Principle of Communications

Angle Modulation

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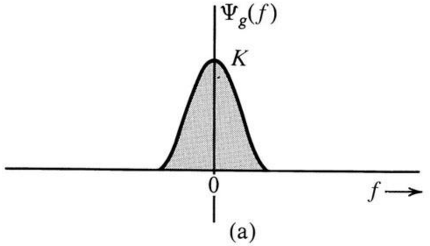
Roadmap

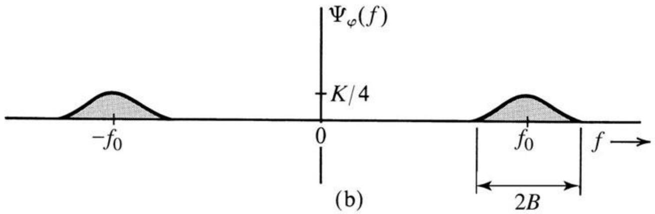
- Angle modulation introduced
- Bandwidth of Angle Modulated signals
- Properties of Angle Modulation
- Generation of FM signals

False Start

Historical comments:

In Amplitude Modulation, bandwidth of the modulated signal is at least the same as the original message signal.





If we modulate $m(t)$ to $A_c \cos[(\omega_c + km(t))t + \phi]$
And $|m(t)| \le m_p$, then the frequencies may locate between $[\omega_c - km_p, \omega_c + km_p]$. Without noise, we can make k as small as possible and then communicate without using any bandwidth at all.

It turned out that the above vision is seriously wrong. But historically, it is why people got interested in angle modulation techniques.

Fallacy Exposed

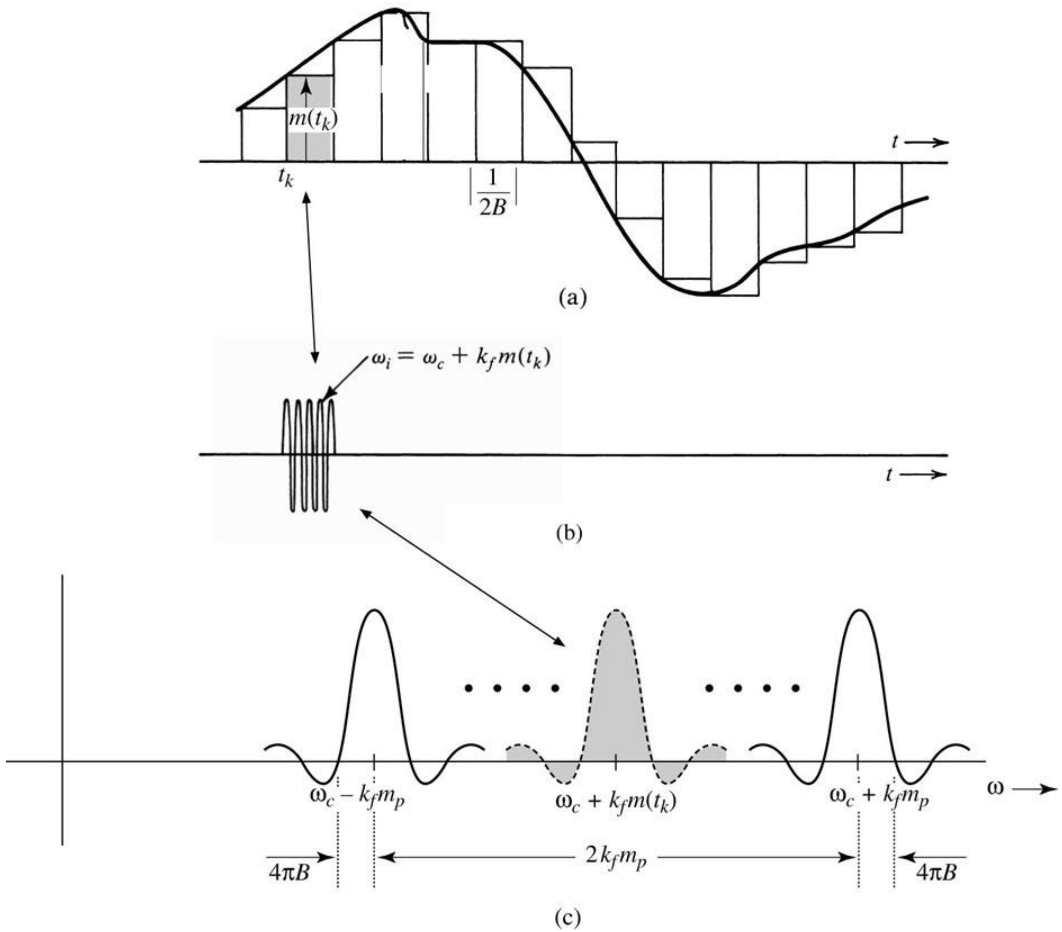


Figure 5.6 Estimation of FM wave bandwidth.

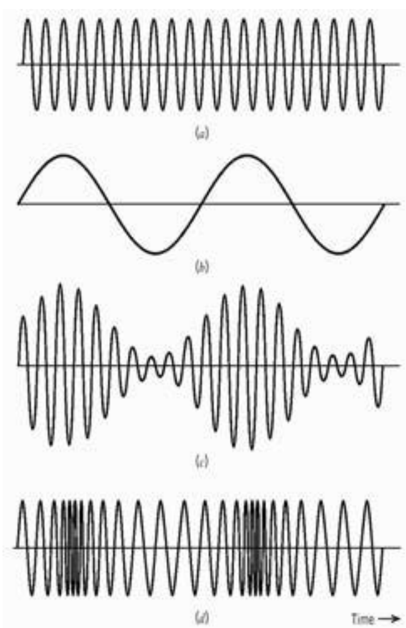
Angle Modulation

Basic Definition

Basic carrier signal: $A_c \cos(2\pi f_c t + \phi)$

Baseband signal: $m(t)$

If we write modulated signal as: $s(t) = A_c \cos(\theta(t))$



Carrier

Message

Amplitude Modulation (AM, DSB, SSB, VSB)

Angle Modulation (FM, PM)

Instantaneous Phase/Frequency

If we write modulated signal as: $s(t) = A_c \cos(\theta(t))$

$\theta(t)$ is defined as the instantaneous phase

$\omega_i(t) = \dfrac{d\theta(t)}{dt}$ is defined as the instantaneous frequency. (slope of the phase change)

$$\theta(t) = \int_0^t \omega_i(\tau) d\tau$$

Two possibilities

Phase Modulation $\Phi(t) = A_c \cos(\omega_c t + \theta_0 + k_p m(t))$

Assume $\theta_0 = 0$ $\Phi(t) = A_c \cos(\omega_c t + k_p m(t))$

Frequency Modulation $\Phi(t) = A_c \cos((\omega_c + k_f m(t))t + \theta_0)$

Assume $\theta_0 = 0$ $\Phi(t) = A_c \cos[(\omega_c + k_f m(t))t]$

PM vs. FM

Phase Modulation $\Phi(t) = A_c \cos(\omega_c t + k_p m(t))$

Instantaneous Frequency $\omega_c + k_p \dfrac{dm(t)}{dt}$

PM with $k_p, m(t)$ $\Leftrightarrow$ FM with $k_p, \dfrac{dm(t)}{dt}$

FM with $k_f, m(t)$ $\Leftrightarrow$ PM with $k_f, \int_0^t m(\tau) d\tau$

Why not consider a more general form $q(t) = f(m(t))$?

There is only a limited number of ways to generate the modulated signal using circuits.

Power of Angle Modulated Signal

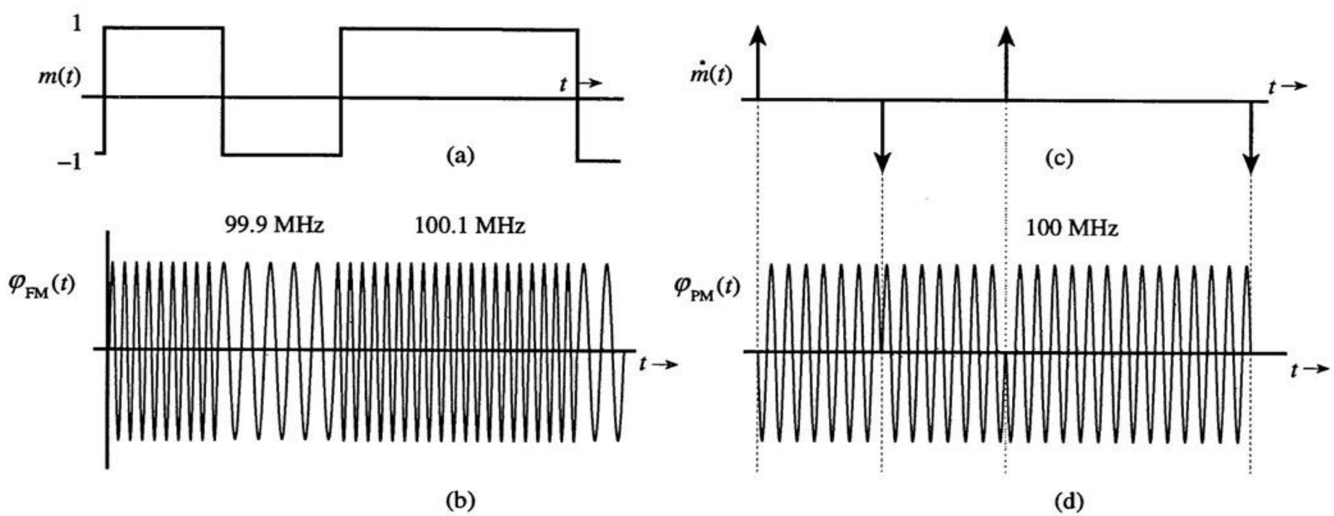
Because the amplitude remains constant

$$P_{FM} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|\Phi_{FM}(t)\right|^2 dt = E\left\{A_c^{\,2}\cos^2\left[\left(\omega_c + k_f m(t)\right)t\right]\right\}$$

