Unemployment Volatility: When Workers Pay Costs upon Accepting Jobs

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May 27, 2024

Abstract

Hiring workers is costly. Firms' costs reduce resources that can go to recruitment and amplify how unemployment responds to changes in productivity. Workers also incur up-front costs. Examples include moving expenses and regulatory fees. Workers' costs lessen unemployment volatility and leave resources available for recruitment unchanged. Their influence is bounded by properties of a matching function. Using adjusted data on job finding, I estimate a bound that ascribes limited influence. The results demonstrate that workers' costs affect outcomes (firms threaten workers with paying the fixed costs again if negotiations fail), but their influence on volatility is less than firms' costs.

Keywords: business cycle, fundamental surplus, job creation, job finding, job search, market tightness, matching function, matching models, Nash wage equation, productivity, search frictions, unemployment, unemployment volatility

JEL Codes: E23, E24, E32, J23, J29, J63, J64

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1 Introduction

Firms recruit workers by posting notices to online job boards and taping help-wanted signs to storefront windows. When the advertisement is answered and a firm decides to hire a worker, the firm must pay the cost of adding the worker to payroll. This one-off cost of creating a job reduces the amount of resources that can be allocated to recruitment. Recruitment determines how job openings respond and therefore how unemployment responds to changes in fundamentals like productivity (Ljungqvist and Sargent, 2017).

Imagine that Acme Corporation wants to hire a worker and Don wants a job. Acme posts their opening to an online job board, which advertises the position for a monthly fee. Acme expects to have trouble filling their position in periods when there are many other firms looking to add workers and few workers looking for jobs. Meanwhile, Don searches for work, collects benefits from unemployment insurance, and enjoys some leisure that in total amounts to z. While scouring online job boards, Don finds the Acme listing. He applies, interviews, and accepts the job. Don uses the technology at Acme to produce y.

The match between Don and Acme generates the surplus $y - z$. But Acme must first pay the fixed cost of adding Don to payroll. If this cost is properly accounted at ξ, then only $y - z - \xi$ can be allocated to vacancy creation. Viewed as a fraction of output, potential resources for vacancy creation are increasing in y: the derivative of $(y - z - \xi)/y$ is positive. A change in productivity will generate a large increase in this fraction if $(z + \xi)/y^2$ is large, which occurs when $z + \xi$ is large. The presence of costs paid by firms upon hiring workers means resources allocated to job creation will respond more to changes in productivity, which will generate larger unemployment responses. Importantly, Ljungqvist and Sargent (2017) establish that Acme's expected cost of posting the vacancy to the online job board is much less important than the cost of adding Don to payroll.

One-off up-front costs paid by firms offer an answer to a widely acknowledged puzzle: the failure of the standard search model to match the observed volatility of unemployment (see, for example, Hall, 2005b; Shimer, 2005; Mortensen and Nagypál, 2007; Costain and Reiter, 2008; Pissarides, 2009; Gertler and Trigari, 2009; Brügemann and Moscarini, 2010; Gomme and Lkhagvasuren, 2015; Kehoe et al., 2023).

That costs paid by firms can explain unemployment volatility suggests these types of costs are worth exploring. Surprisingly, while firms' costs have been partially explored, costs paid by workers upon taking a job have not, despite evidence that the prevalence of these costs has risen.

Costs paid by workers upon accepting a job. In short, here is the main issue: Responses of unemployment to changes in productivity depend on resources that can be allocated to vacancy creation. These resources are reduced by costs paid by firms upon hiring a worker. The effect is larger responses of unemployment to changes in productivity. Do costs paid by workers upon accepting a job have a similar effect? Put another way using the example above, if Don has to relocate for the job at Acme, do Don's costs subtract from resources available for vacancy creation?

Examples of up-front costs paid by workers include not only any costs to relocate for work but also any administrative fees associated with job regulation. I place these costs in an economy that features unemployment. Unemployment arises because some jobs end through separation and labor-market frictions prevent firms from instantaneously hiring workers and workers from instantaneously finding jobs. In a similar Diamond–Mortensen–Pissarides environment, Mortensen and Nagypál (2007) consider turnover costs paid by firms. Turnover costs include the costs of adding a worker to payroll and taxes for firing a worker. Pissarides (2009) uses the cost a firm pays to add a worker to establish that their incorporation into a standard DMP model implies unemployment responds realistically to observed changes in productivity. Ljungqvist and Sargent (2017) identify a common channel through which these features affect the elasticity of market tightness with respect to productivity. Mortensen and Nagypál (2007), Pissarides (2009), and Ljungqvist and Sargent (2017) do not consider fixed costs paid by workers.[1]

In contrast to costs paid by firms, using the common channel identified by Ljungqvist and Sargent (2017), I find that costs paid by workers reduce the response of unemployment to changes in productivity and there is little scope for them to influence unemployment volatility. To preview results, this conclusion is based on two contributions of this paper.

1. I generalize Ljungqvist and Sargent's (2017) fundamental decomposition to show that fixed costs paid by workers are subsumed by a factor that is bounded from above by the elasticity of matching with respect to unemployment, severely limiting the scope for workers' costs to influence volatility.

2. This would be the end of story, except that I show this elasticity is not in fact bounded by "a consensus about reasonable parameter values" (Ljungqvist and Sargent, 2017, 2636). To reconcile the consensus, I estimate a matching function that does not exhibit constant elasticity of matching with respect to unemployment (like in the Cobb–Douglas case), using data on the labor force from the US Bureau of Labor Statistics and hires from the US Bureau of Labor Statistics' Job Openings and Labor Turnover Survey. These data are adjusted for time aggregation—workers can continuously find and separate from a job within a month—and how the JOLTS program records hires—all hires within a month are reported, not only hires that remain employed at the end of the month. Once I adjust for these potential sources of bias, I estimate a nonlinear matching technology and re-establish the "consensus."

The result bolsters the idea that Ljungqvist and Sargent's (2017) fundamental surplus refers to resources *available to firms* that the invisible hand can allocate to vacancy creation. Fixed costs of job creation paid by workers do not reduce these resources.

Of course up-front costs paid by workers affect labor-market outcomes. According to the traditional account, occupational regulation boosts workers' wages by restricting the supply of workers and increasing demand through higher-quality output. In the search environment, however, the firm–worker pair already enjoys a bilateral monopoly over the surplus they generate. The presence of fixed costs paid by workers allows a firm to threaten a worker with paying the fixed costs again if the wage negotiation fails. As a consequence, the negotiated wage is lower. This makes posting a vacancy more valuable, which lowers equilibrium unemployment. While

[1] Silva and Toledo (2013) revisit Pissarides's (2009) model and allow firms' fixed costs to have two components. One component of fixed cost can be partially passed on to workers through lower wages. Cheron (2005) considers the efficiency properties of a similar cost in the context of efficient reallocation (see also Miyamoto, 2011).

Hamermesh (1993, figure 2.2, page 47) provides a thoughtful description of costs faced by firms that employ workers, including hiring costs. In addition, Hamermesh (1993, 208) reports survey results on various components of costs (see also Hinkin and Tracey, 2000; Silva and Toledo, 2009, 2013).

wages are often observed to be higher in regulated occupations (think doctors and lawyers), there are few stories of certified nursing assistants getting rich. In the *Occupational Outlook Handbook*, for example, the BLS reports that "the median annual wage for nursing assistants was $35,760 in May 2022," which is $17.18 per hour.[2] Going beyond the scope of this paper, firms' gains from workers' costs could partly explain the increased prevalence of such costs.

The remainder of the paper is organized as follows. Section 2 provides motivating examples of job-creation costs paid by workers. Section 3 describes the economic environment and section 4 presents the two-factor fundamental decomposition of the elasticity of labor-market tightness with respect to productivity. Because unemployment is a fast-moving state variable, a good approximation of model dynamics is an analysis of steady states indexed by productivity (Hall, 2005a,b). For certain productivity levels the consensus that bounds the influence of job-creation costs paid by workers falls apart. Section 5 presents a calibration that restores the consensus. I calibrate the model to match the labor-market evidence in Pissarides (2009). I then estimate a matching function that does not exhibit constant elasticity of matching with respect to unemployment. The elasticities that vary by the state of the business cycle are close to the single number provided by the Cobb–Douglas evidence. Section 6 presents properties of the model, including the insight that workers' costs can reduce wages and unemployment volatility. The model's properties are discussed within the context of the literature in section 7, which includes a discussion of outstanding questions. Section 8 concludes.

2 Two Examples of Job-Creation Costs Paid by Workers

When a worker accepts a job they may be required to pay fees associated with regulation. For example, a worker may have to pay administrative fees to a government agency to file their name, address, and qualifications before starting work (Kleiner and Krueger, 2013). Administrative fees are often required when a government agency maintains a register. Variants of "Registered Dietitian" can be used by workers meeting certain requirements in California, for example, but anyone can provide nutritional advice.[3] Registration may also require a worker to post a bond in order to practice (Kleiner, 2006). A related example is testing fees. Testing fees are sometimes required when the government or a nonprofit agency issues a certificate to workers who demonstrate skill or knowledge of some tasks. Registration and certification fall under the rubric of occupational regulation (Kleiner, 2000).

In addition to these two forms of regulation, "the toughest form of regulation is licensure" (Kleiner and Krueger, 2013, S175). A licensing policy means a worker cannot legally work in an occupation unless they meet some standard. Recent data from the US Census Bureau's Survey of Income and Program Participation indicate that one in four adults aged 18 through 64 had attained a license or certificate (Gittleman, Klee, and Kleiner, 2018). In an analysis of 102 li-

[2] This information is available in the *Occupational Outlook Handbook* at https://www.bls.gov/ooh/healthcare/nursing-assistants.htm. Accessed January 12, 2024.

[3] The requirements to be a Registered Dietitian in California can be found on websites maintained by the Department of Nutritional Sciences & Toxicology at the University of California in Berkeley and the Department of Family & Consumer Sciences at California State University, Sacramento: https://nst.berkeley.edu/mnsd/how-to-become-a-registered-dietitian-nutritionist and https://www.csus.edu/college/social-sciences-interdisciplinary-studies/family-consumer-sciences/nutrition/becoming-rdn.html. Accessed January 12, 2024.

censed occupations that pay below the average income in the United States, Knepper et al. (2022) document that a worker can expect to pay $295 in licensing fees.

A few notable facts accompany this statistic on workers' costs. First, the amount does not include lost wages from time spent earning a degree or accumulating experience. Second, "licensing burdens often bear little relationship to public health or safety—the purported rationale for much licensing" (Kleiner and Vorotnikov, 2018, 8). For example, only 12 percent of the 102 occupations analyzed by Knepper et al. (2022, 37) are licensed universally across states, "which means workers are safely practicing them in at least one state—and often many more than one—without a government permission slip." Third, the prevalence of occupational licensing is on the rise (Kleiner and Krueger, 2010, 2013; DOT, CEA, and DOL, 2015; Furman and Giuliano, 2016).[4]

A worker may be required to purchase parts of their uniform. For example, a worker may be asked to wear steel-toed boots. If the employer permits them to be worn off the job-site, then the worker may be asked to purchase the boots out of their own pocket.[5]

Regulatory fees are one example of costs borne by workers when a job is created. Another example is the cost of relocation: once a job is accepted, a worker may have to move to begin work. A reasonable inference from the documented fall in worker mobility is a rise in relocation costs. Such a straightforward interpretation, though, may be incomplete. Amior (Forthcoming) provides evidence that workers move in exchange for large salaries that justify the cost of moving. Recent perspectives on mobility are provided by Kennan and Walker (2011), Molloy et al. (2016), Notowidigdo (2020), Schmutz, Sidibé, and Vidal-Naquet (2021), and Zabek (Forthcoming).

Both examples are multifaceted. I take a straightforward approach to analyzing one-off job-creation costs paid by workers. The representative experience for a worker is payment of a one-off fixed amount upon accepting a job.

The amount can be thought of as the average cost. In the model, to capture the feature that not all workers face meaningful fixed costs, each worker who finds a job faces a probability of drawing a fixed cost from a known distribution. Integrating over the idiosyncratic costs returns the average value of unemployment. Only the average value is needed to determine aggregate unemployment dynamics. The elasticity of *aggregate* labor-market tightness with respect to productivity is the fundamental decomposed by the fundamental decomposition.

3 Model: A DMP Environment with One-Off Costs When a Job Is Created

The environment shares the features of a conventional DMP model, including linear utility, workers with identical skills, random search, exogenous separations, wages determined as the outcome of Nash bargaining, and competitive job creation that drives the value of posting a vacancy to zero. In the model, firms' costs to match with a worker include recruitment costs, which are paid each period an ad for a vacancy is posted. When there are fewer unemployed workers, a vacancy

[4] Additional work on occupational licensing includes Kleiner and Todd (2009), Kleiner and Vorotnikov (2017), Johnson and Kleiner (2020), and Kleiner and Timmons (2020).

[5] The policy on steel-toed boots comes from the Occupational Safety and Health Administration, OSHA, Occupational Safety and Health Standards, Standard No. 1910.132(h)(2), United States Department of Labor, https://www.osha.gov/laws-regs/regulations/standardnumber/1910/1910.132.

will take longer to fill, making the cost of recruitment proportional to the ratio of vacancies to unemployment. This ratio is commonly called labor-market tightness.[6]

Job creation may also involve fixed matching costs paid by firms. These costs include "training, negotiation, and one-off administrative costs of adding a worker on the payroll" (Pissarides, 2009, 1363). These costs are often ignored but once they are added to a standard DMP environment, Pissarides (2009) demonstrates how their addition can generate unemployment fluctuations in response to changes in productivity that match the magnitudes observed in data. Some evidence, though, suggests that workers may bear a significant fraction of one-off administrative costs, including relocation costs, training costs, tuition, foregone wages, and testing fees.

Motivated by these features, I add fixed job-creation costs paid by workers to a standard DMP model. In addition to costs paid by workers, firms are required to pay turnover costs: hiring and firing costs. Hiring costs are emphasized by Pissarides (2009) and firing costs are studied by Mortensen and Nagypál (2007).[7]

A synthesis of these features is presented by Ljungqvist and Sargent (2017). And I build directly upon their work. In section 4, I establish that their fundamental decomposition, which reduces the elasticity of tightness with respect to productivity into two terms, holds not only for a Cobb–Douglas matching function but also for any reasonable matching function. One of the two terms subsumes costs paid by workers upon taking a job and this term is bounded by properties of a function that determines how many jobs are created when there are v vacancies and u unemployed workers. When the matching function is not Cobb–Douglas, there is no consensus on its properties. I therefore estimate in section 5 another matching function using data that are adjusted for worker flows and how the JOLTS program records hires. Properties of this function determine the maximal influence that workers' costs can have on how unemployment responds to changes in productivity.

But first I describe the model environment.

3.1 Description of the Model Environment

Time is discrete. A continuum of identical workers populate the economy. Workers are risk neutral and live forever. They are endowed with an indivisible unit of labor and they discount the future using the discount factor $\beta = (1+r)^{-1}$. They derive utility by consuming a single homogeneous good.

Workers are either unemployed or employed. Employed workers produce the consumption good using a technology owned by firms. Unemployed workers meet recruiting firms randomly. The matching process is summarized by a matching function $M(u, v)$, where u is the number of

[6] Pissarides (2000) and Petrosky-Nadeau and Wasmer (2017) provide excellent textbook treatments. Essential contributions in this area are Pissarides (1985) and Mortensen and Pissarides (1994). Diamond (1982a,b) made fundamental earlier contributions. Economic Sciences Prize Committee (2010) provides further background.

[7] Mortensen and Nagypál (2007) also consider hiring costs paid by firms. They present a version of the fundamental decomposition emphasized by Ljungqvist and Sargent (2017). Layoff costs are considered by Hamermesh (1993); Ljungqvist (2002); Cahuc and Zylberberg (2008); Pavosevich (2020); Johnston (2021).

Silva and Toledo (2009) document costs of hiring a worker and present a dynamic, stochastic model where new workers are less productive than experienced employees, which reflects the costs of hiring a worker who does not know how a specific firm operates. The costs documented by Silva and Toledo (2009) are beyond the scope of the model presented here. They rely on numerical work to show how their model fits the facts and the data (see also Silva and Toledo, 2013).

unemployed workers and v is the number of posted vacancies. The value u can be interpreted as the unemployment rate because the labor force is normalized to one.

I assume that M increases in both its arguments and exhibits constant returns to scale in (u, v). The function M determines the rate at which each worker meets a firm. This rate is identical across workers because it does not depend on characteristics of workers or firms. The ratio of vacancies to unemployment is denoted by $\theta \equiv v/u$ and called tightness. The rate at which a recruiting firm meets an unemployed worker is $q(\theta) \equiv M(u, v)/v$. The rate at which an unemployed worker meets a recruiting firm is $\theta q(\theta) \equiv M(u, v)/u$. I assume that q satisfies standard regularity assumptions that ensure that $q(\theta)$ is decreasing in θ, $\theta q(\theta)$ is increasing in θ, $\lim_{\theta\to\infty} q(\theta) = 0$, $\lim_{\theta\to 0} q(\theta) = 1$, $\lim_{\theta\to\infty} \theta q(\theta) = 0$, and $\lim_{\theta\to 0} \theta q(\theta) = 1$. I refer to $q(\theta)$ as the job-filling rate and $f(\theta) \equiv \theta q(\theta)$ as the job-finding rate.

At the beginning of each discrete period, economic activity takes place. This means searching workers look for jobs and recruiting firms post vacancies. By the end of the period, unemployment and vacancies are determined. At the very instant before the next period, labor-market transitions take place: some unemployed workers find jobs and some employed workers separate. In steady state, the number of workers who separate from jobs, $s(1-u)$, equals the number of unemployed workers who find employment, $\theta q(\theta) u$. The steady-state value of u is therefore $s/(s + f(\theta))$.

I focus on steady-state equilibria and drop explicit reference to time.

3.2 Bellman Equations for Firms

The value of a productive firm, J, satisfies the Bellman equation

$$J = y - w + \beta [s(V - \tau) + (1 - s) J]. \tag{1}$$

The value of a productive firm equals flow output, y, less the flow wage payment, w, plus the continuation value. The continuation value is the value of a vacancy in the event of a separation, which occurs with probability s, in which case the firm must pay the layoff tax τ, plus the value of continued production if a separation does not occur. The continuation value needs to be discounted.

The value of a vacancy, V, is

$$V = -c + \beta [q(\theta)(J - h) + (1 - q(\theta)) V]. \tag{2}$$

A recruiting firm becomes productive by posting a vacancy, which entails a flow cost of c. The following period the vacancy is unfilled with probability $1 - q(\theta)$ and filled with probability $q(\theta)$. Upon a match, the firm must pay the fixed cost h, which summarizes one-off administrative costs.

To be precise about timing, J and V are values just after fixed costs have been paid. Fixed costs are incurred at the very beginning of the period. Then workers produce and earn wages and firms post vacancies. Labor-market transitions take place at the end of the period. The layoff tax reduces the value of a productive firm with coefficient βs because the tax is incurred in the event that the worker separates, which occurs at the end of the period with probability s. The hiring cost h reduces the value of a posted vacancy with coefficient $\beta q(\theta)$ because the cost is incurred in the event that the vacancy is filled, which occurs at the end of the period with probability $q(\theta)$.

Competitive efforts by the large measure of firms drive the value of a vacancy to zero. Using the competitive assumption in equation (2) implies

$$J = \frac{c}{\beta q(\theta)} + h. \tag{3}$$

The value of a productive job under competition is driven to the expected recruitment cost, $c/\beta/q(\theta)$, plus the fixed cost of job creation paid by firms.

Substituting the expression for J in (3) into (1) implies a job-creation condition for firms equal to

$$J = \frac{1+r}{r+s}(y - w - \beta s\tau) = \sum_{j=0}^{\infty} \beta^j (1-s)^j (y - w - \beta s\tau) = \frac{c}{\beta q(\theta)} + h. \tag{4}$$

The value of a productive firm equals the present value of flow profit, $y - w$, less the expected present value of the layoff tax faced by the firm, $\beta s\tau$. Discounting includes the discount factor and the job-retention rate, $1 - s$. The amount $\beta s\tau$ is subtracted from flow profits because "the invisible hand can never allocate" these resources to vacancy creation (Ljungqvist and Sargent, 2017, 2642). The right side shows that a firm's expected gain equals the expected cost of job creation.

There are two components to this cost. The first is a proportional cost c that rises in expectation with how long the firm expects the vacancy to be posted before it is filled. The second is the fixed cost emphasized by Pissarides (2009).

In summary, equations (1) and (3) along with the zero-profit condition imply

$$w = y - \beta s\tau - \frac{r+s}{q(\theta)}c - \frac{r+s}{1+r}h. \tag{5}$$

In θ–w space, equation (5) is downward sloping (Pissarides, 2000, chapter 1). A higher wage makes posting vacancies less attractive for a firm. The relationship represents job creation by firms. A higher cost of job creation, h, shifts this job-creation condition downward.

Appendix 11 provides a derivation of these results.

3.3 Bellman Equations for Workers

Not all workers face meaningful costs when accepting jobs, including costs of occupational licensing or moving. In addition, these costs vary by worker. To capture these feature, upon accepting a job, each worker faces a probability of drawing a fixed cost from a known distribution. These idiosyncratic costs can be integrated out to produce average Bellman equations. Average values determine aggregate unemployment dynamics.

When a worker finds a job there is a probability $\pi \in [0, 1]$ that the worker will be required to pay a fixed cost and probability $1 - \pi$ that the worker will not be required to pay a fixed cost. In the event that there is a fixed cost, the worker draws the fixed cost from the known distribution g.

To write the value of unemployment, suppose the following period the unemployed worker knows that they will pay, with probability π, the known fixed cost x upon accepting a job. The value of unemployment depends on x. And because the value of employment depends on the value of unemployment, E also depends on x. The value of unemployment, $U(x)$, is

$$\begin{aligned} U(x) = z + \beta f(\theta)\left[\pi\left(\int E(x')\,g(x')\,dx' - x\right) + (1-\pi)\int E(x')\,g(x')\,dx'\right] \\ + \beta(1-f(\theta))\int U(x')\,g(x')\,dx'. \end{aligned} \tag{6}$$

The unemployed worker experiences z from nonwork. By the following period, with probability $\pi\theta f(\theta)$ the worker finds a job with expected value $\int E(x')\,g(x')\,dx'$ and pays the fixed cost x. With probability $(1-\pi)f(\theta)$ the worker finds a job with expected value $\int E(x')\,g(x')\,dx'$ and no fixed cost is paid. With probability $1-f(\theta)$ the worker does not find a job, which has expected value $\int U(x')\,g(x')\,dx'$.

I define

$$U \equiv \int U(x')\,g(x')\,dx',\ E \equiv \int E(x')\,g(x')\,dx',\ \text{and } \ell \equiv \pi\int x' g(x')\,dx'.$$

The value ℓ is expected value of the fixed cost multiplied by the likelihood that a worker confronts a fixed cost upon accepting a job. At the beginning of the period when z is experienced, the worker does not know if they will find a job and incur a fixed cost. Therefore, the relevant value is the average value of unemployment, which can be produced by integrating (6) with respect to x. The average value of unemployment satisfies the Bellman equation

$$U = z + \beta\left[f(\theta)(E-\ell) + (1-f(\theta))\,U\right]. \tag{7}$$

There are constraints on the distribution g. In order for a worker to willingly accept paying the fixed cost and taking the job, the value of employment less the fixed cost must be greater than the expected value of unemployment. This condition will always be met if

$$E - \bar{h} \geq \int U(x')\,g(x')\,dx',$$

where $\bar{h}$ is the highest fixed cost in the support of g.

The average value of employment for a worker satisfies the Bellman equation

$$E = w + \beta\left[sU + (1-s)E\right]. \tag{8}$$

The value of employment equals the flow wage, w, plus the discounted continuation value, which equals the value of unemployment, U, if a separation occurs, which happens with probability s, and the value of employment, E, if the job remains productive, which happens with probability $1-s$.

The values E and U are values just after fixed costs have been paid. The fixed cost ℓ reduces the value of unemployment with coefficient $\beta f(\theta)$ because the cost is incurred in the event that the worker finds a job, which occurs at the end of the period with probability $f(\theta)$.

