

Stat 243 – Homework 03

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11.31a It is not always the case that 1 out of 10 adults are left-handed although approximately 10% of adults are left-handed because since it is a random selection, we could have a case where most of the 10% of left-handed adults are picked or even none. This randomness means that we can't be sure of our results in taking a simple random sample of 10 people.

11.31b It is given that the probability of each pitch is $1/3$. This is true only when the pitches are equally likely. But, the pitches are not equally likely in baseball.

11.31c It is given that the probability that the two consecutive rolls turn up 1 is $1/6 + 1/6 = 1/3$. But in this case, the two consecutive rolls that give us 1 are independent events. This means that the probability of two consecutive rolls is not found by adding but rather multiplying the probabilities; the multiplication rule should be used here.

Therefore, the probability that the two consecutive rolls that show 1 is $1/6 * 1/6 = 1/36$.

11.31d The probability here is more than 1. This doesn't make sense because the probability of any event can only lie between 0 and 1.

11.32a About 20% of movies are comedies. Therefore the probability is $P(C) = 0.20$

About 9% of movies were produced by Warner Bros. Therefore the probability is $P(W) = 0.09$

About 2% of movies are comedies from Warner Bros. Therefore the probability is $P(C \text{ and } W) = 0.02$

11.32b $P(C \text{ or } W) = P(C) + P(W) - P(C \text{ and } W) = 0.20 + 0.09 - 0.02 = 0.29 - 0.02 = 0.27$

Therefore, the probability that a Warner Bros movie is a comedy is **0.27**

11.32c $P(C|W) = P(C \text{ and } W) / P(W) = 0.02 / 0.09 = 0.222$

Therefore, about **22.2%** of the movies produced by Warner Bros are comedies.

11.32d $P(W|C) = P(C \text{ and } W) / P(C) = 0.02 / 0.20 = 0.1$

Therefore, about **10%** of the movies are comedies which are produced by Warner Bros.

11.32e $P(\text{not } C) = 1 - P(C) = 1 - 0.20 = 0.80$

Therefore, the probability that a movie is not a comedy is **0.80**.

11.32f The events C and W are disjoint means that there are no comedy movies produced by Warner Bros. About 2% of movies are comedies from Warner Bros. Therefore, the events C and W are **not disjoint**.

11.32g The events C and W are independent mean that given that the movie produced by Warner Bros has no information about whether it is comedy and given that the movie is comedy, it has no information about whether it is produced by Warner Bros.

From part (c), $P(C|W) = 0.222$

From the information we have, $P(C) = 0.2$

Therefore, $P(C|W) \neq P(C)$

From part (d), $P(W|C) = 0.10$

From the information given, $P(W) = 0.09$

Therefore, $P(W|C) \neq P(W)$

From all this information, it is clear that neither of the results is equal. This means that it can be concluded that the events C and W are **not independent**.

11.34a $P(MP) = \text{no. of music performers} / \text{total no. of inductees} = 181/273 = 0.633$

Therefore, the probability that an inductee is a performer is **0.663**.

11.34b $P(\text{not } F) = 1 - P(F) = 1 - (\text{no. of female members} / \text{total no. of inductees}) = 1 - 41/273 = 1 - 0.15 = 0.85$

Therefore, the probability that an inductee chosen does not have any female members is **0.85**.

11.34c $P(F|MP) = P(F \text{ and } MP) / P(MP) = (32/273) / (181/273) = 32/181 = 0.177$

Therefore, the probability that an inductee chosen has female members if it is a performer is **0.177**.

11.34d $P(\text{not } MP | \text{not } f) = P(\text{not } MP \text{ and not } F) / P(\text{not } F) = (83/273) / (232/273) = 83/232 = 0.358$

Therefore, the probability that an inductee chosen is not a performer if it has no female members is **0.358**.

11.34e $P(MP \text{ and not } F) = \text{no. of performers and not female members} / \text{total no. of inductees} = 149/273 = 0.546$

Therefore, the probability that an inductee chosen is a performer with no female members is **0.546**.

11.34f $P(\text{not } MP \text{ or } F) = P(\text{not } MP) + P(F) - P(\text{not } MP \text{ and } F) = 92/273 + 41/273 - 9/273 = 124/273 = 0.454$

Therefore, the probability that an inductee chosen is either not a performer or has female members is **0.454**.

