# Homework 7: Monomorphic and Polymorphic Type Systems

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## Formation

This law defines the $\textsc{List}(\tau)$ type, which represents a list which contains elements of type $\tau$. Note that this means that lists can only have one type $\tau$.

$$\frac{\tau \text{ is a type}}{\textsc{List}(\tau) \text{ is a type}} \ (\textsc{List})$$

## Introduction

This law defines a non-empty $\textsc{List}(\tau)$ is of type $\tau$ created using cons.

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \tau \quad \Gamma_{\xi,\phi,\rho} \vdash y : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Cons-List}(x, y) : \textsc{List}(\tau)} \ (\textsc{ConsList})$$

$$\frac{\tau \text{ is a type}}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Empty-List}(\tau) : \textsc{List}(\tau)} \ (\textsc{EmptyList})$$

## Elimination

null?, car, and cdr are all elimination rules, as they are rules which can "observe" information about the list. null? checks if the list is empty, so it returns a boolean given a list of type $\tau$. car returns the first element of the cons cell, which will be the type of the list $\tau$, so car will return type $\tau$. cdr returns the second element of the cons cell, which in a list, is the rest of the list, so cdr will return type $\textsc{List}(\tau)$.

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Is-Null}(x) : \textsc{Bool}} \ (\textsc{IsNull})$$

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Car}(x) : \tau} \ (\textsc{Car})$$

$$\frac{\Gamma_{\xi,\phi,\rho} \vdash x : \textsc{List}(\tau)}{\Gamma_{\xi,\phi,\rho} \vdash \textsc{Cdr}(x) : \textsc{List}(\tau)} \ (\textsc{Cdr})$$