Probability
Part 1 - Introduction

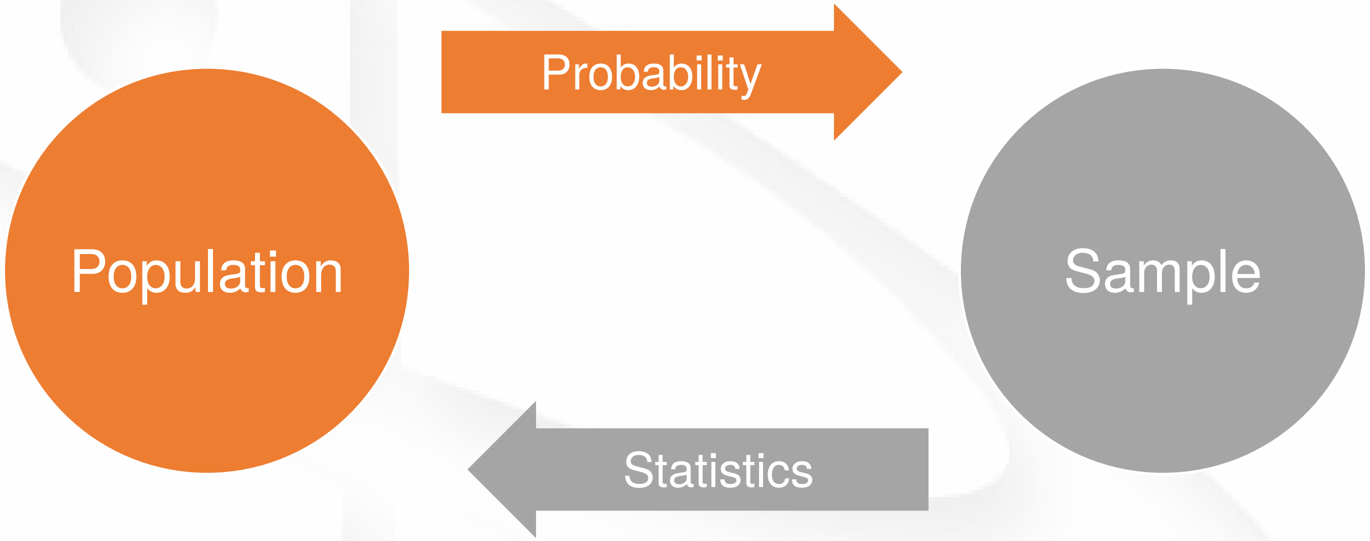
DASC 512

Overview

- Probability and Statistics
- Events
- Sets
- Venn Diagrams
- Rules of Probability

Probability and Statistics

- So far we have used sample data to describe something about the population
  - This is the field of "descriptive statistics"
- Now we'll use assumptions of the population to infer about a sample
  - This is the field of "probability"



Why Probability?

- Questions that we can answer using probability
  - How likely is it that we will destroy a target?
  - What is the chance that the backup system fails if the main system is down?
  - What are the odds of my team winning the big game?
  - Should I attack using my 2D6-damage sword or my 1D12-damage axe?
- Why do we use probability?
  - It allows us to make better decisions
  - Can often exploit known relationships to improve decisions further

Definitions

- Event: A specific set of possible outcomes of an experiment
- Simple Event: Set of a single possible outcome of an experiment
- Compound Event: Set of multiple simple events
- Sample Space: The set of all possible outcomes of an experiment
- p_i: The probability of the ith event in the sample space occurring
- $P(A)$: The probability of any outcome in the event A occurring

Example: Rolling a 6-sided die

	A={6}	A={4,5,6}	A={2,4,6}	A={1,2,3,4,5,6}	A={}
Event					
Simple Event					
Compound Event					
Sample Space					

Example: Rolling a 6-sided die

	A={6}	A={4,5,6}	A={2,4,6}	A={1,2,3,4,5,6}	A={}
Event	X				
Simple Event	X				
Compound Event					
Sample Space					

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Compound Event		X			
Sample Space					

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Event	X	X	X	X	
Simple Event	X				
Compound Event		X	X	X	
Sample Space				X	

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Event	X	X	X	X	X
Simple Event	X				
Compound Event		X	X	X	
Sample Space				X	

What is Probability

Probability is a map of events from a sample space to the interval [0,1]

- The probability of any event must lie between 0 and 1

$$0 \leq p \leq 1$$

- The total probability of the entire sample space is 1

$$\sum_{i=1}^{|S|} p_i = 1$$

Calculating Probability (Discrete Events)

- Steps for calculating probabilities of events:
  - Define the experiment (type of observation, collection method, etc.)
  - List all simple events in the sample space
  - Assign probabilities to those simple events
  - Determine which simple events are contained in the event of interest
  - Add the simple event probabilities to get the event probability

Dice Example

- Example: Roll two fair 6-sided dice. What is the probability of rolling an 8?