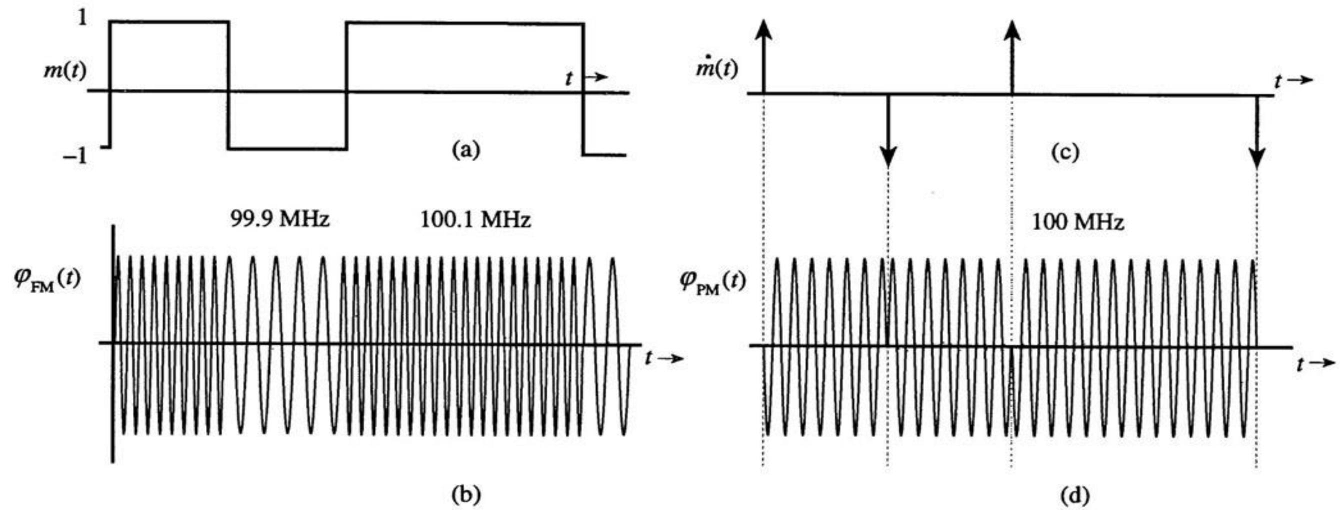
$$= E\left\{\frac{A_c^{\,2}}{2}\left[\cos\left[2\left(\omega_c + k_f m(t)\right)t\right] + 1\right]\right\} = \frac{A_c^{\,2}}{2}$$

Power of the modulated signal does not depend on the message

Example 5.2: Sketch the FM and PM signal for *m*(*t*) given below



Example 5.2: Sketch the FM and PM signal for $m(t)$ given below



FM signal: Instantaneous frequency $\omega_i = \omega_c + k_f m(t)$

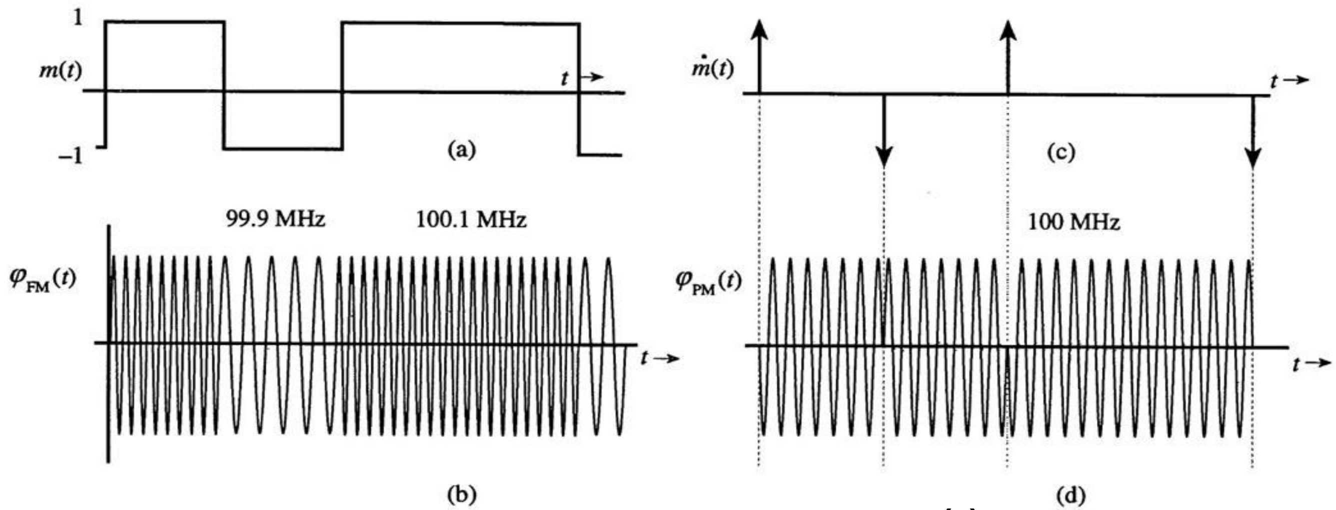
$$\omega_i = \omega_c + k_f m(t) = 2\pi \times 10^8 + 2\pi \times 10^5 m(t) = \begin{cases} 2\pi \times (100.1MHz) & m(t) = 1 \\ 2\pi \times (99.9MHz) & m(t) = -1 \end{cases}$$

$$\theta(t) = \int_0^t \omega_i(\tau) d\tau = \begin{cases} 2\pi \times (100.1MHz)t & m(t) = 1 \\ 2\pi \times (99.9MHz)t & m(t) = -1 \end{cases}$$

We will see later that this is Frequency Shift Keying (FSK) modulation

Note: Range of instantaneous frequency ≠ bandwidth !

Example 5.2: Sketch the FM and PM signal for $m(t)$ given below



PM signal: Instantaneous frequency $\omega_i = \omega_c + \frac{k_p}{1}\frac{dm(t)}{dt} = 2\pi \times 10^8 + \frac{\pi}{2}\frac{dm(t)}{dt}$

$$\Phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right) = A\cos\left(\omega_c t + \frac{\pi}{2} m(t)\right) = \begin{cases} A\sin\omega_c t & m(t) = -1 \\ -A\sin\omega_c t & m(t) = 1 \end{cases}$$

We will see later that this is Phase Shift Keying (PSK) modulation

Note: Need infinite bandwidth to support sharp phase change

Message Continuity and kp

$$\Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$$

Suppose $k_p = \pi$ $\quad \Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t)) = \begin{cases} A\cos(\omega_c t + \pi) & m(t) = 1 \\ A\cos(\omega_c t - \pi) & m(t) = -1 \end{cases}$

But $A\cos(\omega_c t + \pi) \equiv A\cos(\omega_c t - \pi)$, hence can't distinguish $m(t) = 1$ from $m(t) = -1$

When $m(t)$ is not continuous

To avoid ambiguity, we require $k_p m(t) \in [-\pi, \pi)$

Can only identify phase change within a 2π range.

When $m(t)$ is continuous

No need to have $k_p m(t) \in [-\pi, \pi)$

Because a 2π phase change causes discontinuity. The fact that $m(t)$ is continuous can help us to avoid phase jumps.

Roadmap

- Angle modulation introduced
- Bandwidth of Angle Modulated signals
- Properties of Angle Modulation
- Generation of FM signals

Bandwidth of Angle Modulated Waves

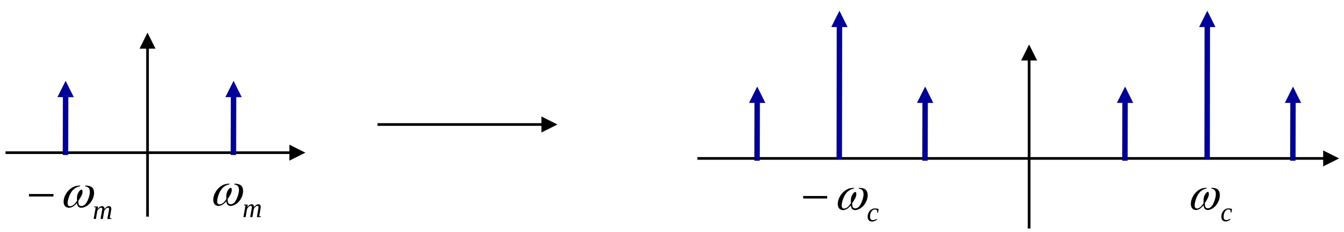
PM, message signal = single tone. $m(t) = A_m \sin \omega_m t$

$$\Phi_{PM}(t) = A\cos(\omega_c t + k_p A_m \sin(\omega_m t))$$

Assume $k_p A_m$ is small. $f(x+\varepsilon) = f(x) + \varepsilon f'(x) + O(\varepsilon^2)$

$$\Phi_{PM}(t) = A\cos(\omega_c t + k_p A_m \sin(\omega_m t)) \approx A\cos(\omega_c t) - A\beta \sin(\omega_m t)\sin(\omega_c t)$$

$$= A\cos(\omega_c t) + \frac{A\beta}{2}\left[\cos([\omega_c + \omega_m]t) - \cos([\omega_c - \omega_m]t)\right]$$



Similar to AM, but requires $k_p A_m$ to be small.