Equations (7) and (8) can be combined to form an expression similar to the one in (4):

$$\sum_{j=0}^{\infty}(1-s)^j\beta^j w_R = z + \beta\left[f(\theta)\left(\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \ell\right) + (1-f(\theta))\,U\right], \tag{9}$$

where $w_R = rU/(1+r)$ and $U = W(w_R)$, which is established in result 1 in the appendix. The expression for the reservation wage indicates that a worker is indifferent between earning the expected value of taking a job that pays the reservation wage starting from the current period and receiving z in unemployment and then, the following period, accepting the expected value of a job, which occurs with probability $f(\theta)$, and the present value of unemployment with probability $1 - f(\theta)$. Upon taking the job, the worker pays the fixed cost ℓ.

By convention, wages are determined by the outcome of asymmetric Nash bargaining. The present value of match surplus is the gain from a productive match, $S = (J - V) + (E - U)$. The outcome of bargaining specifies

$$E - U = \phi S \text{ and } J = (1 - \phi) S, \tag{10}$$

where $\phi \in [0, 1)$ measures workers' bargaining power. The timing specifies that both firms and workers pay fixed costs before bargaining. As such, the parameters h and ℓ do not appear directly in (10).

The expression for surplus along with the Nash sharing rule in (10) and expressions for J and E in (1) and (8) can be rearranged to express the wage as

$$w = (1 - \phi) w_R + \phi (y - \beta s \tau). \tag{11}$$

A worker earns fraction $1 - \phi$ of their reservation wage plus fraction ϕ of flow profit net of the appropriate layoff-tax payment. The expression for wages in (11) is analogous to equation (9) in Ljungqvist and Sargent (2017, 2634) and is derived in appendix 11.

Equation (11) contains the value of unemployment, U, through w_R. Appendix 11 shows how equation (7) and the outcome of Nash bargaining in (10) can be combined to solve for U. Using this result in (11) yields

$$w = z + \phi (y - z - \beta s \tau + \theta c) + \beta f(\theta) [\phi h - (1 - \phi) \ell]. \tag{12}$$

The worker earns the value of nonemployment plus their share of the flow gain generated by a productive match plus $c\theta = c \times v/u$, which is the total cost of hiring, cv, divided by the number of unemployed workers, or the average hiring cost per unemployed worker (Pissarides, 2000, 17).

The fixed cost h increases wages with coefficient $\phi\beta f(\theta)$. Because if the negotiation fails, then the firm has to pay h when it meets another worker. This event takes place the following period with probability $f(\theta)$ and needs to be discounted. By staying in the match, the worker saves the firm an expected cost $\beta f(\theta) h$ and the worker captures fraction ϕ of this amount through bargaining. In contrast, the fixed cost ℓ decreases wages with coefficient $(1 - \phi)\beta f(\theta)$. Because if the negotiation fails, then the worker has to pay ℓ when it meets another firm. This event takes place the following period with probability $f(\theta)$ and needs to be discounted. By staying in the match, the firm saves the worker an expected cost $\beta f(\theta) \ell$ and the firm captures fraction $1 - \phi$ of this amount through bargaining.[8]

Equation (12) is the workers' wage curve. If $h = \ell = 0$, then $w = z + \phi(y - z - \beta s \tau + \theta c)$. In θ–w space the condition is increasing in tightness. A tighter labor market raises the cost of

[8] I have adopted this discussion from Pissarides (2009, 1364), who provides a description of an analogous wage equation in a model set in continuous time, where there are fixed costs paid by firms upon a match but not costs paid by workers. Appendix 11 derives the expression for the wage in equation (12).

hiring a worker, part of which the worker captures through the outcome of bargaining. When h and ℓ are both positive, for the condition to be upward sloping in θ–w space, parameters must be such that $\phi c + \beta f'(\theta)[\phi h - (1-\phi)\ell] > 0$. As shown below, this condition will guarantee a unique equilibrium.

3.4 Equilibrium

A steady-state equilibrium is a list of values $\langle u, \theta, w\rangle$ that satisfy the Bellman equations (1), (2), (8), (7) and the sharing rule in (10) along with the free-entry condition that requires $V = 0$ and the steady-state unemployment rate, $s/(s + f(\theta))$. In u–v space, there is a negative relationship between v and u: "when there are more vacancies, unemployment is lower because the unemployed find jobs more easily" (Pissarides, 2000, 20). The model generates a Beveridge curve.[9]

The equilibrium value of θ is jointly determined by the two expressions for the wage rate in (5) and (12):

$$y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi} = \frac{r + s + \phi\theta q(\theta)}{(1-\phi)q(\theta)}c + \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right). \tag{13}$$

Conditions for existence and uniqueness of an equilibrium are summarized in proposition 1.

Proposition 1 (Existence and uniqueness of θ). Assume $y > z$, which says that workers produce more of the homogeneous consumption good at work than at home, and assume that the initial vacancy yields a positive value, a condition that can be stated as $(1-\phi)(y - z - \beta s\tau)/(r+s) > c + \beta h$. In the canonical DMP search model, which features random search, linear utility, workers with identical capacities for work, exogenous separations, no disturbances in aggregate productivity, and fixed costs paid by workers and firms when a job is created, there exists a $\theta \in (0, \bar{\theta})$ that solves the steady-state relationship in (13), where

$$\bar{\theta} \equiv \frac{1-\phi}{c\phi}\left[y - z - \beta s\tau - \frac{\beta(r+s)}{1-\phi}h + \beta\ell\right] > 0. \tag{14}$$

If the workers' wage curve slopes upwards, then the equilibrium is unique.

The steady-state level of tightness, θ, determines steady-state unemployment, defined as the measure of job creation, $f(\theta)u$, equaling job destruction, $s(1-u)$, which implies $u = s/(s + f(\theta))$.

A detailed proof that uses insights from Ryan (2023) is found in appendix 12. For the purposes of this paper, the most important part of proposition 1 is conditions that permit an investigation into how tightness responds to productivity. In an economy with some unemployment, which is guaranteed with exogenous separations, the value of an initial vacancy can be thought of as the value of $\lim_{\theta\to 0} V$. In this scenario, a vacancy is immediately filled and $-c - \beta h + \beta(1-\phi)(y - z - \beta s\tau)/[1 - \beta(1-s)] > 0$, which is the condition given in proposition 1.

Existence can be expressed as $\mathcal{T}(\theta) = 0$, where

$$\mathcal{T}(x) \equiv y - z - \beta s\tau - \frac{\beta(r+s)}{1-\phi}h - \frac{c}{1-\phi}\left[\frac{r + s + \phi x q(x)}{q(x)} + \frac{\beta x q(x)}{c}(\phi h - (1-\phi)\ell)\right]. \tag{15}$$

[9] Elsby, Michaels, and Ratner (2015) provide an overview.

Then

$$\mathcal{T}(0) = y - z - \beta s\tau - \frac{\beta(r+s)}{1-\phi}h - \frac{c}{1-\phi}(r+s) > 0,$$

where the inequality uses the positive value of posting an initial vacancy.

A firm that posts the initial vacancy pays c in the current period. The vacancy is filled by the following period as there is a sole vacancy and many unemployed workers. Upon the match, the following period the firm pays the fixed cost h, which must be discounted to the present. These two costs must be less than the expected value of a productive match:

$$c + \beta h < \beta \sum_{j=0}^{\infty} \beta^j (1-s)^j (1-\phi)(y - z - \beta s\tau). \tag{16}$$

To understand the expected value of a productive match, note that the firm–worker pair begins production the following period, which requires discounting. The pair generates a net flow equal to flow surplus, $y - z$, less the appropriate resources deducted for the layoff tax, $\beta s\tau$. The firm's bargaining position allows it to collect fraction $1-\phi$ of the flow. Net flow is discounted by both the discount factor and the job-retention rate, $1-s$. The condition in (16) is a rearrangement of the condition $\mathcal{T}(0) > 0$.

In addition, using the definition of $\bar{\theta}$ in proposition 1, it is straightforward to establish that $\mathcal{T}(\bar{\theta}) > 0$. Because $\mathcal{T}$ is the composition of continuous functions, it is continuous on $[0, \bar{\theta}]$. Thus, by the intermediate-value theorem, there exists $\theta \in (0, \bar{\theta})$ such that $\mathcal{T}(\theta) = 0$, which establishes existence.

If $\mathcal{T}$ is everywhere decreasing on $(0, \bar{\theta})$, then the equilibrium will be unique. The derivative of $\mathcal{T}$ is

$$\mathcal{T}'(x) \equiv \frac{c(r+s)}{(1-\phi)[q(x)]^2} q'(x) - \frac{c\phi}{1-\phi} - \beta \frac{f'(x)}{1-\phi}[\phi h - (1-\phi)\ell]. \tag{17}$$

As $q' < 0$, the first term is negative. The remaining terms are also negative, for example, if the workers' wage curve in (12) is increasing in tightness in θ–w space. Then only one θ satisfies $\mathcal{T}(\theta) = 0$. But a larger set of parameters implies a unique equilibrium.

The existence and uniqueness of the equilibrium permits the computation of a comparative static, which is the subject of section 4.

4 The Fundamental Decomposition

While policymakers' interest lies in how unemployment responds to productivity, u is not a fundamental, as Ljungqvist and Sargent (2017) point out. It makes sense to understand how θ, a fundamental, responds to changes in y. A related decomposition of the response of θ to changes in y is provided by Mortensen and Nagypál (2007, 332).

Paralleling the steps in the cited research for determining the implied volatility of the conventional model, I arrive at the following proposition. The proposition provides a minor but important generalization of some results in Ljungqvist and Sargent (2017), who assume the matching function is Cobb–Douglas.

Proposition 2 (Fundamental decomposition). In the economic environment described in proposition 1, the elasticity of market tightness with respect to productivity, $\eta_{\theta,y}$, can be decomposed into two multiplicative factors

$$\eta_{\theta,y} = \Upsilon \frac{y}{y - z - \beta s \tau - \frac{\beta(r+s)h}{1-\phi}}, \tag{18}$$

where the second factor is the inverse of the fundamental surplus fraction and the first factor is

$$\Upsilon \equiv \frac{r + s + \theta q(\theta) \left[\phi + \beta q(\theta) \left(\frac{\phi h - (1-\phi)\ell}{c}\right)\right]}{(r+s)\,\eta_{M,u} + \theta q(\theta) \left[\phi + \beta \left(1 - \eta_{M,u}\right) q(\theta) \left(\frac{\phi h - (1-\phi)\ell}{c}\right)\right]}. \tag{19}$$

The factor Υ is bounded by

$$0 < \Upsilon < \max\left\{\frac{1}{\eta_{M,u}}, \frac{1}{1 - \eta_{M,u}}\right\}, \tag{20}$$

where $\eta_{M,u} \in (0,1)$ is the elasticity of matching with respect to unemployment. The elasticity of unemployment with respect to y is $\eta_{u,y} = -(1-u)\left(1-\eta_{M,u}\right)\eta_{\theta,y}$.

Both parts of proposition 2 are important. The decomposition implies that fixed costs of job creation paid by workers affect volatility through Υ. A main message of Ljungqvist and Sargent (2017), though, is that the upper bound for Υ is small. For example, many parameterizations posit that the matching function is Cobb–Douglas, which exhibits constant elasticity of matching with respect to unemployment. The elasticity is commonly chosen to be around 0.5, implying an upper bound for Υ of 2. This choice is consistent with empirical evidence in Petrongolo and Pissarides (2001) and consonant with efficiency if the bargaining parameter ϕ also equals 0.5, so that Hosios's (1990) condition holds. Notwithstanding some empirical evidence covered in Jäger et al. (2020), this choice of ϕ implies workers and firms have equal bargaining power, a reasonable assumption given limited data to assess. But if Υ is bounded in magnitude by 2, then ℓ cannot matter much for labor-market dynamics.

In general, however, unlike in the Cobb–Douglas parameterization, $\eta_{M,u}$ varies with tightness. Another matching technology adopted in the literature implies the influence of Υ on volatility may be meaningful (Ryan, 2023). For values of tightness observed in US data since December 2000, the bound reached 11.862. This leaves open the question of whether ℓ affects volatility.

Section 5 takes up this question by estimating the alternative, nonlinear matching function. The estimation uses readily available data that must be corrected for the fact that workers can transition between unemployment and employment continuously and the way hires are recorded.

5 Calibration

To gain insights into how costs of job creation affect labor-market volatility, I explore how the unemployment rate responds to changes in productivity. As a shortcut for analyzing model dynamics, I compare steady states, appealing to Shimer (2005, 39–40), who "documented that comparisons of steady states described by [the expression for $\eta_{\theta,y}$] provide a good approximation to average outcomes from simulations of an economy subject to aggregate productivity shocks" (Ljungqvist and Sargent, 2017, 2636).

Steady-state comparisons require assigning values to parameters. All except one are largely agreed upon by convention. The exception is the parameter that determines the elasticity of matching with respect to unemployment, $\eta_{M,u}$. Proposition 2 establishes that $\eta_{M,u}$ bounds Υ, one of the two multiplicative factors that determine volatility.

A matching function that allows the elasticity of matching to vary with tightness is suggested by den Haan, Ramey, and Watson (2000):

$$M(u,v) = \mu \frac{uv}{(u^\gamma + v^\gamma)^{1/\gamma}}. \tag{21}$$

One motivation for this form is random contact among all agents. Imagine that an unemployed worker contacts all agents, including firms posting vacancies, randomly. The probability the other agent is a recruiting firm is $v/(u+v)$, implying matches total $uv/(u+v)$. The nonlinear term can capture externalities from thick and thin markets (den Haan, Ramey, and Watson, 2000). The parameterization in (21) implies that $\eta_{M,u}$, the elasticity of matching with respect to unemployment, is $\theta^\gamma/(1+\theta^\gamma)$.

The nonlinear elasticity may well imply a large bound for Υ. Ryan (2023) documents that taking γ in (21) equal to 1.27, a value found in the literature, implies that the upper bound varies between between 2.006 and 11.862 when θ takes on values observed in US data since December 2000. The number 11.862 suggests there is scope for fixed costs of job creation paid by workers to affect volatility in the labor market. But the value 1.27 was not estimated. Because this value has implications for whether ℓ affects volatility, I take up the task of estimating γ.

The estimation uses readily available data on hires from the US Bureau of Labor Statistics' Job Openings and Labor Turnover Survey. The value for hires records all hires made within a month, even though the worker may not remain employed at the end of the month. The unadjusted data will therefore bias job-finding higher. I adjust the data to account for this bias. As far as I know, this adjustment to the measure of hires has not been done before and this is the first estimate of the matching technology in (21).

Section 5.1 briefly discusses conventional parameter values, section 5.2 discusses the bias adjustment, and section 5.3 covers the estimation of γ.

5.1 Standard Parameters Agreed upon by Convention

Except for the parameter that determines the elasticity of matching with respect to unemployment, choices about parameter values are standard. I adopt many of the values used by Pissarides (2009), who considers job-creation costs paid only by firms.

The value of output, y, produced by each firm's constant-returns-to-scale technology is 1. The value of nonwork, z, which includes leisure and compensation from unemployment insurance, is 0.71. I follow convention and set $\phi = 0.5$, which is a common choice that specifies workers and firms split any surplus generated from a match.

The model period is one day. This choice, as noted by Ljungqvist and Sargent (2017, 2639, FN 6), prevents job-finding and -filling rates from falling outside of 0 and 1. The interest rate is set so that the monthly interest rate is 0.004 and the separation rate is set so that the monthly separation rate is 0.036.

The average level of tightness observed in the US between 1960 and 2006 is 0.72 and the average monthly job-finding probability observed over the same period is 0.594. With $\theta = 0.72$, I target the monthly job-finding probability by adjusting the parameter for matching efficiency, μ in equation (21), using the estimate of γ presented in section 5.3. The implied unemployment rate is 5.7 percent.

The cost of advertising vacancies and recruiting, c, is implied by two features. First, its value reflects the normalization of output. Second, its value is determined by the steady-state condition in (13).

These values agree with Pissarides (2009). In a baseline calibration, where $h = \ell = \tau = 0$, the equilibrium wage is 0.988. Pissarides (2009, 1351) points out that this represents a flow percentage gain of $(0.988/0.71 - 1) \times 100$ percent $= 39$ percent when a worker transitions from unemployment into a job. Which is substantially more than some parameterizations where z nearly equals the wage. These nearly-equal parameterizations rely on the story of competitive markets in which workers are indifferent between work and nonwork.[10]

5.2 A New Measure of Transition Probabilities That Corrects for Time Aggregation and How the JOLTS Program Records Hires

One way to estimate the matching function in (21) uses the homogeneity of M to write the model in terms of job finding and therefore tightness. Measuring rates of job finding, though, is challenging. One challenge is the data, which are available only at discrete, monthly intervals, even though workers can transition between employment and unemployment continuously throughout the month. Another challenge is the way matches or hires are reported. Data on the number of hires are available from the US Bureau of Labor Statistics' Job Openings and Labor Turnover Survey or the JOLTS program. The JOLTS program reports all hires within a month, including hires who are fired before the month ends.

Using the unadjusted hires measure would bias the probability of finding a job upwards. The rate of job finding in the model is $f(\theta) \equiv m(u, v)/u$. Unlike in the model, however, hires recorded by the JOLTS program do not necessarily work the following period. And even though the probability of separating from a job is low, the number of people who find a job each month is large.

To account for this biasing feature of the data, I model the process of job transitions, using techniques developed by Shimer (2012), to uncover instantaneous transition rates between employment and unemployment. The adjusted probabilities of job finding for the month can then be uncovered.

5.2.1 Description of the Data Environment

The interval $[t, t+1)$, for $t \in \{0, 1, 2, \dots\}$, is referred to as "period t." Within period t workers neither exit or enter the labor force. I define the following quantities:

- $f_t \in [0, 1]$ is the probability of finding a job in period t, the probability that a worker who begins period t unemployed finds at least one job during period t;
- $s_t \in [0, 1]$ is the probability a worker separates from a job in period t, the probability that a worker who begins period t employed loses at least one job during period t;

[10] Appendix 10 explores the implications of $z = 0.42$, which is based on Shimer's (2005) choice of parameters.

- $\varphi_t \equiv -\log(1-f_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from unemployment to employment; and
- $\varsigma_t \equiv -\log(1-s_t) \geq 0$ is the arrival rate of the Poisson process that changes a worker's state from employment to unemployment.

The relationship, for example, between φ_t and f_t is $f_t = 1-\exp(-\varphi_t)$, or one minus the probability that no jobs are found during the period. I am interested in uncovering s_t and f_t, using hires data available from the JOLTS program and unemployment data from the Current Population Survey.

5.2.2 Intraperiod Evolution of Stocks

Fix $t \in \{0, 1, 2, \dots\}$ and let $\tau \in [0,1]$ be the elapsed time since the start of the period. The following intraperiod stocks are of interest:

- $e_{t+\tau}(\tau)$ is the number of employed workers at time $t+\tau$,
- $u_{t+\tau}(\tau)$ is the number of unemployed workers at time $t+\tau$, and
- $e^h_{t+\tau}(\tau)$ is the number of workers who were unemployed at t and employed at some time $t' \in [t, t+\tau)$, which corresponds with the number of hires, where hires includes "workers who were hired and separated during the month."[11]

The notation is overkill, but the parenthesis emphasize that the stocks are functions of time within the period, which is indexed by τ, and the subscripts emphasize that data are available for e_t, e^h_t, e_{t+1}, etc.

At the start of each period, there are no hires: $e^h_t(0) = 0$ for all t. And the number of hires at the end of the period is defined as $e^h_{t+1} \equiv e^h_{t+1}(1)$, the number of hires measured and reported by the JOLTS program for period t. Employment within the period evolves according to the system of differential equations

$$\dot{e}_{t+\tau}(\tau) = \varphi_t u_{t+\tau}(\tau) - \varsigma_t e_{t+\tau}(\tau) \tag{22}$$

$$\dot{e}^h_{t+\tau}(\tau) = \varphi_t u_{t+\tau}(\tau). \tag{23}$$

Employment, as described in equation (22), increases as unemployed workers find jobs at instantaneous rate φ_t and decreases as employed workers separate from jobs at instantaneous rate ς_t. Hires, as described in equation (23), cumulates all hires, which corresponds to how the JOLTS program measures new hires and differs from Shimer's (2012) model of short-term unemployment.

To make progress on solving the system of differential equations, I assume that the labor force is constant within the period: $l_t = e_{t+\tau'}(\tau') + u_{t+\tau'}(\tau')$ for all $t' \in [0,1)$. The constant-labor-force assumption can be substituted into the system of differential equations in (22) and (23) to yield two differential equations for e, e^h, and l:

$$\dot{e}_{t+\tau}(\tau) = \varphi_t l_t - (\varphi_t + \varsigma_t) e_{t+\tau}(\tau) \tag{24}$$

$$\dot{e}^h_{t+\tau}(\tau) = \varphi_t l_t - \varphi_t e_{t+\tau}(\tau). \tag{25}$$

[11] This category of workers is listed in the US Bureau of Labor Statistics's *Handbook of Methods* under "Job Openings and Labor Turnover Survey: Concepts," which is accessible at https://www.bls.gov/opub/hom/jlt/concepts.htm. Accessed October 5, 2023.

Using equation (25) to eliminate $\varphi_t l_t$ from equation (24) yields the differential equation $\dot{e}_t(\tau) = -\varsigma_t e_{t+\tau}(\tau) + \dot{e}^h_{t+\tau}(\tau)$. Its general solution involves the integration of the unknown function $e^h_{t+\tau}(\tau)$. Because I do not know of evidence about hires made within a month, to make progress on the solution, I assume new hires are added linearly: $e^h_{t+\tau}(\tau) = e^h_{t+1}\tau$. The model for new hires is depicted in figure 5 in appendix 16.

Once the model for new hires is set, the differential equation for $\dot{e}_t(\tau)$ can be solved as

$$e_{t+1} = e_t(1-s_t) + e^h_{t+1} - e^h_{t+1}\left[1 + \frac{s_t}{\ln(1-s_t)}\right]. \tag{26}$$

The evolution of employment in (26) approximates the evolution of employment in stock–flow models of the labor market:

$$e_{t+1} \approx e_t(1-s_t) + e^h_{t+1} - e^h_{t+1}\left[1 + \frac{s_t}{-s_t}\right] = e_t(1-s_t) + e^h_{t+1}, \tag{27}$$

where I have used the approximation $\ln(1-s_t) \approx -s_t$. Employment the following period is approximately the number of workers who remain employed because they did not separate plus the number of new hires. The approximation, however, does not correct for the fact that JOLTS reports all hires, including people who were hired and then let go within the month. The difference between equations (26) and (27) is a correction for time aggregation. The correction may be quantitatively important. Because the rate of job finding in the United States is high, even a low separation rate could mean the number of hires who remain employed the following month is meaningfully less than the number reported by the JOLTS program. This feature of the data could induce a meaningful upward bias in measures of job finding that rely on unadjusted hiring data.

Equation (26) is a nonlinear equation for s_t, which can be solved for using data on employment and new hires. Appendix 16 provides a derivation. The appendix also discusses the difference between this model of transitions and Shimer's (2012) model, which makes use of a neat cancellation.

Given s_t, I still need to recover f_t. But this is achievable because equation (25) is a linear differential equation for $\dot{e}_{t+\tau}(\tau)$ with constant coefficients. Its solution is

$$e_{t+1} = \frac{\varphi_t l_t}{\varsigma_t + \varphi_t}\left(1 - e^{-(\varsigma_t+\varphi_t)}\right) + e_t e^{-(\varsigma_t+\varphi_t)}. \tag{28}$$

Equation (26) provides a solution for s_t and therefore ς_t. Given this value, equation (28) implicitly defines φ_t. Appendix 16 provides a derivation and shows that equation (28) implies $e_{t+1} \approx e_t$ in steady state, when the number of jobs created equals the number of jobs destroyed.

5.3 Estimate of a Matching Function

Section 5.2 provides a way to measure the probability of job finding from readily available data on employment and hires. The counterpart in the model is the probability, $f(\theta) \equiv M(u,v)/u$. The parameterization in (21) implies $f(\theta) = \mu\theta(1+\theta^\gamma)^{-1/\gamma}$.

There are many concerns when estimating a relationship between job-finding and tightness. One concern is that hires measure employment-to-employment transitions. I will ignore this issue so that I can use readily available data. Another concern is that matching efficiency, μ, can vary over the business cycle (Borowczyk-Martins, Jolivet, and Postel-Vinay, 2013). For example, on uncharacteristically sunny days workers may be more optimistic and apply for more jobs. In addition, matching efficiency may vary over distinct eras of the business cycle, which could reflect different ways of organizing economic activity.