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(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Dice Example

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(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	**(2,6)**
(3,1)	(3,2)	(3,3)	(3,4)	**(3,5)**	(3,6)
(4,1)	(4,2)	(4,3)	**(4,4)**	(4,5)	(4,6)
(5,1)	(5,2)	**(5,3)**	(5,4)	(5,5)	(5,6)
(6,1)	**(6,2)**	(6,3)	(6,4)	(6,5)	(6,6)

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(3,1)	(3,2)	(3,3)	(3,4)	**(3,5)**	(3,6)
(4,1)	(4,2)	(4,3)	**(4,4)**	(4,5)	(4,6)
(5,1)	(5,2)	**(5,3)**	(5,4)	(5,5)	(5,6)
(6,1)	**(6,2)**	(6,3)	(6,4)	(6,5)	(6,6)

$$P(8) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{5}{36} \approx 0.14$$

Kids Example

- I’m talking to another dad at the park the other day, and he tells me he has two kids. What is the probability that at least one of his kids is a boy?

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	Eldest = Girl	Eldest = Boy
Youngest = Girl	¼	¼
Youngest = Boy	¼	¼

Kids Example

- I'm talking to another dad at the park the other day, and he tells me he has two kids. What is the probability that at least one of his kids is a boy?

	Eldest = Girl	Eldest = Boy
Youngest = Girl	¼	¼
Youngest = Boy	¼	¼

$$P(B) = P(BB) + P(BG) + P(GB) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} = 0.75$$

Sets

- Let *A* and *B* be events.
  - <u>Union</u> of *A* and *B* $(A \cup B)$: Occurs if outcome is in *A*, *B*, or both
  - <u>Intersection</u> of *A* and *B* $(A \cap B)$: Occurs if outcome is in both *A* and *B*
  - <u>Complement</u> of *A* (A^c): Occurs if outcome is not in *A*

