Case Study: Computer Graphics in Automotive Design

1 Introduction

This case study investigates how three-dimensional images are rendered effectively in two dimensions. This type of operation is very important in automotive design, for it allows engineers to experiment with images on a computer screen instead of using a three-dimensional model of the automobile.

The data for this project was derived from measurements made on a 1983 Toyota Corolla; all coordinates are measured in feet. The origin is placed at the center of the car. Data points are :
$(-6.5, -2, -2.5), (-6.5, -2, 2.5), (-6.5, .5, 2.5), (-6.5, .5, -2.5), (89 - 2.5, 5, -2.5), (-2.5, .5, 2.5),$
$(-.75, 2, -2.5), (-.75, 2, 2.5), (3.25, 2, -2.5), (3.25, 2, 2.5), (4.5, 5, -2.5), (4.5, .5, 2.5),$
$(6.5, 5, 2.5), (6.5, .5, 2.5), (6.5, -2, 2.5), (6.5, -2, -2.5).$
These data points are collected in a data matrix D: each column contains the z, y, and z coordinates of a particular data point. A fourth row containing all ones is attached to the matrix since homogeneous coordinates are being used

$$D = \begin{bmatrix} -6.5 & -6.5 & -6.5 & -6.5 & -2.5 & -2.5 & -0.75 & -0.75 & 3.25 & 3.25 & 4.5 & 4.5 & 6.5 & 6.5 & 6.5 & 6.5 \\ -2 & -2 & 0.5 & 0.5 & 0.5 & 0.5 & 2 & 2 & 2 & 2 & 0.5 & 0.5 & 0.5 & 0.5 & -2 & -2 \\ -2.5 & 2.5 & 2.5 & -2.5 & -2.5 & 2.5 & -2.5 & 2.5 & -2.5 & 2.5 & -2.5 & 2.5 & -2.5 & 2.5 & 2.5 & -2.5 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

This data matrix D accompanies this case study. In addition to the data points, how these data points are to be connected must be known. An adjacency matrix is used to record how the points are connected. This matrix consists only of 0's and 1's; the (i, j)-entry in the matrix is a 1 if and only if points i and j are connected. The adjacency matrix for the data of Toyota Corolla is

$$C = \begin{bmatrix} 0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&1&0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&1&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&1&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&1&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&1&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \end{bmatrix}.$$

One way to render a three-dimensional image in two dimensions is called a perspective projection, which is studied Section 2.7. In order to perform a perspective projection a center of projection (b, c, d) and a viewing plane must be identified. The center of projection is the position of the viewer's eye; the viewing plane is the plane onto which the image is projected. In the textbook, it is assumed that the center of projection has coordinates $(0, 0, d)$ and that the viewing plane is the $xy-$plane. In this case study it is also assumed that the viewing plane is the xy-plane, but any choice of center of projection (b, c, d) will be allowed.

2 Questions

1. (*15 points*) Find the perspective projection using the Toyota data in matrix D using $(b, c, d) = (-5, 10, 10)$ as center of projection. What is the matrix of the perspective projection? Present the outcome from your Python by attaching the figure of the vehicle viewed from $(-5, 10, 10)$.
2. (*15 points*) Find the perspective projection using the Toyota data in matrix D using $(b, c, d) = (0, 10, 25)$ as the center of projection. What is the matrix of the perspective projection? Present the outcome from your Python by attaching the figure of the vehicle viewed from $(0, 10, 25)$.
3. (*25 points*) Rotate the Toyota 30° about the y-axis, then perform the perspective projection with center of projection $(0, 10, 25)$. What is the matrix of the rotation? Present the outcome from your Python by attaching the figure of the rotated vehicle viewed from $(0, 10, 25)$. How does this figure compare with that in Question 2?
4. (*25 points*) Rotate the Toyota 45° about the z-axis, then perform the perspective projection with center of projection $(0, 10, 25)$. What is the matrix of the rotation? Present the outcome from your Python by attaching the figure of the rotated vehicle viewed from $(0, 10, 25)$. How does this figure compare with that in Question 2?
5. (*25 points*) Zoom in on the Toyota with a zoom factor of 150%, then perform the perspective projection with center of projection $(0, 10, 25)$. What is the matrix of the zoom? Present the outcome from your Python by attaching the figure of the zoomed vehicle viewed from $(0, 10, 25)$. How does this figure compare with that in Question 2?
6. (*5 points*) Include Python codes in plain text at end of your report.

3 Instructions for Submission

1. This is a group project. Each group consists of two students. Each group submit onr report.
2. Answer all questions in order. Write your report into a Word Document or a Google Document. Then convert it into a PDF file and submit the PDF file in Canvas.
3. (*50 points*) Submit your Python codes in a file with extension name py in Canvas. Your Python program will be tested when grading.
4. Each group submits the files before 11:00pm on Saturday, November 5.