11.36a $P(R) = \text{no. of red candies} / \text{total candies} = 11/80 = 0.1375$

Therefore, the probability that the candy selected is red is **0.1375**.

11.36b $P(\text{not } B) = 1 - P(B) = 1 - \text{no. of blue candies} / \text{total candies} = 1 - 20/80 = 1 - 0.25 = 0.75$

Therefore, the probability that the candy selected is not blue is **0.75**.

$$\mathbf{11.36c} \quad P(R \text{ or } O) = P(R) + P(O) - P(R \text{ and } O) = 11/80 + 12/80 - 0 = 23/80 = 0.2875$$

Therefore, the probability that the candy selected is red or orange is **0.2875**.

$$\mathbf{11.36d} \quad P(B1 \text{ and } B2) = P(B1) * P(B2) = 20/80 * 20/80 = 400/6400 = 0.0625$$

Therefore, the probability that both first and second candies selected are blue is **0.0625**.

$$\mathbf{11.36e} \quad P(R \text{ and } G) = P(R) * P(G|R) = 11/80 * 11/79 = 121/6320 = 0.0191$$

Therefore, the probability that the first candy selected is red and the second candy selected is green is **0.0191**.

$$\mathbf{11.39a} \quad P(M) = 7/100 = 0.07$$

Therefore, the probability of selecting a red-green color-blind male is **0.07**.

$$\mathbf{11.39b} \quad P(\text{not } W) = 1 - P(W) = 1 - 0.4/100 = 1 - 0.004 = 0.996$$

Therefore, the probability of selecting a woman who is not red-green color-blind is **0.996**.

$$\mathbf{11.39c} \quad P(\text{not } M) * P(\text{not } F) = [1 - P(M)] * [1 - P(F)] = (1 - 0.07) * (1 - 0.004) = 0.93 * 0.996 = 0.926$$

Therefore, the probability that neither of the two selections are red-green color-blind is **0.926**.

$$\mathbf{11.39d} \quad P(\text{at least one}) = 1 - P(\text{neither}) = 1 - 0.926 = 0.074$$

Therefore, the probability that at least one of the selections is red-green color-blind is **0.074**.

$$\mathbf{11.57} \quad P(\text{person with restless leg had fibromyalgia}) = P(F|R) = P(F \cap R) / P(R) = P(R|F) * P(F) / P(R|F) * P(F) + P(R|F) = (0.33 * 0.02) / ((0.33 * 0.02) + (0.03 * 0.98)) = \mathbf{0.1833}$$

$$\mathbf{11.58} \quad P(\text{Positive if cancer}) = 1 - P(\text{Positive if no cancer}) = 1 - 1.1/1000 = 1 - 0.0011 = 0.9989$$

$$P(\text{Cancer}) = 1/38 = 0.0263$$

$$\begin{aligned} P(\text{Cancer if Positive}) &= \frac{P(\text{Positive and Cancer})}{P(\text{Positive})} \\ &= \frac{P(\text{Cancer}) * P(\text{Positive if cancer})}{[P(\text{No cancer}) * P(\text{Positive if no cancer})] + [P(\text{Cancer}) * P(\text{Positive if Cancer})]} \\ &= \frac{0.0263 * 0.9989}{[(1 - 0.0263) * 0.0866] + [0.0263 * 0.9989]} \\ &= \frac{0.0263 * 0.9989}{(0.9737 * 0.0866) + (0.0263 * 0.9989)} \\ &= \frac{0.02627107}{0.08432242 + 0.02627107} \\ &= \frac{0.02627107}{0.11059349} = 0.2375 \end{aligned}$$

Therefore, the probability that a selected woman has breast cancer if she had positive result is **0.2375**.

$$\mathbf{11.83a} \quad \text{Sum} = 0.217 + 0.363 + 0.165 + 0.145 + 0.067 + 0.026 + 0.018 = 1.001$$

Over here, the sum is slightly greater than one. This happens due to the rounding off the individual probabilities.

$$11.83b \quad P(X=1 \text{ or } X=2) = p(1) + p(2) = 0.217 + 0.363 = 0.58$$

Therefore, the probability that a unit has only one or two people is **0.58**.