Boolean Logic

And / Intercept / $A \cap B$

	B=False	B=True
A=False	False	False
A=True	False	True

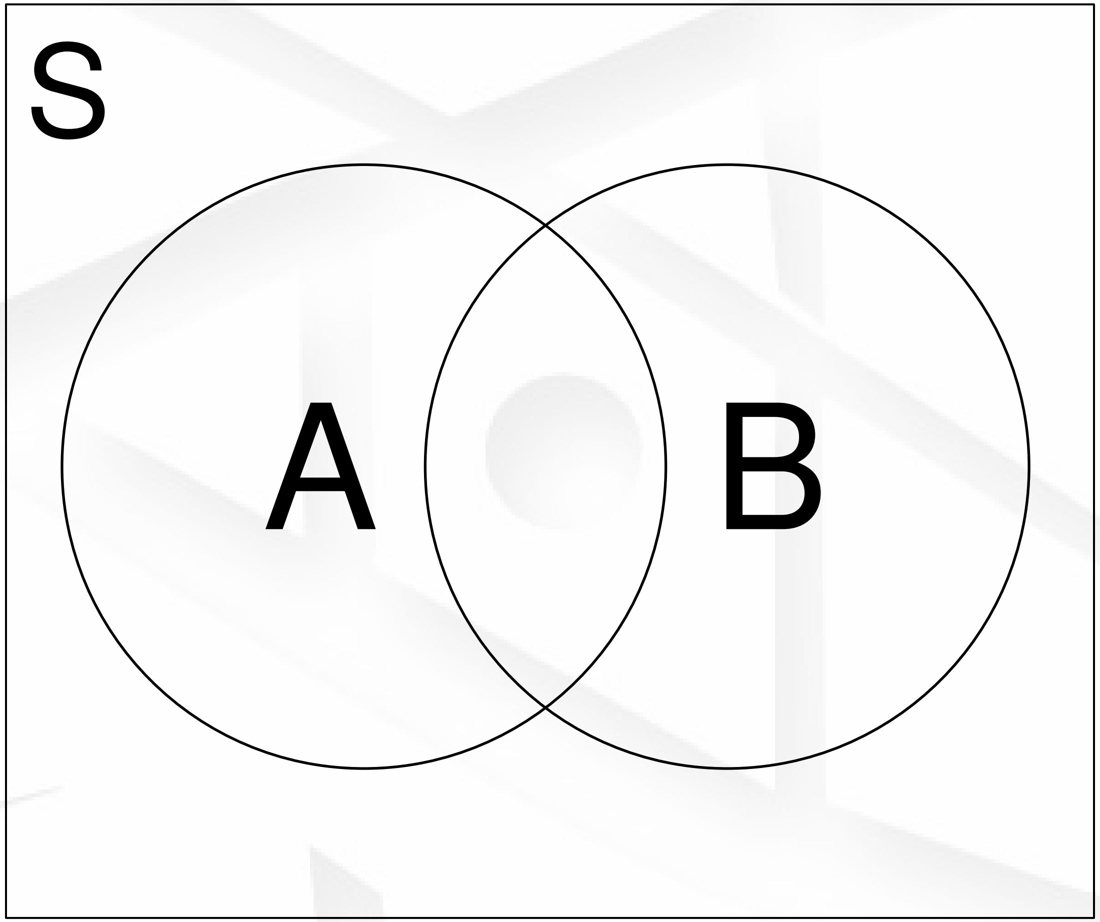
Or / Union / $A \cup B$

	B=False	B=True
A=False	False	True
A=True	True	True

Not / Complement / A^C

