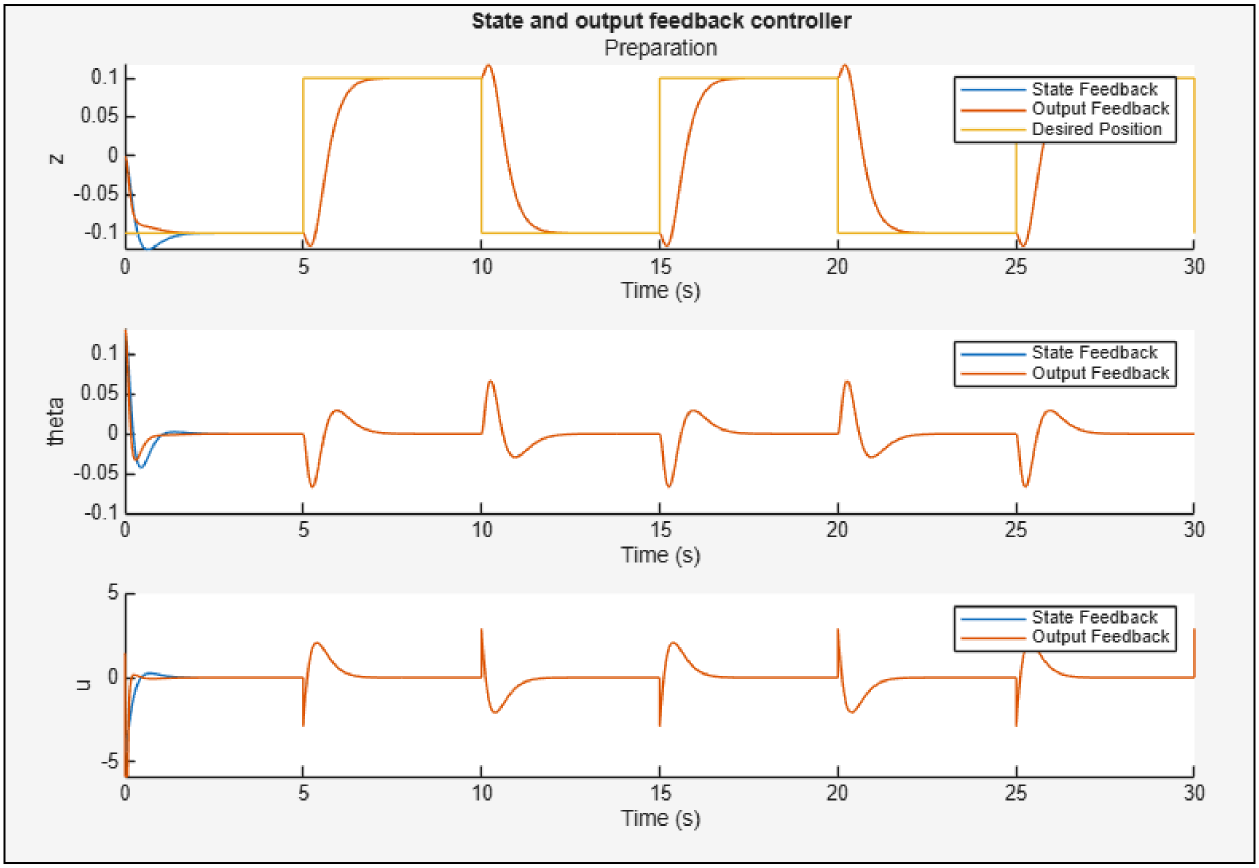

Figure 4. Plots of z, θ, u for $\text{eig}(A + BK)$ at -5 and $\text{eig}(A - LC)$ at -40.

With the change to L, the output feedback controller no longer oscillates in the transient and the overshoot is now less than that of the state feedback controller. We can conclude that moving the poles of $A - LC$ more negative causes damping effects on the transience.

Now, going further, we keep L the same and adjust K to meet the following goals:

- Settling time T_s of the cart position, $z(t)$ is less than 1 second.
- The control signal $u(t)$ does not have excessive saturation.

From Figure 4, we see that excessive saturation already does not occur. The control input rarely reaches the saturation limit of $5.875V$ and only peaks momentarily when the square wave changes position. However, the settling time does not meet the 1 second requirement. To make the controller track the square wave better, we decided to move the eigenvalues of $A+BK$ from -5 to -8, as this improves the convergence time. The new K matrix is:

$$K = \begin{bmatrix} 95.2115 & 53.6876 & -126.9814 & -23.0092 \end{bmatrix}$$

The simulation plots are in Figure 5 below.

