Principle of Communications

Amplitude Modulation

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Outline

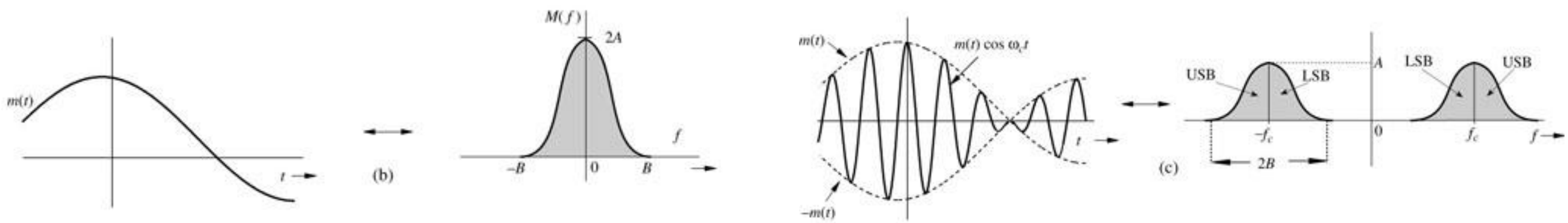
- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
  - DSB-SC modulation and demodulation
  - DSB-SC modulators
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Double-Sideband Amplitude Modulation

- Amplitude of the carrier is varied in proportion to the baseband (message) signal $m(t)$.
- Message Signal: $m(t)$
- Carrier Signal: $\cos(\omega_c t)$
- Modulated Signal: $m(t)\cos(W_c t) = \frac{1}{2}m(t)e^{-jW_c t} + \frac{1}{2}m(t)e^{jW_c t}$
- Spectrum: $M(\omega) \rightarrow \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$



Double Sideband - Suppressed Carrier (DSB-SC)

DSB-SC: Modulation & Demodulation

- Modulation:

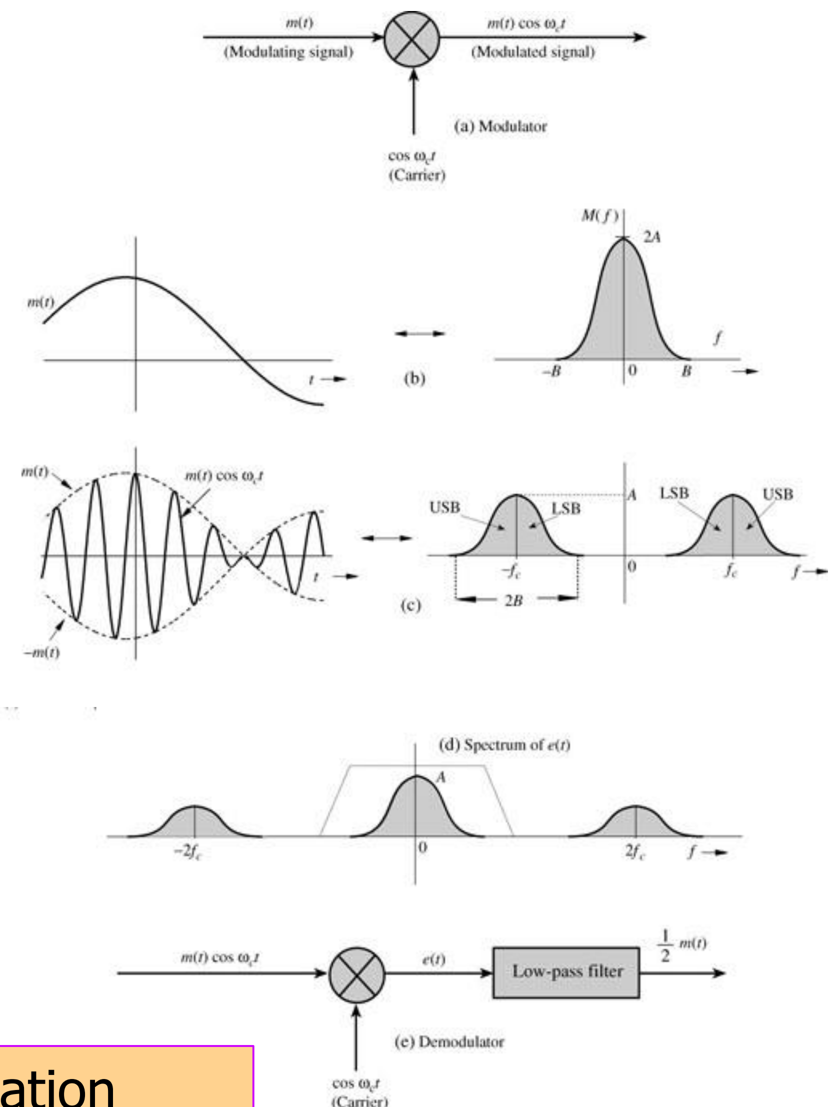
  Linear? Time invariant?

$$[m_1(t)+m_2(t)]\times\cos(\omega_c t)$$
$$=m_1(t)\cos(\omega_c t)+m_2(t)\cos(\omega_c t)$$

- Demodulation:

$$m(t)\cos(\omega_c t)\times\cos(\omega_c t)=m(t)\cos^2(\omega_c t)$$
$$=\frac{A}{2}m(t)[\cos(2\omega_c t)+1]$$
$$=\frac{1}{2}m(t)\cos(2\omega_c t)+\frac{1}{2}m(t)$$

Synchronous/Coherent demodulation



DSB-SC Coherent Demodulation

$$m(t)\cos(\omega_c t)\times\cos(\omega_c t)=m(t)\cos^2(\omega_c t)$$

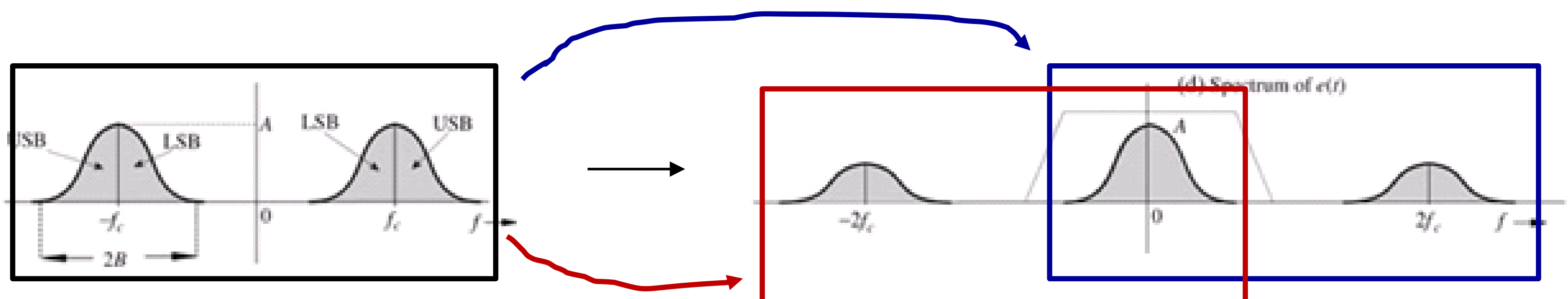
$$=\frac{A}{2}m(t)[\cos(2\omega_c t)+1]$$

$$=\frac{1}{2}m(t)\cos(2\omega_c t)+\frac{1}{2}m(t)$$

$$=\frac{1}{4}m(t)e^{j2\omega_c t}+\frac{1}{4}m(t)e^{-j2\omega_c t}+\frac{1}{2}m(t)$$

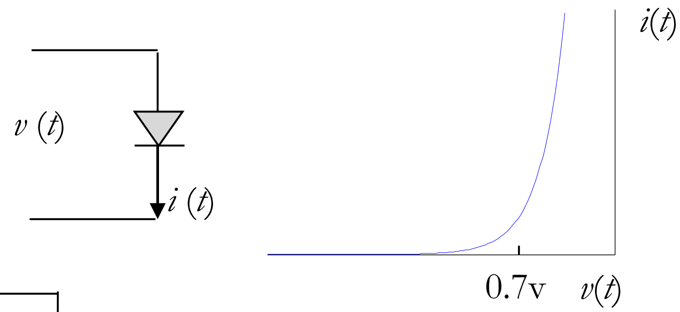
Spectrum $\quad \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]\rightarrow\frac{1}{4}[M(\omega+2\omega_c)+M(\omega-2\omega_c)]+\frac{1}{2}M(\omega)$

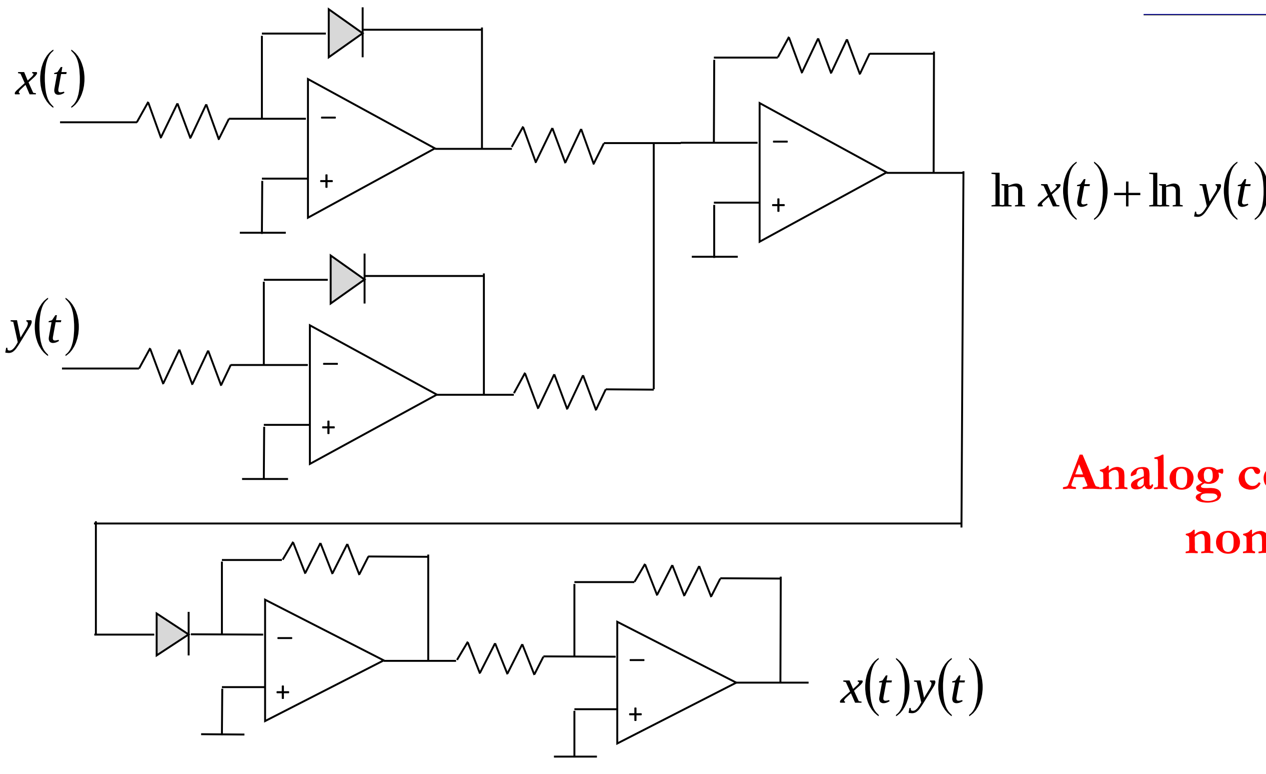
$$\frac{1}{4}[M(\omega+2\omega_c)+M(\omega-2\omega_c)]+\frac{1}{2}M(\omega)\rightarrow\frac{1}{2}M(\omega)$$



Implementation of A Multiplier

$$x(t)y(t) = \exp\big(\ln(x(t)) + \ln(y(t))\big)$$





Analog computing is nontrivial!