I account for these features by modeling μ with two components. The first component is an independent and identically distributed factor $\exp(\varepsilon_t)$. The second component allows matching efficiency to shift after the Great Recession, which began in December 2012, and after the COVID-19 recession, which began in February 2020.[12] These assumptions imply a statistical model for the probability that a worker finds a job in month t:

$$\log f(\theta_t) = \alpha + \log \theta_t - \frac{1}{\gamma} \log\left(1 + \theta_t^{\gamma}\right) + \psi\, G(t) + \xi\, C(t) + \varepsilon_t, \tag{29}$$

where ψ is a component of matching efficiency that captures shifts after the Great Recession, $G(t)$ is a function that equals 1 if the month occurs after November 2007 and before February 2020, ξ is a component of matching efficiency that captures shifts after the COVID-19 recession, $C(t)$ is a function that equals 1 if the month occurs after January 2020, and ε_t is an unobserved, time-varying error capturing shocks to search and recruiting intensity along with other factors.

Using nonlinear least squares on the statistical model in (29), I estimate $\hat{\gamma} = 0.103$. The estimate of γ differs from the value used by den Haan, Ramey, and Watson (2000) and Petrosky-Nadeau and Wasmer (2017). They take γ to be 1.27. The two values imply meaningfully different bounds for Υ, which can be seen in figure 1. The matching function in (21) implies $\eta_{M,u} = \theta^{\gamma}/(1+\theta^{\gamma})$ and the upper bound is given in (20).

Figure 1 depicts the upper bound in (20) for values of tightness observed in the US economy after December 2000 when JOLTS data on vacancies became available. The bound computed using $\gamma = 1.27$ is always above the bound computed using $\hat{\gamma}$. The difference is meaningful: the bound computed using $\hat{\gamma}$ takes on values between 2 and 2.214, whereas the bound computed using 1.27 takes on values between 2.006 and 11.862. The decomposition in (18) implies there is much more scope when $\gamma = 1.27$ for the features of the economic environment subsumed in Υ, including ℓ, to affect labor-market volatility. But when $\hat{\gamma}$ is used, Ljungqvist and Sargent's (2017) synopsis applies: "A consensus about reasonable parameter values bounds [Υ's] contribution to the elasticity of market tightness. Hence, the magnitude of the elasticity of market tightness depends mostly on the second factor in expression (18), i.e., the inverse of what in the introduction we defined to be the fundamental surplus fraction."[13]

In figure 1 the black, horizontal line with y-intercept 2 shows the bound in (20) when the matching function is Cobb–Douglas and the elasticity-of-matching parameter (the exponent) is

[12] There are other concerns when estimating a matching technology, including compositional effects and firms and workers adjusting behavior around periods characterized by high or low rates of job finding. But these concerns fall outside the scope of this paper.

[13] In the quote, I changed the equation number to reflect the decomposition presented in this text. Petrosky-Nadeau and Zhang (2017), in an exercise aimed at uncovering the model dynamics in Hagedorn and Manovskii (2008), use $\gamma = 0.407$; but, use $\gamma = 1.25$ in a more realistic exercise aimed at uncovering the model dynamics in Petrosky-Nadeau, Zhang, and Kuehn (2018). A value of 1.553 is used by Silva and Toledo (2009) who consider a DMP model with turnover costs paid by firms, as in Mortensen and Nagypál (2007), and a boost to productivity from experience earned by workers who transition from inexperienced to experienced at constant rates. Figures 13 and 14 in the appendix show the bound over time.

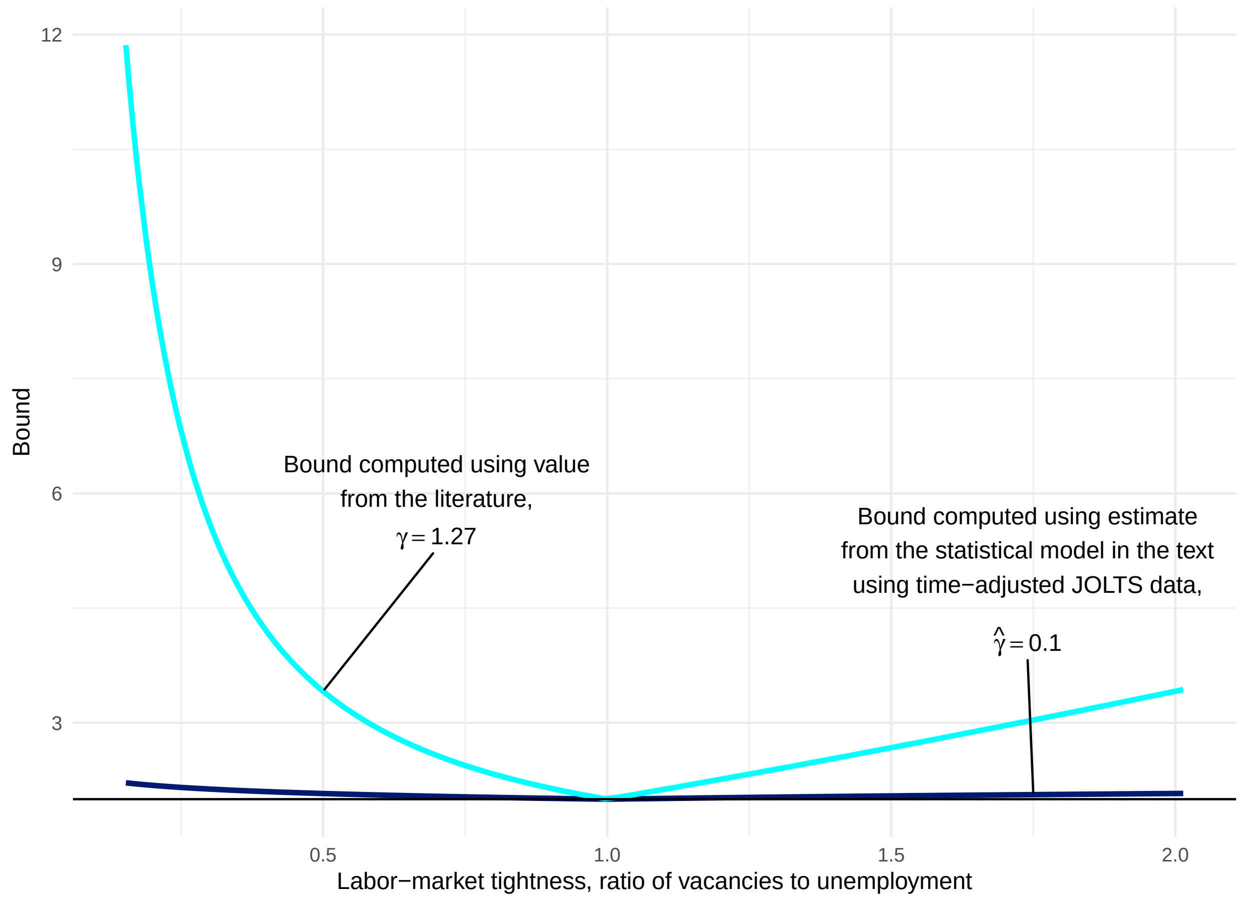

Figure 1: Upper bounds for Υ computed for levels of tightness observed in the US economy after December 2000.

Notes: Tightness in the labor market is $\theta \equiv v/u$. The upper bound for Υ is given in (20). Both series in blue depict the upper bound using the matching function given in (21) for different values of γ. The horizontal, black line depicts the upper bound using the Cobb–Douglas matching function and elasticity parameter equal to 0.5.

Sources: Authors calculations that use data from FRED. Data on vacancies are from the US Bureau of Labor Statistics' series Job Openings: Total Nonfarm [JTSJOL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/JTSJOL. Data on the level of unemployment are from the US Bureau of Labor Statistics' series Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNEMPLOY.

0.5, which is the value used by Ljungqvist and Sargent (2017) and consistent with estimates summarized by Petrongolo and Pissarides (2001). The Cobb–Douglas bound is much closer to the bound computed using the value for γ estimated using data adjusted for sources of bias.

This makes sense: The Cobb–Douglas parameterization is empirically successful (Bleakley and Fuhrer, 1997; Petrongolo and Pissarides, 2001). For the Cobb–Douglas case, the elasticity of matching with respect to unemployment is constant and equal to 0.5. When the matching function takes the form in (21), the elasticity of matching with respect to unemployment estimated here falls between 0.452 and 0.518. In other words, the implied elasticities that can vary by the state of the business cycle summarized by tightness are close to the constant case of Cobb–Douglas matching.

In summary, a literature suggested the bound for Υ in many cases was high. But it is not. This conclusion is based on an estimate of a nonlinear matching function using data adjusted for time aggregation and how the JOLTS program collects data on hires. The estimated elasticities are in line with what is found when matching is parameterized as Cobb–Douglas.

5.4 Result on How Υ Depends on Fixed Job-Creation Costs

What does the estimate of γ mean for volatility in the labor market when workers pay one-off hiring costs upon accepting a job? Because the costs are subsumed in Υ and Υ is bounded above by 2.214 for levels of tightness observed in the data, the result emphasizes that Ljungqvist and Sargent's (2017) fundamental surplus refers to recruitment resources available to firms. There is limited scope for hiring costs paid by workers to affect labor-market volatility and the magnitude of the elasticity of market tightness depends on the fundamental-surplus channel.

The result can be made precise when $\eta_{M,u} \approx 0.5$. This is done in proposition 3.

Proposition 3. Suppose that workers' wage curve increases in θ–w space and $\eta_{M,u} \approx 0.5$. Then increasing workers' fixed cost reduces the elasticity of market tightness with respect to productivity and increasing firms' fixed cost increases the elasticity of market tightness with respect to productivity; that is,

$$\frac{\partial \Upsilon}{\partial \ell} < 0 \text{ and } \frac{\partial \Upsilon}{\partial h} > 0. \tag{30}$$

The proof follows by direct computation of the partial derivatives. Appendix 14 provides the details. When the bargaining parameter $\phi = 0.5$, the matching function is Cobb–Douglas, and the economy satisfies the Hosios condition, then $\eta_{M,u} = 0.5$ and the proposition applies. This is the case considered by Ljungqvist and Sargent (2017). Section 5.3 provides estimates of $\eta_{M,u}$ between 0.452 and 0.518 after December 2000, which makes proposition 3 apply more generally.

Proposition 3 establishes that for reasonable parameter values fixed costs paid by workers upon accepting a job reduce the response of unemployment to changes in productivity. Ljungqvist and Sargent's (2017) fundamental surplus refers to resources *available to firms* that the invisible hand can allocate to vacancy creation. Proposition 3 also establishes that fixed job-creation costs paid by firms increase unemployment volatility for two reasons. A higher h reduces the fundamental surplus in (18) and increases Υ in (19). Both terms in the decomposition in proposition 2 increase.

In addition to influencing fluctuations, one-off costs paid by workers influence the level of equilibrium tightness, unemployment, and wages. These effects are investigated in the next section. While a full accounting of the business cycle is beyond the scope of this paper, a comparative steady-state analysis is undertaken. This is a shortcut to analyzing the fully dynamic model, but goes beyond computing an elasticity for a specific value of θ.

6 Properties of the Model

The costs to create a job affect labor-market outcomes in two primary ways. Costs affect the steady-state equilibrium and they affect labor-market dynamics. Both effects are investigated in this section. Section 6.1 shows that costs paid by workers upon accepting a job reduce wages. Which makes recruiting more profitable for firms, so equilibrium unemployment is lower. Section 6.2 illustrates how the unemployment rate would change if baseline productivity is perturbed. Costs of job creation borne by workers reduce the volatility of unemployment.

6.1 Steady-State Equilibrium

In this section I am interested in how ℓ affects equilibrium. To do this, I begin with a baseline. I set the layoff tax, τ, and costs of job creation paid by firms and workers, h and ℓ, to zero. I then solve equation (13) for c given the choice of tightness. The choice of tightness implies a steady state job-finding and unemployment rate. The steady-state unemployment rate is 5.7 percent. Given the solved-for c, figure 2 depicts the two curves for job creation in θ−w space. They intersect at the economy's equilibrium. The curve that depicts job creation by firms traces out the ordered pairs that satisfy equation (5) in black. A higher wage reduces firms' willingness to post vacancies, so the curve slopes downward. The curve that depicts the wage curve for workers traces out the ordered pairs that satisfy equation (12) in blue. Higher tightness increases the value of unemployment, which increases the bargaining position of workers. The intersection of the curves shows steady-state tightness is $\theta = 0.72$, as discussed in section 5.1. These results are standard (Pissarides, 2000, chapter 1).

Increasing ℓ does not affect firms' decisions to create jobs. The costs are borne by workers and do not affect resources available for recruitment. The costs do, however, lower wages for any given level of tightness through workers' job creation. One-off costs paid by workers upon a match allow a firm to threaten a worker with paying the costs again if the wage negotiation fails. The equilibrium outcome is lower wages and increased tightness, as depicted in figure 2.

Firms are not directly affected by ℓ. Instead, firms are indirectly affected through a lower value of unemployment, U, and thus lower wages. The indirect effect increases the value of a productive firm and encourages recruitment. Unlike the case where $h > 0$, the cost ℓ does not subtract from resources that the invisible hand can allocate to vacancy creation. The decomposition in (18) makes clear that the fundamental surplus applies to resources available *to firms* for recruiting workers.

6.2 Model Dynamics

To understand how costs affect economic dynamics, I index five economies by costs of job creation. In the steady state of each economy, the level of productivity is 1, the level of tightness is 0.72, the probability a worker finds a job is 0.594, and the unemployment rate is 5.7 percent. The steady-state values imply different combinations of c, h, and ℓ that satisfy (13). The approach

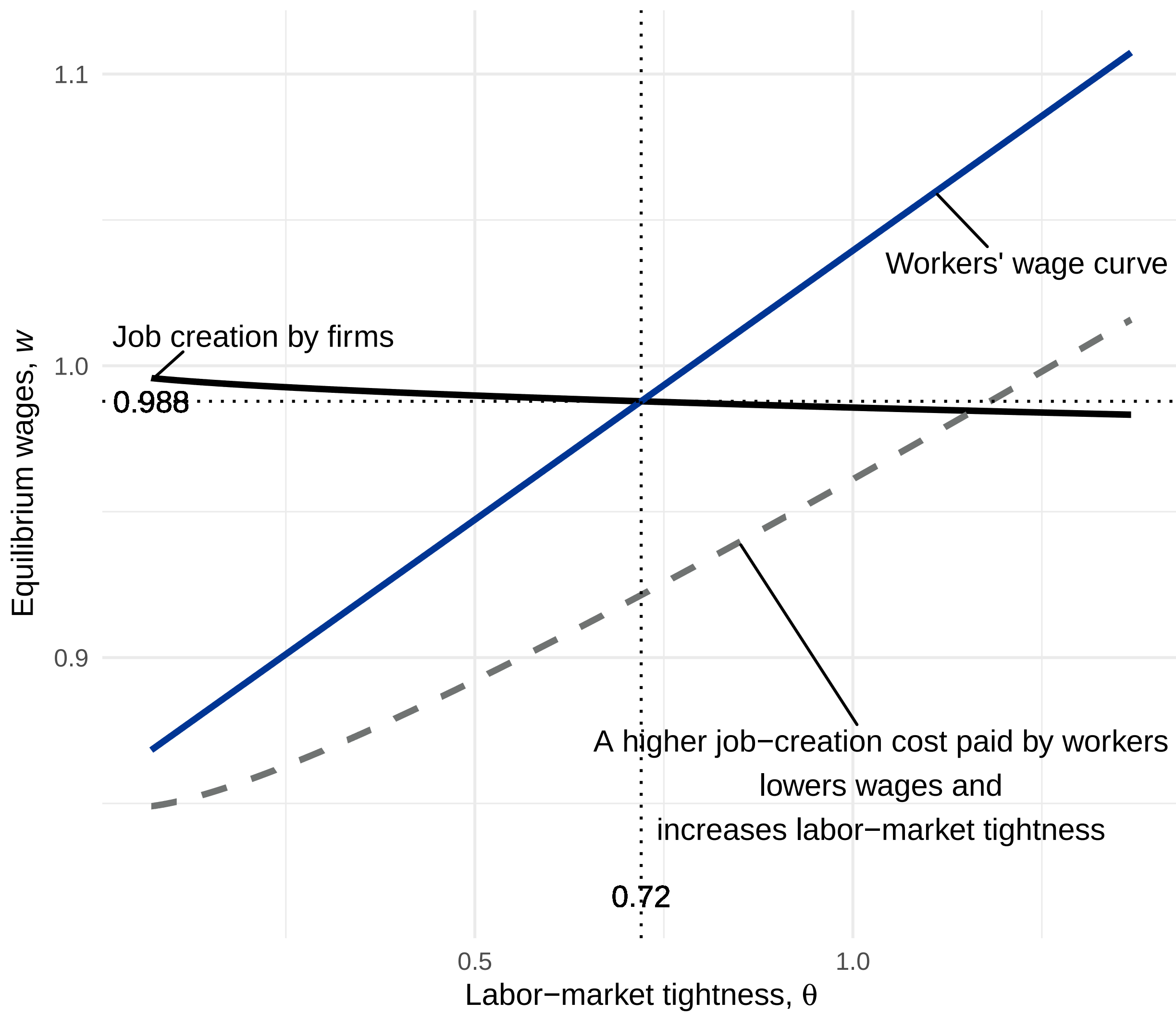

Figure 2: Job creation by firms and workers' wage curve in the presence of costs paid by workers upon accepting a job.

Notes: Job creation by firms traces out the ordered pairs that satisfy equation (5) in black. The wage curve traces out the ordered pairs that satisfy equation (12) in blue. The blue curve represents an economy where $\tau = 0$, $h = 0$, and $\ell = 0$. Costs paid by workers upon accepting a job shift the wage curve lower, which is represented by the dashed gray curve and is drawn for $\ell > 0$. Costs paid by workers upon accepting a job do not affect firms' hiring decisions independently from the effect on wages, so the curve is fixed. Because costs are paid by a worker upon a match, a firm can bargain for a lower wage by threatening the worker with paying the cost again upon a subsequent match if the wage negotiation fails. Lower wages encourage vacancy posting, which increases equilibrium tightness. Higher ℓ means a higher equilibrium θ, lower equilibrium w, and lower equilibrium u.

Table 1: Model results at different combinations of job-creation costs.

Economy	h	ℓ	c	$\eta_{\theta,y}$	$\eta_{w,y}$
Baseline	0	0	0.369	3.602	0.991
Middle ℓ	0	4.488	0.538	2.965	0.877
Split	4.488	4.488	0.354	3.760	0.884
Middle h	4.488	0	0.185	4.846	0.991
High h	8.976	0	4.111×10^{-5}	7.402	0.991

Notes: The column $\eta_{\theta,y}$ is the elasticity of market tightness with respect to productivity. The column $\eta_{w,y}$ is the elasticity of wages with respect to productivity.

uses values found in measurable and available labor-market data to back out possible values of fixed costs. Surveys on fixed costs are in comparison limited (see Hamermesh, 1993; Silva and Toledo, 2009, 2013, for some evidence). Because the value of a day's work is normalized to 1, the magnitudes of fixed costs can be interpreted as days of lost work. For example, $\ell = 4.488$ implies a cost of around 4.5 productive days. Licensing fees in 102 low-wage occupations, for example, are on average $295, which is within this range.

At one extreme is a baseline economy where $h = \ell = 0$. The only cost of job creation is the flow cost of posting a vacancy, c. The value of c is directly determined from equation (13). At the other extreme is an economy where costs of job creation are primarily paid by firms through a fixed cost, instead of the proportional cost. In this high-h economy, c is nearly 0 and h is nearly 9. A value of h a little higher than this amount would make any job posting unprofitable.

Between the two extremes is an economy where the high-h cost is split between firms and workers. I then consider two middle economies: in one, workers incur half the high-h cost and, in the other, firms incur half the high-h cost. (To belabor the point, each economy implies a different flow cost of posting a vacancy.)

These economies are labeled in the first column of table 1. The second, third, and fourth columns report the values of c, h, and ℓ in each economy. The fifth column reports the elasticity of tightness with respect to y in each economy, using the expression in (18). Considering the period over which the steady-state values are calculated, the data suggest that $\eta_{\theta,y}$ is around 7.56.[14]

Consistent with Shimer (2005), the baseline economy fails to generate the observed volatility in tightness. In the baseline economy, $\eta_{\theta,y} = 3.602$. This feature is sometimes referred to as the Shimer puzzle or the unemployment-volatility puzzle. Turning to fixed costs, as Ljungqvist and Sargent's (2017) decomposition proves, shifting firms' costs from proportional costs to fixed costs raises volatility in the labor market. In the high-h economy, $\eta_{\theta,y}$ is 7.402, which is close to the data. Pissarides (2009) emphasizes this result. Propositions 2 and 3 establish that $\eta_{\theta,y}$ is higher because Υ is bigger and fundamental surplus is smaller.

[14] The number comes from values reported by Shimer (2005). Table 1 of Shimer (2005, 28) reports that the correlation between y and θ in the data is 0.396, the standard deviation of θ is 0.382, and the standard deviation of y is 0.020. All of these values are based on series that remove a low-frequency trend. Together, the values imply the correlation between y and θ in the data is around 7.56. Pissarides (2009) notes that this number is the correct comparison and conducts a similar exercise that can be used as a comparison to my results.

The economy in which the high-h cost is split between firms and workers lies between the two extremes. But $\eta_{\theta,y}$ in the split economy is not halfway between 3.602 and 7.402, being closer to $\eta_{\theta,y}$ in the baseline economy.

Propositions 2 and 3 indicate why. In the split economy, firms and workers incur the cost: $h = \ell = 4.488$. Because $\phi = 0.5$, the effects of h and ℓ are canceled in Υ. The effect of h is only happening through the fundamental-surplus channel. When $h = 4.488$ and $\ell = 0$ in the middle-h economy, $\eta_{\theta,y}$ is higher because Υ is higher *and* the fundamental surplus is smaller. In other words, both factors of the two-factor decomposition in 2 contribute to higher $\eta_{\theta,y}$. (This is also true in the high-h economy.) In contrast, when $h = 0$ and $\ell = 4.488$ in the middle-ℓ economy, $\ell > 0$ reduces Υ only and fundamental surplus is unaffected. The effect on $\eta_{\theta,y}$ will be less than an equally large increase in the fixed cost paid by firms.

The sixth column of table 1 reports the elasticity of wages with respect to y. The values of $\eta_{w,y}$ in the last column of table 1 demonstrate that the high-h economy can generate realistic volatility without fixing wages. Hall (2005b) fixes real wages in a DMP model to generate realistic volatility, but data on real wages suggest wages are more procyclical. For example, Solon, Barsky, and Parker (1994) document how wages are substantially procyclical once composition is accounted for, consistent with the values reported in the sixth column of table 1. Gertler and Trigari (2009) stagger wage bargaining, offering a waypoint between period-by-period bargaining considered here and Hall's (2005b) model. Some caution is called for when it comes to wages. Pissarides (2009) shows that wages in new jobs determine volatility and data on wages in new jobs are characterized by procyclicality.

The values of $\eta_{\theta,y}$ report how tightness responds to productivity at the steady state. There are at least two concerns. First, $\eta_{\theta,y}$ applies locally and responses may vary away from the steady state. Second, there may be more interest in how unemployment responds as opposed to θ. To address these concerns, I consider perturbations in productivity around each economy's steady-state value of $y = 1$. I hold other parameters fixed and solve for the implied level of θ from (13), which I use to compute the unemployment rate. The comparative equilibrium analysis is a shortcut for modeling dynamics.

Figure 3 depicts how each economy's unemployment rate responds to productivity perturbations. The two extremes are the middle-ℓ and high-h economies. Unemployment in the middle-ℓ economy responds least to perturbations, whereas unemployment in the high-h economy responds most. The difference in responses is especially pronounced for negative perturbations.

Responses of unemployment in the middle-h economy are, not surprisingly, between the two extremes. Of some surprise, is the similarity between the baseline economy and the economy where the fixed costs of job creation in the high-h economy are split evenly between workers and firms.

Figure 3 shows that responses of unemployment to productivity perturbations are asymmetric. An increase in productivity generates a given fall in unemployment. The magnitude of this fall is less than the increase in unemployment in response to a decrease in productivity of equal magnitude. In steady state, where the unemployment rate is 5.7 percent, there is little way for the unemployment rate to fall, given exogenous separations.

