Homework 7: Monomorphic and Polymorphic Type Systems

Matthew Wong, Kaelen Song

March 2024

Formation

This law defines the $\textsc{List}(\tau)$ type, which represents a list which contains elements of type τ. Note that this means that lists can only have one type τ.

$$\frac{\tau \text{ is a type}}{\textsc{List}(\tau) \text{ is a type}} \; (\textsc{List})$$

Introduction

This law defines a non-empty list is of type τ creat.

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \tau \quad \Gamma_{\xi,\phi,\rho} \vdash y : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Cons-List}(x, y) : \textsc{List}(\tau)} \; (\textsc{ConsList})$$

$$\frac{\tau \text{ is a type}}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Empty-List}(\tau) : \textsc{List}(\tau)} \; (\textsc{EmptyList})$$

Elimination

null?, car, and cdr are all elimination rules, as they are rules which can "observe" information about the list. null? checks if the list is empty, so it returns a boolean given a list of type τ. car returns the first element of the cons cell, which will be the type of the list τ, so car will return type τ. cdr returns the second element of the cons cell, which in a list, is the rest of the list, so cdr will return type $\textsc{List}(\tau)$.

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Is-Null}(x) : \textsc{Bool}} \; (\textsc{IsNull})$$

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Car}(x) : \tau} \; (\textsc{Car})$$

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Cdr}(x) : \textsc{List}(\tau)} \; (\textsc{Cdr})$$