DSB-SC: Nonlinear Modulator

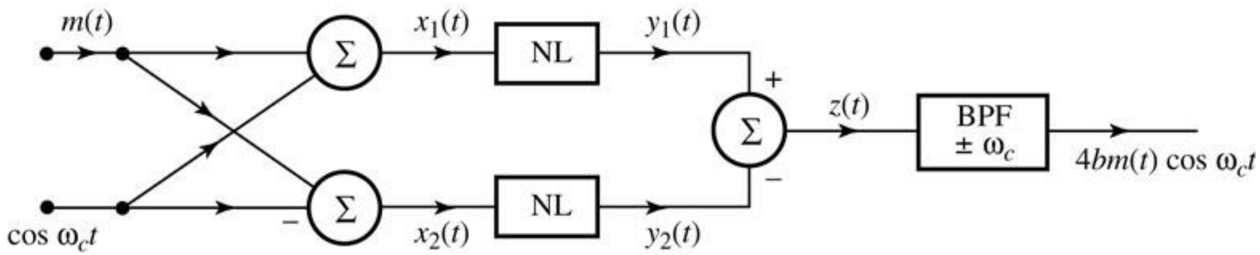
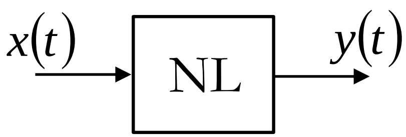


Figure 4.3 Nonlinear DSB-SC modulator.



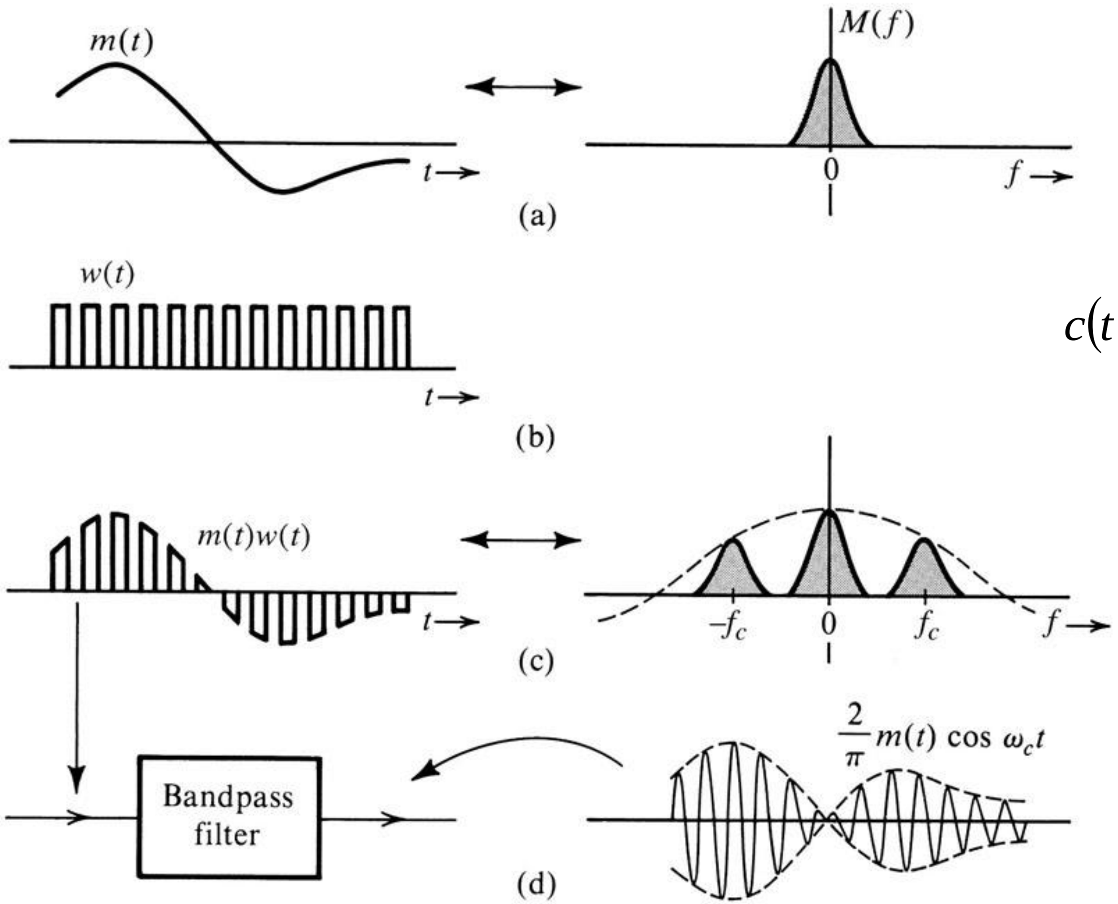
$$y(t) \approx ax(t) + bx^2(t)$$

$$x_1(t) = \cos(\omega_c t) + m(t)$$
$$x_2(t) = \cos(\omega_c t) - m(t)$$

$$z(t) = y_1(t) - y_2(t) = 2am(t) + 4bm(t)\cos\omega_c t$$

can be extracted
using a bandpass filter

DSB-SC: Switching Modulator



Multiply $m(t)$ with a periodic signal with fundamental frequency ω_c

$$c(t) \approx a\cos(\omega_c t) + b\cos(2\omega_c t) + \cdots$$

Have frequency components around ω_c, $2\omega_c$, etc. The components around ω_c can be singled out using a bandpass filter.

DSB-SC: Diode Bridge Modulator

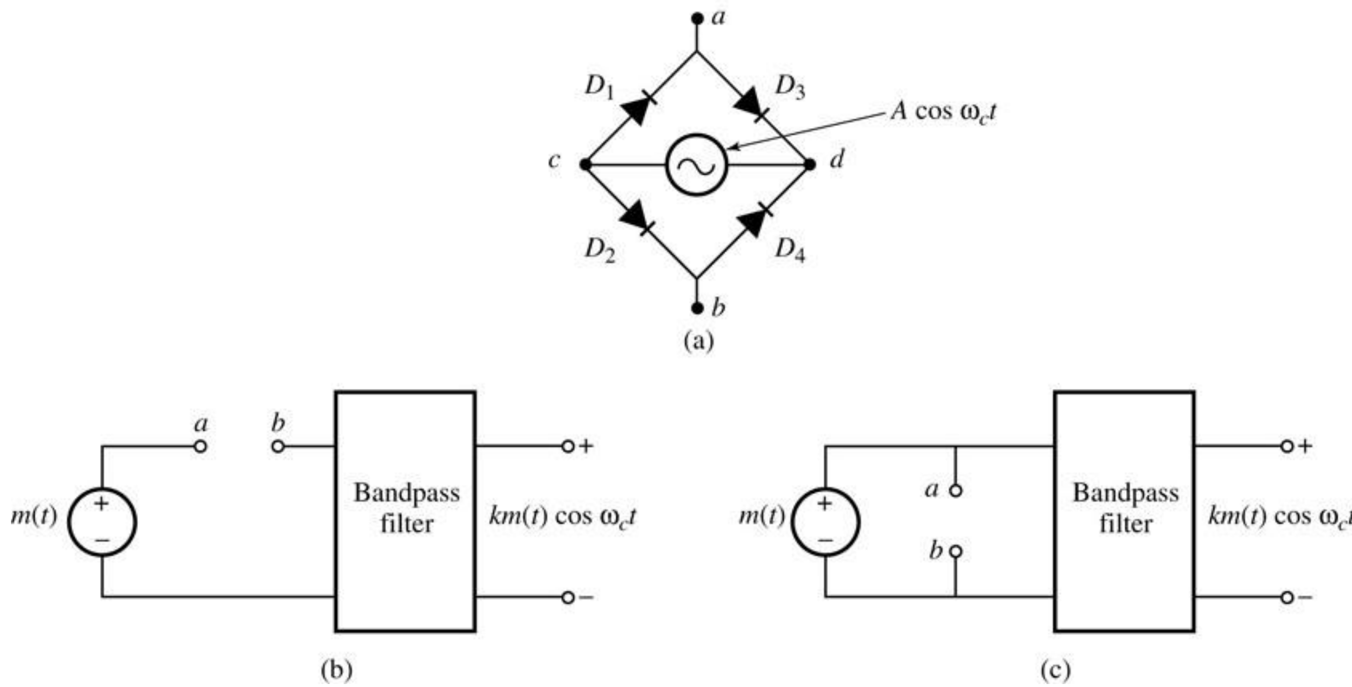
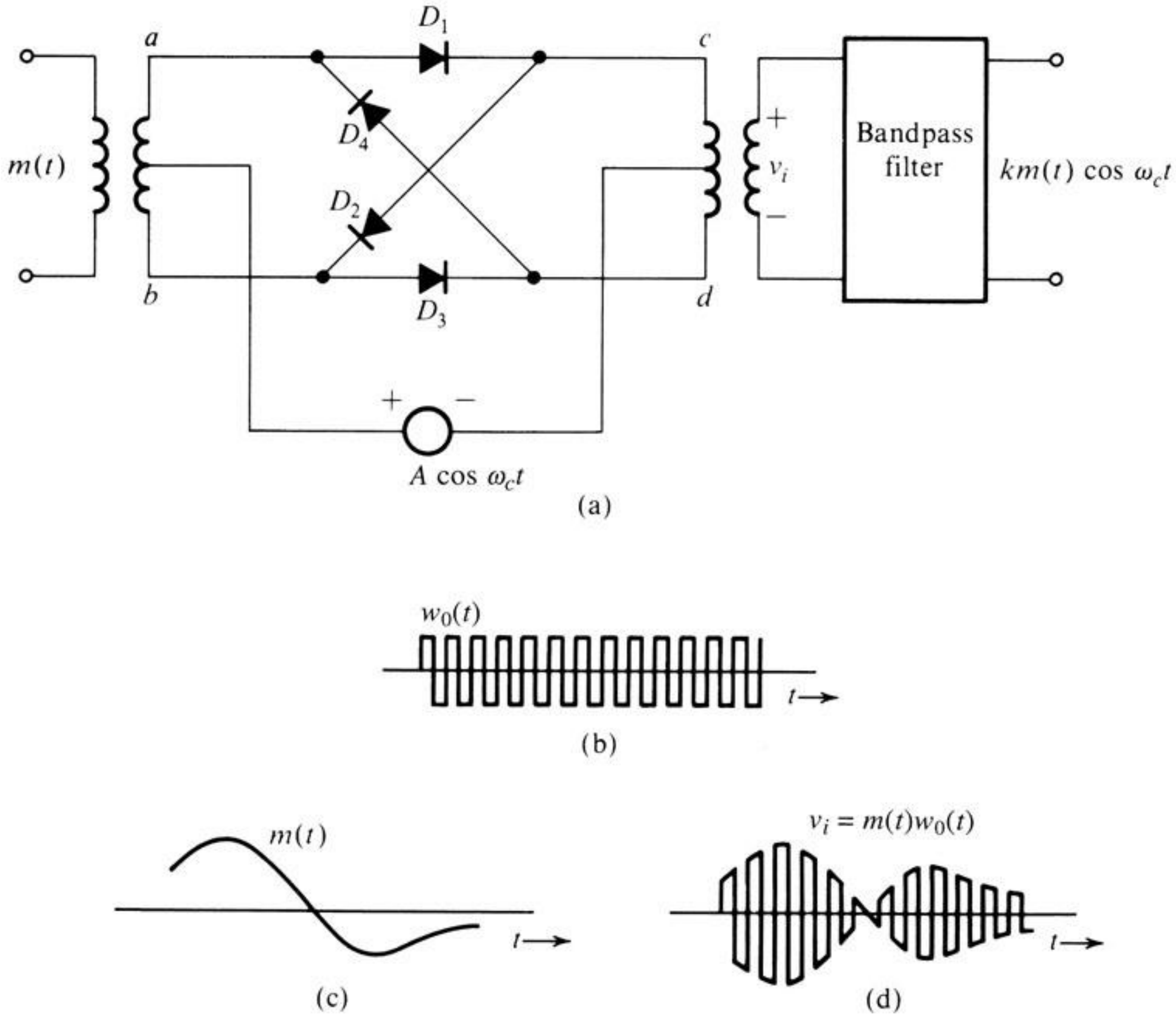


Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

Advantage: Easy to implement (than multiplying with $\cos \omega_c t$)

Example: use crystal oscillator to generate a periodic signal, amplify it to get periodic rectangular waveform, and then use it to control electronic switches.