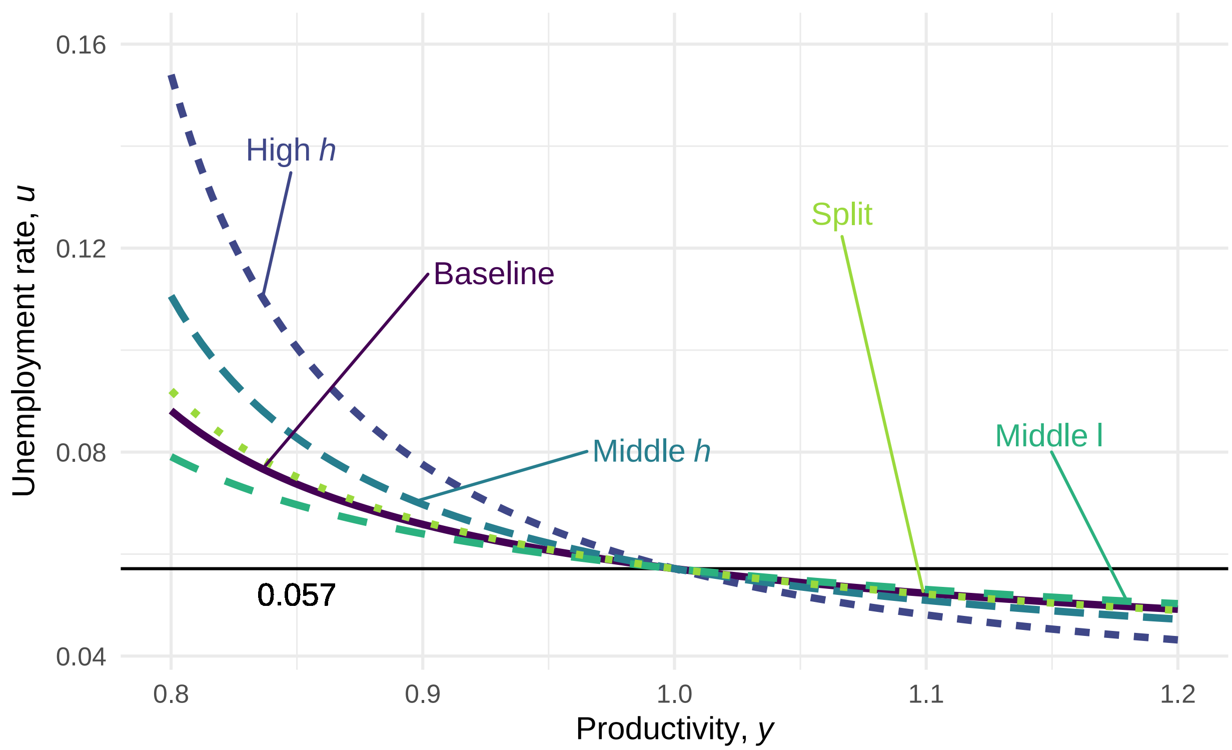

Figure 3: Steady-state unemployment dynamics for economies indexed by costs of job creation.

Notes: Economies are indexed by the job-creation costs listed in table 1. For each economy, costs of job creation are adjusted so that tightness and the probability of finding a job equal their monthly targets, which produces a 5.7 percent unemployment rate when y is 1. Steady-state dynamics are computed by solving for equilibrium tightness in (13) when productivity is perturbed 20 percent above and below $y = 1$.

7 Discussion and Outstanding Questions

The model and numerical work predict that volatility in the labor market should decline as the prevalence and magnitude of ℓ-type costs increase. Evidence supports this prediction. The prevalence of such costs has increased over time (DOT, CEA, and DOL, 2015; Kleiner and Krueger, 2013, figure 1, S177). And Barnichon (2010), looking at data from the period 1948–2008, documents how a positive productivity fluctuation lowered unemployment on average early in the sample and increased unemployment on average later in the sample.[15] The model I presented offers a partial explanation for why the cyclical component of unemployment and the cyclical component of productivity are less negatively correlated; although, at this point, the link is just suggestive and more evidence is needed.

In addition, the perspective of costs paid by workers I have presented raises a number of questions for future research.

Mechanisms that make unemployment respond realistically to realistic changes in productivity will also make unemployment respond meaningfully to changes in unemployment-insurance benefits. [Equation (13) reveals that θ responds to changes in y and changes in z symmetrically.] Yet, data suggest unemployment does not respond as much to the significant differences in benefits observed across countries. This dilemma was pointed out by Costain and Reiter (2008). Rogerson, Visschers, and Wright (2009) offer a solution: a fixed factor of production like managerial talent or home production. If the factor is abundant in certain states but scarce in others, then unemployment responses can be muted for large changes in benefits. Does Rogerson, Visschers, and Wright's (2009) fixed-factor solution interact with fixed costs of job creation?

In general, unemployment insurance raises a trade-off between smoothing consumption, which increases welfare, and distorting search for work, which raises unemployment. How does Andersen's (2016) efficiency–equity locus change in the presence of job-creation costs?

The assumption of Nash bargaining is not essential. As Plotnikov (2019) emphasizes, building on Farmer's (2008) insights, any wage that divides the surplus is feasible. Plotnikov (2019) replaces the Nash sharing rule with a rule that specifies agents' beliefs about their wealth to determine aggregate demand and thus wages, output, and unemployment. Do beliefs interact with turnover costs and job-creation costs paid by workers?

Shifting the focus to costs of occupational regulation, many features of registration, certification, and licensure are left unmodeled. Work by Tumen (2016) on job search using standard means versus social networks and by Flórez (2019) on employment in an informal sector demonstrates how selection matters. In their models composition across sectors is determined endogenously. How would outcomes in the labor market change if workers selected into sectors where work was regulated differently?

Several important features of the labor market are left unmodeled here too. Maury and Tripier (2019) emphasize the importance large firms and disruptions to productivity that induce firing. Does intra-firm bargaining and firing change in the presence of fixed costs? Do policies that encourage participation in the labor market like the earned income tax credit, which Regev and

[15] How can a positive productivity fluctuation increase unemployment? A positive innovation to productivity increases wages and thus demand. But if firms' prices are stuck, then the increase in demand is less than what firms can produce given the increase in productivity and size of the workforce. The value of a worker is low and firms do not recruit workers, which raises unemployment. Barnichon (2010) presents a model with these features.

Strawczynski (2019) study in the presence of risk-averse workers, interact with costs of job creation? Do these costs affect entrepreneurial decisions like those modeled in Gries, Jungblut, and Naudé (2016)? Does the presence of a binding minimum wage, which happens in the economies studied by Brecher and Gross (2019) and Brecher and Gross (2020), interact with costs of job creation? In contrast, a minimum wage may not reduce labor demand in a model that assigns heterogeneous workers to produce with heterogeneous capital, as Correa and Parro (2020) demonstrate, which raises the question: how are workers assigned to tasks when creating a job costs a fixed amount?

Guerrazzi (2023) takes up the question of optimal capital accumulation when "hiring is a labor-intensive activity" (Guerrazzi, 2023, footnote 1, 2). Instead of posting vacancies, large firms allocate part of their workforce to job creation (Shimer, 2010). A wage rule is available that achieves optimal capital accumulation and labor allocation in a decentralized equilibrium. Capital and transitional dynamics are significant parts of the model. How would transitional dynamics change if workers paid a one-off cost upon taking a job or if firms were forced to pay a cost to switch a worker from production to recruitment?

8 Conclusion

Creating jobs takes resources. When workers pay a cost upon accepting a job, wages are lower, which increases the value of posting a vacancy and lowers unemployment. The result exposes a previously unrecognized but important variable that affects wages. Going beyond the scope of this paper, this arrangement benefits firms and could explain why workers more often have to pay costs like fees to take an exam before starting work.

Costs paid by workers also reduce volatility in the labor market; although, there is limited scope for this channel. The conclusion is based on a generalization of Ljungqvist and Sargent's (2017) fundamental decomposition that allows the matching function to take any reasonable form. The decomposition reduces the elasticity of tightness with respect to productivity into two terms. One of the two terms subsumes costs paid by workers upon taking a job. This term is bounded by consensus, but this judgment is based on the estimation of a matching function undertaken in this paper. The estimation uses data that are adjusted for worker flows and how the JOLTS program records hires. The estimated elasticity of matching with respect to unemployment varies with tightness but the variation is small. The updated bound limits the scope of influence that workers' costs can have on unemployment volatility. Nevertheless, updated data used to assess the Shimer puzzle or unemployment-volatility puzzle are consistent with the increased prevalence of costs paid by workers.

Finally, I close by suggesting that costs paid by workers upon accepting a job have far-reaching implications. The regulation of occupations is a growing "phenomenon" (Kleiner and Krueger, 2013, S182). By adding features of regulation into a DMP model, I have taken a step towards understanding how such regulation affects dynamics in the labor market. Likewise, the flourishing of workers depends on their ability to take opportunities. Workers may need help to overcome barriers like the cost of moving to accept jobs.

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9 Appendix

Replication materials are available at:

https://github.com/richryan/funDecomp.

10 Alternative Parameterization of the Flow Value of Nonwork

The value of nonwork may comprise primarily the value of unemployment-insurance benefits. In fact, Shimer (2005, 38) took this approach: "I set the value of leisure to $z = 0.4$. Since mean labor income in the model is 0.993, this lies at the upper end of the range of income replacement rates in the United States if interpreted entirely as an unemployment benefit." Subsequent authors, including Hagedorn and Manovskii (2008) and Hornstein, Krusell, and Violante (2011), have discussed the implications of this parameterization. In this section, I show that treating the value of nonwork primarily as a non-exhaustible UI benefit that is close to 40 percent of mean labor income reproduces the Shimer puzzle or unemployment-volatility puzzle.

The United States Department of Labor Employment and Training Administration provides data on how much unemployment-insurance benefits replace claimants' average weekly wages. The ratio of the weekly benefit amount to weekly labor income is the replacement rate. There is no perfect measure of the replacement rate because labor income can vary week by week for many people. Nevertheless, the Employment and Training Administration reports data on two replacement ratios. The first is a weighted average of claimants' weekly benefit amounts divided by their normal hourly wages times 40 hours:

$$\text{replacement ratio 1} = \text{weighted average}\left(\frac{\text{weekly benefit amount}}{\text{normal hourly wage} \times 40 \text{ hours}}\right). \tag{31}$$

The second is a weighted average of claimants' weekly benefit amounts divided by a weighted average of claimants' normal hourly wage multiplied by 40:

$$\text{replacement ratio 2} = \frac{\text{weighted average (weekly benefit amount)}}{\text{weighted average (normal hourly wage} \times 40 \text{ hours)}}. \tag{32}$$

The analysis in table 2 considers the case where unemployment-insurance benefits are the main component of z. In this analysis, $z = 0.42$. The level of productivity is 1. As in the main text, the level of tightness is 0.72, the probability a worker finds a job is 0.594, and the unemployment rate is 5.7 percent. In the baseline scenario, where the only cost of job creation is the cost of posting a vacancy, the replacement rate is 0.43, which is the average of the two values depicted by horizontal, dotted lines in figure 4. (In the economy listed in line 1 of table 1 in the main text, $z = 0.72$.) The flow percentage gain when a worker earns w instead of z is $(0.976/0.42 - 1) \times 100$ percent $= 132$ percent (compared to 39 in line 1 of table 1 in the main text).

As in the main text, equilibrium unemployment is the same in each economy. Fixing steady-state unemployment implies different combinations of c, h, and ℓ that satisfy equation (13) in the main text. Costs of job creation index economies. For each economy, table 2 reports a row of statistics.

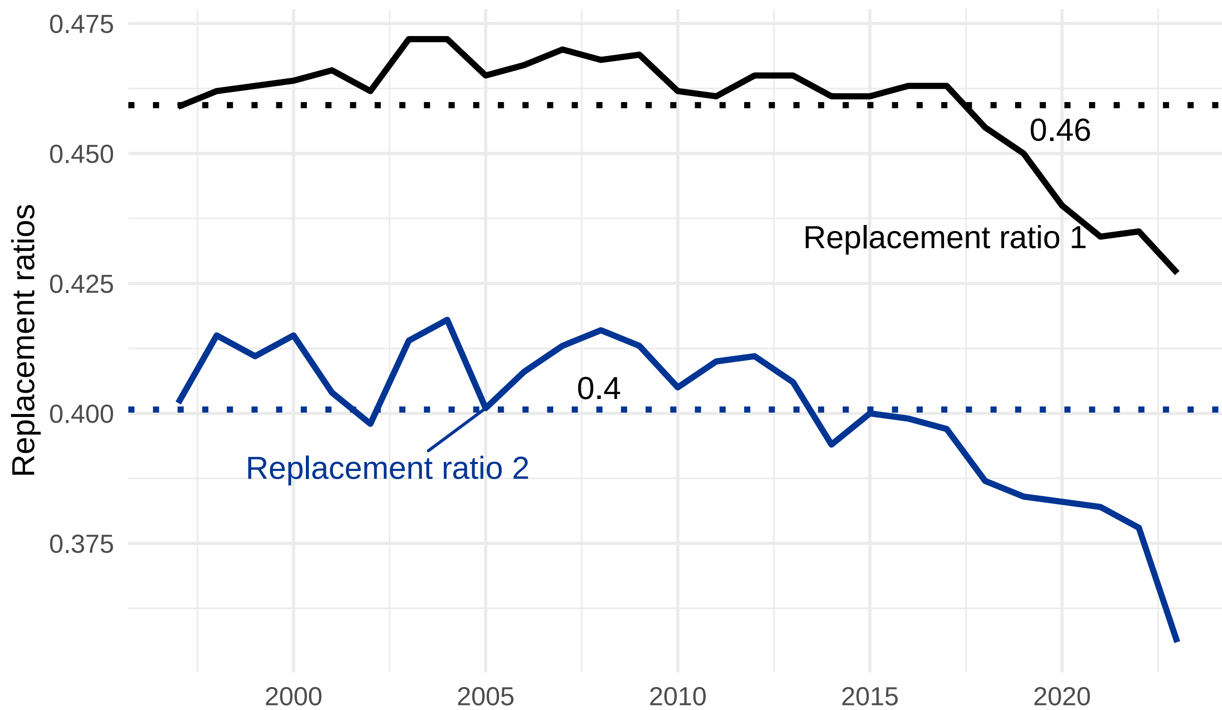

Figure 4: Two measures of how much unemployment-insurance benefits replace claimants' labor income, 1997–2023.

Notes: The two replacement ratios are defined in equations (31) and (32). The horizontal, dotted lines show the averages over the 1997–2023 period.
Source: These data are reported by the Employment and Training Administration of the US Department of Labor, https://oui.doleta.gov/unemploy/ui_replacement_rates.asp.

Table 2: Model results at different combinations of job-creation costs.

Economy	h	ℓ	c	$\eta_{\theta,y}$	$\eta_{w,y}$
Baseline	0	0	0.738	1.801	1.003
Middle ℓ	0	8.976	1.076	1.483	0.795
Split	8.976	8.976	0.707	1.880	0.805
Middle h	8.976	0	0.369	2.423	1.003
High h	17.953	0	4.111×10^{-5}	3.701	1.003

Notes: The column $\eta_{\theta,y}$ is the elasticity of market tightness with respect to productivity. The column $\eta_{w,y}$ is the elasticity of wages with respect to productivity. The value of nonwork is 0.42.

As proposition 2 establishes, the elasticity of market tightness with respect to productivity will be lower when z is lower. This feature of the DMP class of models can be seen in column $\eta_{\theta,y}$ of table 2. Even in the high-h economy, $\eta_{\theta,y}$ is close to or less than all the economies in table 1 in the main text, where $z = 0.71$. When $z = 0.42$, table 2 reproduces the Shimer puzzle or unemployment-volatility puzzle.

There is evidence that z comprises more than unemployment-insurance benefits. Chodorow-Reich and Karabarbounis (2016) model the flow value of nonwork as the sum of two components: $z = b + \xi$. The component b includes unemployment-insurance benefits, supplemental nutritional assistance (SNAP), welfare assistance (Temporary Assistance for Needy Families or TANF), and healthcare (Medicaid). In addition, "UI benefits become partially federally taxable in 1979 and fully taxable as part of the Tax Reform Act of 1986" (Chodorow-Reich and Karabarbounis, 2016, 1583). Importantly, their measure of b is based on received benefits rather than program availability. "This matters because, on average, only about 40 percent of unemployed actually receive UI" (Chodorow-Reich and Karabarbounis, 2016, 1576). Chodorow-Reich and Karabarbounis (2016) find that b is close to 6 percent of $y = 1$.

In other words, ξ dominates the value of nonwork. In Chodorow-Reich and Karabarbounis's (2016) model, the value of ξ depends on how utility is specified, including the disutility of work, and on consumption enjoyed by employed and unemployed people. Using different parameterizations of utility, Chodorow-Reich and Karabarbounis (2016, equation (30), 1596) provide a range of values for ξ between 0.41 and 0.9.

Within this range of values for b and ξ falls the value of nonwork, $z = 0.71$, used to construct table 1 and compute figure 3 in the main text.

11 Derivations of Equations in Section 3

11.1 Bellman Equations for Firms

Key Bellman equations in the economy for firms are

$$J = y - w + \beta\left[s\left(V - \tau\right) + \left(1 - s\right)J\right], \tag{33}$$

$$V = -c + \beta\left[q\left(\theta\right)\left(J - h\right) + \left(1 - q\left(\theta\right)\right)V\right]. \tag{34}$$

Imposing the zero-profit condition in equation (34) implies

$$\begin{aligned} 0 &= -c + \beta\left\{q\left(\theta\right)\left(J - h\right) + \left[1 - q\left(\theta\right)\right] \times 0\right\} \\ \therefore c &= \beta q\left(\theta\right)\left(J - h\right) \\ \therefore \frac{c}{\beta q\left(\theta\right)} &= J - h \end{aligned}$$

or

$$J = \frac{c}{\beta q\left(\theta\right)} + h. \tag{35}$$

Substituting this result into equation (33) and using the fact that vacancy creation exhausts all potential profits yields the job-creation condition:

$$\begin{aligned} J &= y - w + \beta \left[s \left(V - \tau \right) + \left(1 - s \right) J \right] \\ \therefore J &= y - w - \beta s \tau + \beta \left(1 - s \right) J \\ \therefore J \left[1 - \beta \left(1 - s \right) \right] &= y - w - \beta s \tau \\ \therefore J &= \frac{y - w - \beta s \tau}{1 - \beta \left(1 - s \right)} \\ &= \frac{y - w - \beta s \tau}{\frac{1+r}{1+r} - \frac{1-s}{1+r}} \\ &= \frac{1+r}{r+s} \left(y - w - \beta s \tau \right). \end{aligned} \tag{36}$$

Using the expressions for J in (35) and (36), the job-creation condition can be expressed as

$$\begin{aligned} J &= \frac{1+r}{r+s} \left(y - w - \beta s \tau \right) \\ &= \sum_{j=0}^{\infty} \left(\frac{1-s}{1+r} \right)^{j} \left(y - w - \beta s \tau \right) \\ &= \sum_{j=0}^{\infty} \beta^{j} \left(1 - s \right)^{j} \left(y - w - \beta s \tau \right) = \frac{c}{\beta q \left(\theta \right)} + h, \end{aligned} \tag{37}$$

where the second equality uses result 2. The expression in (37) establishes equation (4) in the main text.

What accounts for subtracting $\beta s \tau$ from flow profits? The layoff tax represents resources that "the invisible hand can never allocate" to vacancy creation (Ljungqvist and Sargent, 2017, 2642). For a productive firm, the expected value of the layoff tax is

$$\begin{aligned} &\beta s \tau + \beta^{2} \left(1 - s \right) s \tau + \beta^{3} \left(1 - s \right)^{2} s \beta + \cdots \\ &\qquad = s \tau \left[\beta + \beta \left(1 - s \right) \beta + \beta^{2} \left(1 - s \right)^{2} \beta + \cdots \right] \\ &\qquad\qquad = \sum_{t=1}^{\infty} \beta^{t} \left(1 - s \right)^{t-1} s \tau. \end{aligned}$$

Because the firm is productive, there is no chance of a layoff happening in the current period. In the following period, with probability s the layoff tax τ is paid, which must be discounted using β. Two periods from now, with probability $s\left(1-s\right)$ the layoff tax τ is paid, which must be discounted using β^2. The accounting of the layoff tax continues in a similar manner. A flow amount each period must account for this expected cost each period the firm is in operation. Ljungqvist and Sargent (2017, 2642) call this amount a and note that

$$\sum_{t=0}^{\infty} \beta^{t} \left(1 - s \right)^{t} a = \sum_{t=1}^{\infty} \beta^{t} \left(1 - s \right)^{t-1} s \tau,$$

which implies $a = \beta s\tau$, using

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t (1-s)^t \beta s\tau &= \beta s\tau + \beta (1-s) \beta s\tau + \beta^2 (1-s)^2 \beta s\tau + \cdots \\ &= s\tau \left[\beta + \beta (1-s) \beta + \beta^2 (1-s)^2 \beta + \cdots\right] \\ &= \sum_{t=1}^{\infty} \beta^t (1-s)^{t-1} s\tau.\end{aligned}$$

The last line is: "the future layoff tax on the right side occurs after the initial period of operation. Since the invisible hand can never allocate those resources to vacancy creation, it is appropriate to subtract this annuity value when computing the surplus" Ljungqvist and Sargent (2017, 2642).

Using equations (33) and (35) along with the zero-profit condition imply

$$\begin{aligned} J &= y - w + \beta \left[s (V - \tau) + (1-s) J\right] \\ \therefore \frac{c}{\beta q(\theta)} + h &= y - w - \beta s\tau + \beta (1-s) \left(\frac{c}{\beta q(\theta)} + h\right) \\ \therefore w &= y - \beta s\tau - \left(\frac{c}{\beta q(\theta)} + h\right) + \beta (1-s) \left(\frac{c}{\beta q(\theta)} + h\right) \\ &= y - \beta s\tau + \left[-1 + \beta (1-s)\right] \left(\frac{c}{\beta q(\theta)} + h\right) \\ &= y - \beta s\tau + \left[-\frac{1+r}{1+r} + \frac{1-s}{1+r}\right] \left(\frac{c}{\beta q(\theta)} + h\right) \\ &= y - \beta s\tau - \frac{r+s}{1+r} \left(\frac{c}{\beta q(\theta)} + h\right) \end{aligned}$$

or

$$w = y - \beta s\tau - \frac{r+s}{1+r} \left(\frac{c}{\beta q(\theta)} + h\right).$$

This expression simplifies to

$$w = y - \beta s\tau - \frac{r+s}{q(\theta)} c - \frac{r+s}{1+r} h, \tag{38}$$

which is equation (5) in the main text.

11.2 Workers' Bellman Equations

11.2.1 An Interpretation of U

The key Bellman equations for workers are

$$E = w + \beta \left[sU + (1-s) E\right] \tag{39}$$

$$U = z + \beta \left[\theta q(\theta) (E - \ell) + (1 - \theta q(\theta)) U\right]. \tag{40}$$

These are equations (8) and (7) in the main text.

From equation (39)

$$\begin{aligned} E &= w + \beta\left[sU + (1-s)E\right] \\ \therefore E\left[1-\beta(1-s)\right] &= w + \beta sU \\ \therefore E &= \frac{w+\beta sU}{1-\beta(1-s)}. \end{aligned} \tag{41}$$

Using this result in (40) yields

$$\begin{aligned} U &= z + \beta\left\{\theta q(\theta)(E-\ell) + \left[1-\theta q(\theta)\right]U\right\} \\ &= z + \beta\theta q(\theta)E - \beta\theta q(\theta)\ell + \beta\left[1-\theta q(\theta)\right]U \\ &= z + \beta\theta q(\theta)\frac{w+\beta sU}{1-\beta(1-s)} - \beta\theta q(\theta)\ell + \beta\left[1-\theta q(\theta)\right]U. \end{aligned}$$

Using the definition of $\beta = (1+r)^{-1}$ and result 2, the latter can be written

$$\begin{aligned} U &= z + \beta\theta q(\theta)\frac{(1+r)w+sU}{r+s} - \beta\theta q(\theta)\ell + \beta\left[1-\theta q(\theta)\right]U \\ &= z + \beta\theta q(\theta)\frac{(1+r)w}{r+s} + \frac{sU}{r+s} - \beta\theta q(\theta)\ell + \beta\left[1-\theta q(\theta)\right]U \\ &= z + \beta\theta q(\theta)\sum_{j=0}^{\infty}(1-s)^j\beta^j w + \frac{sU}{r+s} - \beta\theta q(\theta)\ell + \beta\left[1-\theta q(\theta)\right]U. \end{aligned}$$

And collecting U on the left side of the equation yields

$$\begin{aligned} U - \frac{sU}{r+s} - \beta\left[1-\theta q(\theta)\right]U &= z + \beta\theta q(\theta)\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \beta\theta q(\theta)\ell \\ \therefore \left\{1 - \frac{s}{r+s} - \beta\left[1-\theta q(\theta)\right]\right\}U &= z + \beta\theta q(\theta)\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \beta\theta q(\theta)\ell \\ \therefore \left\{\frac{1}{r+s}\frac{1+r}{1}\frac{r}{1+r}U - \frac{1-\theta q(\theta)}{1}\frac{1}{1+r}U\right\} &= z + \beta\theta q(\theta)\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \beta\theta q(\theta)\ell. \end{aligned}$$

Collecting terms and prettying using result 2 yields

$$\begin{aligned} \therefore \sum_{j=0}^{\infty}(1-s)^j\beta^j\frac{r}{1+r}U &= z + \beta\left[\theta q(\theta)\left(\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \ell\right) + \left[1-\theta q(\theta)\right]U\right] \\ \therefore \sum_{j=0}^{\infty}(1-s)^j\beta^j w_R &= z + \beta\left[\theta q(\theta)\left(\sum_{j=0}^{\infty}(1-s)^j\beta^j w - \ell\right) + \left[1-\theta q(\theta)\right]U\right], \end{aligned}$$

where the last equality uses $rU/(1+r) = w_R$, which is established in result 1. This establishes equation (9) in the main text.

