Theory

Reasoning about Calculational Proofs

Prove that `(o ((curry map) f) ((curry map) g)) == ((curry map) (o f g))`
This means we need to show that
`((o ((curry map) f) ((curry map) g)) xs) == (((curry map) (o f g)) xs)`
Thus, we will use induction to show that the two functions are equivalent.

Base Case

Assume that the given list to the function is empty. Then xs = '().

```
((o ((curry map) f) ((curry map) g)) xs)
 = {By assumption that xs is empty}
((o ((curry map) f) ((curry map) g)) '())
 = {By apply-compose law}
(((curry map) f) (((curry map) g) '()))
 = {By apply-curry law}
(((curry map) f) (map g '()))
 = {By map-nil law}
(((curry map) f) '())
 = {By apply-curry law}
(map f '())
 = {By map-nil law}
'()
 = {By map-nil}
(map (o f g) '())
 = {By apply-curry law}
(((curry map) (o f g)) ()')
 = {By assumption that xs is empty}
(((curry map) (o f g)) xs)
```

Inductive Case

Let xs be a non-empty list xs = (cons y ys) and assume that the inductive hypothesis

`((o ((curry map) f) ((curry map) g)) xs) == (((curry map) (o f g)) xs)`

is true for all lists with length shorter than xs (aka that it is true for ys)

```
((o ((curry map) f) ((curry map) g)) xs)
  = {By apply-compose law}
(((curry map) f) (((curry map) g) xs))
  = {By apply-curry law}
(((curry map) f) (map g xs))
  = {By apply-curry law}
(map f (map g xs))
  = {By xs = (cons y ys)}
(map f (map g (cons y ys)))
  = {By map-cons law}
(map f (cons (g y) (map g ys)))
  = {By map-cons law}
(cons (f (g y)) (map f (map g ys)))
  = {By apply-curry law}
(cons (f (g y)) (map f (((curry map) g) ys)))
  = {By apply-curry law}
(cons (f (g y)) (((curry map) f) (((curry map) g) ys)))
  = {By apply-compose law}
(cons (f (g y)) ((o ((curry map) f) ((curry map) g)) ys))
  = {By inductive hypothesis}
(cons (f (g y)) (((curry map) (o f g)) ys))
  = {By appy-curry law}
(cons (f (g y)) (map (o f g) ys))
  = {By apply-compose law}
(cons ((o f g) y) (map (o f g) ys))
  = {By map-cons law}
(map (o f g) (cons y ys))
  = {By xs = (cons y ys)}
(((curry map) (o f g)) xs)
```

Thus, by induction,
`((o ((curry map) f) ((curry map) g)) xs) == (((curry map) (o f g)) xs)`
So the functions are observational equivalent, so
`(o ((curry map) f) ((curry map) g)) == ((curry map) (o f g))`