Bandwidth of Angle Modulated Waves

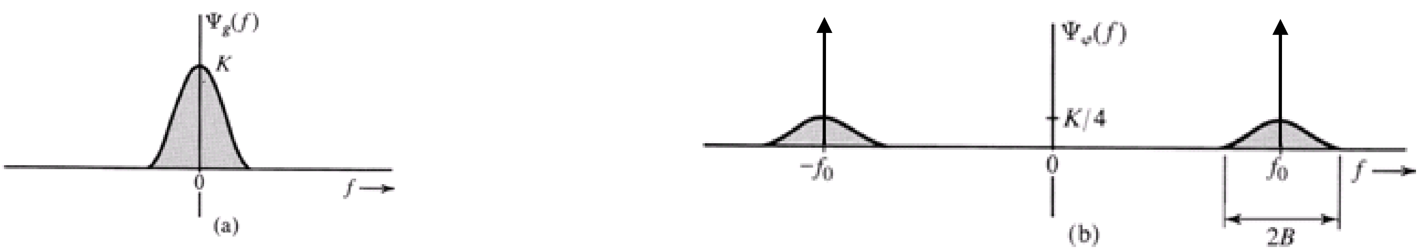
PM, narrow band $\Phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$

Assume $k_p m(t)$ is small. $\sin\left(k_p m(t)\right) \approx k_p m(t),\ \cos\left(k_p m(t)\right) \approx 1$

$$\Phi_{PM}(t) = A\cos(\omega_c t)\cos\left(k_p m(t)\right) - A\sin(\omega_c t)\sin\left(k_p m(t)\right) \approx A\cos(\omega_c t) - k_p m(t) A\sin(\omega_c t)$$

Again similar to AM. Modulated signal = carrier + DSB-SC

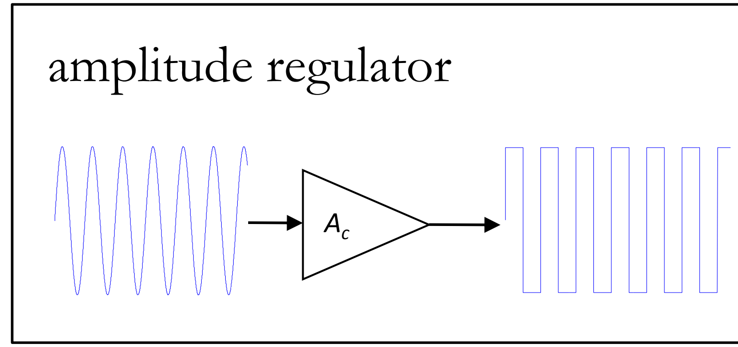
If bandwidth of $m(t)$ is B, then bandwidth of the modulated signal is $2B$.

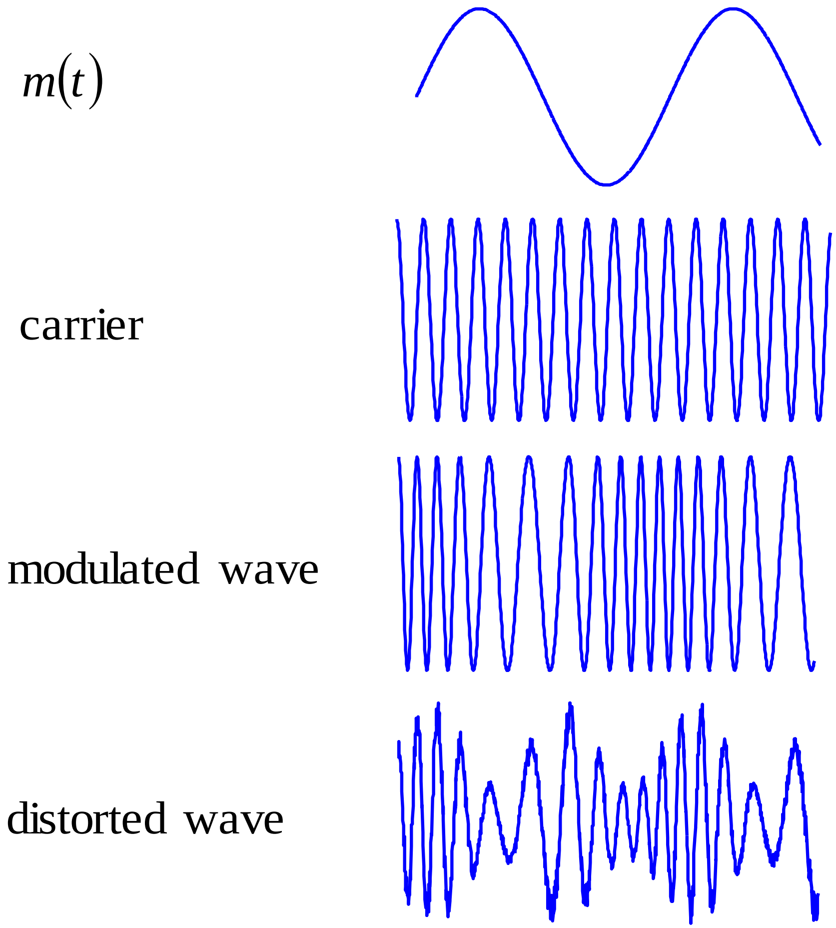


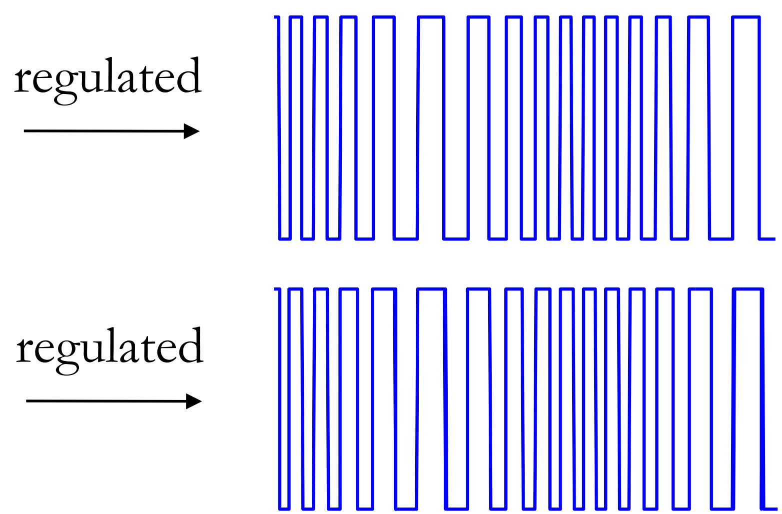
Can't save bandwidth. Then, why we consider PM or FM?

Key Advantage of Angle Modulation

Highly resistant to amplitude distortion







Modulator for Narrowband PM Signal

$$\Phi_{PM}(t) = A\cos(\omega_c t)\cos(k_p m(t)) - A\sin(\omega_c t)\sin(k_p m(t)) \approx A\cos(\omega_c t) - k_p m(t) A\sin(\omega_c t)$$

Suggested modulator

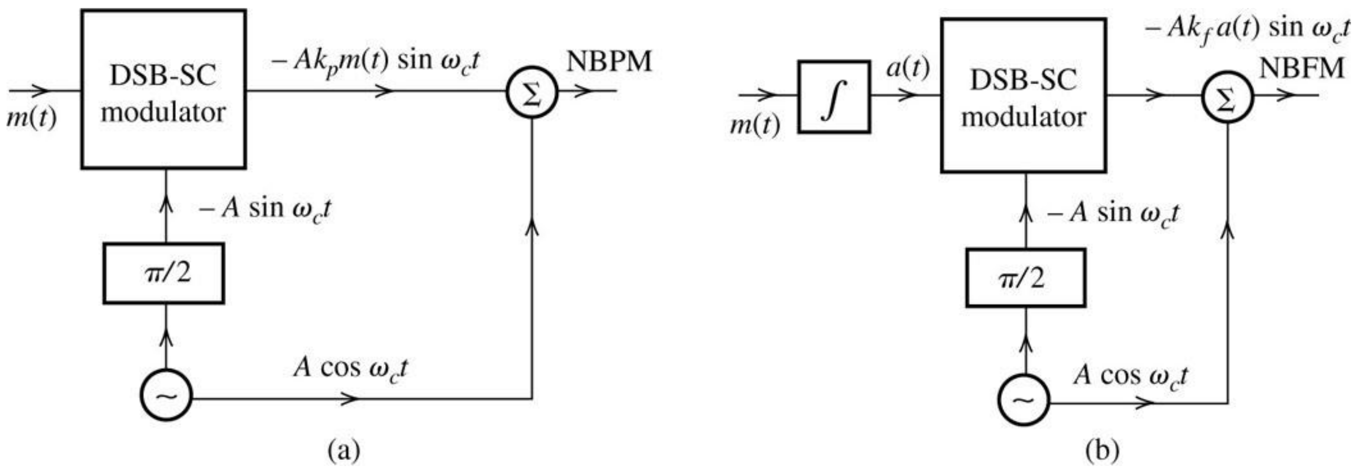


Figure 5.8 (a) Narrowband PM generator. (b) Narrowband FM signal generator.