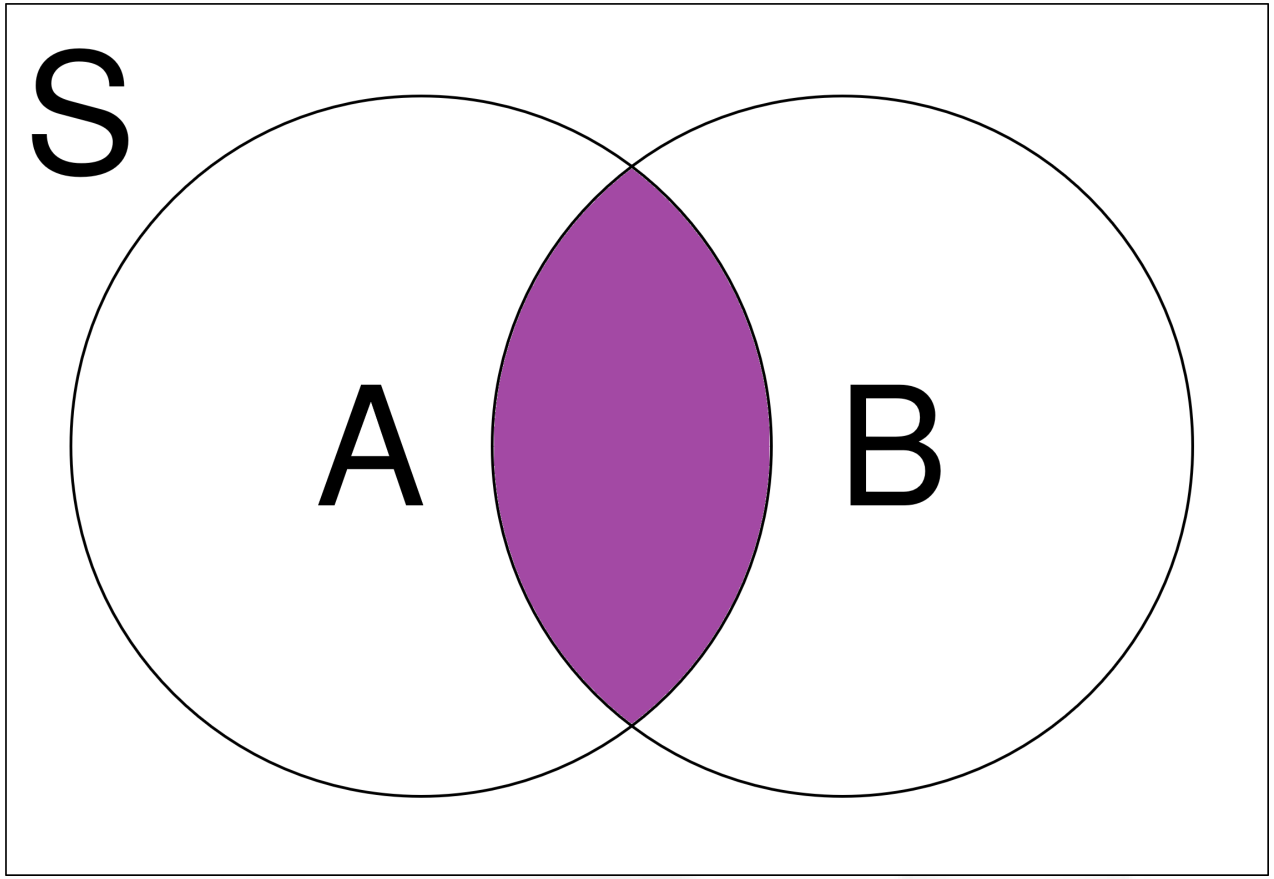
A=False	A^c=True
A=True	A^c=False

Venn Diagrams

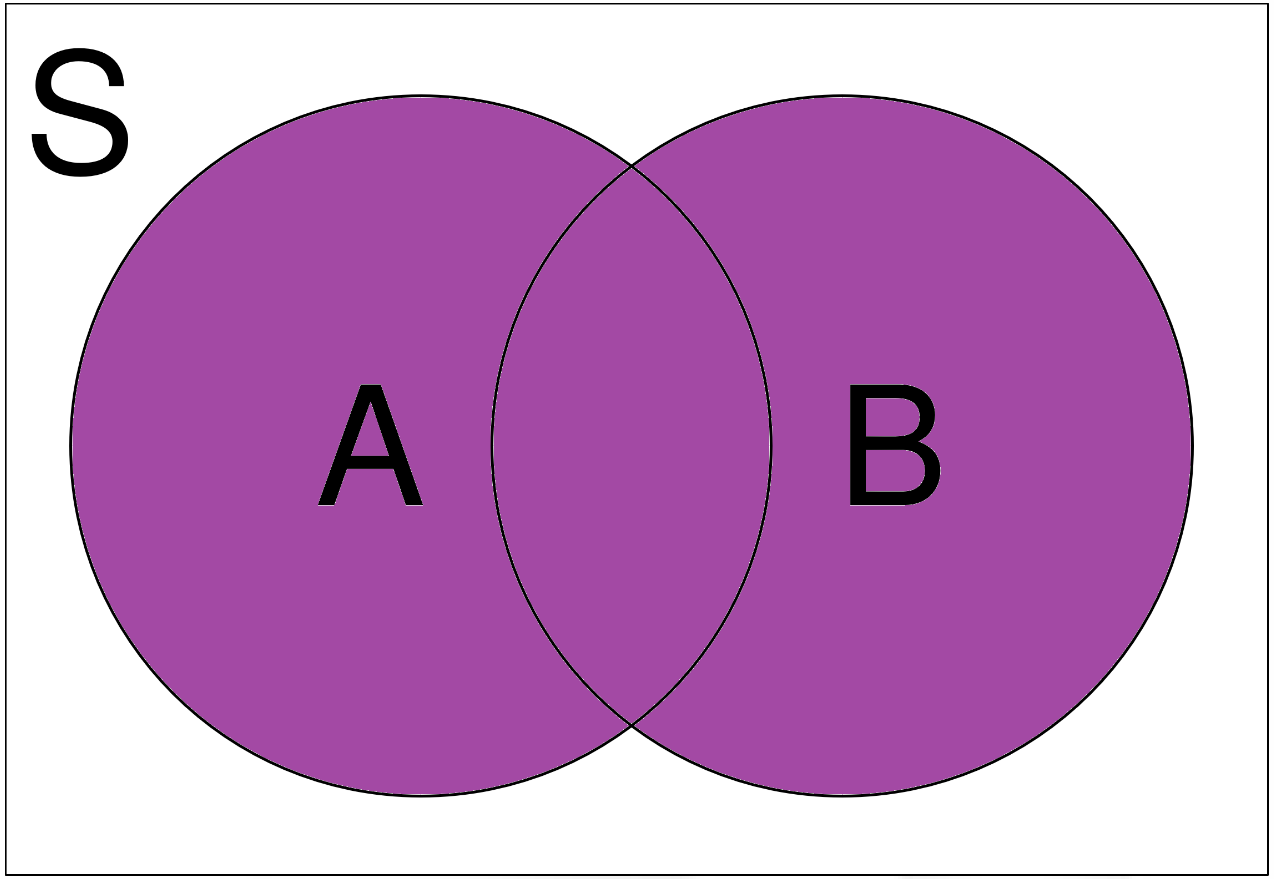


- The area of a section of the Venn diagram represents the probability of that event
- The outermost box represents the sample space S
- $P(S) = 1$