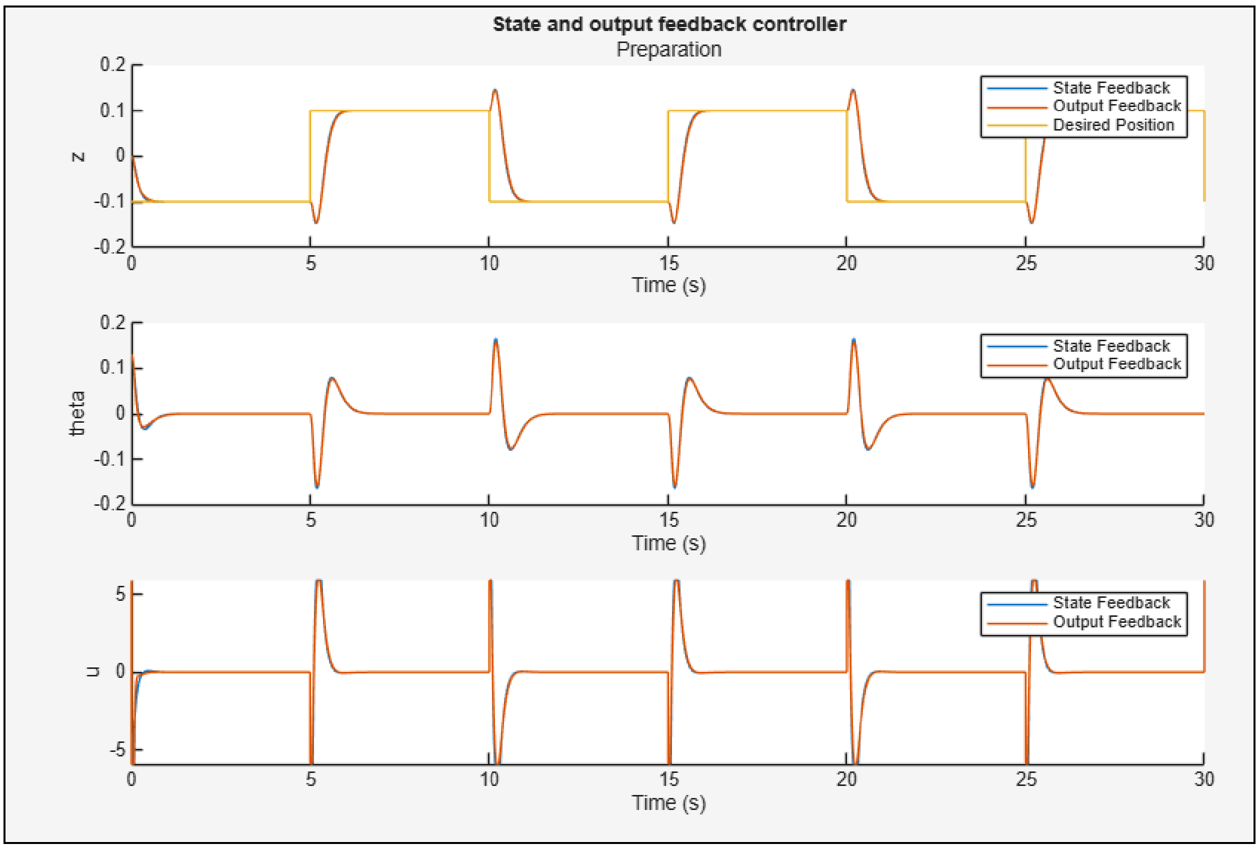

Figure 5. Plots of z, θ, u for $\mathrm{eig}(A+BK)$ at -8 and $\mathrm{eig}(A-LC)$ at -40.

We see that the position tracks the square wave better. The control input is now reaching the saturation limit, however it does not stay saturated for long so we accepted this as satisfying the 2nd goal of no excessive control input saturation. For the settling time, we define it as the time it takes for the error to reach and stay within 2% of the total step variation. The settling time for the behaviour in Figure 5 is $T_s = 0.8966\text{s}$, as we can see from the zoomed in plot of Figure 6.