Wideband Angle Modulation: Rigorous Bandwidth Analysis (1)

PM, wide band $\Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$ Assume $|k_p m(t)| << 1$ does not hold

Assume message signal = single tone. $m(t) = A_m \sin\omega_m t$

$$\Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t)) = A\mathrm{Re}\left\{e^{j(\omega_c t + k_p A_m \sin(\omega_m t))}\right\} = A\mathrm{Re}\left\{e^{j\omega_c t} e^{jk_p A_m \sin(\omega_m t)}\right\}$$

Note that $e^{jk_p A_m \sin(\omega_m t)}$ is periodic $e^{jk_p A_m \sin\left(\omega_m\left(t + \frac{2\pi}{\omega_m}\right)\right)} = e^{jk_p A_m \sin(\omega_m t)}$

$e^{jk_p A_m \sin(\omega_m t)}$ can be expanded using Fourier series

$$e^{jk_p A_m \sin(\omega_m t)} = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_m t}$$

$$\Phi_{PM}(t) = A\mathrm{Re}\left\{e^{j\omega_c t} \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_m t}\right\} = A\mathrm{Re}\left\{\sum_{k=-\infty}^{\infty} C_k e^{j(\omega_c + k\omega_m)t}\right\}$$

Wideband Angle Modulation: Rigorous Bandwidth Analysis (2)

$$e^{jk_p A_m \sin(\omega_m t)} = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_m t}$$

$$C_k = \frac{\omega_m}{2\pi}\int_{-\frac{\pi}{\omega_m}}^{\frac{\pi}{\omega_m}} e^{jk_p A_m \sin(\omega_m t)} e^{-jk\omega_m t} dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(k_p A_m \sin(\tau) - k\tau)} d\tau = J_k(k_p A_m)$$

$J_k(k_p A_m)$ is the kth order Bessel function of the first kind, evaluated at $k_p A_m$

$$\Phi_{PM}(t) = A\,\mathrm{Re}\left\{e^{j\omega_c t}\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_m t}\right\} = A\,\mathrm{Re}\left\{\sum_{k=-\infty}^{\infty} J_k(k_p A_m) e^{j(\omega_c + k\omega_m)t}\right\}$$

Can plot the J_k functions and see the magnitude of frequency components

Problem: too complicated

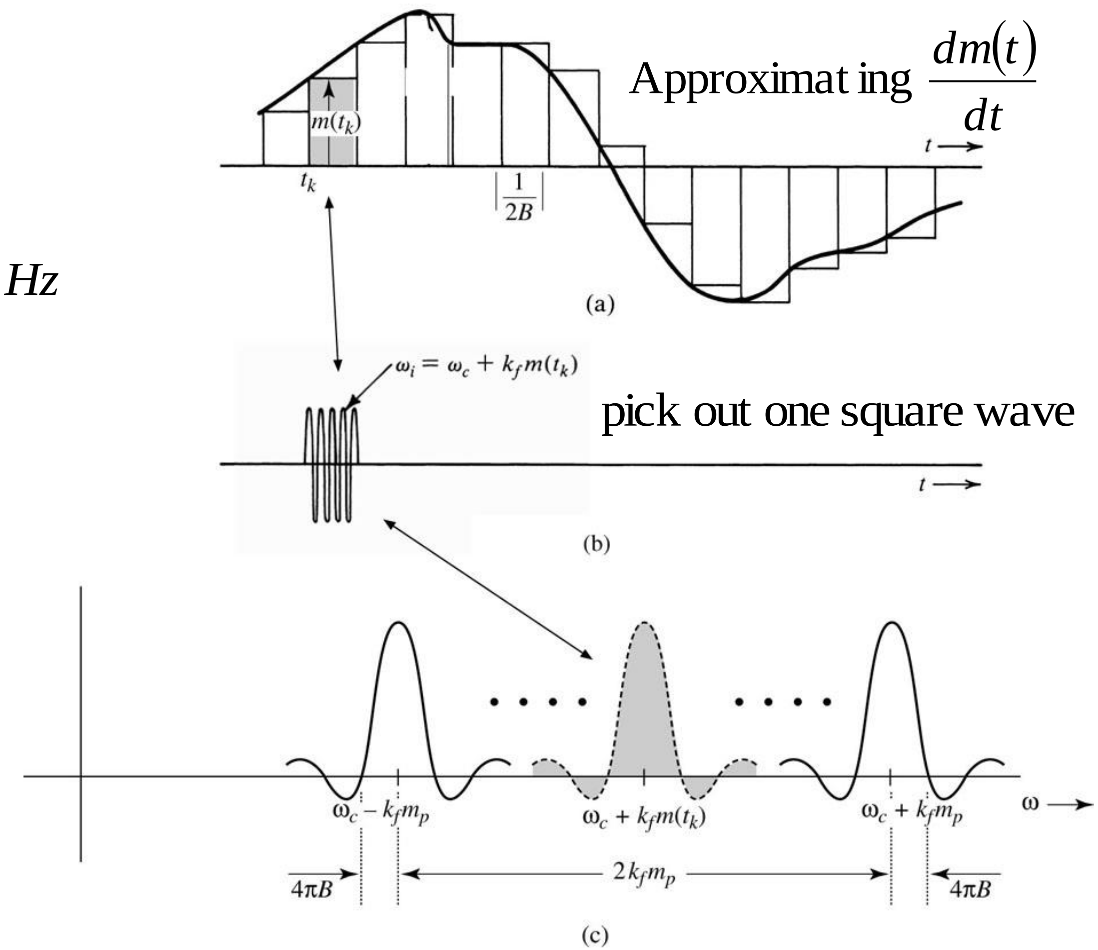
Wideband Angle Modulation: Rough Bandwidth Analysis (1)

PM, wide band $\Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$ Assume $|k_p m(t)| << 1$ does not hold

Precise bandwidth calculation is hard. But there is a simple way to estimate it.

Assume $m(t)$ has bandwidth of B Hz

Approximate $m(t)$ by rectangular waveform each of width $1/2B$.
(Recall Nyquist theorem says, with bandwidth of B, the signal can have $2B$ independent samples per second.



bandwidth union of the square waves

Wideband Angle Modulation: Rough Bandwidth Analysis (2)

With PM, each rectangular wave is converted to

$$A\,\mathrm{rect}(2Bt)\cos\left(\omega_c t + k_p m(t)\right)$$

The corresponding spectrum is

$$\frac{A}{2}\mathrm{sinc}\left(\frac{\omega+\omega_c+k_p m'(t)}{4B}\right)+\frac{A}{2}\mathrm{sinc}\left(\frac{\omega-\omega_c-k_p m'(t)}{4B}\right) \qquad m'(t)=\frac{dm(t)}{dt}$$

Assume $m'(t)$ vary betweem $\left[-m'_p, m'_p\right]$

Bandwidth of the modulated signal can be approximated by

$$B_{\mathrm{PM}} = 2\left(\frac{k_p m'_p}{2\pi}+2B\right)Hz$$

(ignored the ripples of the sinc function)

Define $\beta = \dfrac{k_p m'_p}{2\pi B}$ $\qquad B_{\mathrm{PM}} = 2B(\beta+2)\,Hz$

Note that this is not a solid theoretical analysis

The actual bandwidth is somewhat smaller

Carson's Rule

According to narrow band analysis, when $k_p \to 0$, we should have $B_{\mathrm{PM}} \to 2B$

A more careful analysis shows that $B_{\mathrm{PM}} \approx 2B(\beta+1)\,Hz$ $\quad \beta = \dfrac{k_p m'_p}{2\pi B}$

This is known as Carson's Rule.