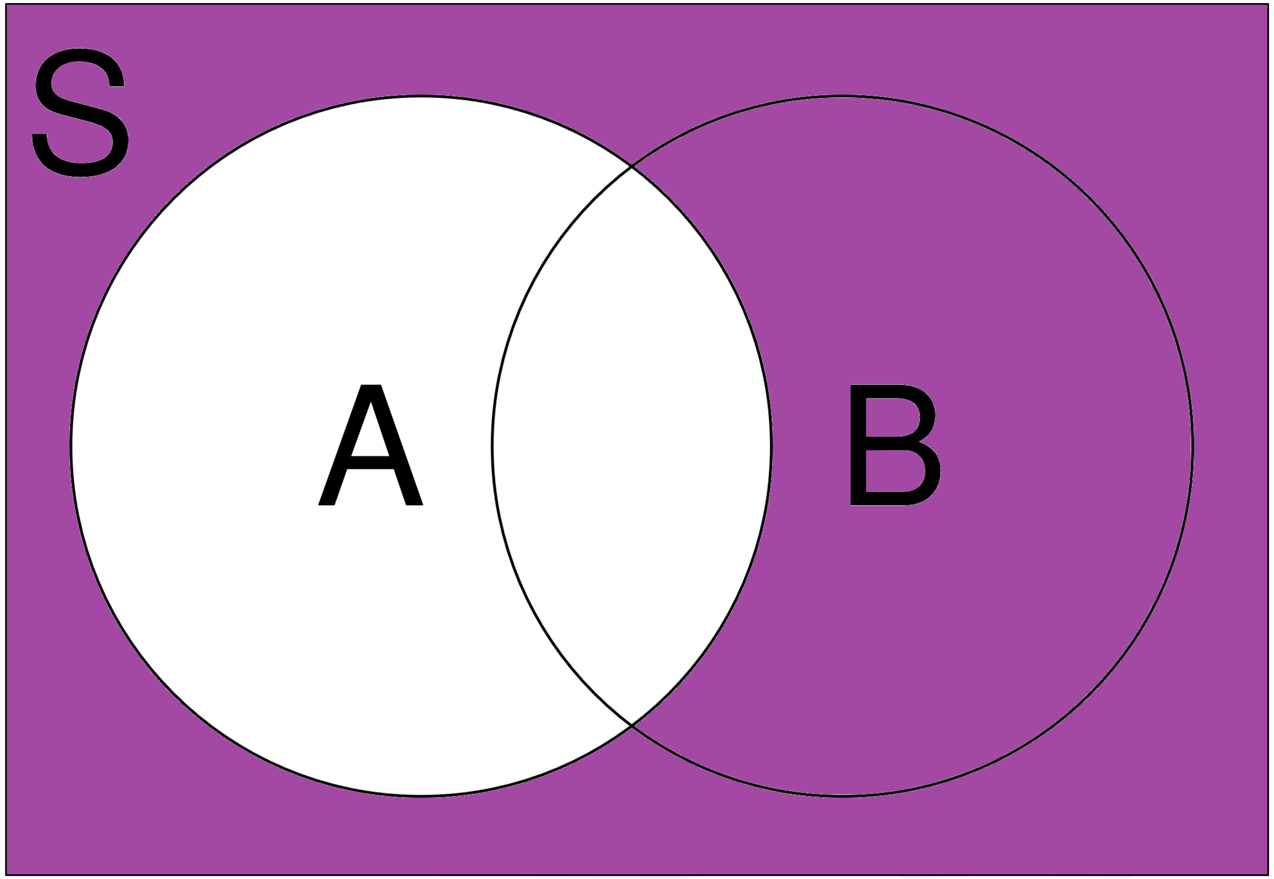
A and B – $A \cap B$ – Intersection



A or B – $A \cup B$ – Union



Not A – A^c – Complement



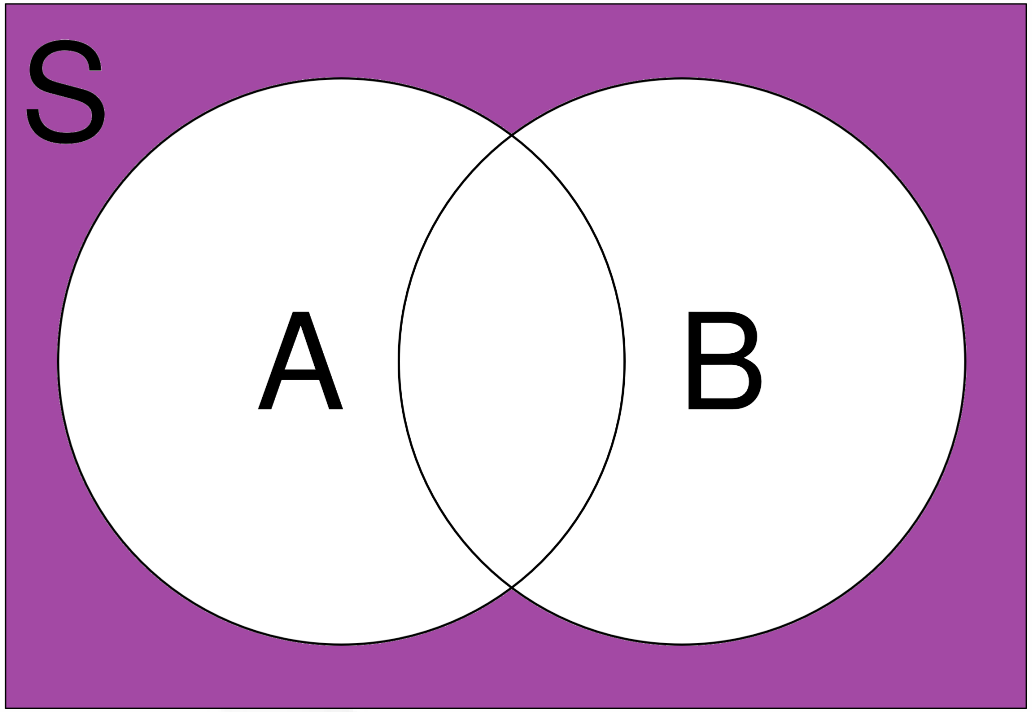
Combining Set Operators

- Try writing the English phrase and drawing Venn diagrams of the following:
  - $(A \cup B)^C$
  - $(A \cap B)^C$
  - $A \cap (B^C)$