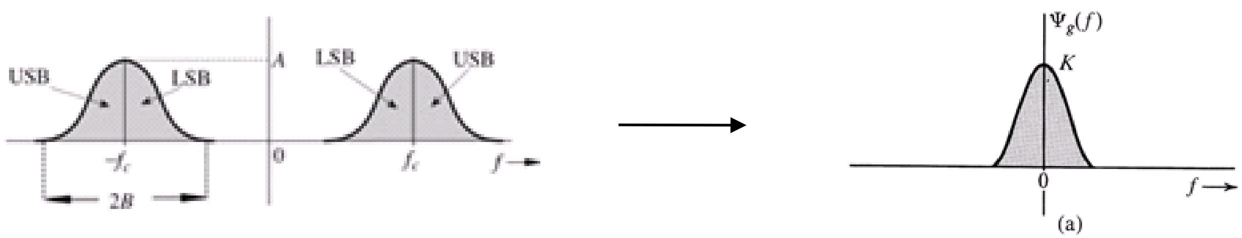
DSB-SC: Ring Modulator



DSB-SC: Switching Demodulator

Multiply $s(t)$ with a periodic signal with fundamental frequency ω_c , then pass the signal through a lowpass filter

$$s(t)\left[a\cos(\omega_c t)+b\cos(2\omega_c t)+\cdots\right]$$



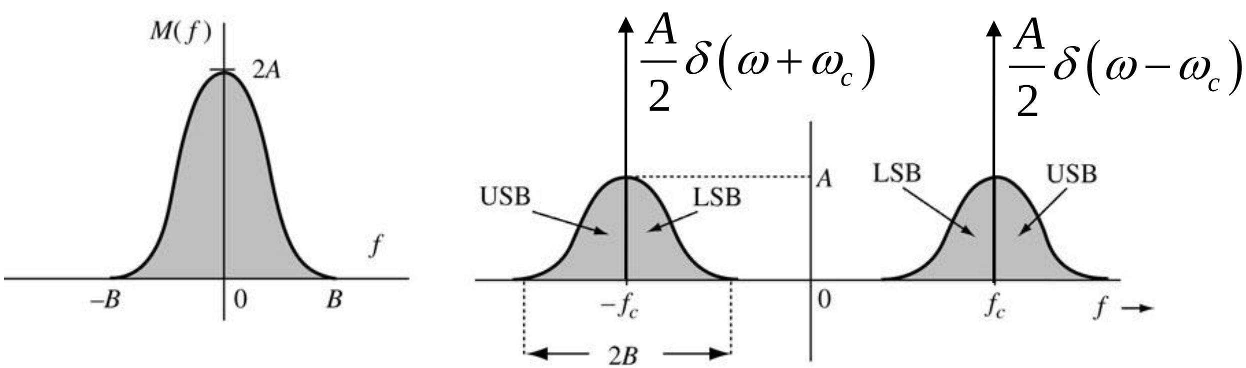
Only multiplying with $\cos(\omega_c t)$ generates low frequency components

Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
  - AM modulation and envelope detector
  - Power efficiency
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Amplitude Modulation (AM)

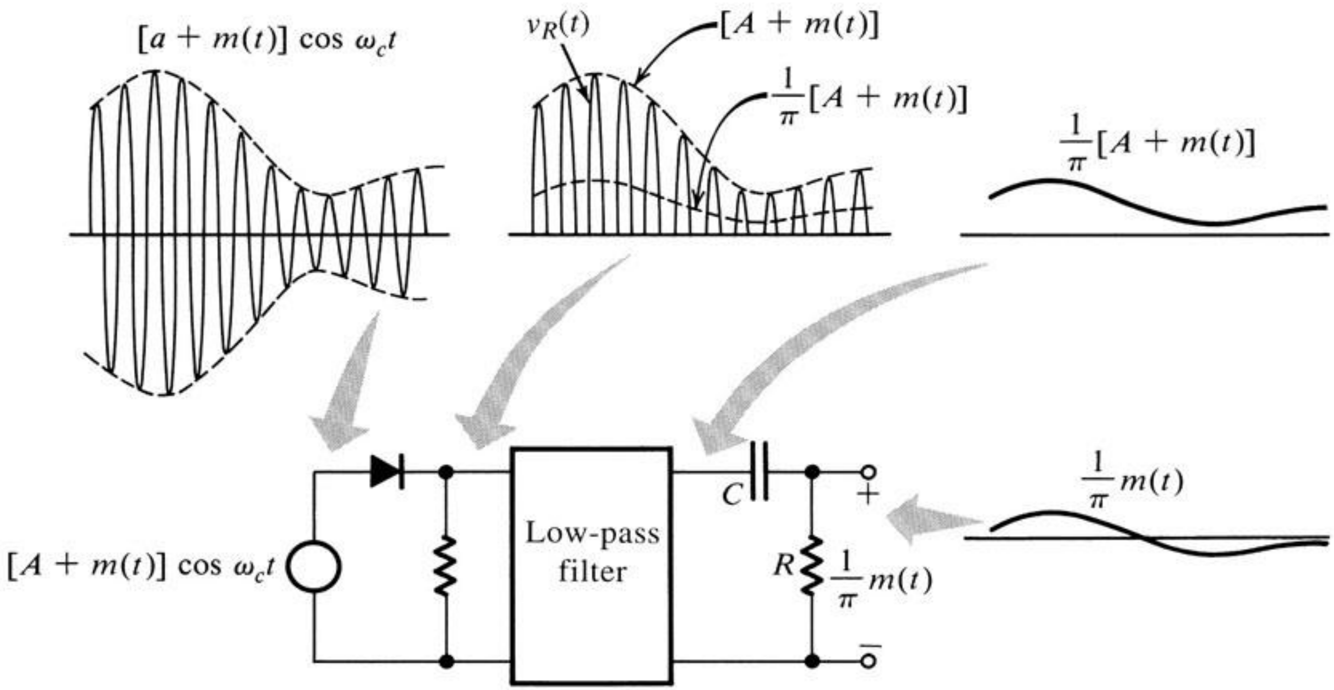
- Motivation: Coherent demodulation of DSB-SC requires the receiver to generate a carrier. To simplify receiver …
- Amplitude of the carrier is varied in proportion to the baseband (message) signal *m*(*t*).
- Message Signal: $m(t)$
- Carrier Signal: $\cos(\omega_c t)$
- Modulated Signal: $(A+m(t))\cos(\omega_c t)$
- Spectrum: $M(\omega) \rightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]+\frac{A}{2}[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)]$



(a) (b)

- Drawback: Extra transmit power

AM Receiver



envelope detector

Key advantages: Only need one diode (can be built using rocks). RC filter does not need to have precise cut-off frequency.

AM: Envelope Detector

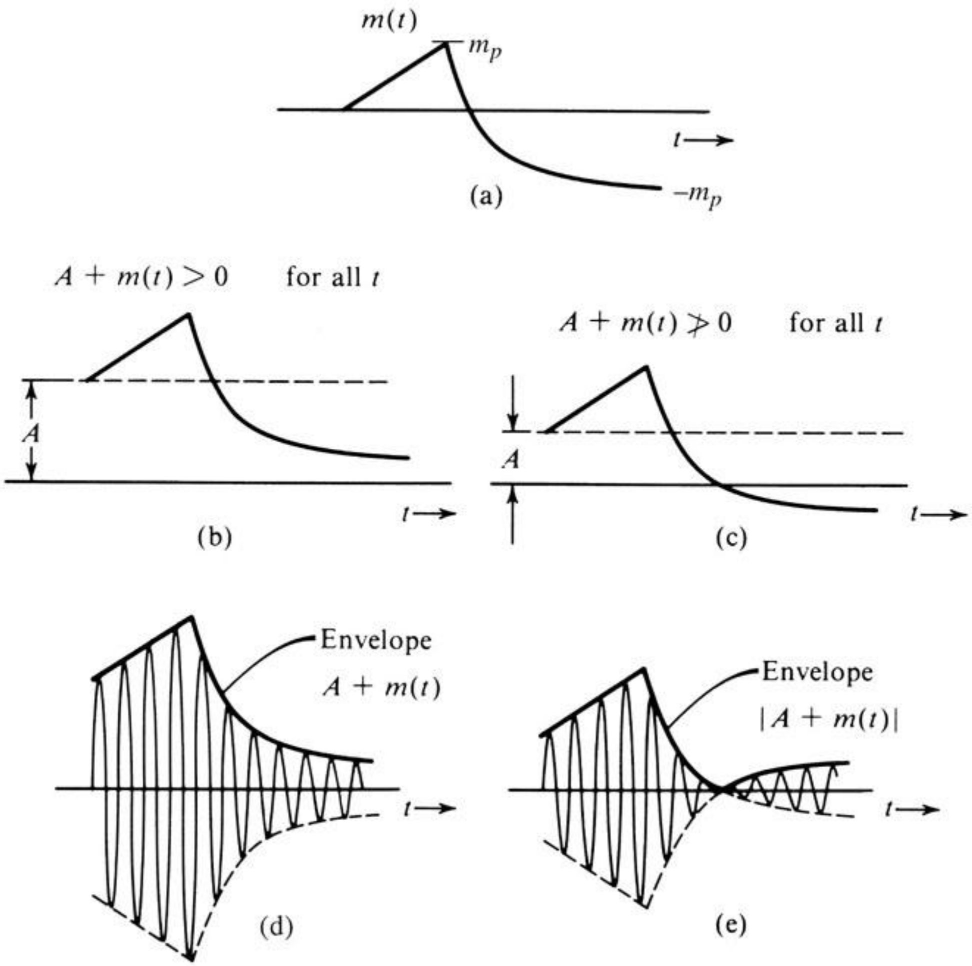
To ensure correct envelope detection, must have $A + m(t) > 0$ for all t