FM: $m(t) \to A\cos\left(\omega_c t + k_f \int_0^t m(\tau)d\tau\right)$

$m(t)$ has bandwidth B, and vary between $[-m_p, m_p]$

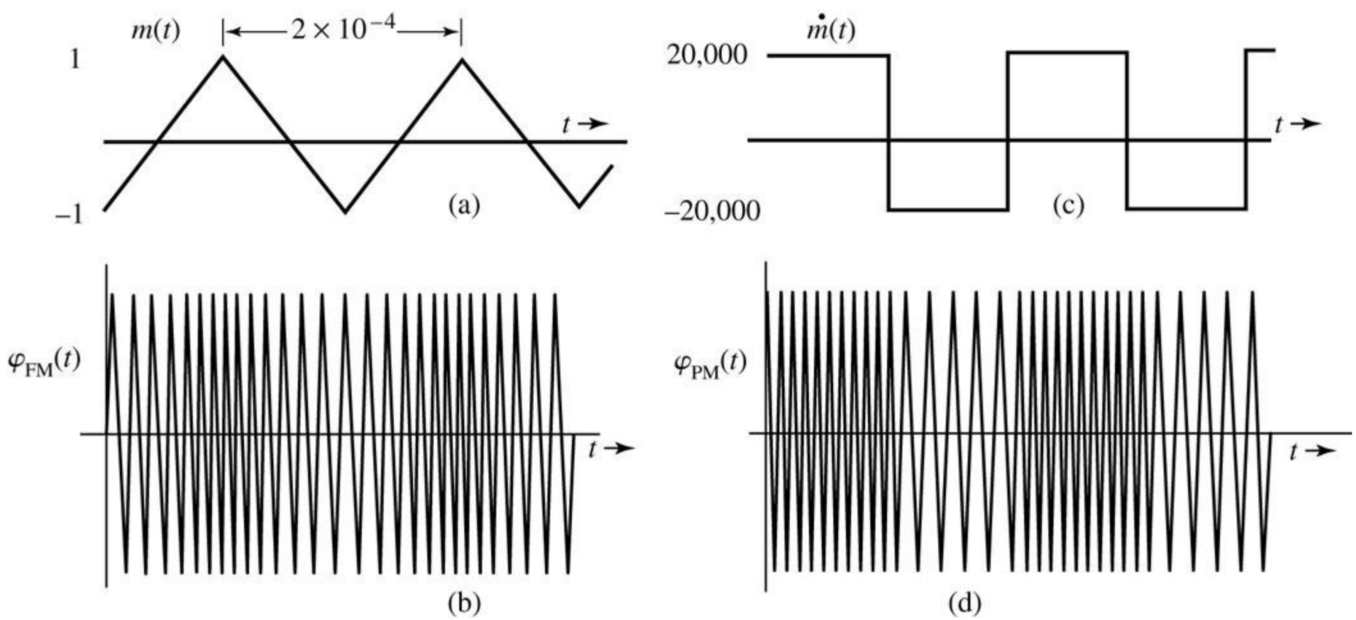
estimated bandwidth $B_{\mathrm{FM}} \approx 2B(\beta+1)\,Hz$ $\quad \beta = \dfrac{k_f m_p}{2\pi B}$

PM: $m(t) \to A\cos(\omega_c t + k_p m(t))$

$m(t)$ has bandwidth B, and $m'(t)$ vary between $[-m'_p, m'_p]$

estimated bandwidth $B_{\mathrm{PM}} \approx 2B(\beta+1)\,Hz$ $\quad \beta = \dfrac{k_p m'_p}{2\pi B}$

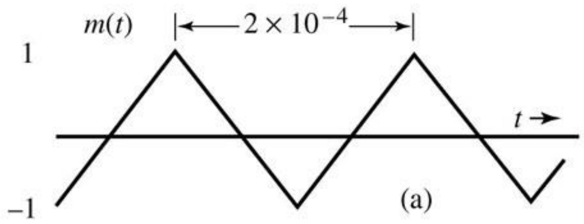
Example 5.3:

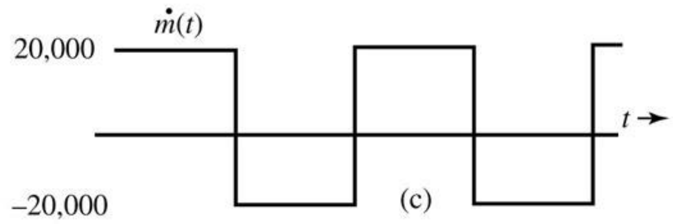


a) Estimate B_{PM} and B_{FM} for the message signal $m(t)$ for $k_f = 2\pi \times 10^5, k_p = 5\pi$. Assume the essential bandwidth of $m(t)$ equals the frequency of its third harmonic

b) Repeat the problem if the amplitude of $m(t)$ is doubled.

c) Repeat the problem is $m(t)$ is time-expanded by a factor of 2.





Peak amplitude is $m_p = 1$ Peak derivative is $m'_p = \dfrac{2}{10^{-4}} = 20000$

To determine the essential bandwidth, expand $m(t)$ using Fourier series

$m(t) = \sum_k C_k e^{jk\omega_0 t}$ $\omega_0 = \dfrac{2\pi}{2\times10^{-4}} = 10^4\pi$ $C_0 = 0$ for $k \neq 0$, we have

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} m(t)e^{jk\omega_0 t}dt = \frac{1}{2}10^4\int_{-10^{-4}}^{0}\left(1+2\times10^4 t\right)e^{jk\omega_0 t}dt + \frac{1}{2}10^4\int_0^{10^{-4}}\left(1-2\times10^4 t\right)e^{jk\omega_0 t}dt$$

$$= 10^8\int_{-10^{-4}}^{0} te^{jk\omega_0 t}dt + 10^8\int_0^{10^{-4}} te^{jk\omega_0 t}dt = 2\times10^8\int_0^{10^{-4}} t\cos\left(k\omega_0 t\right)dt$$

$$= 2\times10^8\int_0^{10^{-4}}\frac{1}{k\omega_0}\sin\left(k\omega_0 t\right)dt = -\frac{2\times10^8}{k^2\omega_0^{\ 2}}\cos\left(k\omega_0 t\right)\Big|_0^{10^{-4}} = \frac{4}{k^2\omega_0^{\ 2}}$$

Because $m(t)$ is real-valued and even, we have $C_k = C_{-k} = \dfrac{4}{k^2 \omega_0^{\;2}}$

Fourier series can be written as $m(t) = 2\sum_{k=1}^{\infty} C_k \cos(k\omega_0 t) = \sum_{k=1}^{\infty} \dfrac{8}{k^2 \omega_0^{\;2}} \cos(k\omega_0 t)$

C_k decreases quickly to 0 in k. If we only consider the third harmonic, the bandwidth of $m(t)$ is $\dfrac{3\omega_0}{2\pi} = \dfrac{3\times 10^4}{2} = 15kHz$

For FM, $B_{FM} = 2B(\beta+1) = 2\left(\dfrac{k_f m_p}{2\pi} + B\right) = 2(100kHz + 15kHz) = 230kHz$

For PM, $B_{PM} = 2B(\beta+1) = 2\left(\dfrac{k_p m'_p}{2\pi} + B\right) = 2(50kHz + 15kHz) = 130kHz$

Note: This does not mean FM is worse than PM. We did not talk about their performance on noise resistance yet.

If $m(t)$ is doubled to $2m(t)$, we have $m_p = 2$, $m'_p = \frac{4}{10^{-4}} = 40000$

Essential bandwidth of $m(t)$ remains $\frac{3\omega_0}{2\pi} = \frac{3\times 10^4}{2} = 15kHz$

For FM, $B_{FM} = 2B(\beta+1) = 2\left(\frac{k_f m_p}{2\pi} + B\right) = 2(200kHz + 15kHz) = 430kHz$

For PM, $B_{PM} = 2B(\beta+1) = 2\left(\frac{k_p m'_p}{2\pi} + B\right) = 2(100kHz + 15kHz) = 230kHz$

Doubling the signal roughly doubles the bandwidth of both FM and PM waves

If $m(t)$ is time expanded to $m(t/2)$, we have $m_p = 1$, $m'_p = \dfrac{2}{2\times 10^{-4}} = 10000$

Essential bandwidth of $m(t)$ becomes $\dfrac{3\times 10^4 \pi}{2\times 2\pi} = \dfrac{3\times 10^4}{4} = 7.5kHz$

For FM, $B_{FM} = 2B(\beta+1) = 2\left(\dfrac{k_f m_p}{2\pi} + B\right) = 2(100kHz + 7.5kHz) = 215kHz$

For PM, $B_{PM} = 2B(\beta+1) = 2\left(\dfrac{k_p m'_p}{2\pi} + B\right) = 2(25kHz + 7.5kHz) = 65kHz$

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- Angle modulation introduced
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Properties of Angle Modulation

Usually, bandwidth of FM depends significantly on the amplitude of $m(t)$, but has little to do with the bandwidth of $m(t)$.
The bandwidth of PM depends on both amplitude and bandwidth of $m(t)$.

In angle modulation, one can tradeoff between message signal power and bandwidth by adjusting $\frac{k_f m_p}{2\pi}$ or $\frac{k_p m_p'}{2\pi}$.