- Pause the video and work this on your own

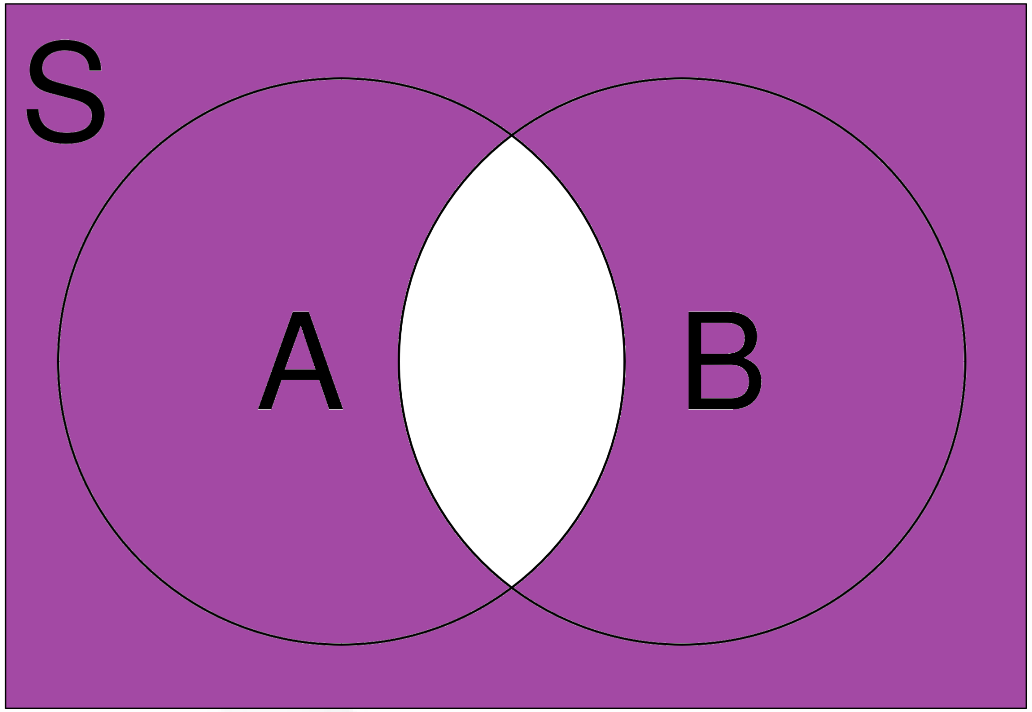
$(A \cup B)^C$

- Neither A nor B
- This is equivalent to $A^C \cap B^C$



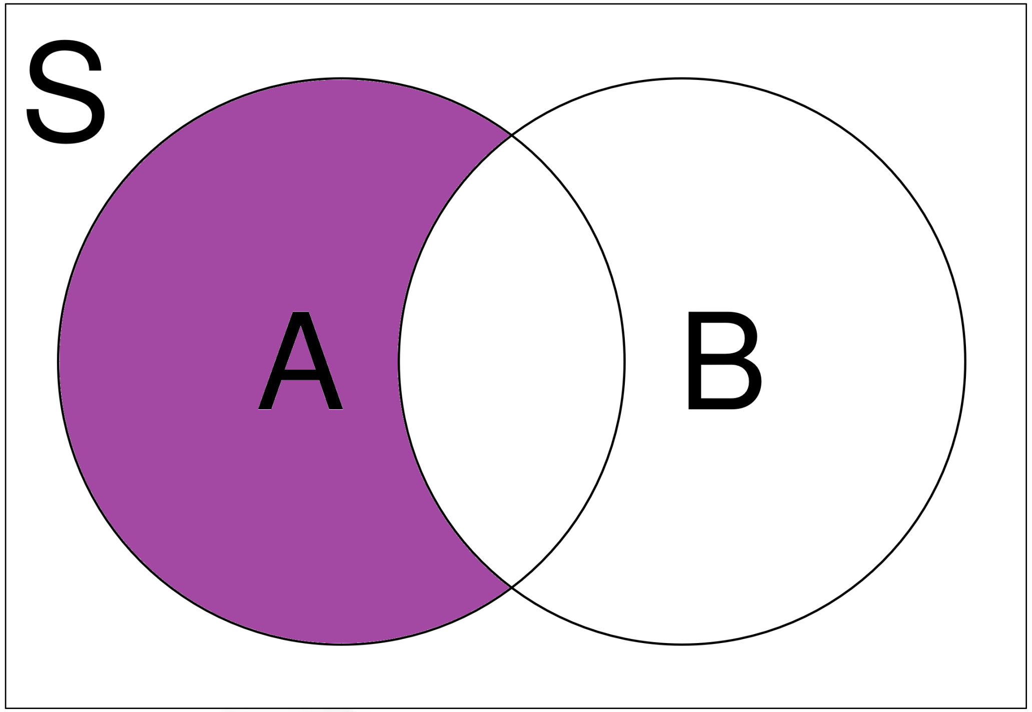
$(A \cap B)^C$

- Not both A and B
- This is equivalent to $A^C \cup B^C$



$$A \cap (B^C)$$

- A but not B



More Definitions

- Marginal Probability: Probability as a function of one event:

$$P(A = A_i) = P(A_i) = p_i$$

