Hypothesis Tests
Part 4: Two-Sample Tests about Central Tendency

DASC 512

Overview

- Paired samples tests
- Independent 2-sample test about mean (2-sample *t* test)
- Independent 2-sample test about median (Mann-Whitney U test)

Paired samples tests

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Independent vs. Paired Sampling

- Paired Samples are equally sized samples for which data points can be matched in a way that may be expected to reduce variance
  - Samples are correlated somehow, typically using the same observational unit under differing conditions
- Independent samples are two, non-correlated samples, possibly of different sizes

Independent vs. Paired Sampling

Independent

Example:

30 patients are given a placebo and 30 patients are given an experimental treatment. After a set time, insulin rates are measured.

Paired

Example:

30 patients’ insulin rates are measured before and after being given an experimental treatment.

Independent vs. Paired Sampling

Independent

Example:

New humidity sensors are installed a 54 locations across Colorado Springs. Humidity readings are measured at those and 96 legacy locations.

Paired

Example:

New humidity sensors are installed at 54 existing weather stations in Colorado Springs. Humidity readings are taken from both old and new sensors at each location.

How to perform a paired test

Given observations $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$

1. Let $d_i = x_i - y_i$.
2. Perform a one-sample test on $d_1, \ldots, d_n$.

It's that simple.

Example

- A random sample is taken of 5 wine companies in 2007. Sales volume is measured. The same measurement is taken again in 2008.

- Have sales volumes increased?

- $H_0: \mu_{2007} - \mu_{2008} = 0$
  $H_a: \mu_{2007} - \mu_{2008} < 0$

2007	2008
13,457	15,473
42,389	41,989
25,690	28,795
17,500	19,300
21,742	22,317

Two-sample *t* test

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Assumptions – 2-sample t-tests

- Type of data – Still quantitative
- Randomization – Data gathered randomly (iid)
- Population distribution – Assumes normality, but varies depending on
  - Pooled variance: Assumption that samples come from populations with the same variance (<u>homoscedasticity</u>)
  - Different variance: No such assumption (<u>heteroscedasticity</u>)
- Sample size – same concerns regarding Central Limit Theorem

Hypotheses

Let sample one be random variables $X_1, \ldots, X_{n_1}$ and sample two be random variables $Y_1, \ldots, Y_{n_2}$.

Null hypothesis

$$H_0: \mu_X - \mu_Y = D_0$$

Alternative hypothesis

$$H_a: \mu_X - \mu_y < D_0, \qquad H_a: \mu_X - \mu_Y > D_0, \qquad H_a: \mu_X - \mu_Y \neq D_0$$

Test Statistic – Independent Samples

Given observations $x_1, \ldots, x_{n_1}$ and $y_1, \ldots, y_{n_2}$,

$$\mathrm{t} = \frac{(\bar{x} - \bar{y}) - D_0}{s_{\bar{x}-\bar{y}}}, \qquad \text{where } s_{\bar{x}-\bar{y}} = \sqrt{\frac{s_x^2}{n_1} + \frac{s_y^2}{n_2}}$$

The sampling distribution has degrees of freedom equal to

$$\nu = \frac{\left(s_{\bar{x}-\bar{y}}^2\right)^2}{\frac{((s_x^2)/n_1)^2}{n_1 - 1} + \frac{\left((s_y^2)/n_2\right)^2}{n_2 - 1}},$$

rounded down.

Pooled Variance

If the variance of each sample is approximately equal, we can improve test precision by pooling the variance measure. The standard error becomes:

$$s_{\bar{x}-\bar{y}} = \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Where

$$s_p^2 = \frac{(n_1 - 1)s_x^2 + (n_2 - 1)s_y^2}{n_1 + n_2 - 2}$$

This sampling distribution has $\nu = n_1 + n_2 - 2$.

Example

A random sample is taken of wine companies in 2007 and 2008. Sales volume is measured.

Have sales volumes increased?

$$H_0: \mu_{2007} - \mu_{2008} = 0$$
$$H_a: \mu_{2007} - \mu_{2008} < 0$$

2007	2008
21,742	15,473
13,457	41,989
25,690	28,795
17,500	19,300
42,389	22,317
	27,315

Mann-Whitney U test

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Nonparametric 2-sample test

- When we have ordinal data or cannot make distributional assumptions, we can use the Mann-Whitney U Test
  - Sometimes confusingly called the Wilcoxon Rank-Sum Test
- Null hypothesis: Distributions of both populations are equal
- Alternative hypothesis: Distributions are not equal
- Can be interpreted as a test of difference in medians.

Mann-Whitney U Test

1. Rank order all values
2. Add up ranks from Sample 1
$$R_1 = 33$$
3. Calculate U_1, U_2
$$U_1 = R_1 - \frac{n_1(n_1+1)}{2} = 33 - 15 = 18$$
$$U_2 = n_1 n_2 - U_1 = 12$$
4. Test statistic $U = \min(U_1, U_2)$
$$U = 12$$
5. Compare to tables.
6. Just use Python from the start next time.

Year	Sales	Rank
2007	21,742	7
2007	13,457	11
2007	25,690	5
2007	17,500	9
2007	42,389	1
2008	15,473	10
2008	41,989	2
2008	28,795	3
2008	19,300	8
2008	22,317	6
2008	27,315	4

Recap

- Paired samples tests
- Independent 2-sample test about mean (2-sample *t* test)
- Independent 2-sample test about median (Mann-Whitney U test)