Angle modulation is highly resistant to nonlinear distortions

$$\Phi(t) = A\cos(\theta(t)) \qquad f(\Phi(t)) = \beta_1 \Phi(t) + \beta_2 \Phi^2(t) + \cdots$$

$$\beta_1 \Phi(t) = \beta_1 A\cos(\theta(t)) \qquad \beta_2 \Phi^2(t) = \beta_2 A^2 \cos^2(\theta(t)) = \frac{\beta_2 A^2}{2}\left[\cos(2\theta(t)) + 1\right]$$

$$f(\Phi(t)) = \sum_{k=-\infty}^{\infty} \gamma_k \cos(k\theta(t))$$

High frequency terms can be removed

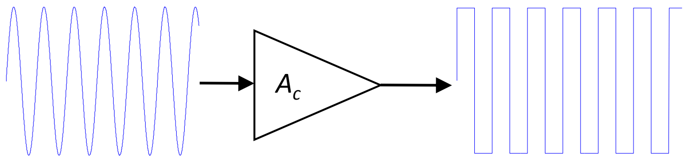
Nonlinear distortions can be recovered by bandpass filtering

Frequency Multiplier

Can double or triple the frequency of angle modulated signals easily.

$$\Phi(t) = A\cos(\theta(t))$$

Pass signal through a nonlinear device (such as amplitude regulator)



$$f(\Phi(t)) = \beta_1\Phi(t) + \beta_2\Phi^2(t) + \cdots = \sum_{k=-\infty}^{\infty} \gamma_k \cos(k\theta(t))$$

If $\Phi(t) = \cos(\omega_c t + k_p m(t))$, can get $\cos(2\omega_c t + 2k_p m(t))$, $\cos(3\omega_c t + 3k_p m(t))$, etc

Can pick up the desired component using an appropriate bandpass filter.

A device that takes $\cos(\theta(t))$ and outputs $\cos(k\theta(t))$ is called a frequency multiplier

Comparison: DSB w/ Nonlinear Distortion

Look at what nonlinear distortion does to DSB signal

$$m(t) = \cos(\omega_m t) \qquad \Phi_{DSB}(t) = \cos(\omega_m t)\cos(\omega_c t) = \frac{1}{2}\cos([\omega_c + \omega_m]t) + \frac{1}{2}\cos([\omega_c - \omega_m]t)$$

$$f(\Phi(t)) = \beta_1\Phi(t) + \beta_2\Phi^2(t) + \beta_2\Phi^3(t) + \cdots$$

$$\beta_2\Phi^2(t) = \beta_2\cos^2(\omega_c t)\cos^2(\omega_m t) = \frac{\beta_2}{4}\left[\cos([\omega_c + \omega_m]t) + \cos([\omega_c - \omega_m]t)\right]^2$$

$$= \frac{\beta_2}{4}\left[\cos^2([\omega_c + \omega_m]t) + \cos^2([\omega_c - \omega_m]t) + 2\cos([\omega_c + \omega_m]t)\cos([\omega_c - \omega_m]t)\right]$$

Look at the term $\cos^2([\omega_c + \omega_m]t)\cos([\omega_c - \omega_m]t)$ in $\beta_3\Phi^3(t)$

$$\cos^2([\omega_c + \omega_m]t)\cos([\omega_c - \omega_m]t) = \frac{1}{2}\left[\cos(2[\omega_c + \omega_m]t) + 1\right]\cos([\omega_c - \omega_m]t)$$

$$= \frac{1}{4}\left[\cos([3\omega_c + \omega_m]t) + \cos([\omega_c + 3\omega_m]t) + \cos([\omega_c - \omega_m]t)\right]$$

$\cos([\omega_c + 3\omega_m]t)$ has freq. around ω_c and differs from $\omega_c \pm \omega_m$

In other words, distortion can't be removed by bandpass filtering

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- Angle modulation introduced
- Bandwidth of Angle Modulated signals
- Properties of Angle Modulation
- Generation of FM signals

Generation of FM Signals

Two methods, indirect and direct

Indirect (Indirect method of Edwin H. Armstrong)

Narrow band FM $\left(k_f \int m(t)\right.$ is small)

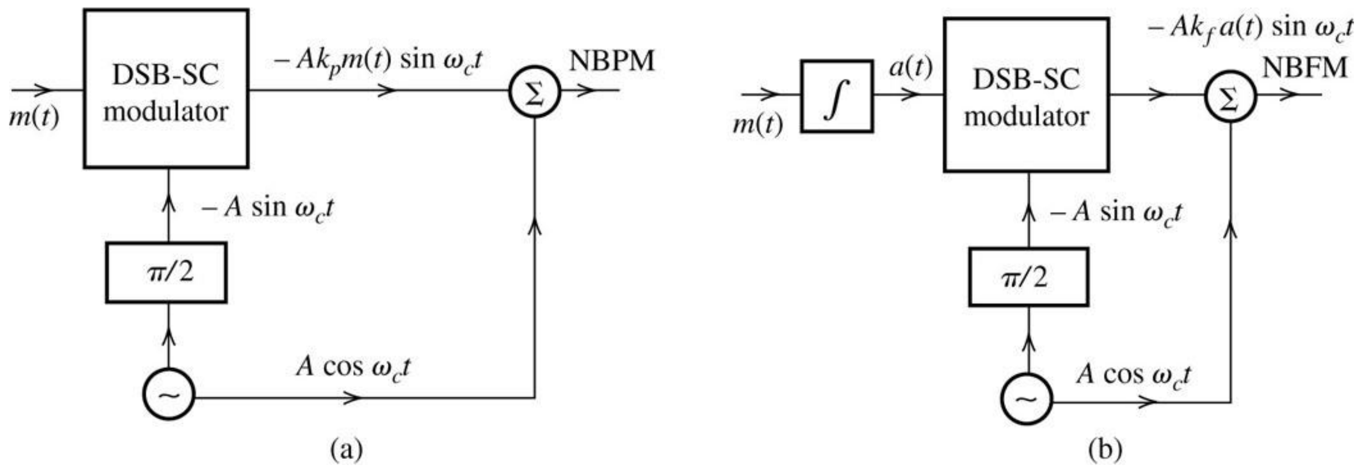


Figure 5.8 (a) Narrowband PM generator. (b) Narrowband FM signal generator.

Assume $k_p m(t)$ is small. $\sin(k_p m(t)) \approx k_p m(t)$, $\cos(k_p m(t)) \approx 1$ $\quad \Phi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$

$$\Phi_{PM}(t) = A\cos(\omega_c t)\cos(k_p m(t)) - A\sin(\omega_c t)\sin(k_p m(t)) \approx A\cos(\omega_c t) - k_p m(t) A\sin(\omega_c t)$$

Generation of FM Signals

Wide band FM $\left(k_f \int m(t)\right.$ is not small)

Modulate at a low carrier frequency, $\tilde{\Phi}_{PM}(t) = A\cos\left(\frac{\omega_c}{N}t + \frac{k_p}{N}m(t)\right)$ $\frac{k_p}{N}m(t)$ is small

Then pass through frequency multiplier to get $\tilde{\Phi}_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$

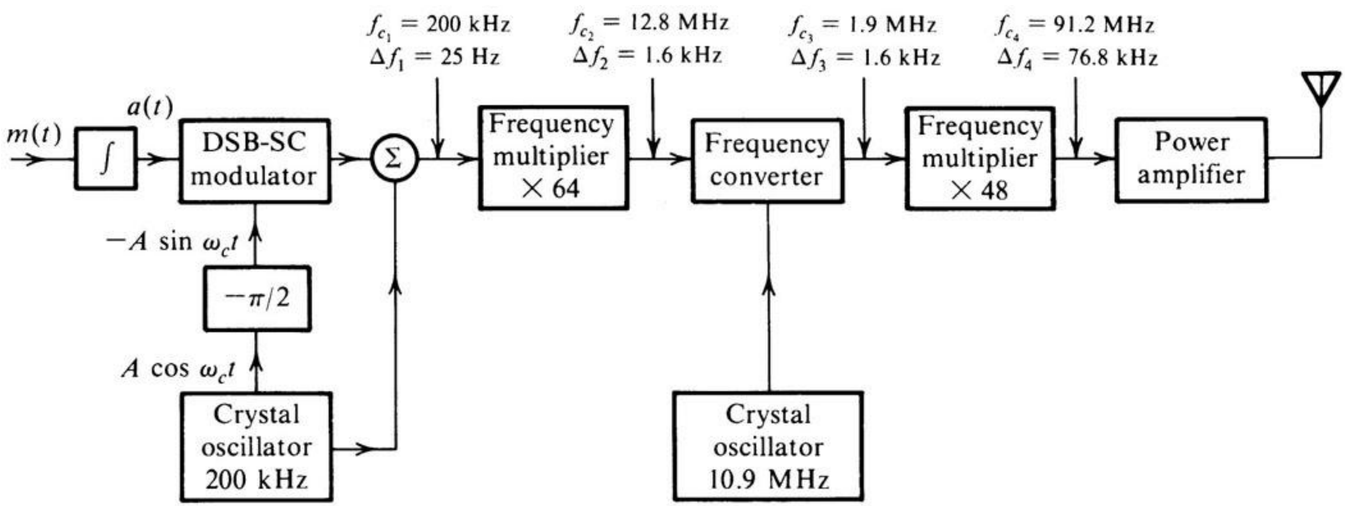


Figure 5.10 Block diagram of the Armstrong indirect FM transmitter.