11.2.2 Outcome of Asymmetric Nash Bargaining

Wages, by convention in the economy, are determined by asymmetric Nash bargaining, where the worker has bargaining parameter ϕ. As Hall (2005b) demonstrates in extreme by use of a fixed wage, Nash bargaining is not essential, but some form of rent sharing of the surplus generated from a productive match is essential. The Nash-bargaining convention is adopted here to agree with the previous literature. The outcome of asymmetric Nash bargaining is specified in (10).

Solving equation (33) for J yields

$$\begin{aligned} J\left[1-\beta\left(1-s\right)\right] &= y - w - \beta s\tau \\ \therefore J &= \frac{y-w-\beta s\tau}{1-\beta\left(1-s\right)}. \end{aligned} \tag{42}$$

And solving (39) for E yields

$$\begin{aligned} E &= w + \beta\left[sU + \left(1-s\right)E\right] \\ \therefore E\left[1-\beta\left(1-s\right)\right] &= w + \beta sU \\ \therefore E &= \frac{w}{1-\beta\left(1-s\right)} + \frac{\beta sU}{1-\beta\left(1-s\right)}. \end{aligned}$$

Developing the expressions in (10) yields

$$\begin{aligned} E - U &= \phi S \\ &= \phi \frac{J}{1-\phi} \end{aligned}$$

and using the just-derived expressions for J and E yields

$$\underbrace{\left[\frac{w}{1-\beta\left(1-s\right)} + \frac{\beta sU}{1-\beta\left(1-s\right)}\right]}_{E} - U = \frac{\phi}{1-\phi}\underbrace{\left[\frac{y-w-\beta s\tau}{1-\beta\left(1-s\right)}\right]}_{J}.$$

Developing this expression yields

$$\begin{aligned} w + \beta sU - \left[1-\beta\left(1-s\right)\right]U &= \frac{\phi}{1-\phi}\left(y - w - \beta s\tau\right) \\ \therefore w + \beta sU - U + \beta U - s\beta U &= \frac{\phi}{1-\phi}\left(y - w - \beta s\tau\right) \\ \therefore w &= \frac{\phi}{1-\phi}\left(y - w - \beta s\tau\right) + \left(1-\beta\right)U \\ \therefore \left(1-\phi\right)w &= \phi\left(y - w - \beta s\tau\right) + \left(1-\phi\right)\left(1-\beta\right)U \\ \therefore w &= \phi\left(y - \beta s\tau\right) + \left(1-\beta\right)U - \phi\left(1-\beta\right)U. \end{aligned}$$

Using the fact that

$$1 - \beta = 1 - \frac{1}{1+r} = \frac{1+r-1}{1+r} = \frac{r}{1+r},$$

the latter expression can be written as

$$w = \frac{r}{1+r}U + \phi\left(y - \beta s\tau - \frac{r}{1+r}U\right), \tag{43}$$

which is equation (12) in Ljungqvist and Sargent (2017).

$$\begin{aligned} w &= \frac{r}{1+r}U + \phi\left(y - \beta s\tau - \frac{r}{1+r}U\right) \\ \therefore\ w &= w_R + \phi\left(y - \beta s\tau\right) - \phi w_R \\ \therefore\ w &= \left(1-\phi\right) w_R + \phi\left(y - \beta s\tau\right). \end{aligned}$$

The worker earns a fraction of their reservation wage plus a fraction of their output that accounts for the eventual layoff tax.

To get an expression for the value $rU/(1+r)$, I solve equation (40) for $E-U$ and substitute this expression and equation (35) into equation (10). Turning to equation (40):

$$\begin{aligned} U &= z + \beta\left\{\theta q\left(\theta\right)\left(E-\ell\right) + \left[1-\theta q\left(\theta\right)\right]U\right\} \\ \therefore\ U &= z + \beta\theta q\left(\theta\right)E + \beta U - \beta\theta q\left(\theta\right)U - \beta\theta q\left(\theta\right)\ell \\ \therefore\ U &= z + \beta\theta q\left(\theta\right)\left(E-U\right) + \beta U - \beta\theta q\left(\theta\right)\ell \\ U\left(1-\beta\right) - z &= \beta\theta q\left(\theta\right)\left(E-U\right) - \beta\theta q\left(\theta\right)\ell \\ \therefore\ U\left(1-\beta\right) - z + \beta\theta q\left(\theta\right)\ell &= \beta\theta q\left(\theta\right)\left(E-U\right) \\ \therefore\ E-U &= \frac{1}{\beta\theta q\left(\theta\right)}\left[\left(1-\beta\right)U - z + \beta\theta q\left(\theta\right)\ell\right] \\ &= \frac{1}{\beta\theta q\left(\theta\right)}\left[\left(1-\beta\right)U - z\right] + \ell \\ &= \frac{1+r}{\theta q\left(\theta\right)}\left[\left(1-\beta\right)U - z\right] + \ell \\ &= \frac{1+r}{\theta q\left(\theta\right)}\left[\left(1-\frac{1}{1+r}\right)U - z\right] + \ell \\ &= \frac{r}{\theta q\left(\theta\right)}U - \frac{1+r}{\theta q\left(\theta\right)}z + \ell. \end{aligned}$$

Using the expression for $E-U$ in (10) yields

$$\begin{aligned} E-U &= \phi S \\ \therefore\ \frac{r}{\theta q\left(\theta\right)}U - \frac{1+r}{\theta q\left(\theta\right)}z + \ell &= \phi S \\ &= \phi\left(\frac{J}{1-\phi}\right) \\ &= \frac{\phi}{1-\phi}\left(\frac{c}{\beta q\left(\theta\right)} + h\right), \end{aligned}$$

where the last equality uses the expression for J in equation (35). Developing this expression yields

$$\begin{aligned}\frac{r}{\theta q(\theta)}U - \frac{1+r}{\theta q(\theta)}z + \ell &= \frac{\phi}{1-\phi}\left(\frac{c}{\beta q(\theta)} + h\right)\\ \therefore\ rU - (1+r)z + \theta q(\theta)\ell &= \frac{\phi}{1-\phi}\frac{1}{\beta}c\theta + \theta q(\theta)\frac{\phi}{1-\phi}h\\ \therefore\ rU - (1+r)z &= \frac{\phi}{1-\phi}(1+r)c\theta + \theta q(\theta)\left(\frac{\phi}{1-\phi}h - \ell\right)\\ \therefore\ \frac{r}{1+r}U &= z + \frac{\phi c\theta}{1-\phi} + \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right),\end{aligned} \tag{44}$$

which is comparable to equation (10) in Ljungqvist and Sargent (2017). Substituting equation (44) into equation (43) yields

$$\begin{aligned}w &= \frac{r}{1+r}U + \phi\left(y - \beta s\tau - \frac{r}{1+r}U\right)\\ &= \left[z + \frac{\phi c\theta}{1-\phi} + \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right)\right] + \phi\left[y - \beta s\tau - z - \frac{\phi c\theta}{1-\phi} - \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right)\right]\\ &= (1-\phi)z + \frac{\phi c\theta}{1-\phi}(1-\phi) + (1-\phi)\frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right) + \phi(y - \beta s\tau)\\ &= (1-\phi)z + \phi c\theta + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell] + \phi(y - \beta s\tau)\end{aligned}$$

or

$$w = z + \phi(y - z - \beta s\tau + \theta c) + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell]. \tag{45}$$

Equation (45) can be re-arranged to read

$$\begin{aligned}w &= z + \phi(y - z - \beta s\tau + \theta c) + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell]\\ \therefore\ w &= (1-\phi)[z - \beta\theta q(\theta)\ell] + \phi[y - \beta s\tau + \theta c + \beta\theta q(\theta)h].\end{aligned} \tag{46}$$

The two expressions for the steady-state wage rate in (38) and (45) jointly determine the equilibrium value of θ. Setting the two expressions for the steady-state wage rate equal to each other implies

$$y - \beta s\tau - \frac{r+s}{q(\theta)}c - \frac{r+s}{1+r}h = z + \phi(y - z - \beta s\tau + \theta c) + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell].$$

Developing this expression yields

$$\begin{aligned}y - \beta s\tau - \frac{r+s}{q(\theta)}c - \frac{r+s}{1+r}h &= z + \phi(y - z - \beta s\tau + \theta c) + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell]\\ \therefore\ (1-\phi)(y - z - \beta s\tau) - \frac{r+s}{q(\theta)}c - \frac{r+s}{1+r}h &= \phi\theta c + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell]\\ \therefore\ (1-\phi)(y - z - \beta s\tau) - \frac{r+s}{1+r}h &= \left[\frac{r+s+\phi\theta q(\theta)}{q(\theta)}\right]c + \frac{\theta q(\theta)}{1+r}[\phi h - (1-\phi)\ell]\\ \therefore\ y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi} &= \frac{c}{1-\phi}\left(\frac{r+s+\phi\theta q(\theta)}{q(\theta)}\right) + \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right)\end{aligned}$$

Or

$$y-z-\beta s\tau-\frac{\beta\left(r+s\right)h}{1-\phi}=\frac{r+s+\phi\theta q\left(\theta\right)}{\left(1-\phi\right)q\left(\theta\right)}c+\frac{\theta q\left(\theta\right)}{1+r}\left(\frac{\phi}{1-\phi}h-\ell\right) \tag{47}$$

Expression (47) establishes equation (13) in the main text.

12 Proof of Proposition 1

12.1 Value of an Initial Vacancy

In order for firms to post vacancies, it must the the case that the value of the initial vacancy is positive. Imagine an economy where the initial vacancy is instantly filled. The hired worker is just like all other workers productivity-wise. If the initial vacancy is not profitable, then no other vacancies will be profitable. While intuitively clear within the DMP class of models, the check yields a requirement for parameters. Ljungqvist and Sargent (2021) in their discussion of Christiano, Eichenbaum, and Trabandt (2021, 2020) point out that failure to check may lead to downplaying key channels.

When the number of vacancies is vanishingly small compared to the number of workers searching for a job, the job-filling probability is 1 and the job-finding probability is 0:

$$\lim_{\theta\to 0} q\left(\theta\right)=1 \text{ and } \lim_{\theta\to 0} f\left(\theta\right)=0.$$

To compute the value of an initial vacancy, I need to know the wage rate paid to the initial worker and the value of a productive match in the extreme initial case. The value of the wage comes from (46):

$$\begin{aligned}\lim_{\theta\to 0} w &= \lim_{\theta\to 0}\left(1-\phi\right)\left[z-\beta\theta q\left(\theta\right)\ell\right]+\phi\left[y-\beta s\tau+\theta c+\beta\theta q\left(\theta\right)h\right]\\ &=\left(1-\phi\right)z+\phi\left(y-\beta s\tau\right)\\ &=z+\phi\left(y-z-\beta s\tau\right).\end{aligned}$$

With the wage, the value of a productive firm, using (42), is

$$\begin{aligned}\lim_{\theta\to 0} J &= \lim_{\theta\to 0}\frac{y-w-\beta s\tau}{1-\beta\left(1-s\right)}\\ &=\lim_{\theta\to 0}\frac{y-\left[z+\phi\left(y-z-\beta s\tau\right)\right]-\beta s\tau}{1-\beta\left(1-s\right)}\\ &=\frac{\left(1-\phi\right)\left(y-z-\beta s\tau\right)}{1-\beta\left(1-s\right)}.\end{aligned}$$

The expression for J in (42) has used the idea that once the initial vacancy yields a positive value, the recruitment efforts of competitive firms will immediately drive the value of a vacancy to 0. The requirement for parameters will guarantee profitability.

The value of the initial vacancy is then

$$
\begin{aligned}
\lim_{\theta\to 0} V &= \lim_{\theta\to 0} -c + \beta \left[q(\theta)(J-h) + (1-q(\theta))V\right] \\
&= \lim_{\theta\to 0} -c + \beta \left[q(\theta)\left(\frac{(1-\phi)(y-z-\beta s\tau)}{1-\beta(1-s)} - h\right) + (1-q(\theta))V\right] \\
&= -c + \beta \frac{(1-\phi)(y-z-\beta s\tau)}{1-\beta(1-s)} - \beta h \\
&> 0,
\end{aligned}
$$

where the third equality uses the fact that the opening is filled instantaneously. The expression for the initial vacancy states that

$$
\beta \sum_{j=0}^{\infty} \beta^j (1-s)^j (1-\phi)(y-z-\beta s\tau) > c + \beta h, \tag{48}
$$

which is equation (16) in the main text.

This expression for $\lim_{\theta\to 0} V$ can be rearranged as

$$
\begin{aligned}
\beta \frac{(1-\phi)(y-z-\beta s\tau)}{1-\beta(1-s)} &> c + \beta h \\
\therefore \frac{1}{1+r} \frac{(1-\phi)(y-z-\beta s\tau)}{1-\frac{1}{1+r}(1-s)} &> c + \beta h
\end{aligned}
$$

or

$$
\frac{(1-\phi)(y-z-\beta s\tau)}{r+s} > c + \beta h. \tag{49}
$$

The requirement in (49) can be rearranged as

$$
\begin{aligned}
\frac{(1-\phi)(y-z-\beta s\tau)}{r+s} &> c + \beta h \\
\therefore (1-\phi)(y-z-\beta s\tau) &> (r+s)(c+\beta h) \\
\therefore y - z - \beta s\tau &> \frac{(r+s)c}{1-\phi} + \frac{\beta(r+s)h}{1-\phi} \\
\therefore y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi} &> \frac{(r+s)c}{1-\phi},
\end{aligned}
$$

which establishes that

$$
y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi} > 0.
$$

The above inequality guarantees that $\bar{\theta} > 0$, which can be verified through the definition of $\bar{\theta}$ in (14).

12.2 Existence and Uniqueness of Equilibrium Tightness θ

The equilibrium level of labor-market tightness in (47) can be written $\mathcal{T}(\theta) = 0$, where

$$\mathcal{T}(x) \equiv y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}\left[\frac{r+s+\phi x q(x)}{q(x)} + \beta x q(x)\left(\frac{\phi h - (1-\phi)\ell}{c}\right)\right] \tag{50}$$

An application of the intermediate value theorem establishes existence. I will apply the theorem by establishing $\mathcal{T}(0) > 0$ and $\mathcal{T}(\bar{\theta}) < 0$, where $\bar{\theta}$ is defined in (14).

To establish $\mathcal{T}(0) > 0$, using $q(0) = 1$ and $0 \times q(0) = 0$, I note that

$$\mathcal{T}(0) = y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}(r+s) > 0,$$

where the inequality comes from the rearrangement of

$$\begin{aligned} y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}(r+s) &> 0 \\ \therefore y - z - \beta s \tau &> \frac{\beta(r+s)}{1-\phi} h + \frac{c}{1-\phi}(r+s) \\ \therefore \frac{(1-\phi)(y - z - \beta s \tau)}{r+s} &> \beta h + c, \end{aligned}$$

which is true from (49), the requirement that the expected value of an initial vacancy yields a positive value.

To establish $\mathcal{T}(\bar{\theta}) < 0$, the definition of $\mathcal{T}$, given in (50), implies

$$\begin{aligned} \mathcal{T}(\bar{\theta}) &= y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}\left[\frac{r+s}{q(\bar{\theta})} + \phi\bar{\theta} + \beta\phi\bar{\theta} q(\bar{\theta})\frac{h}{c} - \beta(1-\phi)\bar{\theta} q(\bar{\theta})\frac{\ell}{c}\right] \\ &= y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \frac{\phi c}{1-\phi}\bar{\theta} - \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})\left[\phi h - (1-\phi)\ell\right]. \end{aligned}$$

The definition of $\bar{\theta}$, given in (14), implies the right side of the latter evaluates to

$$\begin{aligned} \mathcal{T}(\bar{\theta}) = {} & y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h - \frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \left[y - z - \beta s \tau - \frac{\beta(r+s)}{1-\phi} h + \beta\ell\right] \\ & - \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})\left[\phi h - (1-\phi)\ell\right] \\ = {} & -\frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})\left[\phi h - (1-\phi)\ell\right] - \beta\ell \\ = {} & -\frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})\phi h + \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})(1-\phi)\ell - \beta\ell \\ = {} & -\frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \frac{\beta}{1-\phi}\bar{\theta} q(\bar{\theta})\phi h + \beta\bar{\theta} q(\bar{\theta})\ell - \beta\ell \\ = {} & -\frac{c}{1-\phi}\frac{r+s}{q(\bar{\theta})} - \frac{\beta}{1-\phi} f(\bar{\theta})\phi h - \beta\ell\left(1 - f(\bar{\theta})\right) \\ < {} & 0, \end{aligned}$$

Table 3: Sets of parameters guaranteeing a unique equilibrium.

Relative bargaining strength	Fixed cost of job creation $h = \ell$	$h > \ell$	$h < \ell$
$\phi > 1 - \phi$	Unique equilibrium	Unique equilibrium	Numerical check
$\phi = 1 - \phi$	Unique equilibrium	Unique equilibrium	Numerical check
$\phi < 1 - \phi$	Numerical check	Numerical check	Numerical check

where the last inequality uses the fact that $0 < f\left(\bar{\theta}\right) < 1$.

In addition, $\mathcal{T}$ is the composition of continuous functions and therefore continuous on $\left[0, \bar{\theta}\right]$. Thus, by the intermediate-value theorem, there exists $\theta \in \left(0, \bar{\theta}\right)$ such that $\mathcal{T}(\theta) = 0$, which establishes existence. For any θ that satisfies this condition, the equilibrium level of unemployment is

$$u = \frac{s}{s + \theta q(\theta)}, \tag{51}$$

which is the steady state of

$$u_{t+1} = (1 - f(\theta))\, u_t + s\,(1 - u_t),$$

where $1 - u_t$ is employment given the constant-labor-force assumption.

If $\mathcal{T}$ is everywhere decreasing, then only one θ will satisfy $\mathcal{T}(\theta) = 0$. The derivative of $\mathcal{T}$ can be computed directly from (50). It is stated in equation (17) in the main text and repeated here for convenience:

$$\mathcal{T}'(x) \equiv \frac{c\,(r+s)}{(1-\phi)\,[q(x)]^2} q'(x) - \frac{c\phi}{1-\phi} - \beta\frac{f'(x)}{1-\phi}\,[\phi h - (1-\phi)\,\ell]\,. \tag{52}$$

A tighter labor market makes it harder to fill a job and easier to find a job: $q' < 0$ and $f' > 0$. The first term is therefore negative. The term $c\phi/(1-\phi)$ is positive. The sign of $\mathcal{T}'$ therefore depends on the magnitudes of first two terms compared to the magnitude of

$$\frac{1}{1-\phi}\,[c\phi + \beta f'(x)\,[\phi h - (1-\phi)\,\ell]]\,. \tag{53}$$

The comparison can be easily checked for particular numerical values.

Yet, other features of the economic environment can help rule out certain combinations of parameters. If the job-creation condition of workers given in equation (12) is upward sloping in θ–w space, then the term in (53) will be positive and a unique equilibrium is guaranteed. When there are no fixed costs of job creation, for example, the wage curve of workers is upward sloping and this guarantees a unique equilibrium (Pissarides, 2000, chapter 1).

Table 1 describes sets of parameters guaranteeing a unique equilibrium. Straightforward manipulation of the term listed in (53) establish these cases. The economies considered in section 6 are all characterized by a unique equilibrium.

13 Proof of Proposition 2

There are three parts of proposition 2. The first part establishes that the elasticity of tightness with respect to productivity can be decomposed into two terms. One of the terms establishes that the fundamental surplus is an essential object. This part of the proof is established in appendix 13.1. The second part establishes that the other term in the two-factor decomposition is bounded by a nonlinear function of the elasticity of matching with respect to unemployment. This part of the proof is established in appendix 13.2. The third part derives the elasticity of unemployment with respect to y. This part of the proof is established in appendix 13.3.

Related decompositions are undertaken by Zanetti (2011) and Kokonas (2023). Zanetti (2011) considers an environment with taxes and firing costs. Kokonas (2023) considers an environment where intermediate-good-producing firms hire labor and sell output to monopolistically competitive firms that produce the final good.

13.1 The Fundamental Decomposition: A Decomposition of the Elasticity of Market Tightness

One way to understand how market tightness responds to changes in productivity is through the elasticity of market tightness with respect to productivity, $\eta_{\theta,y} \equiv (d\theta/dy)(y/\theta)$. The computation of $\eta_{\theta,y}$ involves $d\theta/dy$. The variable θ is defined implicitly by (47). The expression in (47) can be rearranged as

$$\begin{aligned} y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi} &= \frac{r+s+\phi\theta q(\theta)}{(1-\phi)q(\theta)}c + \frac{\theta q(\theta)}{1+r}\left(\frac{\phi}{1-\phi}h - \ell\right) \\ \therefore\ \frac{1-\phi}{c}\left[y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi}\right] &= \frac{r+s+\phi\theta q(\theta)}{q(\theta)} + \frac{\theta q(\theta)}{1+r}\frac{\phi h - (1-\phi)\ell}{c} \\ &= \frac{r+s}{q(\theta)} + \phi\theta + \frac{\theta q(\theta)}{1+r}\frac{\phi h - (1-\phi)\ell}{c}. \end{aligned} \tag{54}$$

This expression motivates defining the function F as

$$F(\theta, y) \equiv \frac{1-\phi}{c}\left(y - z - \beta s\tau - \frac{\beta(r+s)h}{1-\phi}\right) - \frac{r+s}{q(\theta)} - \phi\theta - \frac{\theta q(\theta)}{1+r}\frac{\phi h - (1-\phi)\ell}{c}. \tag{55}$$

The implicit-function theorem then implies

$$\begin{aligned} \frac{d\theta}{dy} &= -\frac{\partial F/\partial y}{\partial F/\partial\theta} \\ &= -\frac{\frac{1-\phi}{c}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi - \beta\left(\frac{\phi h-(1-\phi)\ell}{c}\right)(q(\theta)+\theta q'(\theta))}. \end{aligned}$$

Using the expression in (54) for $(1-\phi)/c$, the latter can be written

$$\frac{d\theta}{dy} = -\frac{\left[\frac{r+s}{q(\theta)} + \phi\theta + \beta\theta q(\theta)\frac{\phi h-(1-\phi)\ell}{c}\right]\frac{1}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi - \beta\left(\frac{\phi h-(1-\phi)\ell}{c}\right)(q(\theta)+\theta q'(\theta))}$$
$$= -\frac{\left[\frac{r+s}{q(\theta)} + \phi\theta + \beta\theta q(\theta)\frac{\phi h-(1-\phi)\ell}{c}\right]\frac{1}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi - \beta\left(\frac{\phi h-(1-\phi)\ell}{c}\right)(q(\theta)+\theta q'(\theta))} \times \frac{\theta q(\theta)}{\theta q(\theta)}.$$

Developing this expression further yields

$$\frac{d\theta}{dy} = -\frac{\left[\frac{r+s}{q(\theta)} + \phi\theta + \beta\theta q(\theta)\frac{\phi h-(1-\phi)\ell}{c}\right]\frac{1}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}}{\frac{r+s}{[q(\theta)]^2}q'(\theta) - \phi - \beta\left(\frac{\phi h-(1-\phi)\ell}{c}\right)(q(\theta)+\theta q'(\theta))} \times \frac{\theta q(\theta)}{\theta q(\theta)}$$
$$= \frac{\frac{r+s}{q(\theta)} + \phi\theta + \beta\theta q(\theta)\frac{\phi h-(1-\phi)\ell}{c}}{-\frac{r+s}{[q(\theta)]^2}q'(\theta) + \phi + \beta\left(\frac{\phi h-(1-\phi)\ell}{c}\right)(q(\theta)+\theta q'(\theta))} \times \frac{\theta q(\theta)}{\theta q(\theta)}\frac{1}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}$$
$$= \frac{(r+s) + \phi\theta q(\theta) + \beta\theta q(\theta)\left[q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{-(r+s)\frac{\theta q'(\theta)}{q(\theta)} + \phi\theta q(\theta) + \beta\theta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)[q(\theta)+\theta q'(\theta)]} \times \frac{\theta}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}$$
$$= \frac{r+s+\phi\theta q(\theta) + \beta\theta q(\theta)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)}{(r+s)\eta_{M,u} + \phi\theta q(\theta) + \beta\theta q(\theta)\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)} \times \frac{\theta}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}},$$

where the last line uses the expression for matching elasticities in (65) and (66). Tidying the latter expression yields

$$\frac{d\theta}{dy} = \frac{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{(r+s)\eta_{M,u} + \theta q(\theta)\left[\phi + \beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \times \frac{\theta}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}}. \tag{56}$$

Thus

$$\eta_{\theta,y} = \frac{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{(r+s)\eta_{M,u} + \theta q(\theta)\left[\phi + \beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \times \frac{y}{y-z-\beta s\tau-\frac{\beta(r+s)h}{1-\phi}},$$

which establishes equation (18) of proposition 2. Defining the term Υ as

$$\Upsilon \equiv \frac{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{(r+s)\eta_{M,u} + \theta q(\theta)\left[\phi + \beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}. \tag{57}$$

establishes (19) of proposition 2.