Modulation index μ

$$\mu = \frac{m_p}{A}$$

Envelope detection requires $0 \le \mu \le 1$

Note that receiver does not need to know ω_c



Side Band and Carrier Power

- Advantage of envelope detection is achieved under the expense of extra energy

$$\phi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sideband}}$$

$$P_c = \frac{A^2}{2} = \frac{1}{T_c}\int_{-T_c/2}^{T_c/2} A^2\cos^2\omega_c t dt \qquad P_s = \frac{1}{2}\overline{m^2(t)} = \frac{1}{2}P_m$$

- Power efficiency $\eta = \dfrac{P_s}{P_c + P_s} = \dfrac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$

- Example: Calculate the maximum possible η for single tone AM modulation.

$$m(t) = \cos\omega_m t \qquad \phi_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t \qquad A + m(t) \geq 0 \Rightarrow A \geq 1$$

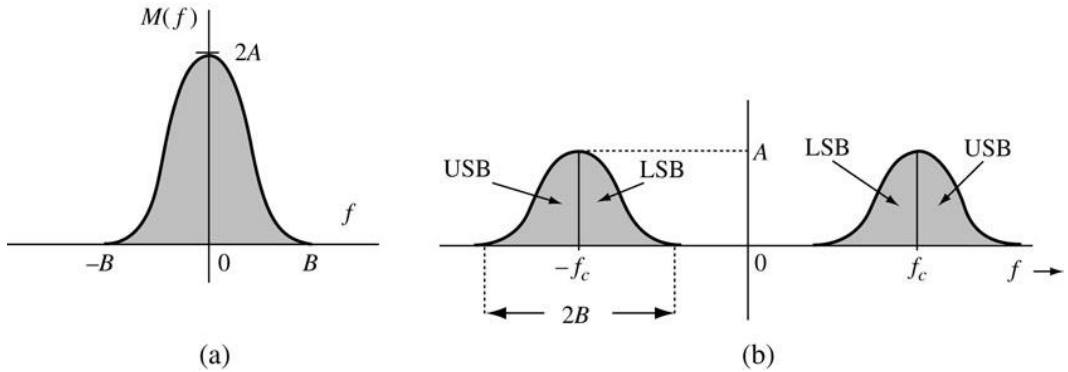
$$\text{Hence}\ \eta = \frac{P_s}{P_c + P_s} = \frac{\frac{1}{2}}{A^2 + \frac{1}{2}} \leq \frac{1}{3} = 33\%$$

Efficiency of a practical AM system is often much worse.

Further thoughts on DSB-SC?

- Envelope detection possible?
  - At receiver, first add carrier signal and then use envelope detector

- Is DSB-SC efficient?

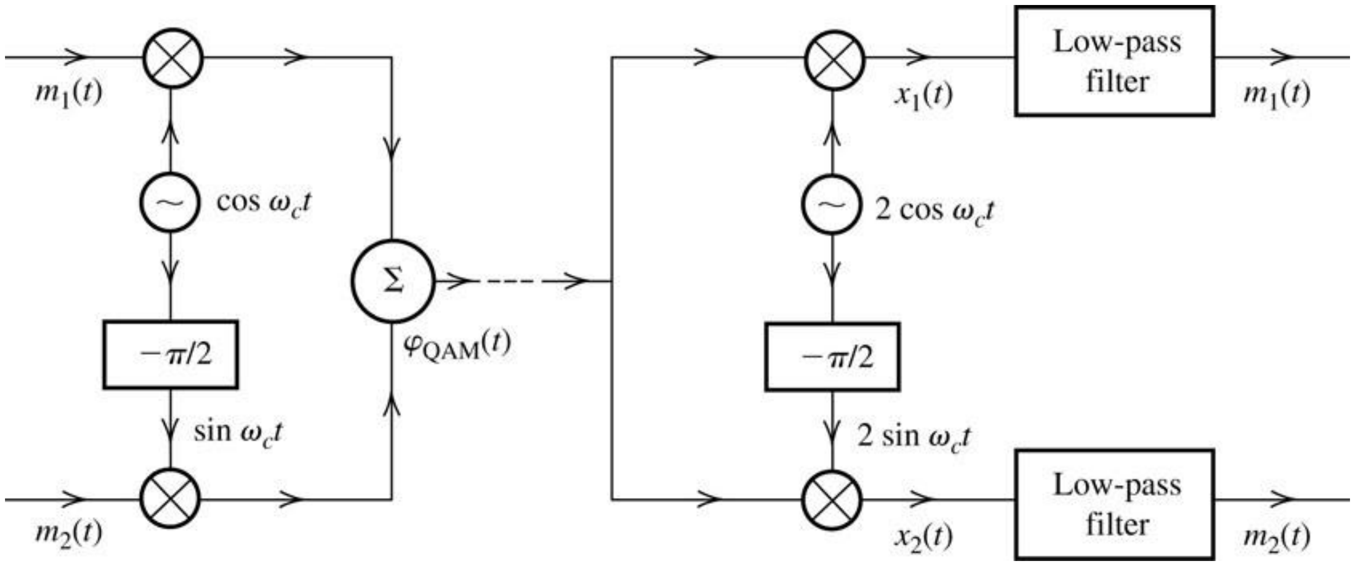


  - If *m*(*t*) is real-valued, we have $M(\omega)=M^*(-\omega)$
    - ➔ Only half of the frequency spectrum carries information
  - Modulated signal $\Phi(t)$ is real-valued, we also have $\Phi(\omega)=\Phi^*(-\omega)$
    - ➔ After modulation, only ¼ of the frequency spectrum carries information
  - Redundancy may not be completely avoidable. But, FOUR copies?

Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
  - QAM modulation and demodulation
  - Complex message signals and their (de-)modulation
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization
- NSTC analog TV broadcasting

Quadrature Amplitude Modulation (QAM)



Two baseband signals $m_1(t), m_2(t)$

Modulated signal $\Phi(t) = m_1(t)\cos \omega_c t + m_2(t)\sin \omega_c t$

$$\Phi(\omega) = \frac{1}{2\pi} M_1(\omega) * \pi(\delta(\omega - \omega_c) + \delta(\omega + \omega_c)) + \frac{1}{2\pi} M_2(\omega) * j\pi(\delta(\omega + \omega_c) - \delta(\omega - \omega_c))$$

$$= \frac{1}{2}(M_1(\omega - \omega_c) + M_1(\omega + \omega_c)) + \frac{j}{2}(M_2(\omega + \omega_c) - M_2(\omega - \omega_c))$$

QAM Demodulation

Channel 1:

$$\Phi(t)2\cos\omega_c t = 2m_1(t)\cos^2\omega_c t + 2m_2(t)\cos\omega_c t\sin\omega_c t$$
$$= m_1(t) + \underbrace{m_1(t)\cos 2\omega_c t + m_2(t)\sin 2\omega_c t}$$

high frequency components,
can be removed by lowpass filter

Channel 2:

$$\Phi(t)2\sin\omega_c t = 2m_1(t)\sin\omega_c t\cos\omega_c t + 2m_2(t)\sin^2\omega_c t$$
$$= \underbrace{m_1(t)\sin 2\omega_c t - m_2(t)\cos 2\omega_c t} + m_2(t)$$

high frequency components,
can be removed by lowpass filter

A Complex View: Modulation

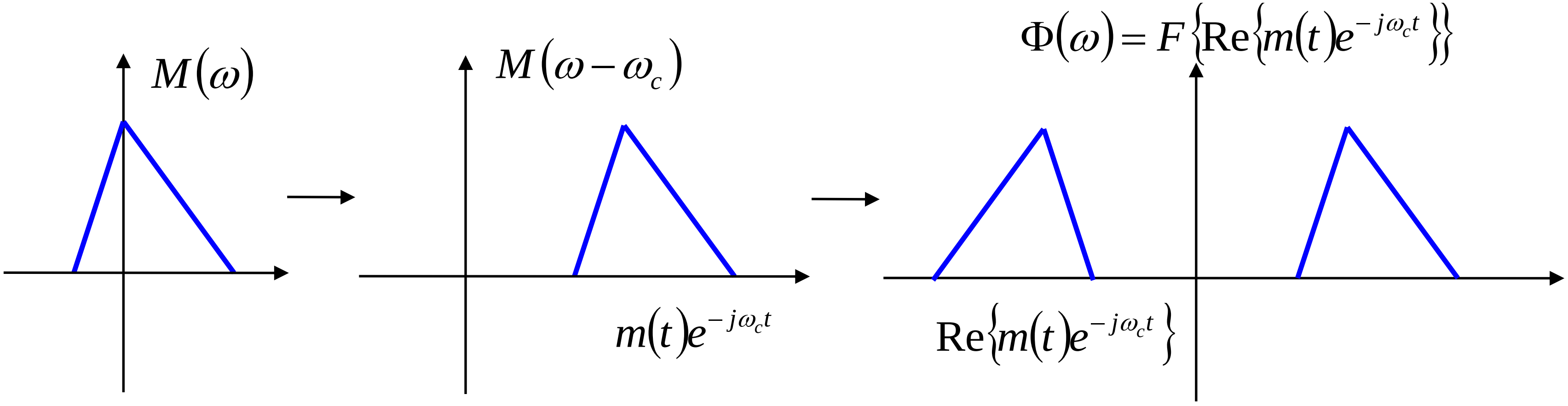
Message Signal $m(t) = m_1(t) + jm_2(t)$

- $m(t)$ is complex → $M(\omega) \neq M^*(-\omega)$
- Both positive and negative spectra carry information

Carrier Signal $\cos(\omega_c t) = \mathrm{Re}\{e^{-j\omega_c t}\}$

Modulated Signal $\mathrm{Re}\{m(t)e^{-j\omega_c t}\} = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$

Spectrum $\Phi(\omega) = F\{\mathrm{Re}\{m(t)e^{-j\omega_c t}\}\}$



A Complex View: Demodulation

Modulated Signal $m(t)\cos(\omega_c t) = \mathrm{Re}\{m(t)e^{-j\omega_c t}\}$

Spectrum $\Phi(\omega) = F\{\mathrm{Re}\{m(t)e^{-j\omega_c t}\}\}$

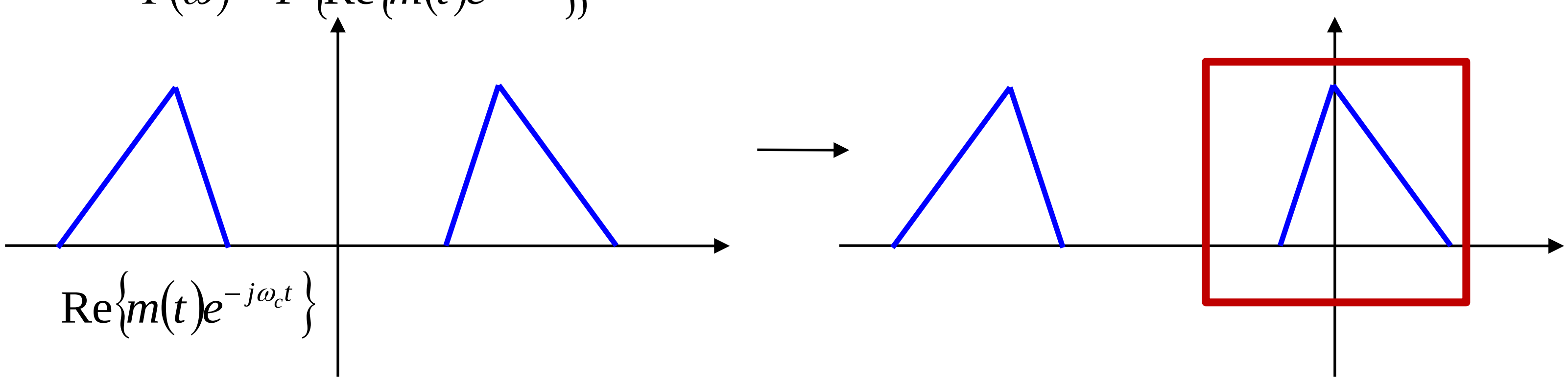
Demodulation $\Phi(t)e^{j\omega_c t}$ and then go through a lowpass filter

$$\Phi(\omega) = F\{\mathrm{Re}\{m(t)e^{-j\omega_c t}\}\} \rightarrow \text{shift to the left by } \omega_c$$

$$\text{after lowpass filtering} \rightarrow \frac{1}{2}M(\omega)$$

$\Phi(\omega) = F\{\mathrm{Re}\{m(t)e^{-j\omega_c t}\}\}$

low-pass filtering



Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
  - Hilbert transform
  - SSB modulators
  - SSB demodulators
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Single Sideband (SSB) Modulation