Distortion Inherent to the Amstrong Method

Two kinds of distortions

1. Amplitude distortion
2. Frequency distortion

Note: narrow band assumption, $k_f m(t)$ is small

Armstrong method

$$\Phi_{FM}(t) = A\cos(\omega_c t) - \left(k_f \int m(t)\right) A\sin(\omega_c t) = AE(t)\cos(\omega_c t + \theta(t))$$

$$E(t) = \sqrt{1 + \left(k_f \int m(t)\right)^2} \neq 1 \qquad \theta(t) = \tan^{-1}\left[k_f \int m(t)\right] \neq k_f \int m(t)$$

Distortion caused by $E(t) \neq 1$ can be solved by passing the signal through an amplitude regulator (and then pass it through a bandpass filter)

Frequency distortion caused by $\theta(t) \neq k_f \int m(t)$ can be analyzed as follows

$$\frac{d\theta(t)}{dt} = \frac{d\tan^{-1}\left[k_f \int m(t)\right]}{dt} = \frac{k_f m(t)}{1 + \left(k_f \int m(t)\right)^2} = k_f m(t)\left[1 - \left(k_f \int m(t)\right)^2 + \left(k_f \int m(t)\right)^4 - \cdots\right]$$

Distortion Inherent to the Amstrong Method

Suppose $m(t) = \alpha \cos \omega_m t$

$$\frac{d\theta(t)}{dt} \approx k_f m(t)\left[1 - \left(k_f \int m(t)\right)^2\right] = k_f \alpha \cos \omega_m t - \frac{k_f{}^3 \alpha^3}{\omega_m{}^2} \cos \omega_m t \sin^2 \omega_m t$$

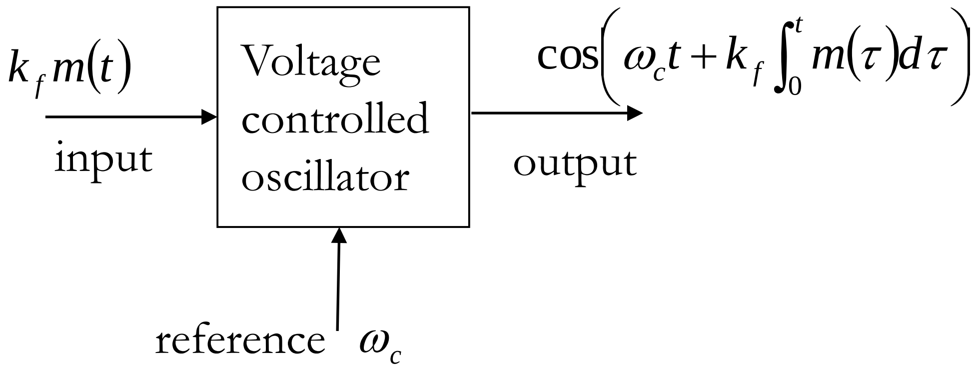
$$= k_f \alpha \cos \omega_m t - \frac{k_f{}^3 \alpha^3}{2\omega_m{}^2} \cos \omega_m t (1 - \cos 2\omega_m t) = k_f \alpha \cos \omega_m t - \frac{k_f{}^3 \alpha^3}{4\omega_m{}^2} (\cos \omega_m t - \cos 3\omega_m t)$$

$$= \underbrace{k_f \alpha \left(1 - \frac{k_f{}^2 \alpha^2}{4\omega_m{}^2}\right) \cos \omega_m t}_{\text{Scaled version of desired signal}} + \underbrace{\frac{k_f{}^3 \alpha^3}{4\omega_m{}^2} \cos 3\omega_m t}_{\text{distortion}}$$

Direct Generation of FM Signal

For FM, the instantaneous frequency is $\omega_i = \omega_c + k_f m(t)$

Hence can use VCO to generate the modulated waveform



The circuit is often more complicated than the indirect method.

Demodulation of FM Signals

Key idea $\omega_i = \omega_c + k_f m(t)$

$$\frac{d\Phi_{FM}(t)}{dt} = A[\omega_c + k_f m(t)]\sin\left(\omega_c t + k_f \int m(t)\right)$$

If $k_f m(t) < \omega_c$, which is often the case can get *m*(*t*) using envelope detection

If the amplitude of the received signal is not a constant, can pass it through a amplitude regulator and then a bandpass filter.

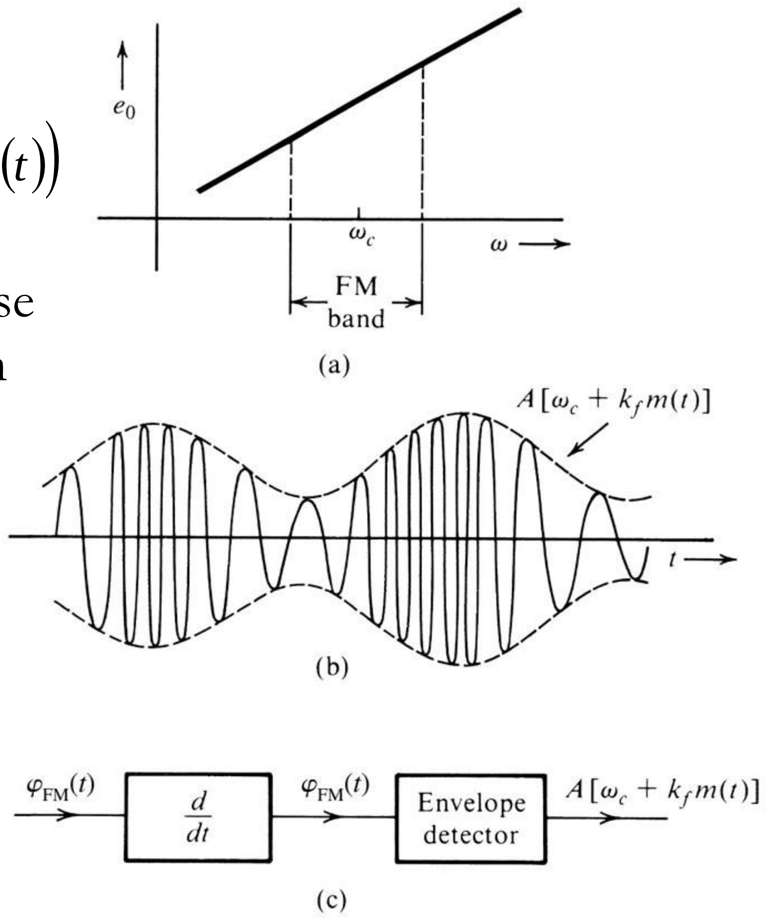
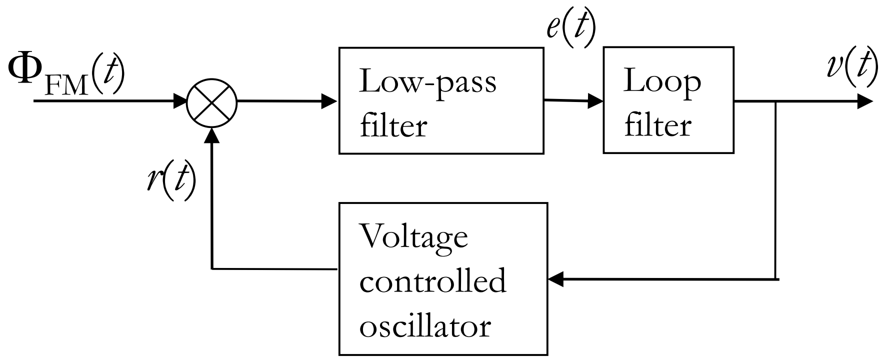
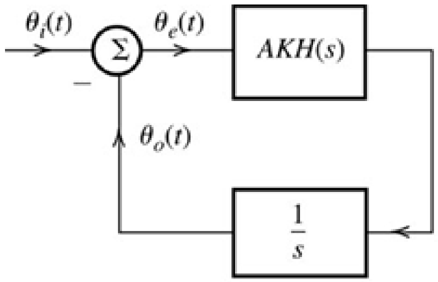


Figure 5.12 (a) FM demodulator frequency response. (b) Output of a differentiator to the input FM wave. (c) FM demodulation by direct differentiation.