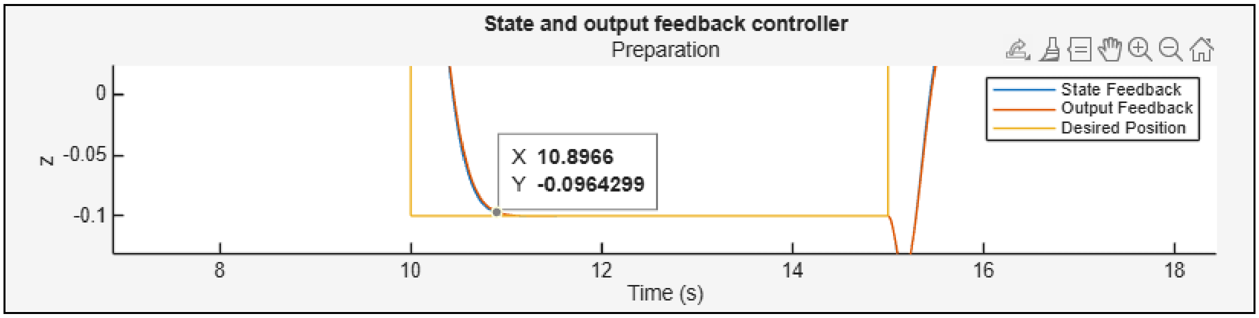

Figure 6. Zoomed in plot of z for $\text{eig}(A + BK)$ at -8 and $\text{eig}(A - LC)$ at -40, showing T_s.

Part 2

In this part, we performed the physical experiment to tune the gains for our LQR controller. The following are our LQR parameters:

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

,where $q_1 = 1$ and $q_2 = 300$, and $R = 0.02$ in the LQR computation. It is important to note here what each of the parameters represent in the physical world. In general, the matrix Q penalizes state errors. In our case, we have two state parameters, cart position z and angle θ. These correspond to q_1 and q_2, respectively. Thus from our control parameters, we noticeably prioritize maintaining the pendulum angle in a relatively stable place, whereas the error in cart position is penalized less (300 >> 1). For the parameter R, we set a very low value as it generally causes more aggressive control inputs.

In terms of our experimental procedure to tune these gains, we relied on a trial and error method to settle on these final LQR parameters. We first set the R value very low, as we wanted a fast system response in general. With the initial parameters we used, which were $q_1 = 50$ and $q_2 = 1$, we noticed that the cart would move to the desired position with the pendulum falling over immediately. This lines up with expectation, as here the cart position error is penalized much more than the pendulum angle. We then changed the order of magnitude of both q_1 and q_2 to see the opposite effect, where $q_1 = 1$ and $q_2 = 50$. With these parameters, the cart remained stable for consistently longer, but tracked the reference signal worse. Because the pendulum still dropped after some period of time, we continued increasing q_2 incrementally until we reached a value of 300 where the system then stayed stable consistently. Below are the graphs for the cart position $z(t)$, pendulum angle $\theta(t)$, and the control input $u(t)$, respectively.