13.2 A Bound for Υ

To establish a bound for Υ, I start from

$$\frac{1}{\Upsilon}=\frac{(r+s)\,\eta_{M,u}+\theta q(\theta)\left[\phi+\beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}.$$

There are two cases to consider. The case where $\eta_{M,u}\geq 0.5$ and the case where $\eta_{M,u}<0.5$.

I first consider the case where $\eta_{M,u}\geq 0.5$. Thus $-\eta_{M,u}\leq -0.5$ and adding 1 to both sides yields

$$1-\eta_{M,u}\leq \eta_{M,u}. \tag{58}$$

Starting from $1/\Upsilon$, I have

$$\begin{aligned}\frac{1}{\Upsilon}&=\frac{(r+s)\,\eta_{M,u}+\theta q(\theta)\left[\phi+\beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}\\&\geq\frac{(r+s)\left(1-\eta_{M,u}\right)+\theta q(\theta)\left[\phi+\beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]},\end{aligned}$$

where the last inequality uses the assumption in (58). Developing the expression on the right side by adding and subtracting $\left(1-\eta_{M,u}\right)\phi\theta q(\theta)$ to the numerator yields

$$\begin{aligned}\frac{1}{\Upsilon}\geq&\frac{\left(1-\eta_{M,u}\right)\left\{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]\right\}}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}\\&+\frac{\theta q(\theta)\,\phi-\left(1-\eta_{M,u}\right)\phi\theta q(\theta)}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}.\end{aligned}$$

Developing the latter by canceling terms in the first expression and collecting terms in the second yields

$$\begin{aligned}\frac{1}{\Upsilon}&\geq\left(1-\eta_{M,u}\right)+\frac{\phi\theta q(\theta)\,\eta_{M,u}}{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}\\&>1-\eta_{M,u}.\end{aligned}$$

Thus,

$$\Upsilon<\left(1-\eta_{M,u}\right)^{-1}. \tag{59}$$

Now consider the case where $\eta_{M,u}<0.5$. Thus $-\eta_{M,u}>-0.5$ and adding 1 to both sides yields

$$1-\eta_{M,u}>0.5>\eta_{M,u}. \tag{60}$$

Starting from $1/\Upsilon$, I have

$$\begin{aligned}\frac{1}{\Upsilon} &= \frac{(r+s)\,\eta_{M,u} + \theta q(\theta)\left[\phi + \beta\left(1-\eta_{M,u}\right) q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \\ &> \frac{(r+s)\,\eta_{M,u} + \theta q(\theta)\left[\phi + \beta \eta_{M,u} q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]},\end{aligned}$$

where the last inequality uses the assumption in (60). Developing the expression on the right side by adding and subtracting $\eta_{M,u}\phi\theta q(\theta)$ to the numerator yields

$$\begin{aligned}\frac{1}{\Upsilon} &> \frac{\eta_{M,u}\left\{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]\right\}}{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \\ &\quad+\frac{\theta q(\theta)\,\phi - \eta_{M,u}\phi\theta q(\theta)}{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}.\end{aligned}$$

Developing the latter yields

$$\begin{aligned}\frac{1}{\Upsilon} &> \eta_{M,u} + \frac{\phi\theta q(\theta)\left(1-\eta_{M,u}\right)}{r+s+\theta q(\theta)\left[\phi + \beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \\ &> \eta_{M,u}.\end{aligned}$$

Thus,

$$\Upsilon < \left(\eta_{M,u}\right)^{-1}. \tag{61}$$

The results in (59) and (61) establish (20) of proposition 2.

The statement in (20) can be strengthened to

$$0 < \Upsilon < \max\left\{\frac{1}{\eta_{M,u}}, \frac{1}{1-\eta_{M,u}}\right\}$$

if conditions are met for a unique equilibrium; namely, if the term listed in (49) is positive. Result 4 establishes the equivalency. In other words, the result establishes the other direction too: if Υ is unambiguously positive, then the equilibrium is unique.

13.3 The Elasticity of Unemployment with Respect to Productivity

The elasticity of unemployment with respect to y is $\eta_{u,y} = (du/dy)\,(y/u)$. Starting from the expression for steady-state unemployment, $u = s/(s+f(\theta))$, it is true that

$$\begin{aligned}\frac{du}{dy} &= -s\,(s+f(\theta))^{-2} f'(\theta)\,\frac{d\theta}{dy} \\ &= -\frac{u}{s+f(\theta)}\,(q(\theta)+\theta q'(\theta))\,\frac{d\theta}{dy},\end{aligned}$$

where the second equality uses the definition of steady-state u and f. Further development yields

$$\begin{aligned}
\frac{du}{dy} &= -\frac{u}{s+f(\theta)}\left(q(\theta)+\theta q'(\theta)\right)\frac{d\theta}{dy}, \\
&= -\frac{uq(\theta)}{s+f(\theta)}\left(1+\frac{\theta q'(\theta)}{q(\theta)}\right)\frac{d\theta}{dy} \\
&= -\frac{uq(\theta)}{s+f(\theta)}\left(1-\eta_{M,u}\right)\frac{d\theta}{dy},
\end{aligned}$$

where the last line uses the definition of the elasticity of matching with respect to unemployment. The elasticity of unemployment with respect to productivity is therefore

$$\begin{aligned}
\frac{du}{dy}\frac{y}{u} &= -\frac{q(\theta)}{s+f(\theta)}\left(1-\eta_{M,u}\right)\frac{d\theta}{dy}y \\
&= -\frac{\theta q(\theta)}{s+f(\theta)}\left(1-\eta_{M,u}\right)\frac{d\theta}{dy}\frac{y}{\theta} \\
&= -\frac{f(\theta)}{s+f(\theta)}\left(1-\eta_{M,u}\right)\frac{d\theta}{dy}\frac{y}{\theta} \\
&= -(1-u)\left(1-\eta_{M,u}\right)\frac{d\theta}{dy}\frac{y}{\theta} \\
&= -(1-u)\left(1-\eta_{M,u}\right)\eta_{\theta,y},
\end{aligned}$$

where the last equality uses the definition of $\eta_{\theta,y}$.

A proof that $\eta_{M,u} \in (0,1)$ can be found in Ryan (2023).

14 Proof of Proposition 3

The proof of proposition 3 uses direct computation of the partial derivatives. To compute the partial derivatives, it will be useful to define two terms for the numerator and denominator of Υ in (19) of the main text:

$$\Upsilon \equiv \frac{r+s+\theta q(\theta)\left[\phi+\beta q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]}{(r+s)\,\eta_{M,u}+\theta q(\theta)\left[\phi+\beta\left(1-\eta_{M,u}\right)q(\theta)\left(\frac{\phi h-(1-\phi)\ell}{c}\right)\right]} \equiv \frac{T_1}{T_2}. \tag{62}$$

14.1 How Υ changes when ℓ changes

Using the expression for Υ in (62), the derivative of Υ with respect to ℓ is

$$\frac{\partial \Upsilon}{\partial \ell} = \frac{\frac{\partial T_1}{\partial \ell}T_2 - \frac{\partial T_2}{\partial \ell}T_1}{T_2^2}.$$

The sign of $\partial\Upsilon/\partial\ell$ depends on the sign of the numerator. I compute the terms

$$\begin{aligned}\frac{\partial T_1}{\partial \ell} &= -\theta q\left(\theta\right)\frac{\beta\left(1-\phi\right)q\left(\theta\right)}{c}\\ \frac{\partial T_2}{\partial \ell} &= -\left(1-\eta_{M,u}\right)\theta q\left(\theta\right)\frac{\beta\left(1-\phi\right)q\left(\theta\right)}{c}\\ &= \left(1-\eta_{M,u}\right)\frac{\partial T_1}{\partial \ell}.\end{aligned}$$

Then

$$\begin{aligned}\frac{\partial\Upsilon}{\partial\ell} &\propto \frac{\partial T_1}{\partial \ell}T_2-\frac{\partial T_2}{\partial \ell}T_1\\ &= \frac{\partial T_1}{\partial \ell}T_2-\left(1-\eta_{M,u}\right)\frac{\partial T_1}{\partial \ell}T_1\\ &= -\theta q\left(\theta\right)\frac{\beta\left(1-\phi\right)q\left(\theta\right)}{c}\left[T_2-\left(1-\eta_{M,u}\right)T_1\right].\end{aligned} \tag{63}$$

The term in square brackets, $T_2-\left(1-\eta_{M,u}\right)T_1$, is

$$\begin{gathered}\left(r+s\right)\eta_{M,u}+\theta q\left(\theta\right)\left[\phi+\beta\left(1-\eta_{M,u}\right)q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\\ -\left(1-\eta_{M,u}\right)\left\{r+s+\theta q\left(\theta\right)\left[\phi+\beta q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\right\}.\end{gathered}$$

This expression can be simplified to

$$\begin{gathered}\left(r+s\right)\left(\eta_{M,u}-1+\eta_{M,u}\right)\\ +\theta q\left(\theta\right)\left[\phi+\beta\left(1-\eta_{M,u}\right)q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\\ -\left(1-\eta_{M,u}\right)\theta q\left(\theta\right)\left[\phi+\beta q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\\ =\left(r+s\right)\left(\eta_{M,u}-1+\eta_{M,u}\right)\\ +\theta q\left(\theta\right)\left[\phi+\beta\left(1-\eta_{M,u}\right)q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\\ -\theta q\left(\theta\right)\left[\phi-\phi\eta_{M,u}+\beta\left(1-\eta_{M,u}\right)q\left(\theta\right)\left(\frac{\phi h-\left(1-\phi\right)\ell}{c}\right)\right]\\ =\left(r+s\right)\left(\eta_{M,u}-1+\eta_{M,u}\right)\\ +\theta q\left(\theta\right)\phi\eta_{M,u}.\end{gathered}$$

Because $\eta_{M,u}\approx 1/2$, the latter provides the result

$$T_2-\left(1-\eta_{M,u}\right)T_1\approx\theta q\left(\theta\right)\phi\eta_{M,u}. \tag{64}$$

Using the result in (64) for the term $T_2 - \left(1-\eta_{M,u}\right) T_1$ in equation (63) yields

$$\begin{aligned}
\frac{\partial \Upsilon}{\partial \ell} &\propto -\theta q(\theta) \frac{\beta(1-\phi) q(\theta)}{c} \left[T_2 - \left(1-\eta_{M,u}\right) T_1\right] \\
&\approx -\theta q(\theta) \frac{\beta(1-\phi) q(\theta)}{c} \theta q(\theta) \phi \eta_{M,u} \\
&= -\phi \eta_{M,u} \frac{\beta(1-\phi) q(\theta)}{c} [\theta q(\theta)]^2 \\
&< 0,
\end{aligned}$$

establishing the inequality in equation (30) in proposition 3 of the main text.

14.2 How Υ changes when h changes

Using the expression for Υ in (62), the derivative of Υ with respect to h is

$$\frac{\partial \Upsilon}{\partial h} = \frac{\frac{\partial T_1}{\partial h} T_2 - \frac{\partial T_2}{\partial h} T_1}{T_2^2}.$$

The sign of $\partial \Upsilon / \partial h$ depends on the sign of the numerator. I compute the terms

$$\begin{aligned}
\frac{\partial T_1}{\partial h} &= \theta q(\theta) \frac{\beta \phi q(\theta)}{c} \\
\frac{\partial T_2}{\partial h} &= \left(1-\eta_{M,u}\right) \theta q(\theta) \frac{\beta \phi q(\theta)}{c} \\
&= \left(1-\eta_{M,u}\right) \frac{\partial T_1}{\partial h}.
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial \Upsilon}{\partial h} &\propto \frac{\partial T_1}{\partial h} T_2 - \frac{\partial T_2}{\partial h} T_1 \\
&= \frac{\partial T_1}{\partial h} T_2 - \left(1-\eta_{M,u}\right) \frac{\partial T_1}{\partial h} T_1 \\
&= \frac{\partial T_1}{\partial h} \left[T_2 - \left(1-\eta_{M,u}\right) T_1\right] \\
&= \theta q(\theta) \frac{\beta \phi q(\theta)}{c} \left[T_2 - \left(1-\eta_{M,u}\right) T_1\right] \\
&\approx \phi \eta_{M,u} \frac{\beta \phi q(\theta)}{c} [\theta q(\theta)]^2 \\
&> 0,
\end{aligned}$$

where the last equality uses the result in (64).

15 Auxiliary Results

Result 1. The reservation wage, $U = W(w_R)$ is

$$w_R = \frac{r}{1+r} U.$$

Proof. Starting from equation (41):

$$
\begin{aligned}
E\left(w_R\right) &= \frac{w_R + \beta s U}{1 - \beta\left(1 - s\right)} = U \\
\therefore\ w_R + \beta s U &= U\left[1 - \beta\left(1 - s\right)\right] \\
\therefore\ w_R &= U\left[1 - \beta\left(1 - s\right)\right] - \beta s U \\
&= U\left[1 - \frac{1}{1+r}\left(1 - s\right) - \frac{1}{1+r} s\right] \\
&= U\left[\frac{1 + r - 1 + s - s}{1+r}\right] \\
&= U\frac{r}{1+r},
\end{aligned}
$$

establishing what was set out to be shown. □

Result 2. The geometric series $\left(1+x\right)/\left(x+y\right)$ can be expressed as

$$
\begin{aligned}
\frac{1+x}{x+y} &= \sum_{j=0}^{\infty}\left(\frac{1-y}{1+x}\right)^{j} \\
&= \frac{1}{1 - \frac{1-y}{1+x}} \\
&= \frac{1+x}{1 + x - \left(1 - y\right)} \\
&= \frac{1+x}{x+y}.
\end{aligned}
$$

This result uses

$$\left|\frac{1-y}{1+x}\right| < 1 \quad \text{or} \quad -1 < \frac{1-y}{1+x} < 1,$$

which requiems

$$\frac{1-y}{1+x} < 1 \quad \text{or} \quad 0 < x + y$$

and

$$\frac{1-y}{1+x} > -1 \quad \text{or} \quad y - x < 2.$$

Result 3 (Elasticity of matching with respect to unemployment). The elasticity of matching with respect to unemployment, $\eta_{M,u}$, is

$$\begin{aligned}
\eta_{M,u} &\equiv \left(\frac{dM}{du}\right)\frac{u}{M} \\
&= \left[\frac{d}{du}q(\theta)v\right]\frac{u}{M} = \left[\frac{d}{du}q(\theta)\right]\frac{vu}{M} \\
&= \left[q'(\theta)\frac{d\theta}{du}\right]\frac{vu}{M} = \left[q'(\theta)\frac{-v}{u^2}\right]\frac{vu}{M} \\
&= \left[q'(\theta)\frac{-v}{u}\right]\frac{v}{M} = -q'(\theta)\theta\frac{v}{M} \\
&= -\frac{q'(\theta)\theta}{q(\theta)}
\end{aligned} \tag{65}$$

where the second line uses the fact that $d\theta/du = -v/u^2$ and the last equality uses $q(\theta) = M/v$. It follows that

$$1 - \eta_{M,u} = 1 - \left[-\frac{q'(\theta)\theta}{q(\theta)}\right] = \frac{q(\theta) + \theta q'(\theta)}{q(\theta)}. \tag{66}$$

Result 4. The slope of the job-creation condition for workers in $\theta{-}w$ space is

$$c\phi + \beta f'(x)\left[\phi h - (1-\phi)\ell\right] \text{ for } x \in \left(0, \bar{\theta}\right),$$

which can be derived from equation (12). A positive slope is equivalent to

$$\begin{aligned}
c\phi + \beta\left(q(x) + xq'(x)\right)\left[\phi h - (1-\phi)\ell\right] > 0,&\ \text{ using } f(x) = xq(x) \\
\therefore \frac{c\phi}{q(x)} + \beta\left(\frac{q(x) + \theta q'(x)}{q(x)}\right)\left[\phi h - (1-\phi)\ell\right] > 0&\\
\therefore \frac{c\phi}{q(x)} + \beta\left(1 - \eta_{M,u}\right)\left[\phi h - (1-\phi)\ell\right] > 0,&\ \text{ using result (66)}, \\
\therefore \phi + q(x)\beta\left(1 - \eta_{M,u}\right)\left[\frac{\phi h - (1-\phi)\ell}{c}\right] > 0,&
\end{aligned}$$

which is the denominator of Υ.

16 Derivation of a New Measure of Transition Probabilities That Corrects for Time-Aggregation and How JOLTS Records Hires

Here is a derivation of transition probabilities that corrects for data being available at discrete intervals even though workers can transition between employment and unemployment continuously. In addition, the adjustment corrects for how the JOLTS program records hires. All hires are reported, even "workers who were hired and separated during the month." See footnote 11 in the main text.

Definitions of variables are included in the main text. Some of the equations in the main text are reproduced here for convenience.

Section 16.1 contains the derivations and section 16.3 considers the hypothetical model that would correspond to new hires being measured as net hires. Section 16.3 highlights how this model differs from the model in Shimer (2012). 16.2 presents the adjusted data.

16.1 Derivations

In the model on transitions, employment and new hires evolve according to

$$\dot{e}_{t+\tau}(\tau) = \varphi_t u_{t+\tau}(\tau) - \varsigma_t e_{t+\tau}(\tau) \tag{67}$$

$$\dot{e}^h_{t+\tau}(\tau) = \varphi_t u_{t+\tau}(\tau). \tag{68}$$

These are equations (22) and (23) in the main text. With the assumption that the labor force is constant within the period, $l_t = e_{t+\tau'}(\tau') + u_{t+\tau'}(\tau')$ for all $t' \in [0, 1)$, equations (67) and (68) yield two differential equations for e and e^h:

$$\dot{e}_{t+\tau}(\tau) = \varphi_t l_t - (\varphi_t + \varsigma_t) e_{t+\tau}(\tau) \tag{69}$$

$$\dot{e}^h_{t+\tau}(\tau) = \varphi_t l_t - \varphi_t e_{t+\tau}(\tau). \tag{70}$$

These are equations (24) and (25) in the main text.

Using equation (70) to eliminate $\varphi_t l_t$ from equation (69) yields the differential equation:

$$\begin{aligned}
\dot{e}_t(\tau) &= \varphi_t l_t - (\varphi_t + \varsigma_t) e_{t+\tau}(\tau) \\
&= \dot{e}^h_{t+\tau}(\tau) + \varphi_t e_{t+\tau}(\tau) - (\varphi_t + \varsigma_t) e_{t+\tau}(\tau) \\
&= -\varsigma_t e_{t+\tau}(\tau) + \dot{e}^h_{t+\tau}(\tau).
\end{aligned}$$

The general solution is (Acemoglu, 2009, 923):

$$\begin{aligned}
e_t(\tau) &= \left[\varrho_1 + \int_0^\tau \dot{e}^h_{t+x}(x)\, e^{\int_0^x \varsigma_t dv} dx\right] e^{\int_0^\tau -\varsigma_t dz} \\
&= \left[\varrho_1 + \int_0^\tau \dot{e}^h_{t+x}(x)\, e^{\varsigma_t x} dx\right] e^{-\varsigma_t \tau},
\end{aligned}$$

where ϱ_1 is a constant of integration. The integral in brackets can be evaluated using integration by parts:

$$\begin{aligned}
\int_0^\tau \dot{e}^h_{t+x}(x)\, e^{\varsigma_t x} dx &= e^{\varsigma_t x} e^h_{t+x}(x) \Big|_{x=0}^{x=\tau} - \int_0^\tau \varsigma_t e^{\varsigma_t x} e^h_{t+x}(x)\, dx \\
&= e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - \varsigma_t \int_0^\tau e^{\varsigma_t x} e^h_{t+x}(x)\, dx.
\end{aligned}$$

Therefore

$$e_{t+\tau} = \left[\varrho_1 + e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - \varsigma_t \int_0^\tau e^{\varsigma_t x} e^h_{t+x}(x)\, dx\right] e^{-\varsigma_t \tau}. \tag{71}$$

Shimer (2012) at this point uses an immaculate cancellation. As discussed below, this neat cancellation isn't available. To make progress on the problem, I therefore assume new hires are added linearly within the period. Put another way, I posit a particular form for the hires function within the period: $e^h_{t+\tau}(\tau) = e^h_{t+1}\tau$. The model for new hires is depicted in figure 5.

With linearity, it is possible to make progress on the equation in (71):

$$\begin{aligned}
e_{t+\tau} &= \left[\varrho_1 + e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - \varsigma_t \int_0^\tau e^{\varsigma_t x} e^h_{t+1} x dx\right] e^{-\varsigma_t \tau} \\
&= \left[\varrho_1 + e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - \varsigma_t e^h_{t+1} \int_0^\tau e^{\varsigma_t x} x dx\right] e^{-\varsigma_t \tau}.
\end{aligned}$$

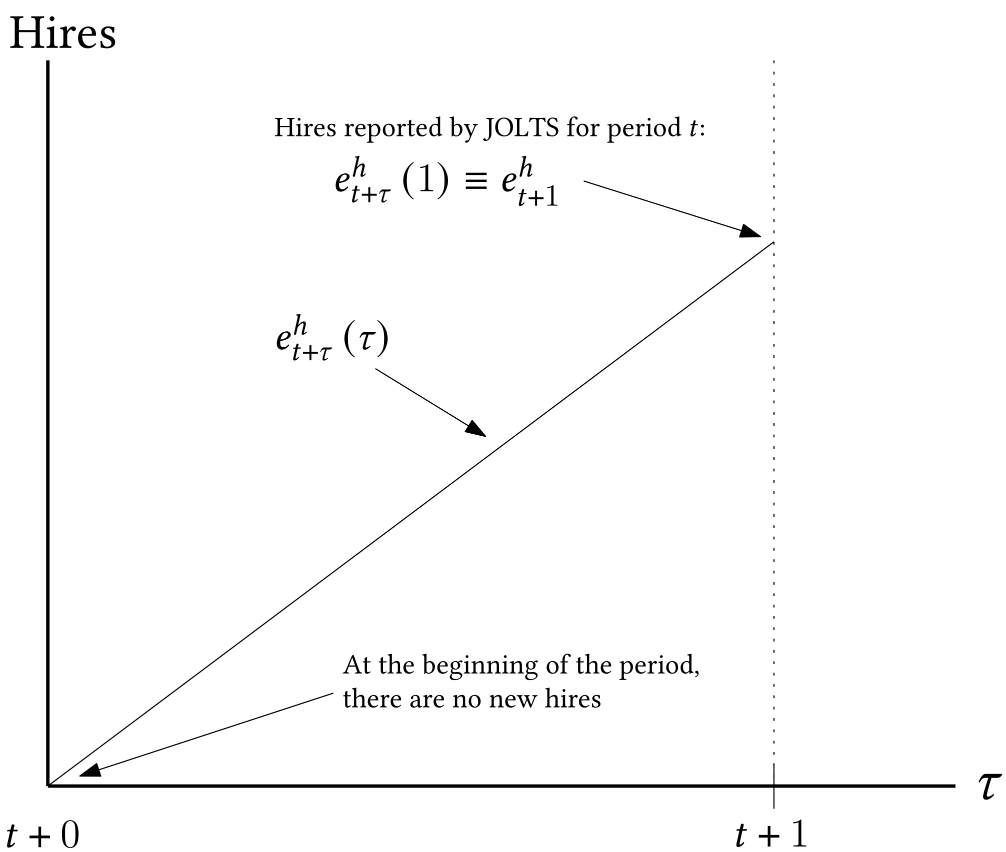

Figure 5: Parameterized function for hires within a period.