- Joint Probability: Joint probability is a function of two events:

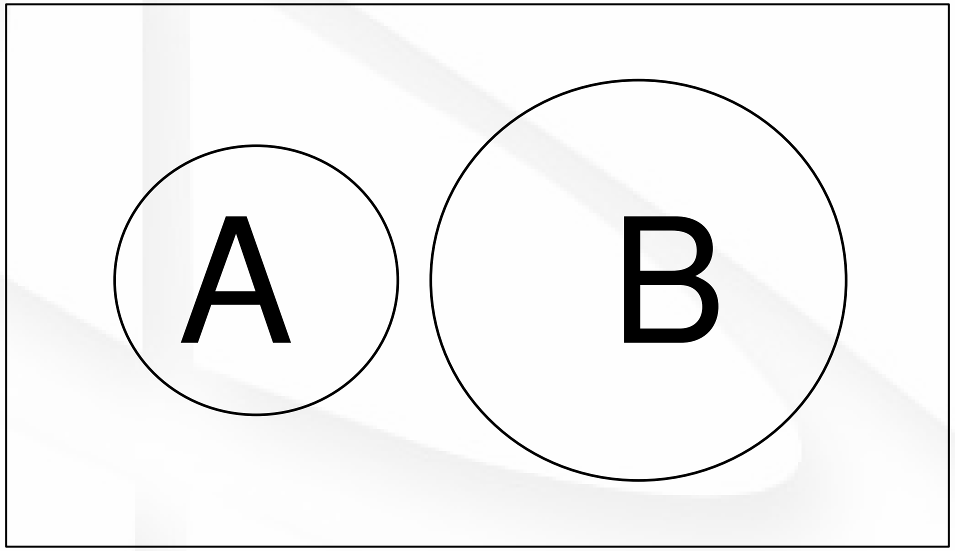
$$P\left((A = A_i) \cap \left(B = B_j\right)\right) = P\left(A_i \cap B_j\right) = p_{i,j}$$

More Definitions

- Disjoint (Mutually Exclusive): Events that have no overlap are disjoint. Mathematically, let A be a set of n disjoint events A_i, $i = 1, \ldots, n$.

$$A_i \cap A_j = \{\ \}, i \neq j$$

- Example: A and B below are disjoint events.