Demodulate FM Signals using PLL (1)





(a)

Phase-shifted version to simplify computation

Suppose $\Phi_{FM}(t) = A_c \sin\left(\omega_c t + k_f \int m(t)\right) = A_c \sin\left(\omega_c t + \phi_1(t)\right)$

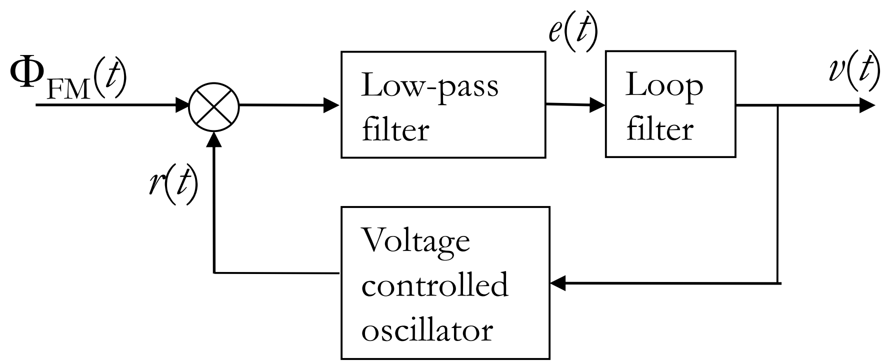
$$r(t) = A_v \cos\left(\omega_c t + \phi_2(t)\right) = A_v \cos\left(\omega_c t + k_v \int v(t)\right)$$

$$\Phi_{FM}(t) r(t) = A_c A_v \sin\left(\omega_c t + \phi_1(t)\right)\cos\left(\omega_c t + \phi_2(t)\right)$$

$$= \frac{1}{2} A_c A_v \sin\left(2\omega_c t + \phi_1(t) + \phi_2(t)\right) + \frac{1}{2} A_c A_v \sin\left(\phi_1(t) - \phi_2(t)\right)$$

After lowpass filter $\quad e(t) = \frac{1}{2} A_c A_v \sin\left(\phi_1(t) - \phi_2(t)\right)$

Demodulate FM Signals using PLL (2)



From the feedback path, we know

$$r(t) = A_v \cos(\omega_c t + \phi_2) = A_v \cos\left(\omega_c t + k_v \int v(t)\right)$$

$$v(t) = \frac{1}{k_v}\left(\frac{d\phi_2(t)}{dt}\right)$$

If $e(t)$ is kept roughly constant. $\phi_1(t) - \phi_2(t) \approx \text{const}, \frac{d\phi_1(t)}{\text{dt}} \approx \frac{d\phi_2(t)}{\text{dt}}$

Hence $v(t) \approx \frac{1}{k_v}\left(\frac{d\phi_1(t)}{dt}\right) = \frac{k_f}{k_v} m(t)$

Superheterodyne Analog FM/AM Receiver

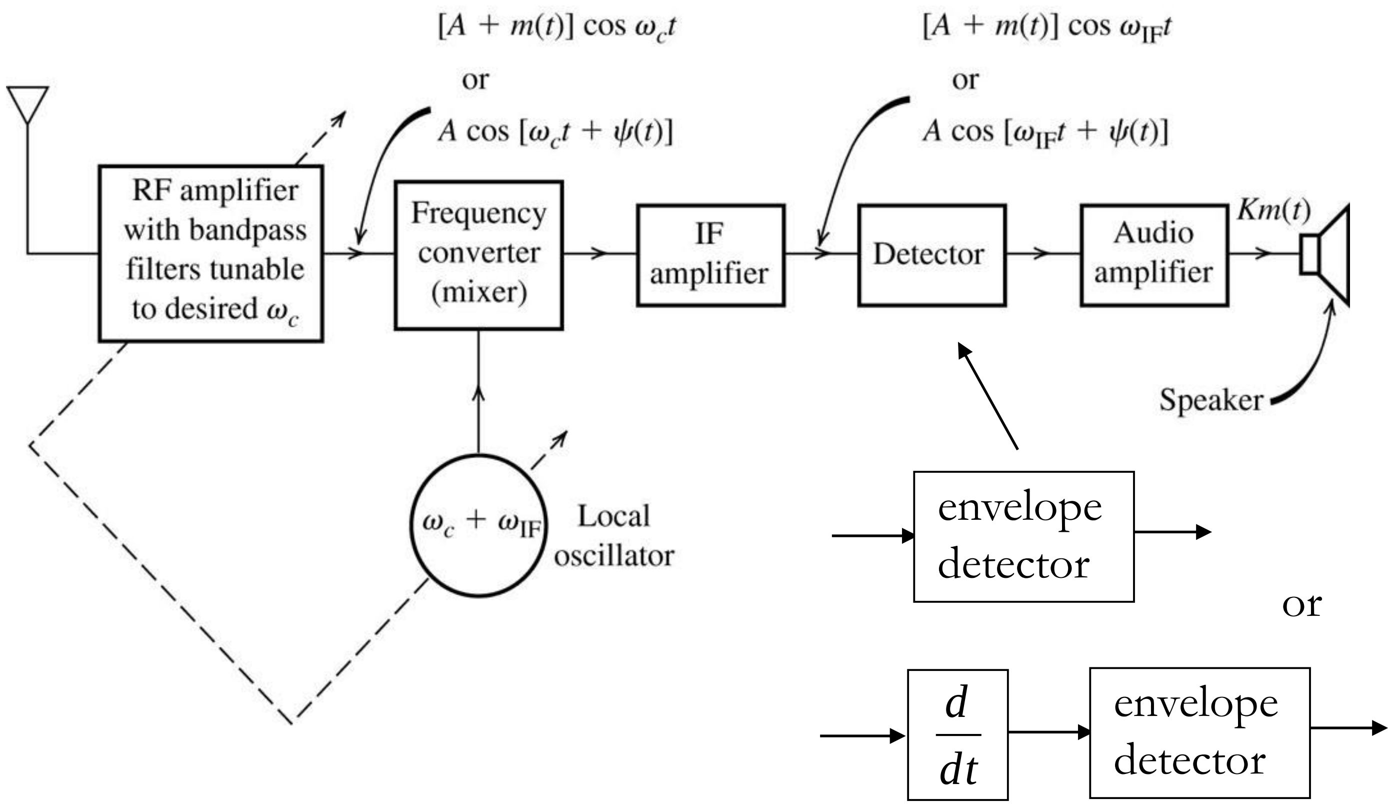


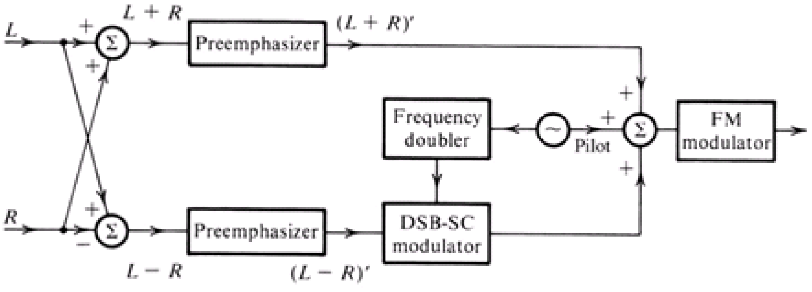
Figure 5.17 Superheterodyne receiver.

FM Stereo Radio Transmitter

General FM radio assumes signal in each channel has a bandwidth of 15KHz

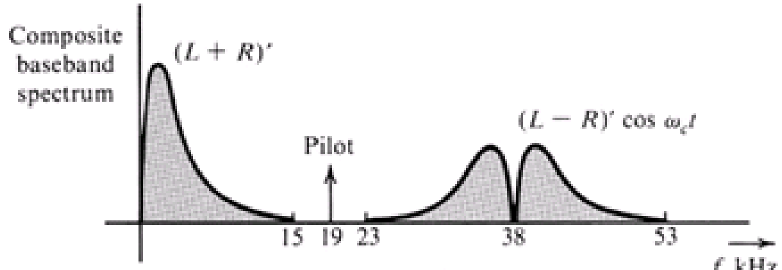
The mono signal is $m_l(t)+m_r(t)$. Use a sub-carrier of 38KHz to carry $m_l(t)-m_r(t)$

Then transmits $m_l(t)+m_r(t)$ & the modulated $m_l(t)-m_r(t)$ as the baseband signal



(a)

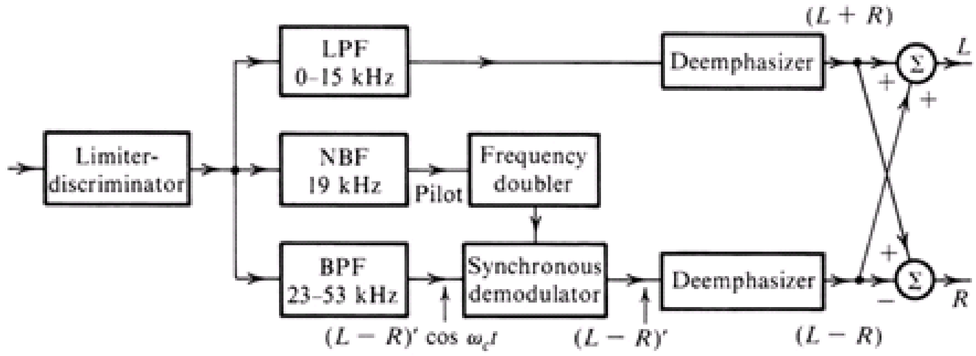
Note that the spectrum of $m_l(t)+m_r(t)$ & modulated $m_l(t)-m_r(t)$ do not overlap



(b)

FM Stereo Radio Transmitter

Receiver



(c)

Practical concerns:

1. Use $m_l(t)+m_r(t)$ as one baseband signal, to be compatible to mono FM receiver.

2. Provided pilot signal at 38KHz so that the receiver does not need to generate its own carrier signal.

(Think about receiving radio in a moving vehicle. Due to Doppler effect, the carrier frequency of the received signal may shift slightly)

Homework

- 5.1.-3, 5.2-1, 5.2-7