Notes: A the beginning of the period there are no new hires. At the end of the period there are e^h_{t+1} new hires, a value that corresponds to data reported by the JOLTS program. I assume new hires are added linearly within the month.

The integral in the bracket is

$$\begin{aligned}\int_0^{\tau} e^{\varsigma_t x} x dx &= \frac{1}{\varsigma_t} x e^{\varsigma_t x}\Big|_{x=0}^{x=\tau} - \frac{1}{\varsigma_t}\int_0^{\tau} e^{\varsigma_t x} dx \\ &= \frac{1}{\varsigma_t}\tau e^{\varsigma_t \tau} - \frac{1}{\varsigma_t}\int_0^{\tau} e^{\varsigma_t x} dx \\ &= \frac{1}{\varsigma_t}\tau e^{\varsigma_t \tau} - \frac{1}{\varsigma_t}\frac{1}{\varsigma_t} e^{\varsigma_t x}\Big|_{x=0}^{x=\tau} \\ &= \frac{1}{\varsigma_t}\tau e^{\varsigma_t \tau} - \frac{1}{\varsigma_t}\frac{1}{\varsigma_t} e^{\varsigma_t \tau} + \frac{1}{\varsigma_t}\frac{1}{\varsigma_t} \\ &= \frac{1}{\varsigma_t}\left[\tau e^{\varsigma_t \tau} + \frac{1}{\varsigma_t}\left(1 - e^{\varsigma_t \tau}\right)\right].\end{aligned}$$

Therefore

$$e_{t+\tau} = \left[\varrho + e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - e^h_{t+1}\left(\tau e^{\varsigma_t \tau} + \frac{1 - e^{\varsigma_t \tau}}{\varsigma_t}\right)\right] e^{-\varsigma_t \tau}.$$

When $\tau = 0$, the latter express evaluates to

$$\begin{aligned} e_{t+0} &= \left[\varrho + e^{\varsigma_t 0} e^h_{t+0} - e^h_{t+1}\left(0 e^{\varsigma_t 0} + \frac{1 - e^{\varsigma_t 0}}{\varsigma_t}\right)\right] e^{-\varsigma_t 0} \\ \therefore\ e_t &= \varrho, \end{aligned}$$

using the fact that $e^h_{t+0} = 0$. Therefore

$$\begin{aligned} e_{t+\tau} &= \left[e_t + e^{\varsigma_t \tau} e^h_{t+\tau}(\tau) - e^h_{t+1}\left(\tau e^{\varsigma_t \tau} + \frac{1 - e^{\varsigma_t \tau}}{\varsigma_t}\right)\right] e^{-\varsigma_t \tau} \\ &= e_t e^{-\varsigma_t \tau} + e^h_{t+\tau}(\tau) - e^h_{t+1}\left(\tau + \frac{e^{-\varsigma_t \tau} - 1}{\varsigma_t}\right) \\ &= e_t e^{-\varsigma_t \tau} + e^h_{t+\tau}(\tau) - e^h_{t+1}\left(\tau - \frac{1 - e^{-\varsigma_t \tau}}{\varsigma_t}\right) \end{aligned}$$

Evaluated at $\tau = 1$ yields

$$\begin{aligned} e_{t+1} &= e_t e^{-\varsigma_t} + e^h_{t+1} - e^h_{t+1}\left(1 - \frac{1 - e^{-\varsigma_t}}{s_t}\right) \\ &= e_t\left(1 - s_t\right) + e^h_{t+1} - e^h_{t+1}\left(1 - \frac{s_t}{\varsigma_t}\right). \end{aligned}$$

Using $\varsigma_t = -\ln\left(1 - s_t\right)$, the latter can be written

$$e_{t+1} = e_t\left(1 - s_t\right) + e^h_{t+1} - e^h_{t+1}\left[1 + \frac{s_t}{\ln\left(1 - s_t\right)}\right], \tag{72}$$

which is equation (26) in the main text.

Equation (72) is an equation only in the separation rate. In addition, the approximation implies:

$$\begin{aligned} e_{t+1} &\approx e_t \left(1 - s_t\right) + e^h_{t+1} \\ \therefore\ e_{t+1} &\approx e_t - e_t s_t + e^h_{t+1} \\ \therefore\ e_t s_t &\approx e_t + e^h_{t+1} - e_{t+1} \\ \therefore\ s_t &\approx 1 - \frac{e_{t+1} - e^h_{t+1}}{e_t}. \end{aligned} \tag{73}$$

As discussed in the main text, I still need to recover the monthly job-finding rate. To recover f_t, I note that equation (69) is a linear differential equation for $\dot{e}_{t+\tau}(\tau)$ with constant coefficients. The solution is

$$e_{t+\tau} = \frac{\varphi_t l_t}{\varsigma_t + \varphi_t} + \varrho_2 e^{-(\varsigma_t + \varphi_t)\tau},$$

where ϱ_2 is a constant of integration. Evaluating the latter at $\tau = 0$ yields

$$\begin{aligned} e_t &= \frac{\varphi_t l_t}{s_t + \varphi_t} + \varrho_2 \\ \therefore\ \varrho_2 &= e_t - \frac{\varphi_t l_t}{s_t + \varphi_t}. \end{aligned}$$

With the constant of integration, the solution is

$$\begin{aligned} e_{t+\tau} &= \frac{\varphi_t l_t}{\varsigma_t + \varphi_t} + \left(e_t - \frac{\varphi_t l_t}{s_t + \varphi_t}\right) e^{-(\varsigma_t + \varphi_t)\tau} \\ &= \frac{\varphi_t l_t}{\varsigma_t + \varphi_t} \left(1 - e^{-(\varsigma_t + \varphi_t)\tau}\right) + e_t e^{-(\varsigma_t + f_t)\tau}. \end{aligned}$$

Evaluated at $\tau = 1$ yields

$$e_{t+1} = \frac{\varphi_t l_t}{\varsigma_t + \varphi_t} \left(1 - e^{-(\varsigma_t + \varphi_t)}\right) + e_t e^{-(\varsigma_t + \varphi_t)}. \tag{74}$$

To understand (74), note that in or near steady state, the employment rate, e_t/l_t, is approximately $\varphi_t/(\varsigma_t + \varphi_t)$, a standard formula derived from the number of jobs created totaling the number of jobs destroyed. The approximation implies

$$l_t - \frac{u_t}{l_t} = \frac{e_t}{l_t} \approx 1 - \frac{\varsigma_t}{\varsigma_t + \varphi_t} = \frac{\varphi_t}{\varsigma_t + \varphi_t}.$$

Using the latter in (74) yields the steady-state approximation $e_{t+1} \approx e_t$:

$$\begin{aligned} e_{t+1} &\approx \frac{e_t}{l_t} l_t \left(1 - e^{-(\varsigma_t + \varphi_t)}\right) + e_t e^{-(\varsigma_t + \varphi_t)} \\ &= e_t \left[\left(1 - e^{-(\varsigma_t + \varphi_t)}\right) + e^{-(\varsigma_t + \varphi_t)}\right] \\ &= e_t \left\{\left[1 - \left(1 - s_t\right)\left(1 - f_t\right)\right] + \left(1 - s_t\right)\left(1 - f_t\right)\right\} \\ &= e_t \left\{\left[1 - 1 + s_t + f_t - f_t s_t\right] + 1 - s_t - f_t + f_t s_t\right\} \\ &= e_t \left\{s_t + f_t - f_t s_t + 1 - s_t - f_t + f_t s_t\right\} \\ &= e_t. \end{aligned}$$

But when the economy is away from steady state, equation (74) allows workers to transition between labor-market states continuously. Equation (74) "captures the fact that a worker who loses her job is more likely to find a new one without experiencing a measured spell of unemployment" (Shimer, 2012, 131).

16.2 Adjusted Data

The adjusted series are reported in figures 6, 7, and 8. The data used in the figures are readily accessible using FRED. All the series are monthly.

Figure 6 shows that the correction for the probability of finding a job is meaningful. The uncorrected series sometimes goes above 1; whereas, the corrected series is closer to 0.7. As discussed in section 16.1, the correction adjusts the probability of finding a job lower.

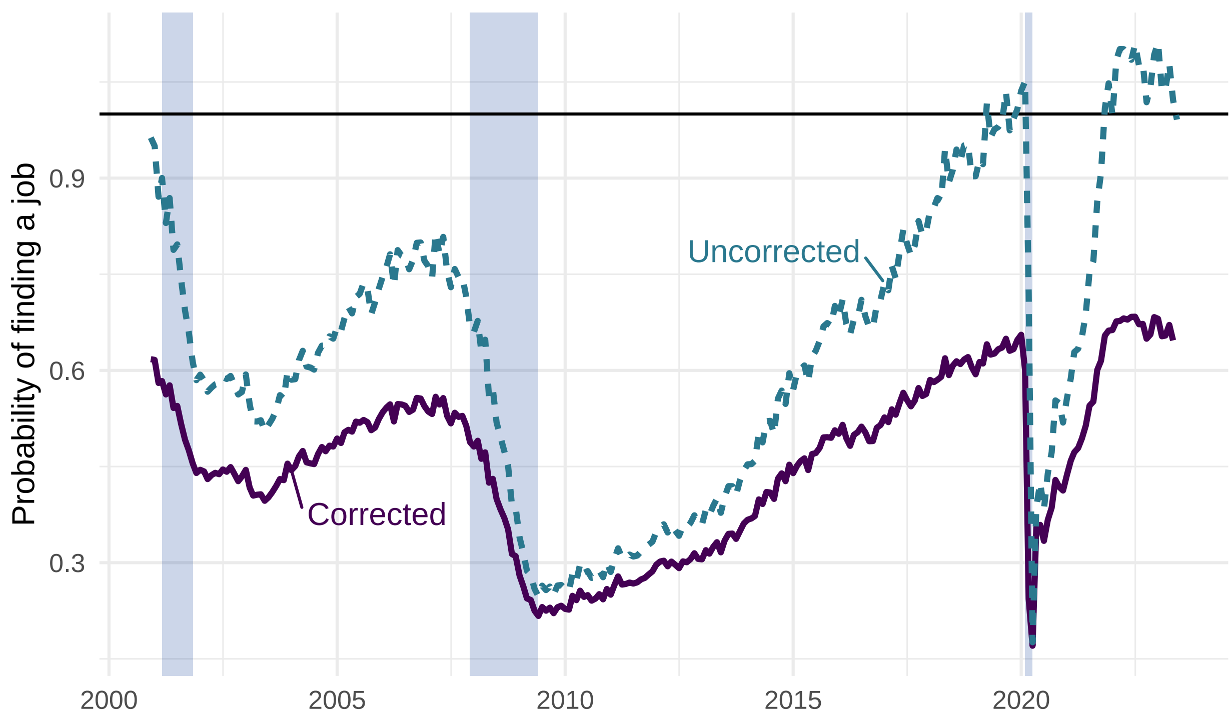

Figure 6: Corrected and uncorrected probabilities of finding a job, December 2000 through May 2023

Notes: The uncorrected series reports the number of hires in the month divided by the level of unemployment, which corresponds to "M/u" in the model economy. The corrected series reports $f_t = 1 - \exp(-\varphi_t)$, where φ_t is implicitly defined in equation (74).

Sources: Authors calculations that use data from FRED. Data on hires are from the US Bureau of Labor Statistics' series Hires: Total Nonfarm [JTSHIL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/JTSHIL. Data on the level of employment are from the US Bureau of Labor Statistics' series All Employees, Total Nonfarm [PAYEMS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PAYEMS. Data on the level of unemployment are from the US Bureau of Labor Statistics' series Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNEMPLOY. The labor force comprises employed and unemployed people.

Figure 7 shows the the approximate probability of separating from a job in (73) as well as the corrected probability of separating from a job, defined implicitly in (72). The approximate and corrected series are close. Equation (72) shows that the correction involves the separation rate, which is small in magnitude. The difference may be easier to see in figure 8, which shows the same series but truncates the vertical axis.

Figures 7 and 8 also report an uncorrected probability of separating from a job. This series corresponds to the separation rate that would make the approximation to the unemployment rate, $\tilde{s}_t / \left(\tilde{s}_t + \tilde{f}_t\right)$, hold with equality. The unemployment rate comes from the monthly BLS series and I use the uncorrected finding rate depicted in figure 6 for $\tilde{f}_t$.

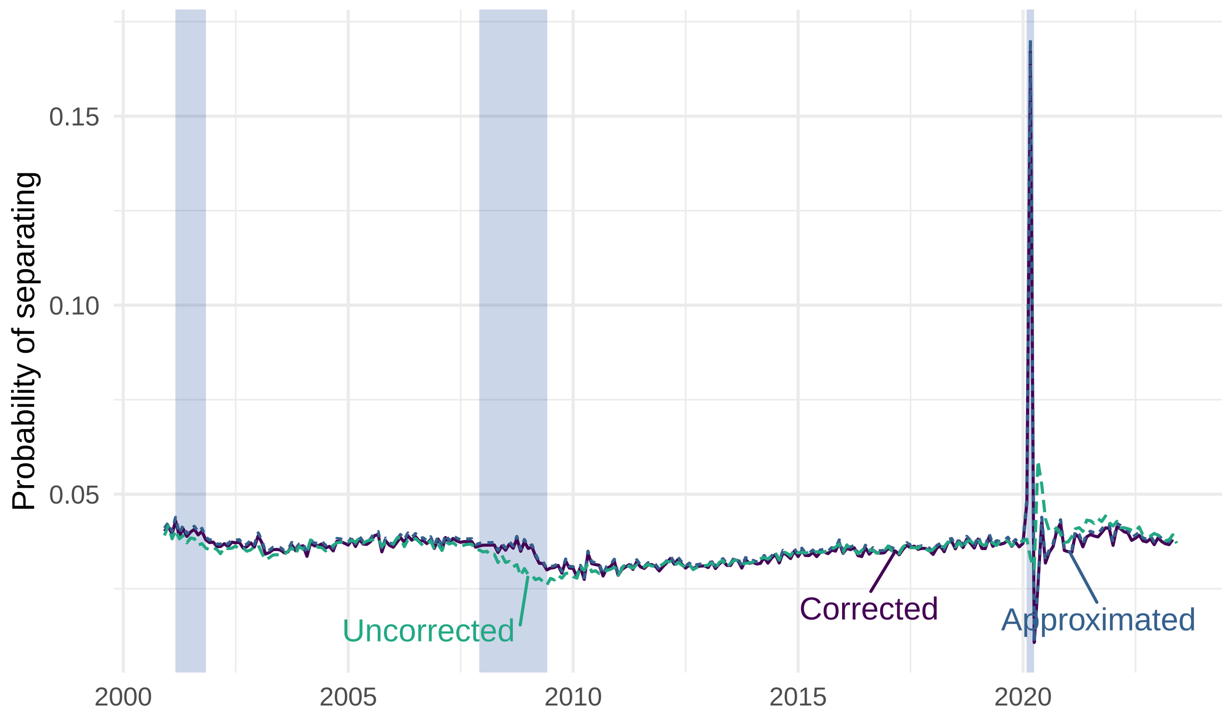

Figure 7: Corrected and uncorrected probabilities of separating from a job, December 2000 through May 2023

Notes: The series labeled "Approximated" reports the separation rate in (73). The uncorrected series uses an approximation of the unemployment rate. If $\tilde{s}_t$ is the uncorrected separation rate in month t and $\tilde{f}_t$ is the uncorrected finding rate in month t, then the unemployment rate is roughly $\tilde{s}_t / \left(\tilde{s}_t + \tilde{f}_t\right)$. Given the monthly unemployment rate and $\tilde{f}_t$ (from figure 6), I report the $\tilde{s}_t$ that would make the approximation hold with equality. The corrected series reports $s_t = 1 - \exp\left(-\varsigma_t\right)$, where ς_t is implicitly defined in equation (72).

Sources: Authors calculations that use data from FRED. The series for $\tilde{f}_t$ is described in the notes to figure 6. Data on the unemployment rate are from the US Bureau of Labor Statistics' series Unemployment Rate [UNRATE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE.

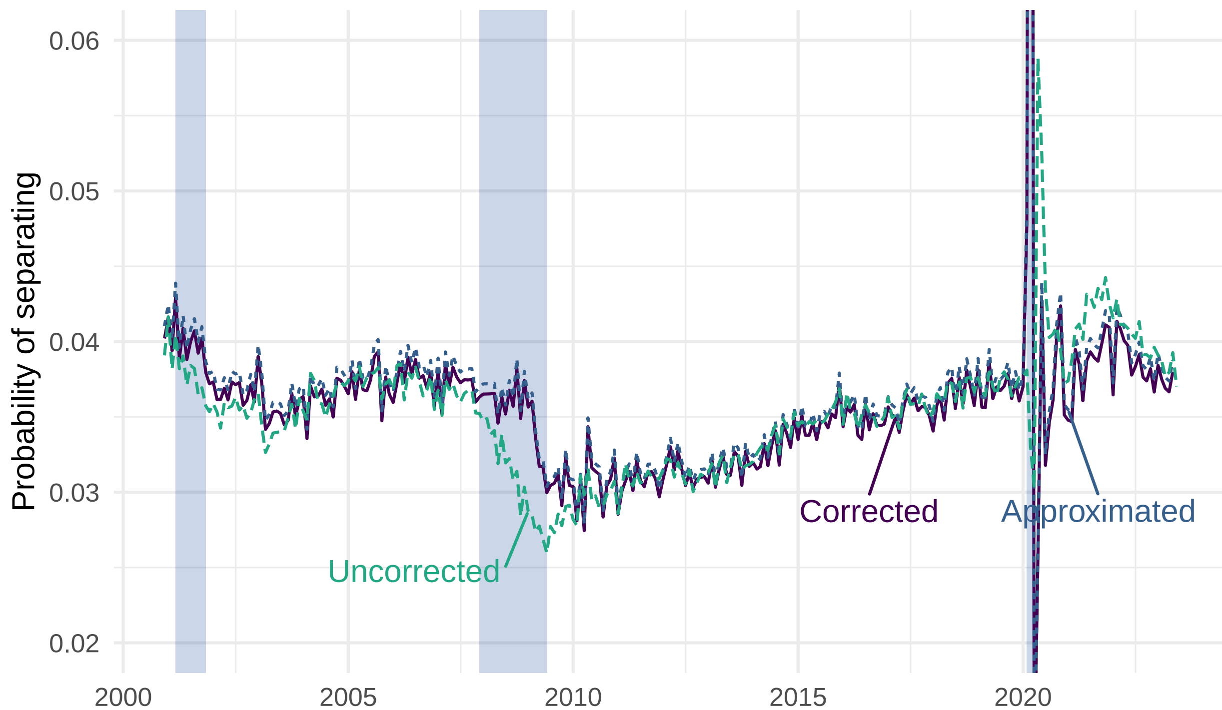

Figure 8: Corrected and uncorrected probabilities of separating from a job, December 2000 through May 2023. The series are the same as in figure 7. The vertical axis is reduced to facilitate inspection.

16.3 A Hypothetical Model of Net Hires and the Immaculate Cancellation in Shimer (2012)

The JOLTS program measures cumulative hires. If the JOLTS program measured net hires instead, then equation (68) would be

$$\dot{e}^h_{t+\tau} = \varphi_t u_{t+\tau} - \varsigma_t e^h_{t+\tau}. \tag{75}$$

Solving equation (75) for $\varphi_t u_{t+h}$ and substituting this into equation (67) yields

$$\begin{aligned}\dot{e}_{t+\tau} &= \varphi_t u_{t+\tau} - \varsigma_t e_{t+\tau} \\ &= \left(\dot{e}^h_{t+\tau} + \varsigma_t e^h_{t+\tau}\right) - \varsigma_t e_{t+\tau} \\ &= \dot{e}^h_{t+\tau} + \varsigma_t e^h_{t+\tau} - \varsigma_t e_{t+\tau}.\end{aligned}$$

This is a linear first-order equation for $\dot{e}_{t+\tau}$ with $\tau \in [0, 1)$. The general solution is

$$\begin{aligned}e_{t+\tau} &= \left[c_0 + \int_0^\tau \left(\dot{e}^h_{t+z} + \varsigma_t e^h_{t+z}\right) e^{-\int_0^z -\varsigma_t dv} dz\right] e^{\int_0^\tau -\varsigma_t dz} \\ &= \left[c_0 + \int_0^\tau \left(\dot{e}^h_{t+z} + \varsigma_t e^h_{t+z}\right) e^{\varrho_t z} dz\right] e^{-\varsigma_t \tau},\end{aligned} \tag{76}$$

where c_0 is a constant of integration. Expanding the integral in the bracketed term in (76) yields

$$\begin{aligned}\int_0^\tau \left(\dot{e}^h_{t+z} + \varsigma_t e^h_{t+z}\right) e^{\varsigma_t z} dz &= \int_0^\tau e^{\varsigma_t z} \dot{e}^h_{t+z} dz + \int_0^\tau e^{\varsigma_t z} \varsigma_t e^h_{t+z} dz \\ &= \int_0^\tau e^{\varsigma_t z} \dot{e}^h_{t+z} dz + \varsigma_t \int_0^\tau e^{\varsigma_t z} e^h_{t+z} dz.\end{aligned} \tag{77}$$

The first integral on the right side of the latter equation can be integrated by parts:

$$\begin{aligned}\int_0^\tau e^{\varsigma_t z} \dot{e}^h_{t+z} dz &= e^{\varsigma_t z} e^h_{t+z} \Big|_{z=0}^{z=\tau} - \int_0^\tau \varsigma_t e^{\varsigma_t z} e^h_{t+z} dz \\ &= \left(e^{\varsigma_t \tau} e^h_{t+\tau} - e^{\varsigma_t 0} e^h_t\right) - \varsigma_t \int_0^\tau e^{\varsigma_t z} e^h_{t+z} dz \\ &= e^{\varsigma_t \tau} e^h_{t+\tau} - \varsigma_t \int_0^\tau e^{\varsigma_t z} e^h_{t+z} dz,\end{aligned}$$

where the last equality uses $e^h_t = e^h_t(0) = 0$. Substituting this result into equation (77) yields

$$\begin{aligned}\int_0^\tau \left(\dot{e}^h_{t+z} + \varsigma_t e^h_{t+z}\right) e^{\varsigma_t z} dz &= e^{\varsigma_t \tau} e^h_{t+\tau} - \varsigma_t \int_0^\tau e^{\varsigma_t z} e^h_{t+z} dz + \varsigma_t \int_0^\tau e^{\varsigma_t z} e^h_{t+z} dz \\ &= e^{\varsigma_t \tau} e^h_{t+\tau}.\end{aligned}$$

The cancellation yields a simpler expression than the one in (71). The result is an expression for next period's employment that does not account for the hires number reporting cumulative hires.

This can be seen by substituting the result into equation (76), which yields

$$\begin{aligned} e_{t+\tau} &= \left[c_0 + \int_0^\tau \left(\dot{e}^h_{t+z} + \varsigma_t e^h_{t+z}\right) e^{\varsigma_t z} dz\right] e^{-\varsigma_t \tau} \\ &= \left[c_0 + e^{\varsigma_t \tau} e^h_{t+\tau}\right] e^{-\varsigma_t \tau} \\ &= c_0 e^{-\varsigma_t \tau} + e^h_{t+\tau}. \end{aligned}$$

The determination of c_0 comes from evaluating the latter at $\tau = 0$:

$$e_t = c_0 + e_t^h (0) = c_0,$$

as there are no new hires at the beginning of the period and $e_t^h(0) = 0$. Therefore

$$e_{t+\tau} = e_t e^{-\varsigma_t \tau} + e^h_{t+\tau}.$$

Evaluating the latter at $\tau = 1$ yields

$$\begin{aligned} e_{t+1} &= e_t e^{-\varsigma_t} + e^h_{t+1} \\ &= e_t (1 - s_t) + e^h_{t+1}, \end{aligned} \tag{78}$$

which indicates that the level of employment in the following survey period equals the employed who do not separate from their jobs plus new hires. Comparing (78) with (73) and (72) makes this point. In addition, the comparison shows how Shimer (2012)'s adjustment for time aggregation, which is meant for use with short-term unemployment data from the CPS, differs from the adjustment for JOLTS data.