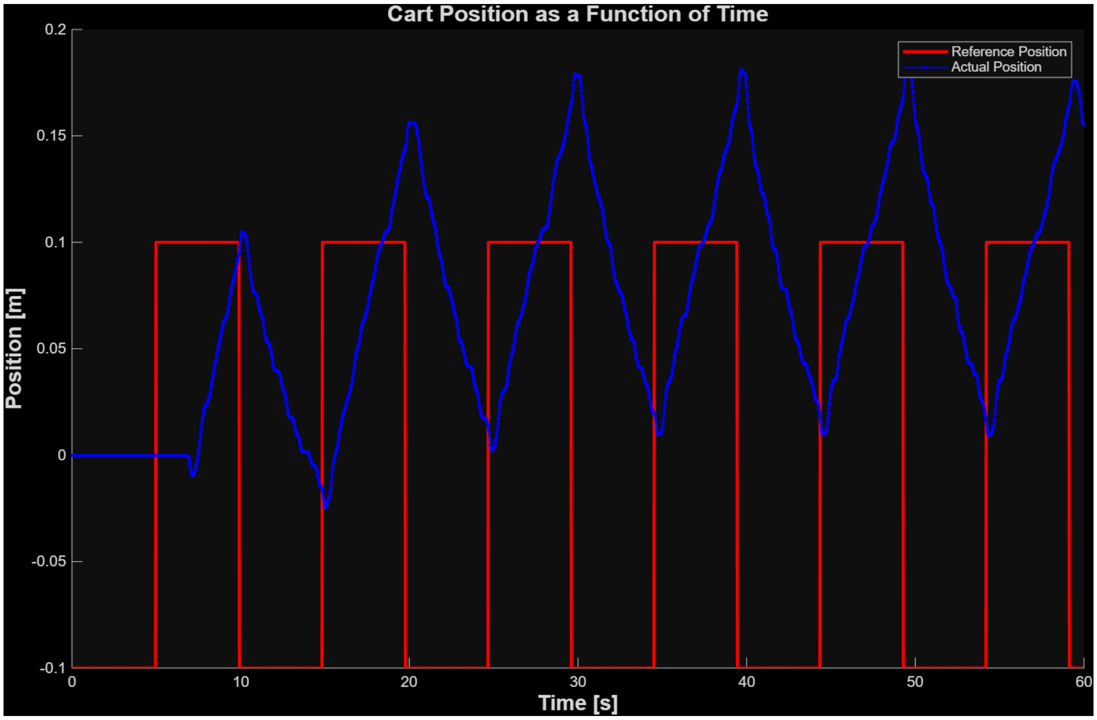

Figure 7. Graph plotting reference and actual cart position against time for the LQR controller.

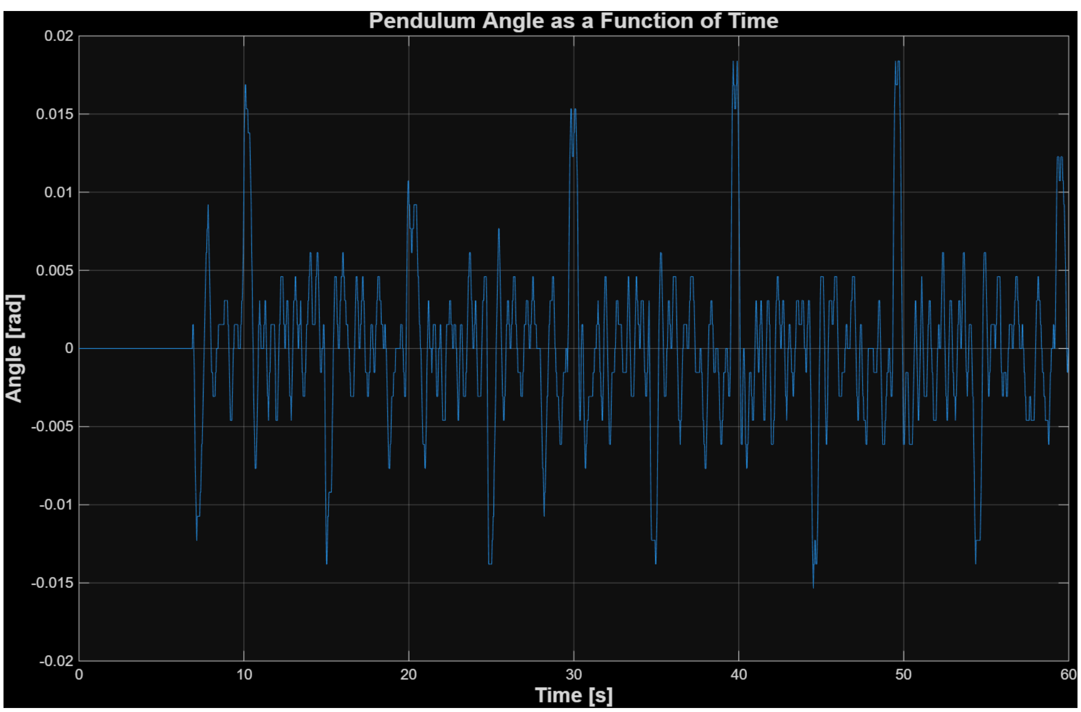

Figure 8. Graph plotting pendulum angle against time. Note that the spikes correspond with changes in the reference cart position signal.