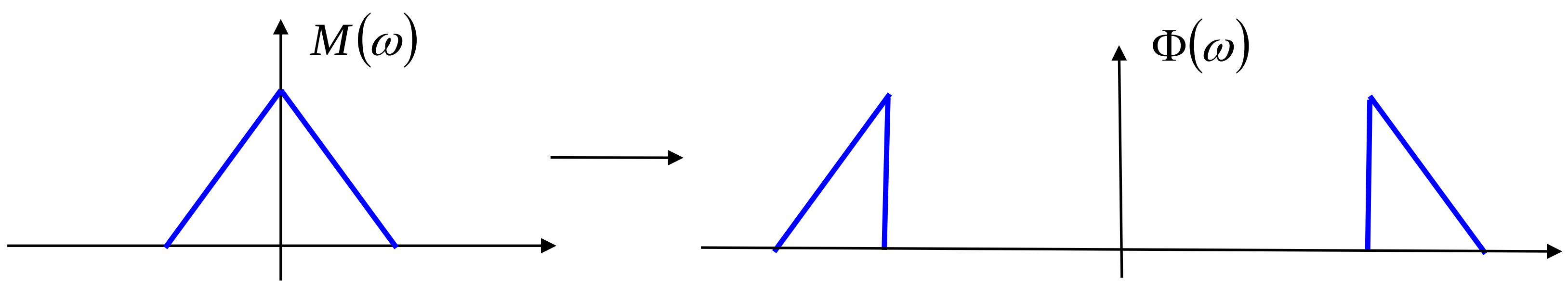
Message Signal: $m(t)$ (real-valued)

Carrier Signal: $\cos(\omega_c t)$

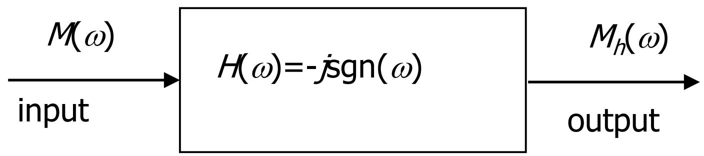
Modulation:

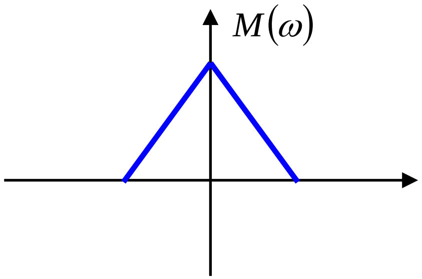
Step 1 $m(t)\cos(\omega_c t)$ Spectrum $\Phi(\omega) \to \frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right]$

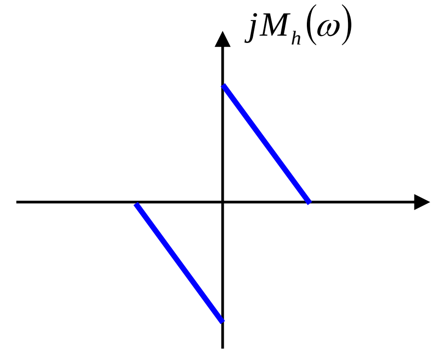
Step 2 Use bandpass filter to remove either the LSB or the USB

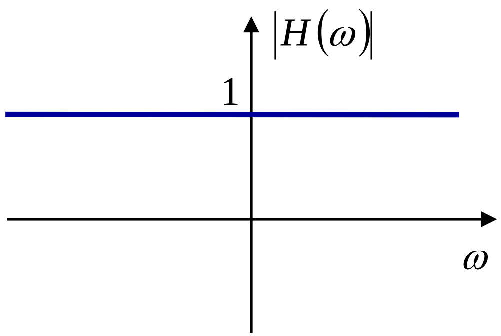


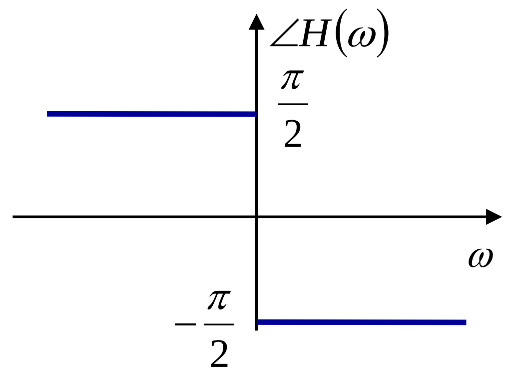
Hilbert Transform











$$h(t)=F^{-1}(-j\,\mathrm{sgn}(\omega))=\frac{1}{\pi t}$$

Non-causal, can't be implemented exactly. But can be approximated

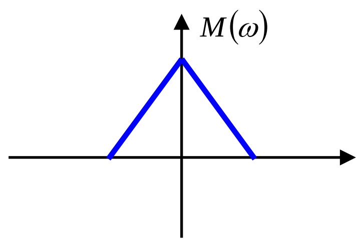
SSB Modulation via Hilbert Transform

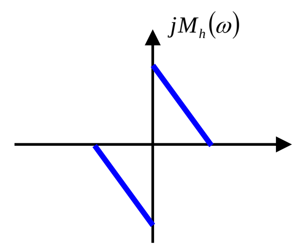
$$\Phi(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

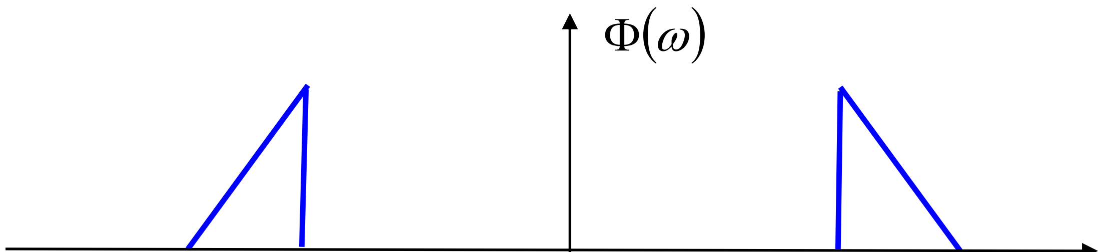
$m_h(t)$ is the Hilbert transform of $m(t)$

$$\Phi(\omega) = \frac{1}{2}\left[M(\omega+\omega_c) + M(\omega-\omega_c)\right] + \frac{j}{2}\left[M_h(\omega-\omega_c) - M_h(\omega+\omega_c)\right]$$

$$= \frac{1}{2}\left[M(\omega+\omega_c) - jM_h(\omega+\omega_c)\right] + \frac{1}{2}\left[M(\omega-\omega_c) + jM_h(\omega-\omega_c)\right]$$







$$\frac{1}{2}\left[M(\omega+\omega_c) - jM_h(\omega+\omega_c)\right] \quad \Big| \quad \frac{1}{2}\left[M(\omega-\omega_c) + jM_h(\omega-\omega_c)\right]$$

SSB Demodulation

$$\Phi(t) = m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)$$

$$\Phi(t)\cos(\omega_c t) = m(t)\cos^2(\omega_c t) + m_h(t)\sin(\omega_c t)\cos(\omega_c t)$$
$$= \frac{1}{2}m(t) + \underbrace{\frac{1}{2}m(t)\cos(2\omega_c t) + \frac{1}{2}m_h(t)\sin(2\omega_c t)}_{\text{remove by lowpass filter}}$$

Q: SSB signal differs from DSB-SC signal, but why the same demodulator?

A: $\cos(\omega_c t)\sin(\omega_c t) = \sin(2\omega_c t)$ will be blocked by the lowpass filter.

Phase Shift SSB Modulator

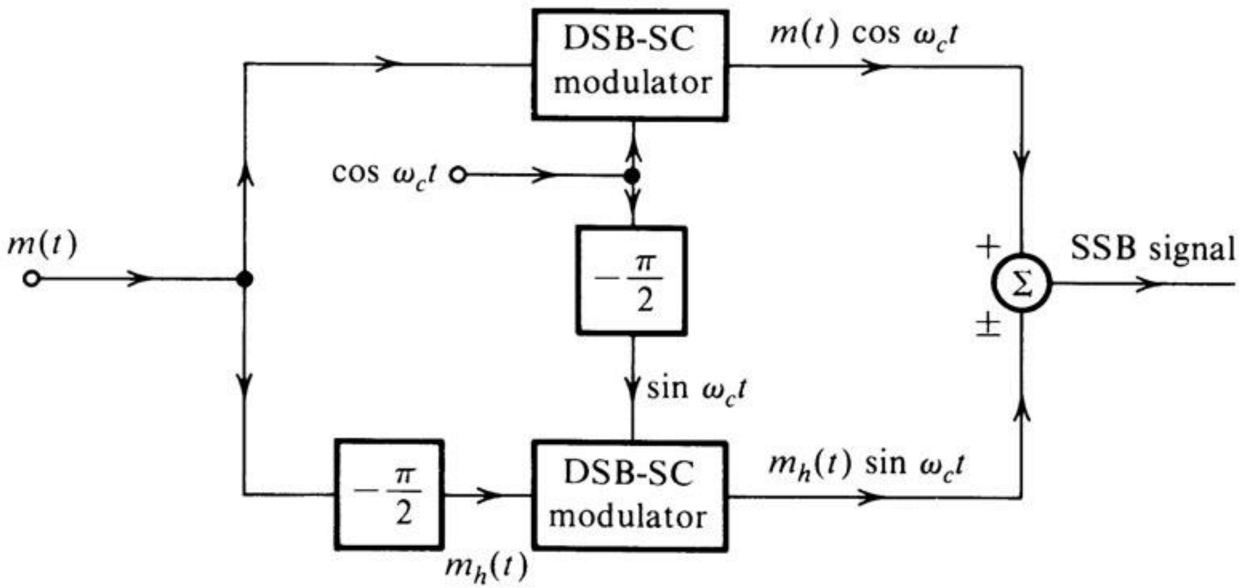
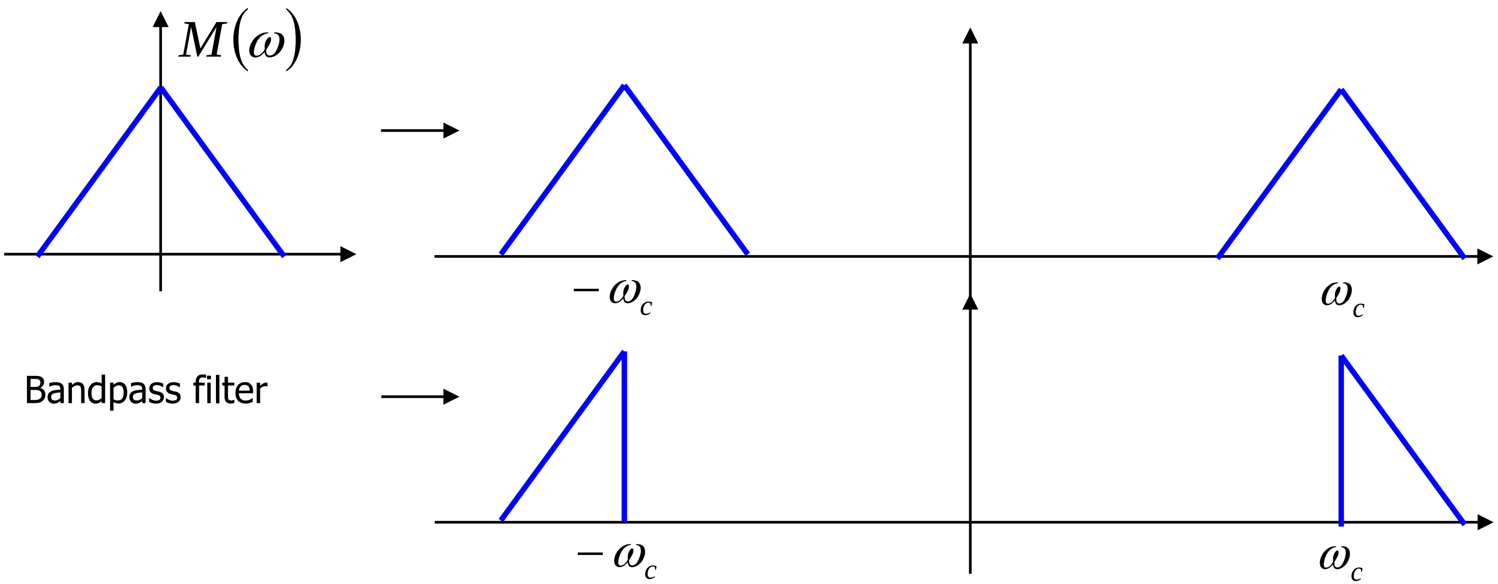