$$11.83c \quad P(X \geq 5) = p(5) + p(6) + p(7) = 0.067 + 0.026 + 0.018 = 0.111$$

Therefore, the probability that a unit has five or more people is **0.111**.

$$11.83d \quad P(X > 1) = 1 - 0.217 = 0.783$$

Therefore, the probability that more than one person lives in a housing unit is **0.783**.

$$11.87a \quad p(1) + p(2) + p(3) + p(4) + p(5) + p(6) = 1$$

$$0.30 + p(2) + 0.20 + 0.15 + 0.10 + 0.05 = 1$$

$$p(2) = 1 - 0.80 = 0.20$$

Therefore, the proportion of fruit flies that die in their second month is **0.20**.

$$11.87b \quad P(X > 4) = p(5) + p(6) = 0.10 + 0.05 = 0.15$$

Therefore, the probability of fruit flies that live more than 4 months is **0.15**.

$$11.89a \quad P(A|B) = P(A \text{ and } B)/P(B) = P(A)/P(X=1 \text{ or } X=2) = \frac{P(X=1)}{P(X=1)+P(X=2)} = \frac{0.30}{0.30+0.20} = 0.30/0.50 = 0.6$$

Therefore, the probability that the fruit fly died in its first month if it died before the end of the second month is **0.60**.

$$11.89b \quad P(C|D) = P(C \text{ and } D)/P(D) = P(C)/P(X \geq 3) = \frac{P(X=5)+P(X=6)}{P(X=3)+P(X=4)+P(X=5)+P(X=6)} \\ = \frac{0.10+0.05}{0.20+0.15+0.10+0.05} = 0.15/0.5 = 0.3$$

Therefore, the probability that the fruit fly lies more than four months if it is past the second month is **0.30**.

$$11.117 \quad n = 3, p = 0.49$$

$$P(X=k) = (nCk)p^k(1-p)^{n-k}$$

$$P(X=0) = (3C0) * (0.49)^0 * (0.51)^3 = (\frac{3!}{0!*(3-0)!}) * (0.49)^0 * (0.51)^3 = (\frac{3!}{(0!)*(3!)}) * (0.49)^0 * (0.51)^3 \\ = (\frac{3*2*1}{(1)*(3*2*1)}) * (1) * (0.133) = 1 * 1 * 0.133 = 0.133$$

$$P(X=1) = (3C1) * (0.49)^1 * (0.51)^{3-1} = (\frac{3!}{1!*(3-1)!}) * (0.49)^1 * (0.51)^2 = (\frac{3!}{(1!)*(2!)}) * (0.49)^1 * (0.51)^2 \\ = (\frac{3*2*1}{(1)*(2*1)}) * (0.49) * (0.2601) = 3 * 0.49 * 0.2601 = 0.382$$

Therefore, the probability that one of the babies is a girl is **0.382**.

$$P(X=2) = (3C2) * (0.49)^2 * (0.51)^{3-2} = (\frac{3!}{2!*(3-2)!}) * (0.49)^2 * (0.51)^1 = (\frac{3!}{(2!)*(1!)}) * (0.49)^2 * (0.51)^1 \\ = (\frac{3*2*1}{(2*1)*(1)}) * (0.2401) * (0.51) = 3 * 0.2401 * 0.51 = 0.367$$

Therefore, the probability that two babies are girls is **0.367**.

$$P(X=3) = (3C3) * (0.49)^3 * (0.51)^{3-3} = (\frac{3!}{3!*(3-3)!}) * (0.49)^3 * (0.51)^0 = (\frac{3!}{(3!)*(0!)}) * (0.49)^3 * (0.51)^0$$

$$= \left(\frac{3*2*1}{(3*2*1)*(1)}\right) * (0.118) * (1) = 1 * 0.118 * 1 = 0.118$$