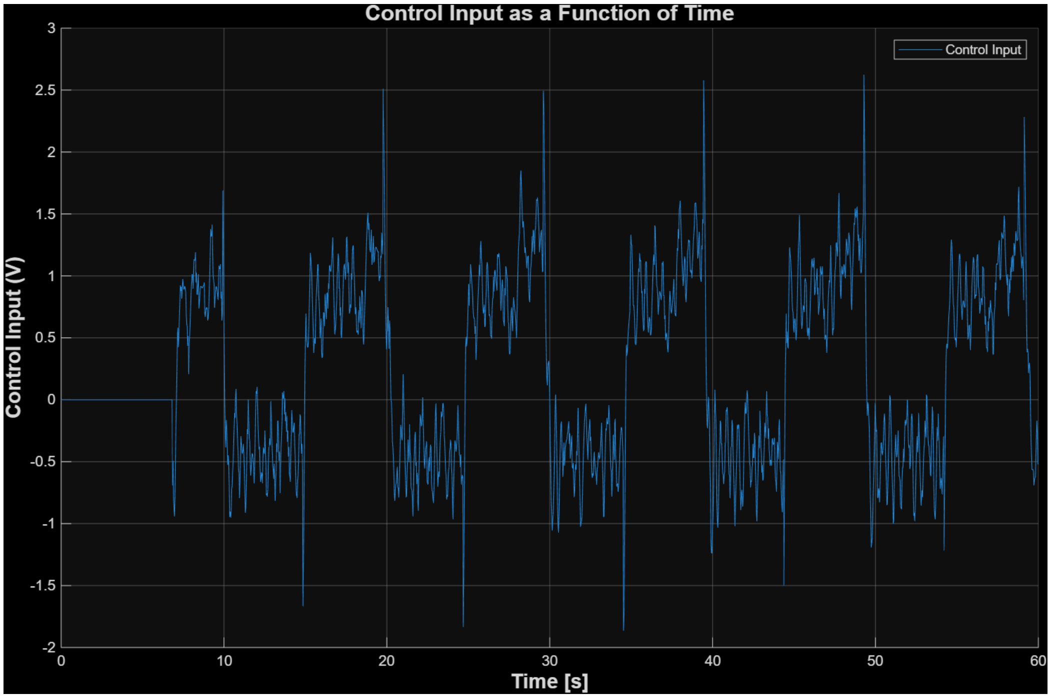

Figure 9. Graph plotting the motor voltage control input against time. Note that the voltage on the motor is limited to 5.875V.

The results from figures 7 through 9 conform to our expectations from the parameters we chose in our gain tuning. We can see in Figure 8 that the pendulum angle is very stable, fluctuating by less than 0.02 radians (~2 degrees). It is difficult to calculate the settling time in θ as the average fluctuation in angle is a large proportion of the maximum angle (indicating our system is very stable). On the other hand, the cart position tracking is quite poor. This is because we designed our controller to greatly prioritize keeping the error in θ small. In the real world it is quite difficult to simultaneously track the position well and prevent the pendulum from falling over, due to various nonlinear effects present in the system dynamics (such as friction, and imperfections in the actual system compared to the ideal simulation). Thus, it may be very time consuming or even be impossible to tune a perfect LQR gain in the actual system. As an additional note, our control input was tuned quite well, as there was nearly no saturation (the motor voltage did not approach $\pm$5.875V at any point). Below are the simulated graphs for $z(t)$, $\theta(t)$, and $u(t)$.