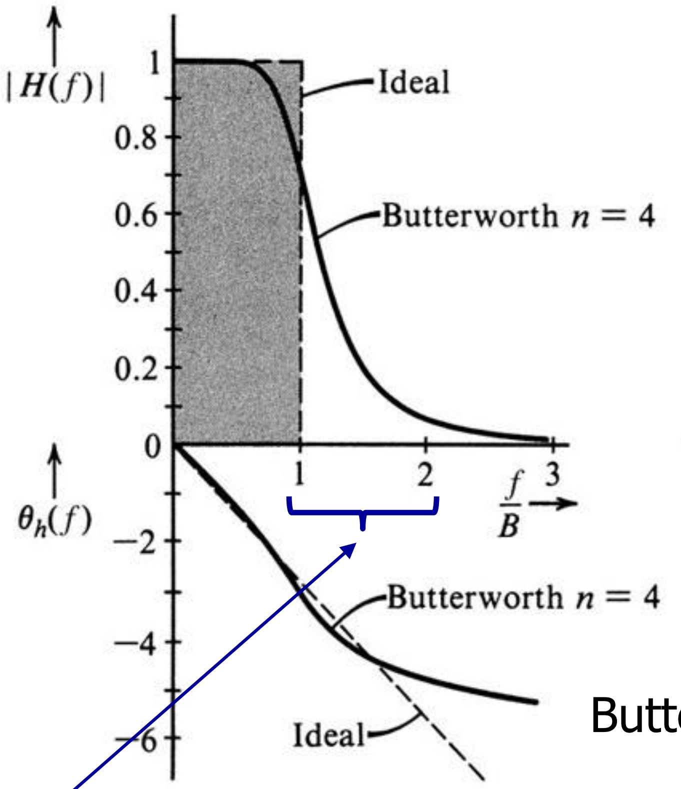
Figure 4.17 Using the phase-shift method to generate SSB.

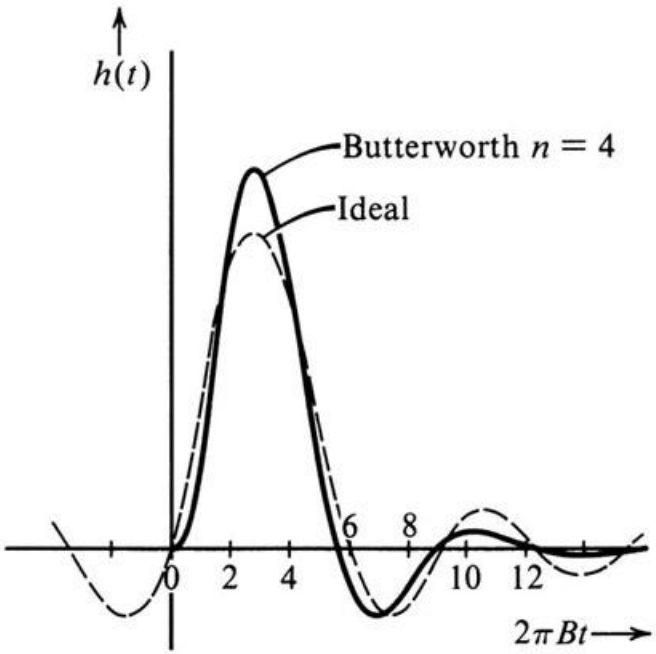
Selective Filtering SSB Modulator



Problem: Sharp frequency cut of is very difficult to implement at high frequency

Practical Lowpass Filters



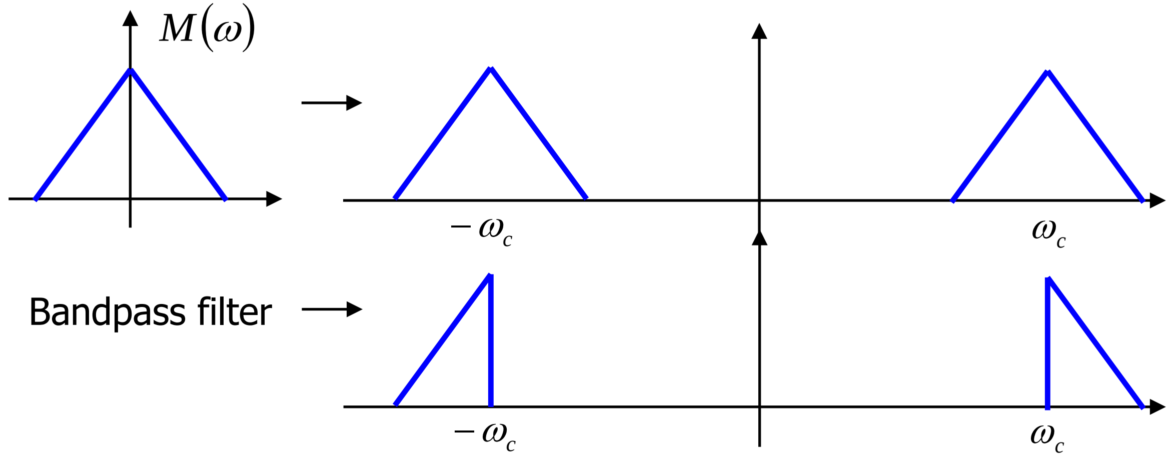


Butterworth filter

$$\left|H(\omega)\right|^2 = H(\omega)H^*(\omega) = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}$$

The size of this interval is proportional to the cutoff frequency (=1 in the figure)
High order filter is needed if we want the gain to drop faster.

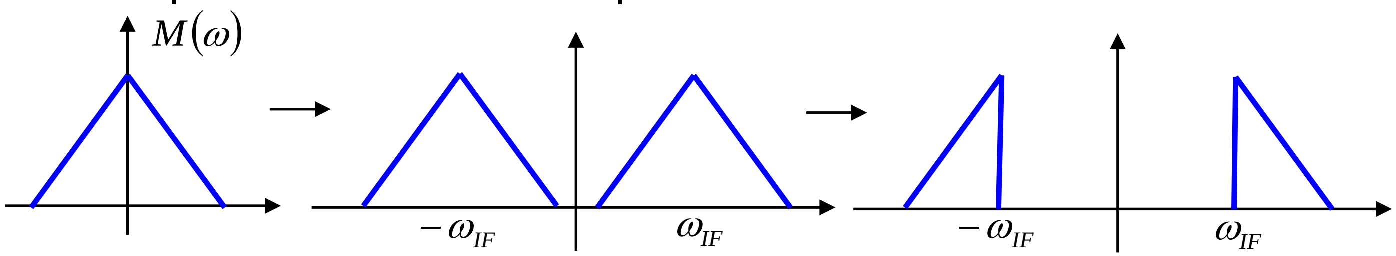
Selective Filtering SSB Modulator



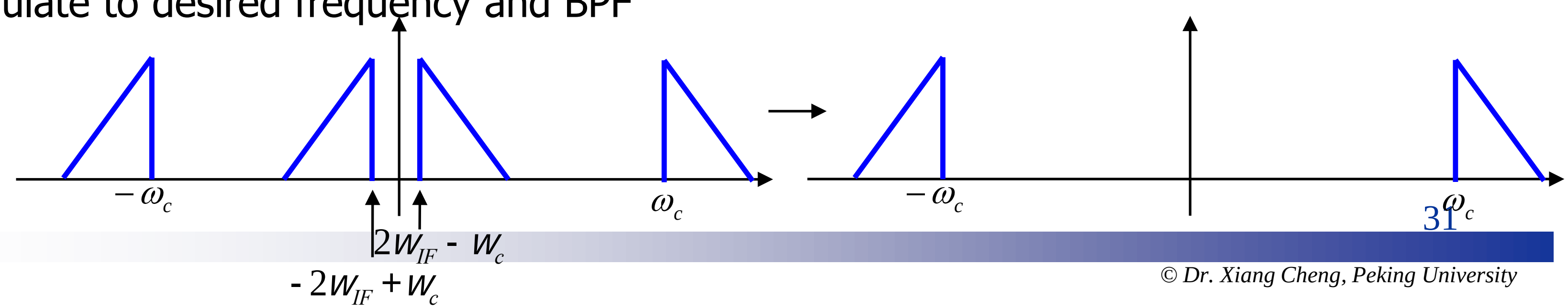
Problem: Sharp frequency cut of is very difficult to implement at high frequency

Solution: Two-step approach.