16.4 Details of the Estimation of the Matching Function

From the parameterization in (21), the implied job-finding probability, $M(u, v)/u$, is

$$\begin{aligned} f(\theta) &= \mu \frac{v}{(u^\gamma + v^\gamma)^{1/\gamma}} \times \frac{1/u}{1/(u^\gamma)^{1/\gamma}} = \mu \frac{\theta}{(1+\theta^\gamma)^{1/\gamma}} \\ \therefore \log(f(\theta)) &= \log(\mu) + \log(\theta) - \log\left((1+\theta^\gamma)^{1/\gamma}\right) \\ &= \log(\mu) + \log(\theta) - \frac{1}{\gamma}\log(1+\theta^\gamma). \end{aligned}$$

This derivation is used to produce the estimating equation in the main text in (29), which I repeat here for convenience:

$$\log f(\theta_t) = \alpha + \log \theta_t - \frac{1}{\gamma} \log\left(1 + \theta_t^\gamma\right) + \psi\, G(t) + \xi\, C(t) + \varepsilon_t. \tag{79}$$

A description of the statistical model is provided in the main text. I assume that ε_t is independently and identically distributed according to the distribution F.

The untransformed expectation is

$$E\left[f\left(\theta_{t_0}\right)\right] = \int \exp\left(\alpha + \log \theta_{t_0} - \frac{1}{\gamma}\log\left(1+\theta_{t_0}^\gamma\right) + \psi_{G(t_0)} + \xi_{C(t_0)} + \varepsilon_{t_0}\right) dF(\varepsilon).$$

The empirical cdf is $\widehat{F}(x) = T^{-1}\sum_{t=1}^{T}\mathbf{1}\{\hat{\varepsilon}_t \leq x\}$, where $\mathbf{1}\{A\}$ is an indicator function that takes the value 1 if the event A is true and 0 otherwise. Duan (1983) shows how the unknown cdf F can be replaced by its empirical estimate, $\widehat{F}_T$:

$$\begin{aligned}
\widehat{E}\left[f\left(\theta_{t_0}\right)\right] &= \int \exp\left(\hat{\alpha} + \log\theta_{t_0} - \frac{1}{\hat{\gamma}}\log\left(1+\theta_{t_0}^{\hat{\gamma}}\right) + \hat{\psi}_{G(t_0)} + \hat{\xi}_{C(t_0)} + \varepsilon\right) d\widehat{F}_T(\varepsilon) \\
&= \frac{1}{T}\sum_{t=1}^{T}\int \exp\left(\hat{\alpha} + \log\theta_{t_0} - \frac{1}{\hat{\gamma}}\log\left(1+\theta_{t_0}^{\hat{\gamma}}\right) + \hat{\psi}_{G(t_0)} + \hat{\xi}_{C(t_0)} + \hat{\varepsilon}_t\right) \\
&= \hat{\mu}\frac{\theta_{t_0}}{\left(1+\theta_{t_0}^{\hat{\gamma}}\right)^{1/\hat{\gamma}}}\frac{1}{T}\sum_{t=0}^{T}\exp\left(\hat{\varepsilon}_t\right). \qquad (80)
\end{aligned}$$

Duan (1983) establishes the result for a transformation that yields a linear model and refers to the result as the smearing estimate. I posit that the smearing estimate works for the nonlinear model in (79). Miller (1984) provides an example where F is lognormal. The smearing estimate in (80) is visually indistinguishable from the naive estimate $\hat{\mu}\theta_{t_0}\left(1+\theta_{t_0}^{\hat{\gamma}}\right)^{-1/\hat{\gamma}}$.

Figure 9 depicts the corrected probability of finding a job between December 2000 and May 2023 in blue. This corrected series is labeled Data and is the same as the corrected series in figure 6. It is used to construct the left side of (79). The broken series is the prediction based on the estimation of the statistical model in (79) and conditional on dates that shift matching efficiency. The predicted level of finding a job uses the estimator in (80).

Figures 10 and 11 show tightness in the labor market and probabilities of finding a job. The figures compare the corrected data versus predictions based on the estimated statistical model in (79).

16.5 Elasticities of Matching and Bounds

Figure 12 shows the elasticity of matching with respect to unemployment for values of θ observed in the US economy after December 2000. The functional form is based on the matching function in (21). To compute the elasticities I take the value of γ to be 0.103, which comes from the estimated statistical model in (29).

A remarkable part of figure 12 is how close the elasticities are to 0.5. This value would be the elasticity of matching with respect to unemployment if the matching function were Cobb–Douglas and the exponent was 0.5.

Figures 13 and 14 show upper bounds for Υ, which are given in (20). In figure 13 the series in bright blue shows the bound computed using $\gamma = 1.27$, which is a value found in the literature. The series in dark blue shows the bound computed using $\gamma = 0.103$. The series from the literature suggests terms in Υ matter more for dynamics in the labor market in periods characterized by low tightness.

Figure 14 removes the series in bright blue from figure 13. The removal facilitates inspection.

Figure 15 shows the same idea in a different way. The upper bound for Υ is computed for the economies indexed by costs of job creation in table 1. These bounds are much different than the bounds in 14.

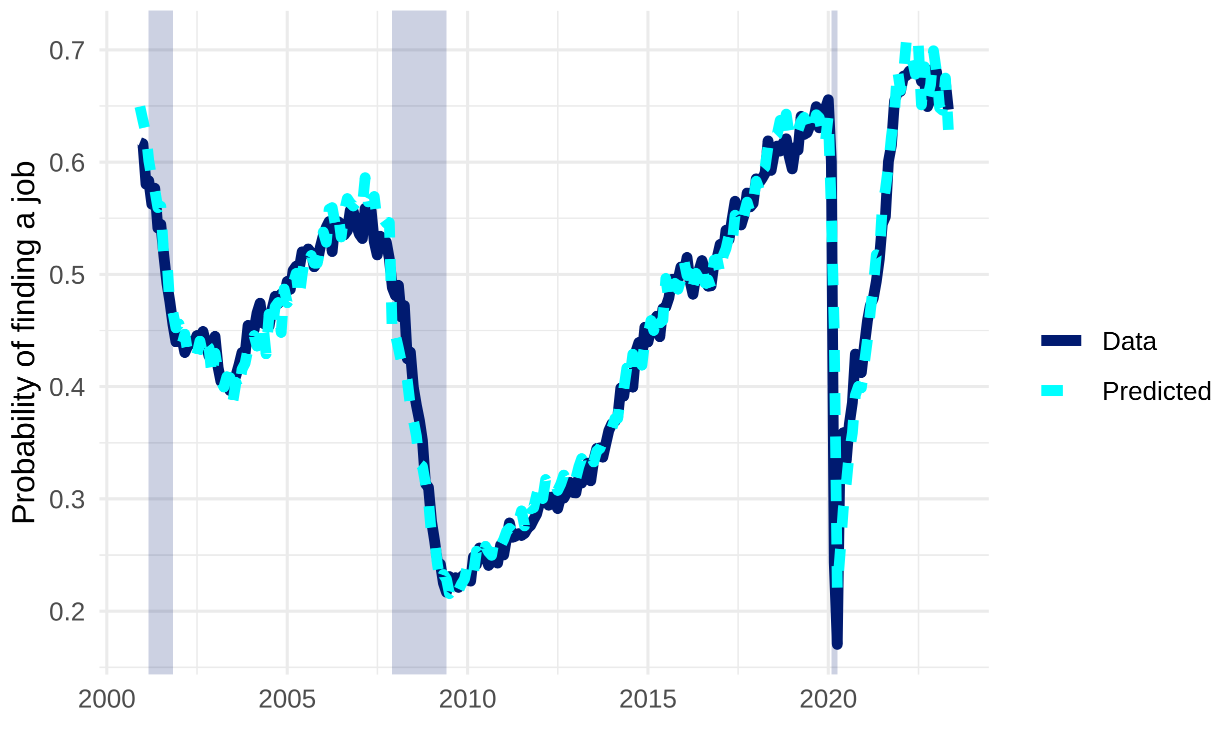

Figure 9: Comparison of the corrected probability of finding a job and the estimated probability of finding a job, December 2000 through May 2023. Shaded areas indicate US recessions.

Notes: Predictions are based on the estimated statistical model found in (79). The predicted level (as opposed to the log) uses the smear estimator in (80). Predictions are conditional on shifters of matching efficiency.

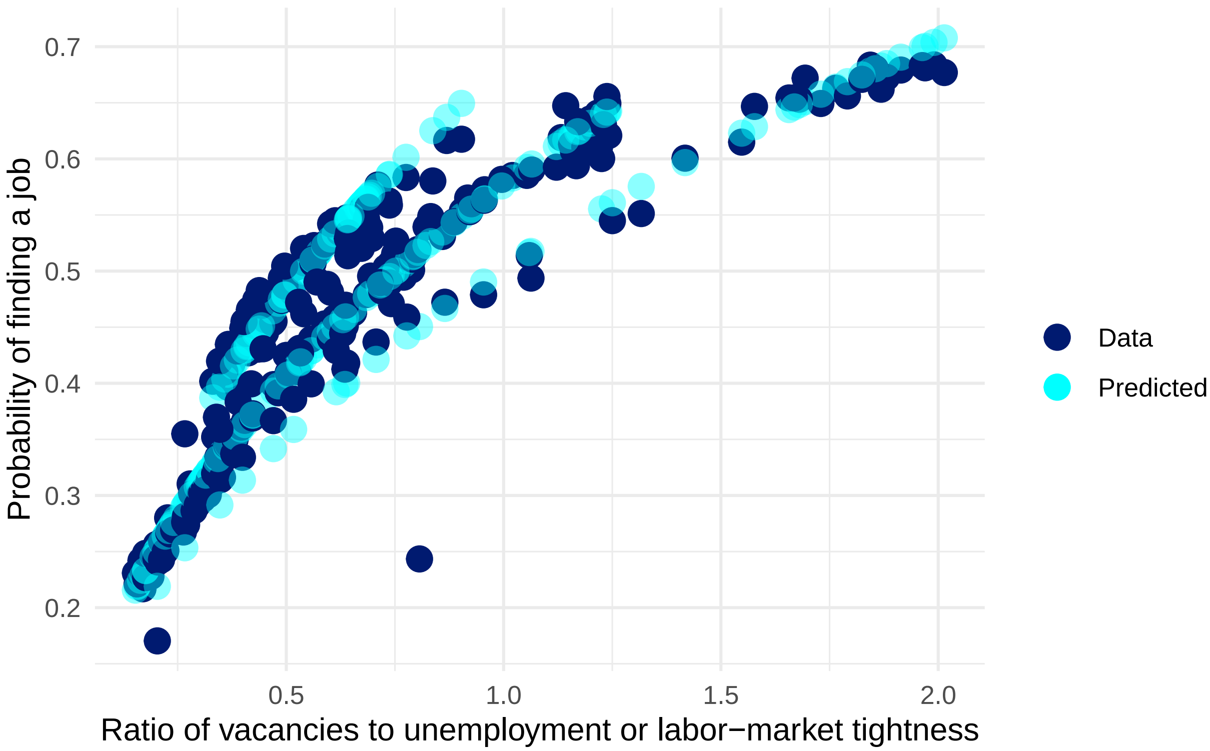

Figure 10: Tightness, θ, versus the monthly probability of finding a job.

Notes: The ordered pairs labeled Data refer to probabilities corrected for worker transitions and how the JOLTS program records hires. The corrected monthly probabilities of finding a job are reported in figure 6. The ordered pairs labeled Predicted refer to predictions based on the estimated statistical model found in (79). The predicted level (as opposed to the log) uses the smear estimator in (80). Predictions are conditional on shifters of matching efficiency.

Figure 11: Tightness, θ, versus the log of the monthly job-finding probability.

Notes: The ordered pairs labeled Data refer to probabilities corrected for worker transitions and how the JOLTS program records hires. The corrected monthly probabilities of finding a job are reported in figure 6. The ordered pairs labeled Predicted refer to predictions based on the estimated statistical model found in (79). Predictions are conditional on shifters of matching efficiency.

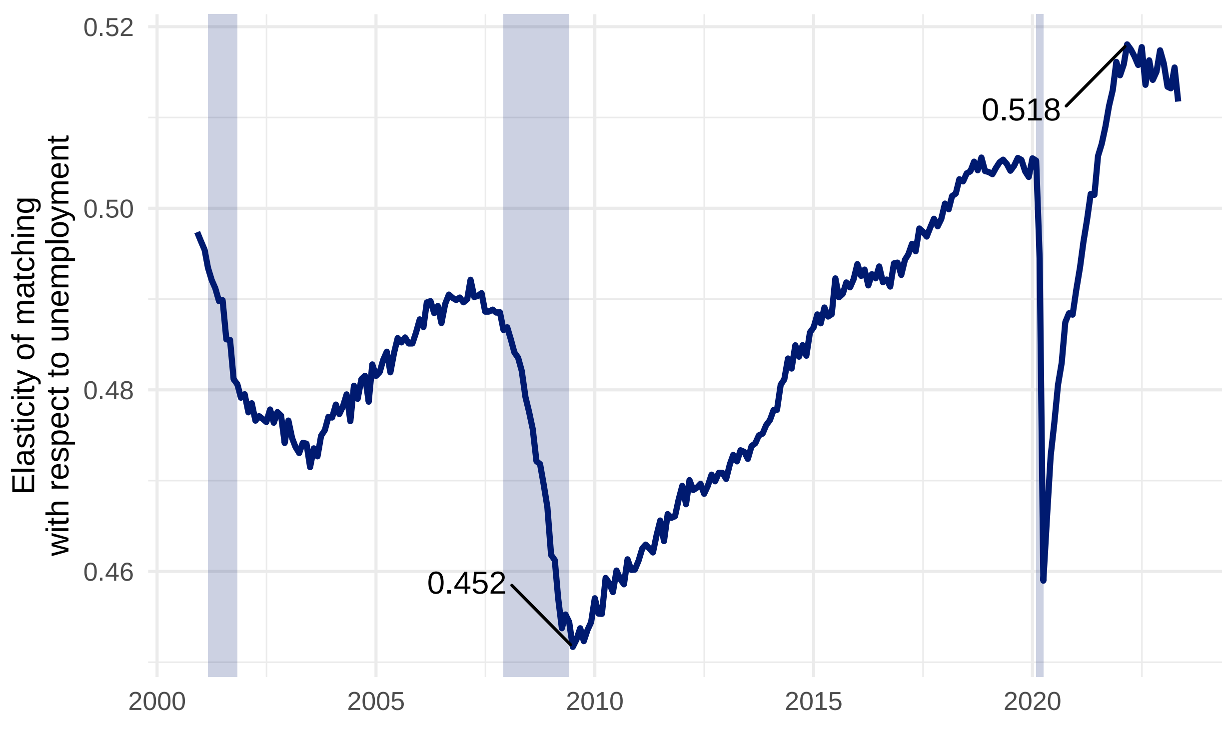

Figure 12: Elasticity of matching with respect to u, December 2000 through May 2023.

Notes: The elasticity of matching with respect to unemployment is computed using the matching function in (21) and equals $\theta^{\gamma}/(1+\theta^{\gamma})$. The elasticity is computed for values of θ observed in the US economy after December 2000. The value of γ is 0.103, the estimate from the statistical model in (79).

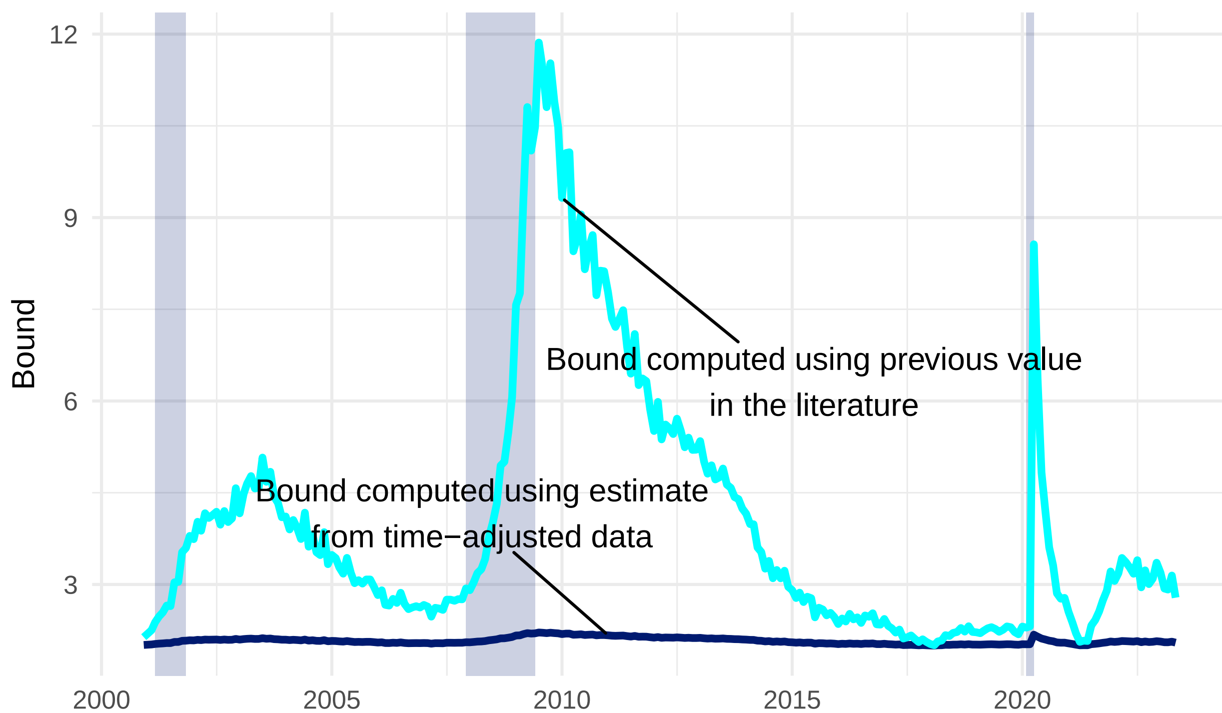

Figure 13: Implied upper bounds for Υ based on the estimate of γ compared to a value found in the literature, December 2000 through May 2023.

Notes: The upper bounds are computed using (20). The elasticity of matching with respect to unemployment, $\theta^{\gamma}/(1+\theta^{\gamma})$, is based on the matching function in (21) and uses values of θ observed in the US economy after December 2000.

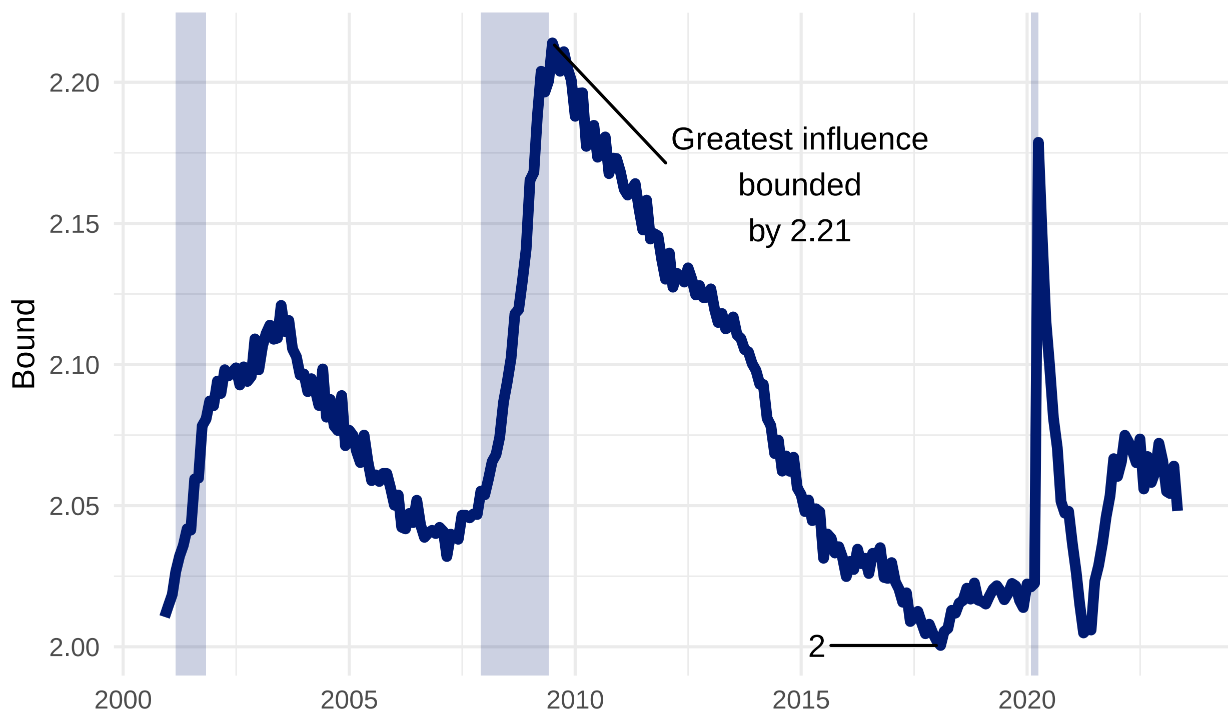

Figure 14: Upper bound for Υ computed using $\gamma = 0.103$ for values of tightness observed in the data after December 2000.

Notes: The choice of γ comes from the estimated statistical model in (29). The bounds are computed using (20).

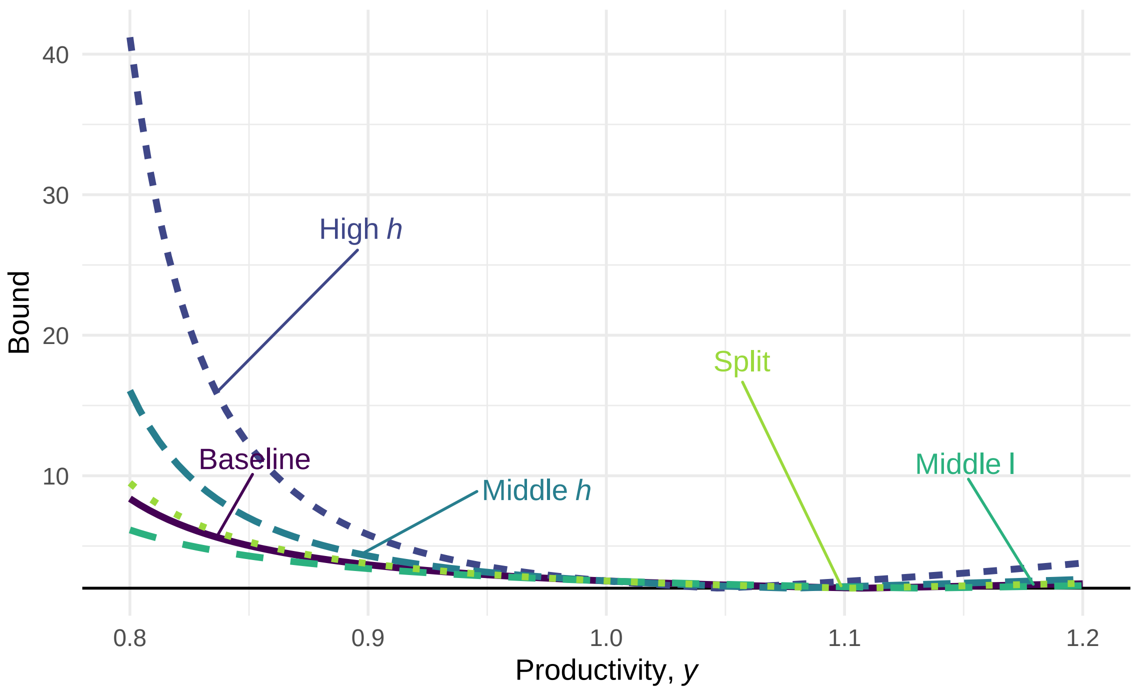

Figure 15: Upper bound for Υ computed using $\gamma = 1.27$ in economies indexed by job-creation costs. The upper bound is computed using (20). Market tightness varies as productivity is perturbed around $y = 1$.