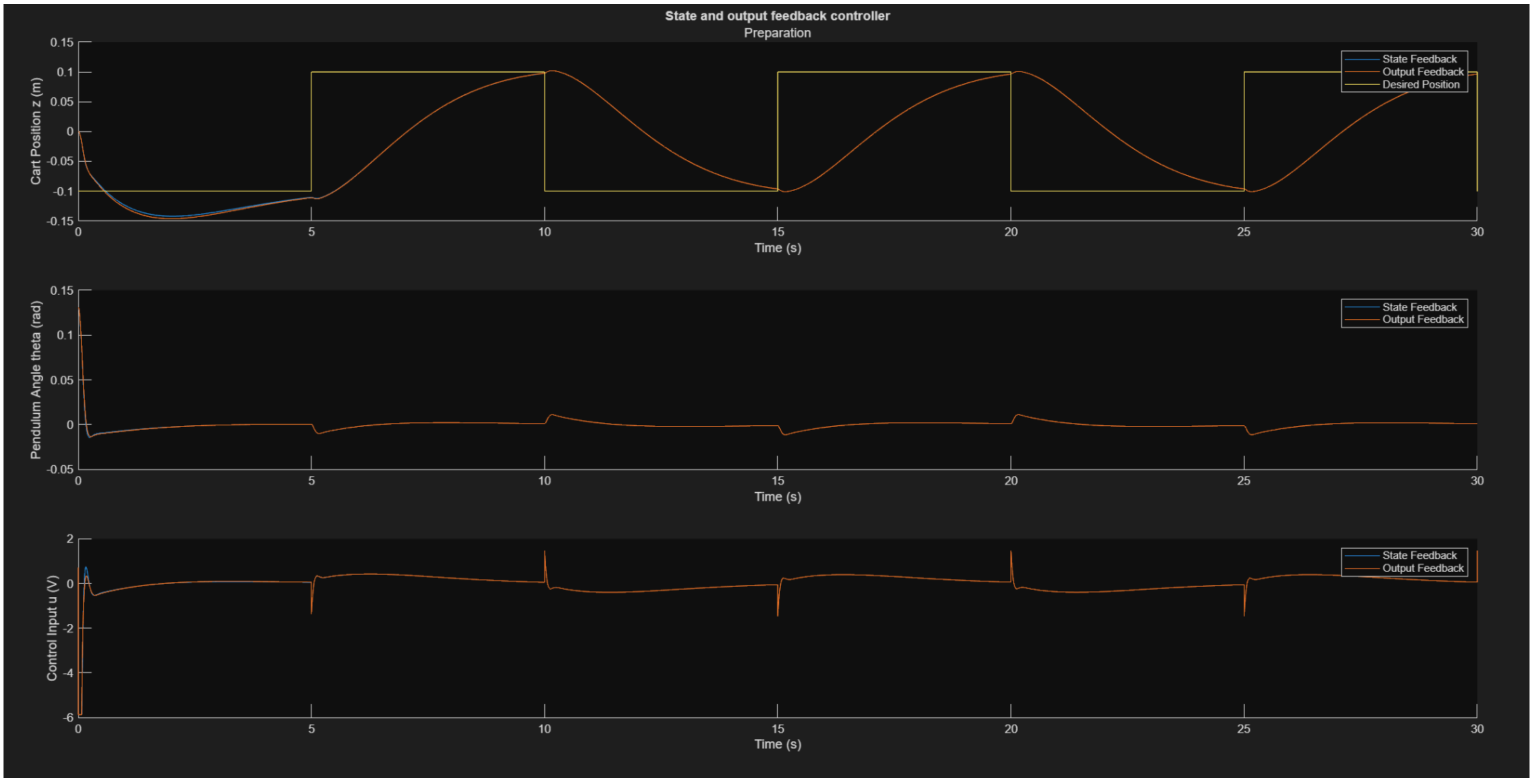

Figure 10. Simulated graphs of cart position, pendulum angle, and motor control input, as calculated in MatLab. The same LQR parameters found in the physical lab were used to produce these diagrams.

From figure 10 we can see the simulated results, and thus compare them to the real world experiment. In general, we can notice how all three graphs match up quite well to their simulated values. For example, both simulated and experimental cart positions are roughly the same. In the graphs for $u(t)$ and $\theta(t)$, there is noticeably more noise in the outputs for the experimental values, but the peak to peak values are about the same. Again, this can mostly be attributed to sensor noise and real world system dynamics such as rail friction or motor inductance.

Part 3

In this part, we augmented our original linearized cart pendulum model with an extra integral state, the tracking error, defined as the difference between the cart's desired position on the track, and its current position. We tuned it by continuously increasing the integral gain until we removed the bias in our controller we observed in part 2. We only did a very minor modification to the gains of the original LQR controller by setting $q_1 = 2$, with $q_2 = 300$,

$R = 0.01$, as before. We found that our steady state bias was gone with an integral gain, $q_3 = 10$.

As we were satisfied with the stability of the system in part 2, we focused our tuning efforts in this part on reducing the tracking error in the cart position, as we sacrificed position error to ensure the pendulum angle always stayed close to zero, and thus maintaining the assumptions needed for the linear model. Due to this tradeoff, we found that while our pendulum was stable, our cart was oscillating about a position that was biased from zero, which can be seen in Figure 7 as the cart oscillates about $z = 0.1$ instead of $z = 0$. We very slightly increased q_2, which we noted had slightly better performance, but otherwise we tuned the new q_3 integral term by setting it to a value of 1, and then increasing it slowly until we were satisfied that the cart was oscillating about $z = 0$. Thus, we found that $q_3 = 10$ was the smallest term that enables us to achieve this zero bias in the cart's position on the track. This can be observed in Figure 11 below.