Therefore, the probability that two babies are girls is **0.118**.

```
ProbabilityTable <- data.frame("X" = "p(x)",
                                "0" = 0.133,
                                "1" = 0.382,
                                "2" = 0.367,
                                "3" = 0.118)

ProbabilityTable
```

```
##      X      X0      X1      X2      X3
## 1 p(x) 0.133 0.382 0.367 0.118
```

$$11.120 \quad P(X=k) = (nCk)p^k(1-p)^{n-k}$$

$$P(X=3) = (10C3) * (0.13)^3 * (1-0.13)^{10-3} = \left(\frac{10!}{3!(10-3)!}\right) * (0.13)^3 * (0.87)^7$$

$$= \left(\frac{10*9*8*7*6*5*4*3*2*1}{(3*2*1)*(7*6*5*4*3*2*1)}\right) * (0.002197) * (0.377255) = \frac{720}{6} * 0.0008288 = \frac{0.596736}{6} = 0.099$$

Therefore, the probability that 3 people are 65 or older is **0.099**.

$$P(X=4) = (10C4) * (0.13)^4 * (1-0.13)^{10-4} = \left(\frac{10!}{4!(10-4)!}\right) * (0.13)^4 * (0.87)^6$$

$$= \left(\frac{10*9*8*7*6*5*4*3*2*1}{(4*3*2*1)*(6*5*4*3*2*1)}\right) * (0.00028561) * (0.433626) = \frac{5040}{24} * 0.00012385 = \frac{0.624204}{24} = 0.026$$

Therefore, the probability that 4 people are 65 or older is **0.026**.

$$11.127a \quad P(X=k) = (nCk)p^k(1-p)^{n-k}$$

$$P(X \geq 7) = P(X=7) + P(X=8) = (8C7) * (0.881)^7 * (1-0.881)^{8-7} + (8C8) * (0.881)^8 * (1-0.881)^{8-8}$$

$$= \left(\frac{8!}{7!(8-7)!}\right) * (0.881)^7 * (0.119)^1 + \left(\frac{8!}{8!(8-8)!}\right) * (0.881)^8 * (0.119)^0$$

$$= \left(\frac{8*7!}{7!(1)!}\right) * (0.411938) * (0.119) + \left(\frac{8!}{8!(0)!}\right) * (0.36291696) * (1) = (8 * 0.411938 * 0.119) + (1 * 0.36291696 * 1)$$

$$= 0.3922 + 0.3629 = 0.7551$$

$$11.127b \quad P(X \geq 70) = 1 - P(X < 70) = 1 - 0.3531 = 0.6469$$

Therefore, the probability that at least 70 free throws are made in a game is **0.6469**.

5.1a HH, HT, TH, TT

5.1b 12, 13, 14, 15, 21, 23, 24, 25, 31, 32, 34, 35, 41, 42, 43, 45, 51, 52, 53, 54 = **16 ways**

5.3 A reasonable listing would be inbetween [4'00" and 6'3"]. This is a guess based on typical heights for women between ages of 18 and 25.

5.4 He is stating that 70% of the Grand Rapids area will experience rain, frequentest interpretation would be that some but not all will be rained on, and subjectiveist would say there may be rain in some areas but not all.

5.5a HHHH HHHT HHTT HHTHHHTH HTHT HTTH HTTT THHH THHT THTH THTT TTHH TTHT TTTH TTTT = **16 options**

5.5b $p(0) = 0.02777$, $p(1) = 0.13888$, $p(2) = 0.27777$, $p(3) = 0.27777$, $p(4) = 0.13888$, $p(5) = 0.02777$

5.6a There are 1024 outcomes

5.6b Exactly 10

5.6c $p(1) = 0.00976562$

5.6d $p(9)$

5.9 $p(2B) = (4/10) * (3/9)$

5.11a 505

```
n = 1000
flips = sample (c(0,1), replace = True, size = n)
str_count(paste(flips, collapse=""), '1')
= 504
```

5.11b When the sample size was increased, so was the proportion of heads that were flipped (5062)

5.15a It is unreasonable because we can visually see that they are all different sizes and probabilities.

5.15b The equal likelihood idea is that things are equally likely to happen, so an estimate that 14 will be landed on is 1 and 14 or $1/14$.

5.18a $\text{pbinom}(4500, 10000, 0.45) = 0.5041431$

5.18b $\text{pbinom}(900, 2000, 0.45) = 0.5092631$

5.19a $p(w) = 0.1631301$

5.19b $p(w \text{ out}) = 0.3231301$