

Part A: Theory

1. Calculational Proof

Base Case: $xs = '()$

$$\begin{aligned} & (\text{append } (\text{append } xs \text{ } ys) \text{ } zs) \\ &= \{ \text{By assumption that } xs = '() \} \\ & (\text{append } (\text{append } '() \text{ } ys) \text{ } zs) \\ &= \{ \text{By append-nil law} \} \\ & (\text{append } ys \text{ } zs) \\ &= \{ \text{By append-nil law (in reverse)} \} \\ & (\text{append } '() \text{ } (\text{append } ys \text{ } zs)) \\ &= \{ \text{By } xs = '() \} \\ & (\text{append } xs \text{ } (\text{append } ys \text{ } zs)) \end{aligned}$$

Inductive Case: $xs = (\text{cons } w \text{ } ws)$ [Non-empty list]

$$\begin{aligned} & (\text{append } (\text{append } xs \text{ } ys) \text{ } zs) \\ &= \{ \text{By } xs = (\text{cons } w \text{ } ws) \} \\ & (\text{append } (\text{append } (\text{cons } w \text{ } ws) \text{ } ys) \text{ } zs) \\ &= \{ \text{By append-cons law} \} \\ & (\text{append } (\text{cons } w \text{ } (\text{append } ws \text{ } ys)) \text{ } zs) \\ &= \{ \text{By append-cons law} \} \\ & (\text{cons } w \text{ } (\text{append } (\text{append } ws \text{ } ys) \text{ } zs)) \\ &= \{ \text{By the inductive hypothesis, } (\text{append } (\text{append } ws \text{ } ys) \text{ } zs) = (\text{append } ws \text{ } (\text{append } ys \text{ } zs)) \} \\ & (\text{cons } w \text{ } (\text{append } ws \text{ } (\text{append } ys \text{ } zs))) \\ &= \{ \text{By append-cons law (in reverse)} \} \\ & (\text{append } (\text{cons } w \text{ } ws) \text{ } (\text{append } ys \text{ } zs)) \\ &= \{ \text{By } xs = (\text{cons } w \text{ } ws) \} \\ & (\text{append } xs \text{ } (\text{append } ys \text{ } zs)) \end{aligned}$$

Thus, by calculations proof, $(\text{append } (\text{append } xs \text{ } ys) \text{ } zs) = (\text{append } xs \text{ } (\text{append } ys \text{ } zs))$