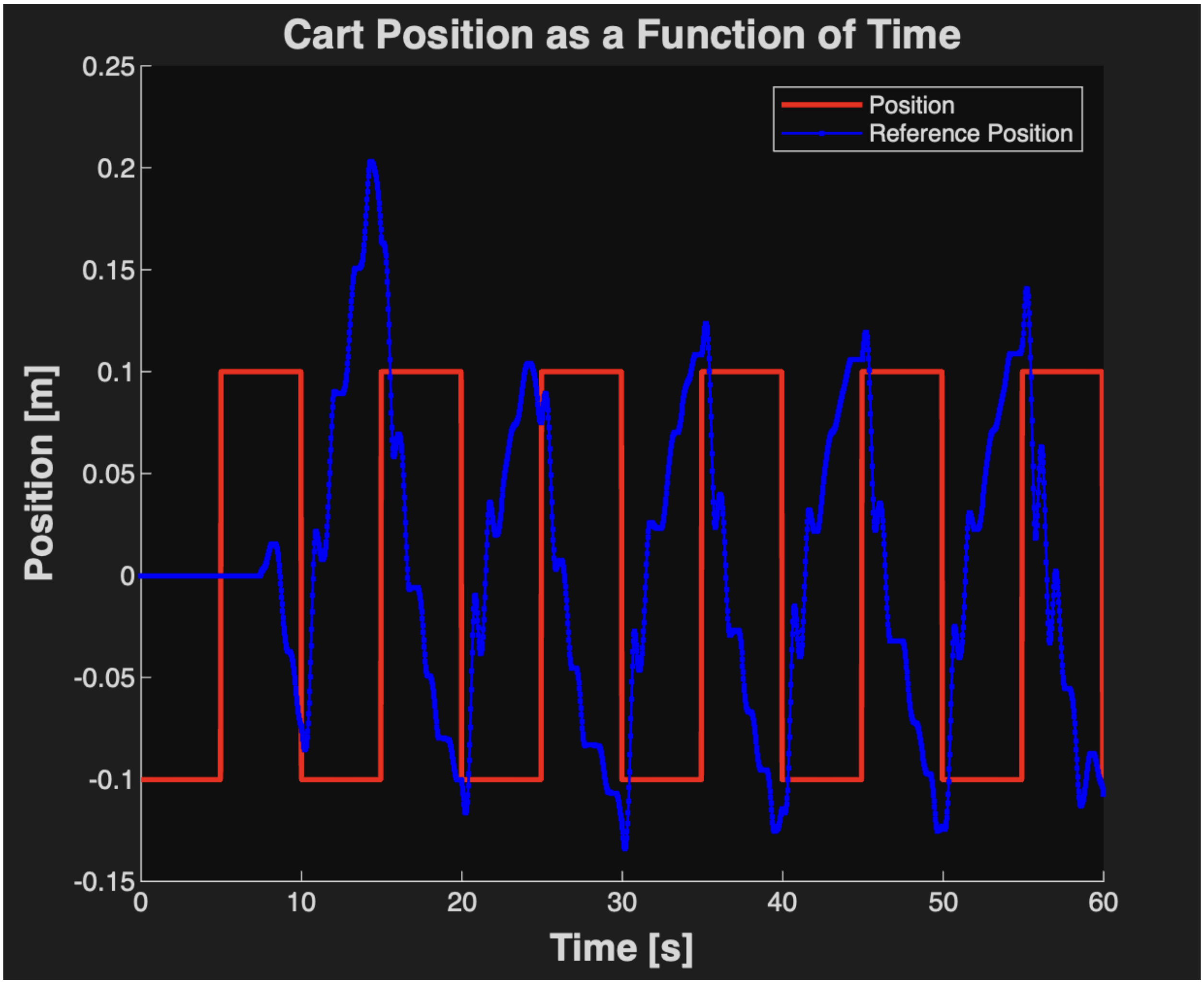

Figure 11. Graph plotting reference and actual cart position against time for the LQR controller augmented with an integral term.

Comparing the plot in Figure 11 to Figure 7, we see that the midline of the oscillation is at $$z=0$$, which indicates good tracking. The first peak has a weird shape, which is a combination of the position initially being zero while the reference is non-zero, along with the fact we have to push the pendulum up to get it started, which is a force we do not account for. We also note that once the system reaches a stable equilibrium following the first peak, its peaks have a period which matches the input wave, which indicates that the period of the position input is not too aggressive for this controller. The amplitudes slightly overshoot the position input, but are within 50% of the original input value. While this seems large, it should be noted that we are dealing with small changes in position, and thus would expect the percent error to be a bit higher. We also note that whereas the plot in Figure 7 has smooth position references, ours are jagged, and the slope slightly decreases as it reaches the reference. This is visual evidence of the integral controller, whose input increases relative to the tracking error, and thus the control input applied near a reference is lower than when it's far away.