1. Modulate to a pre-determined low freq. carrier and BPF

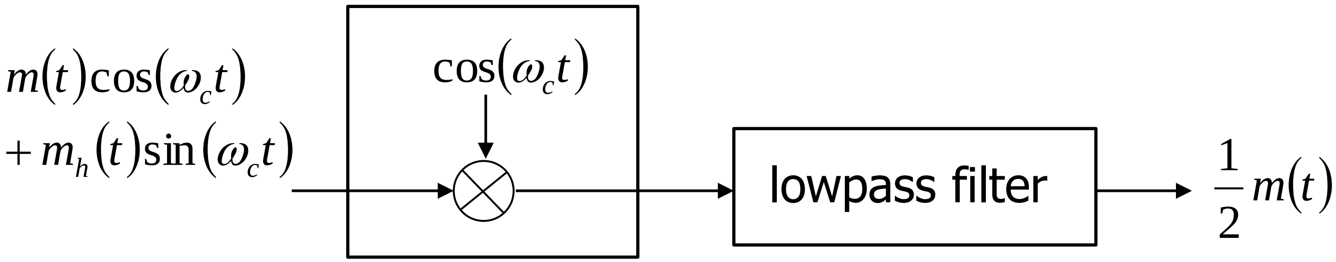


2. Modulate to desired frequency and BPF

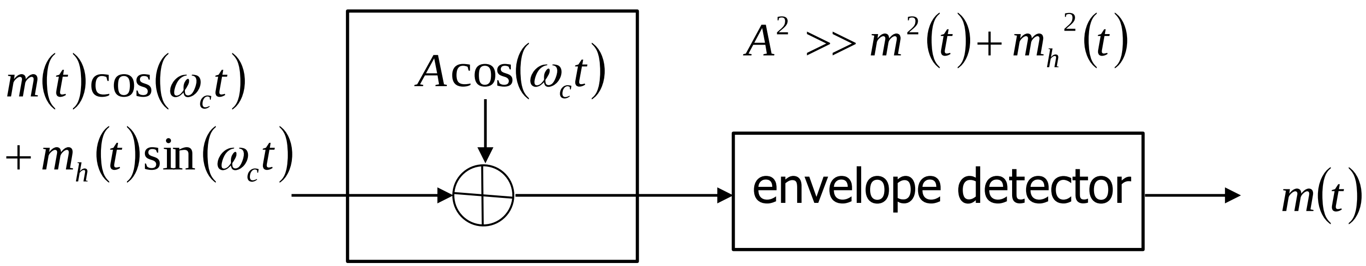


SSB Demodulation

Coherent demodulation



Envelope demodulation



$$A\cos(\omega_c t)+m(t)\cos(\omega_c t)+m_h(t)\sin(\omega_c t)=(A+m(t))\cos(\omega_c t)+m_h(t)\sin(\omega_c t)$$
$$=K(t)[\cos\theta_t\cos(\omega_c t)+\sin\theta_t\sin(\omega_c t)]$$

$$K^2(t)=(A+m(t))^2+m_h{}^2(t)=A^2+2Am(t)+m^2(t)+m_h{}^2(t)=A^2\left[1+\frac{2m(t)}{A}+\frac{m^2(t)+m_h{}^2(t)}{A^2}\right]$$

Envelope $\approx\sqrt{1+2m(t)/A}-1\approx m(t)/A$

Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Vestigial Sideband Modulation

Motivation: It is hard to implement a bandpass filter with sharp edges

Solution: use a more practical sideband shaping filter.

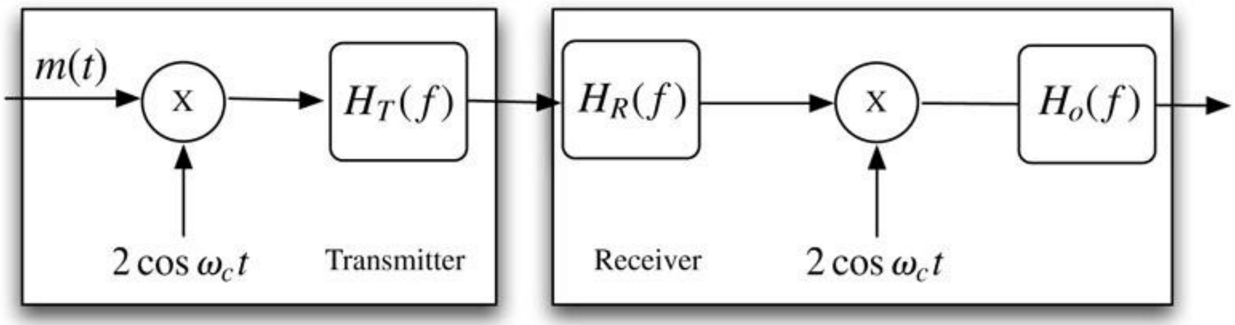
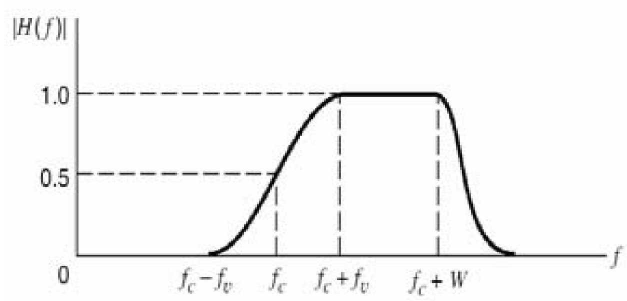
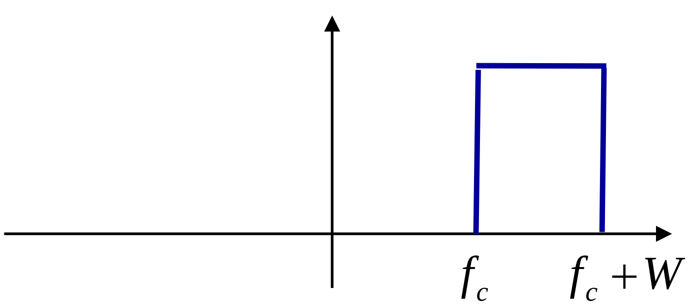


Figure 4.23 Transmitter filter $H_T(f)$, receiver front-end filter $H_R(f)$, and the receiver output low-pass filter $H_o(f)$ in VSB Television systems.

Sideband Shaping filter



SSB



DSB, SSB and VSB

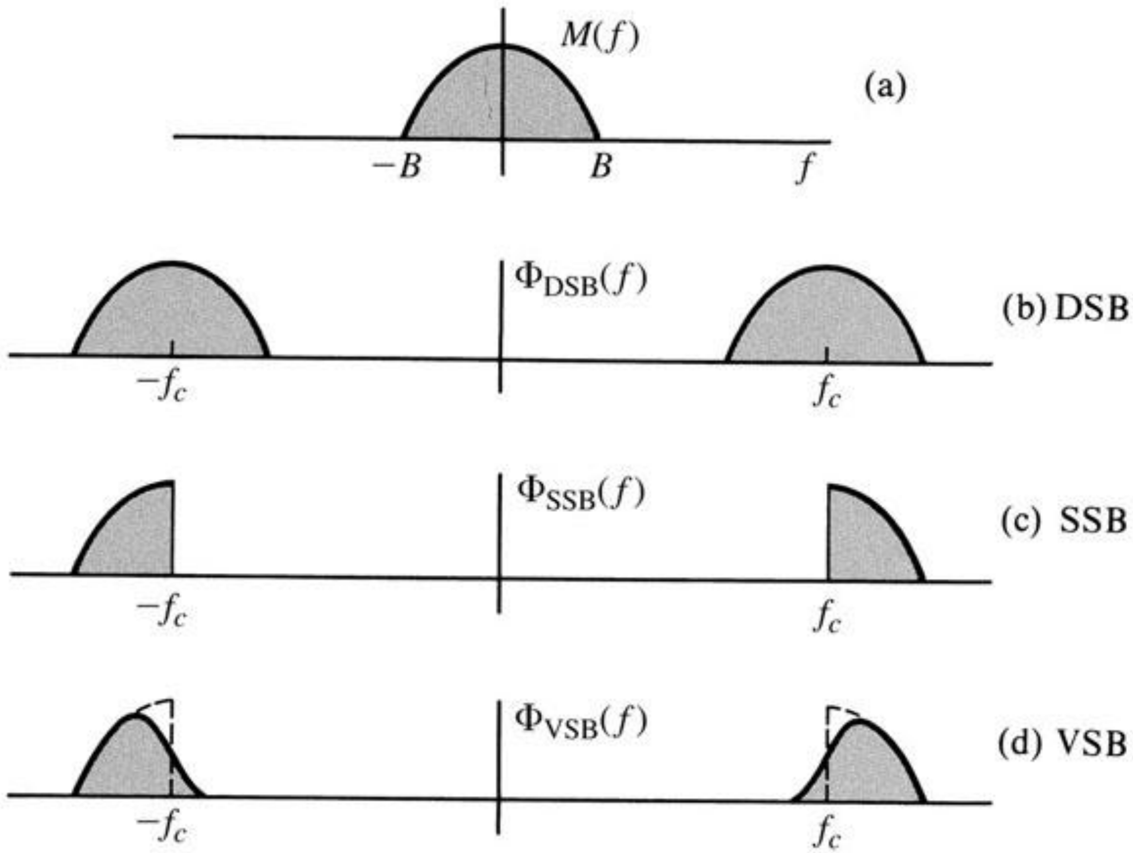


Figure 4.20 Spectra of the modulating signal and corresponding DSB, SSB, and VSB signals.

VSB Modulation and Demodulation

Modulation

$$\Phi_{VSB}(\omega)=[M(\omega-\omega_c)+M(\omega+\omega_c)]H_i(\omega)$$

$H_i(\omega)$ is the transmitter shaping filter

Demodulation

pass $2\Phi_{VSB}(t)\cos(\omega_c t)$ through filter $H_o(\omega)$

$H_o(\omega)$ is the receiver shaping filter

Spectrum of the demodulated signal

$$[\Phi_{VSB}(\omega+\omega_c)+\Phi_{VSB}(\omega-\omega_c)]H_o(\omega)$$
$$=\{[M(\omega)+M(\omega+2\omega_c)]H_i(\omega+\omega_c)+[M(\omega-2\omega_c)+M(\omega)]H_i(\omega-\omega_c)\}H_o(\omega)$$

If only keep the low frequency terms (look at the M terms)

$$M(\omega)\{H_i(\omega+\omega_c)+H_i(\omega-\omega_c)\}H_o(\omega)$$

Hence require

$$\{H_i(\omega+\omega_c)+H_i(\omega-\omega_c)\}H_o(\omega)=1$$

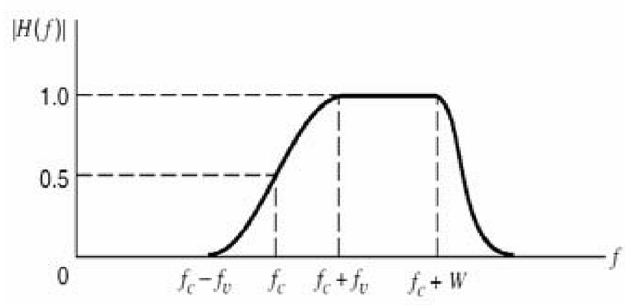
Complimentary VSB Filter

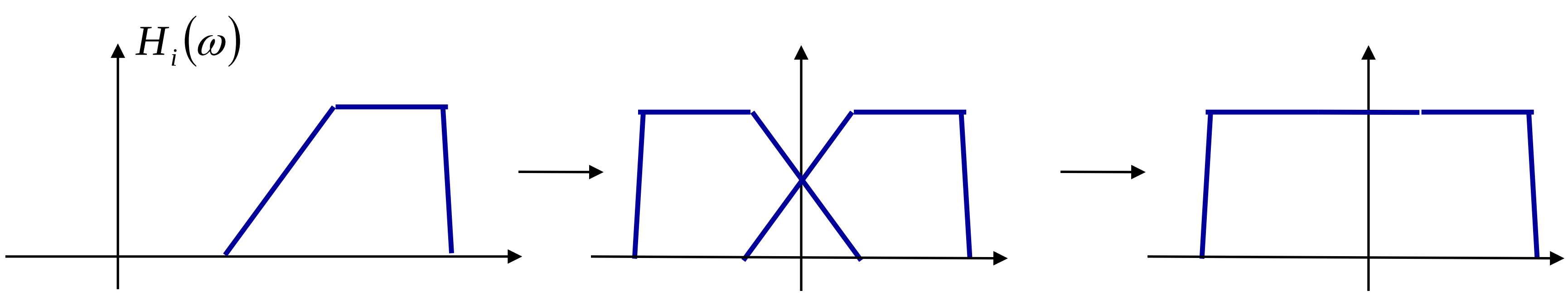
We require:

$$\{H_i(\omega+\omega_c)+H_i(\omega-\omega_c)\}H_o(\omega)=1$$

If $H_i(\omega-\omega_c)+H_i(\omega+\omega_c)=1$ for $|\omega_c|<2\pi B$, then $H_o(\omega)=1$.