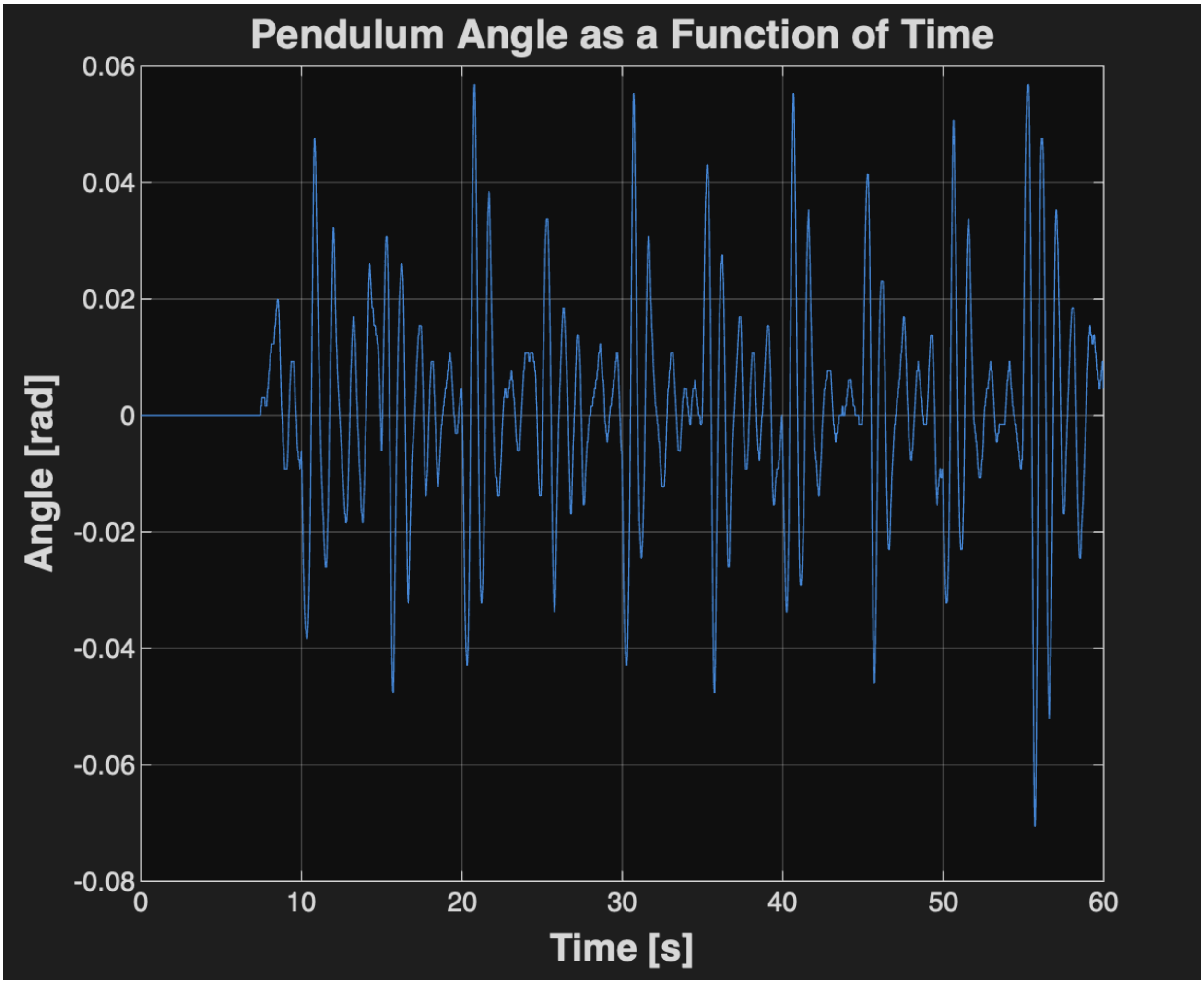

Figure 12. Graph plotting pendulum angle against time for the LQR controller augmented with integral control. Note that the spikes correspond with changes in the reference cart position signal.

Comparing the plot in Figure 12 to Figure 8, we note that the peak amplitudes are 3x greater in Figure 12. This makes sense as the cart's movement is less smooth, and less predictable, requiring more aggressive angular control to stabilize. But despite that, this still means we have a max angular deviation of 3.44 degrees, which is quite small and within the range where a linear approximation of sines and cosines is valid.

Noting the observations we have made in Figure 11 and 12, it is easy to conclude that the LQR controller augmented with integral control is superior to the controller that only uses LQR, since the LQR only controller could not correct for the bias in its position midline, whereas the integral augmented controller managed to correct the bias in its midline in one period, resulting in vastly improved position tracking. Although the error in the pendulum angle increased, it remains stable, and is unlike the situation with the LQR only controller where we had to sacrifice asymptotic stability in the position error for stability in the pendulum angle.