Receiver only needs to do lowpass



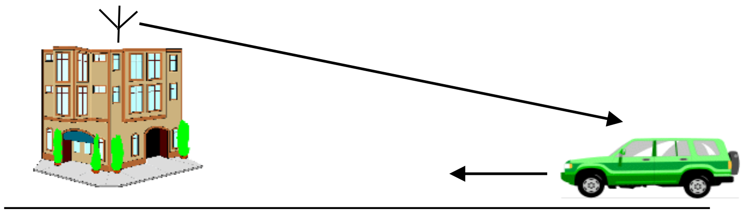


Roadmap

- Double sideband suppressed carrier (DSB-SC) modulation
- Amplitude modulation (AM)
- Quadrature amplitude modulation (QAM)
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Local Carrier Synchronization

Demodulation (other than AM) requires $\cos\omega_c t$, with precise frequency and phase



Doppler effect can cause frequency shift

Propagation delay can cause phase shift

Suppose DSB, and the received signal is

$$\Phi(t) = m(t)\cos[(\omega_c + \Delta\omega)t + \delta]$$

Demodulation

$$2\Phi(t)\cos\omega_c t = 2m(t)\cos[(\omega_c + \Delta\omega)t + \delta]\cos\omega_c t = m(t)\cos[\Delta\omega t + \delta] + m(t)\cos[(2\omega_c + \Delta\omega)t + \delta]$$

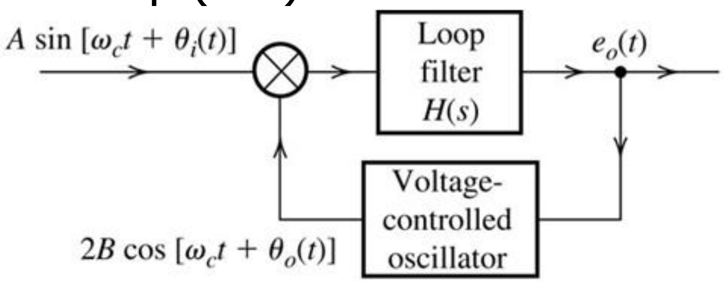
After lowpass filtering

$$m(t)\cos[\Delta\omega t + \delta]$$

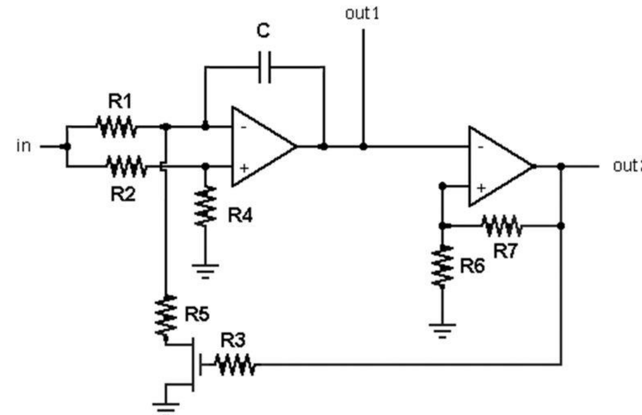
Assume $\Delta\omega$ is small, magnitude of output signal fluctuates in time.

Phase Locked Loop (PLL)

Phase Locked Loop (PLL)



(a)

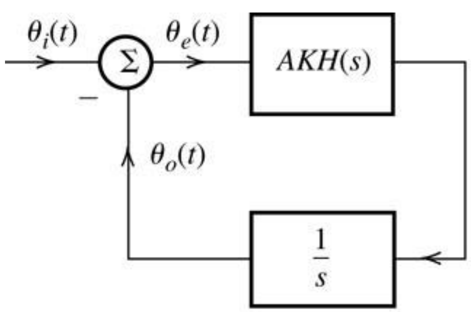


Voltage Controlled Oscillator (VCO)

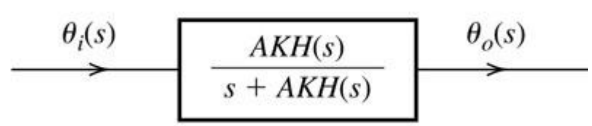
VCO schematic

Instantaneous frequency = $\omega_c + ce_o(t)$

Look at the phase only

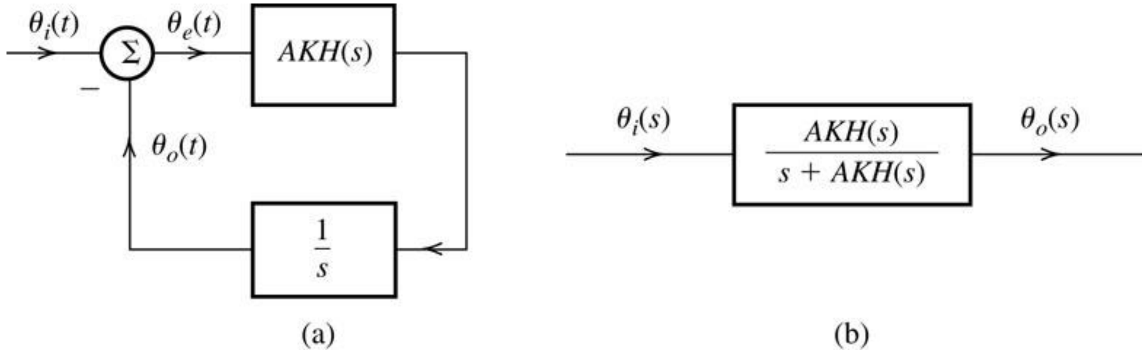


(a)



(b)

PLL



Assume $H(s)=1$ $\quad \theta_o(s)=\frac{AK}{s+AK}\theta_i(s) \qquad \theta_e(s)=\theta_i(s)-\theta_o(s)=\frac{s}{s+AK}\theta_i(s)$

Assume $\quad \theta_i(t)=\Delta\omega t+\phi_0 \qquad \theta_i(s)=\frac{\Delta\omega}{s^2}+\frac{\phi_0}{s}$

$$\theta_e(s)=\frac{s}{s+AK}\theta_i(s)=\frac{\Delta\omega}{s(s+AK)}+\frac{\phi_0}{s+AK}=\frac{\phi_0-\frac{\Delta\omega}{AK}}{s+AK}+\frac{\Delta\omega}{AKs}$$

$$\theta_e(t)=\left(\phi_0-\frac{\Delta\omega}{AK}\right)e^{-AKt}+\frac{\Delta\omega}{AK} \qquad \lim_{t\to\infty}\theta_e(t)=\frac{\Delta\omega}{AK}=\mathrm{const}$$

PLL can extract carrier frequency & lock the phase with a small constant error.

Costas Loop

If modulated signal is $\Phi(t) = m(t)\cos(\omega_c t + \theta)$, and θ is unknown.

Coherent Demodulation

$$\Phi(t)\cos\omega_c t = m(t)\cos(\omega_c t + \theta)\cos\omega_c t$$

$$= \frac{1}{2}m(t)\cos\theta + \frac{1}{2}m(t)\cos(2\omega_c t + \theta)$$

After lowpass filtering

$$\frac{1}{2}m(t)\cos\theta$$

To track the phase

$$\Phi(t)\sin\omega_c t = m(t)\cos(\omega_c t + \theta)\sin\omega_c t$$

$$= \frac{1}{2}m(t)\sin(2\omega_c t + \theta) - \frac{1}{2}m(t)\sin\theta \rightarrow m(t)\sin\theta$$

If phase is locked and $\theta = 0$, we should have

$$m(t)\sin\theta = 0$$

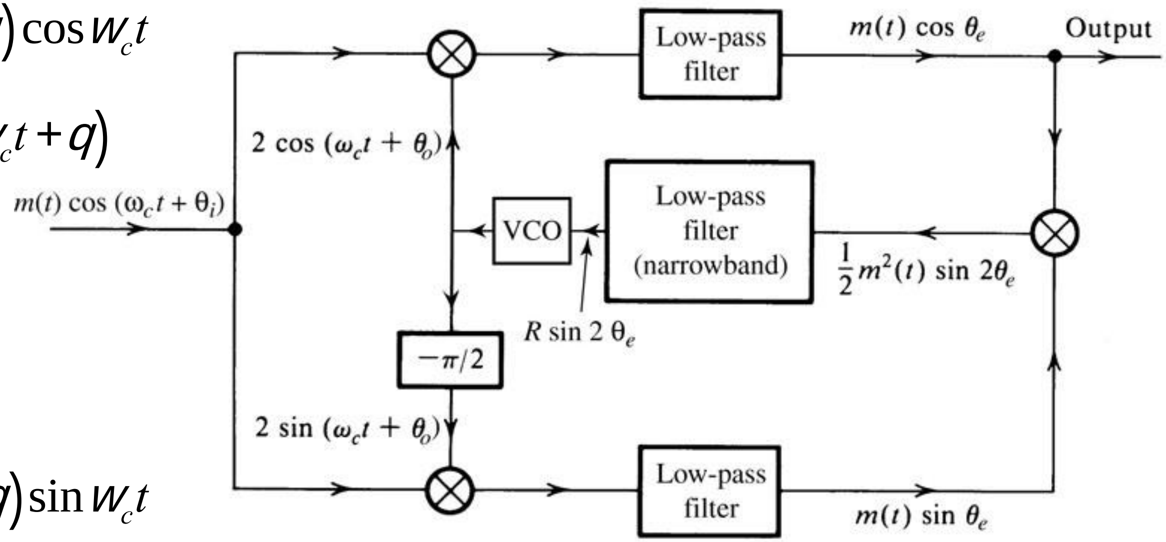


Figure 4.30 Costas phase-locked loop for the generation of a coherent demodulation carrier.

Summary

- Double sideband suppressed carrier (DSB-SC) modulation
  - DSB-SC modulation and demodulation
  - DSB-SC modulators
- Amplitude modulation (AM)
  - AM modulation and envelope detector
  - Power efficiency
- Quadrature amplitude modulation (QAM)
  - QAM modulation and demodulation
  - Complex message signals and their (de-)modulation
- Single sideband (SSB) modulation
  - Hilbert transform
  - SSB modulators
  - SSB demodulators
- Vestigial sideband (VSB) modulation
- Local carrier synchronization

Homework

- 4.2-3, 4.2-4, 4.2-8, 4.3-2, 4.3-5, 4.4-6, 4.5-2