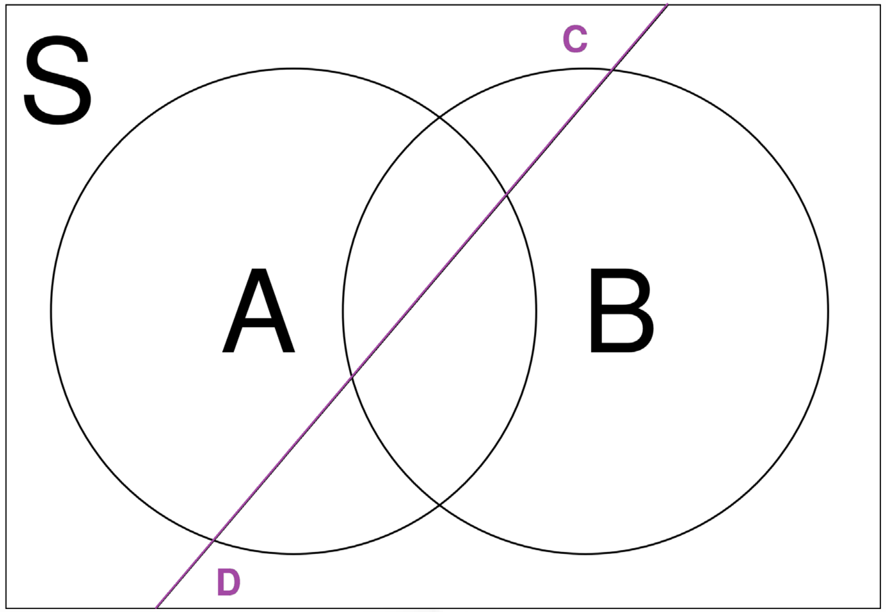


More Definitions

- Collectively Exhaustive: Events that completely cover the sample space are collectively exhaustive.
  Mathematically, let A be a set of n collectively exhaustive events A_i.

$$\bigcup_{i=1}^{n} A_i = S$$

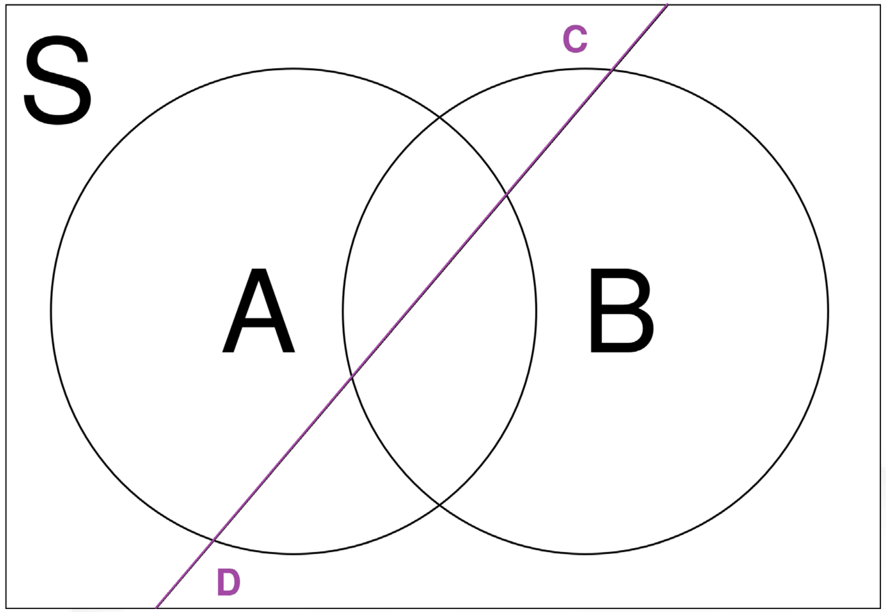
- Example: $E = \{A, B, C, D\}$ is a collectively exhaustive set of events.



More Definitions

- Partition: A set of events that is both disjoint and collectively exhaustive is said to form a partition of the sample space.

- Example: C and D form a partition of the sample space S.

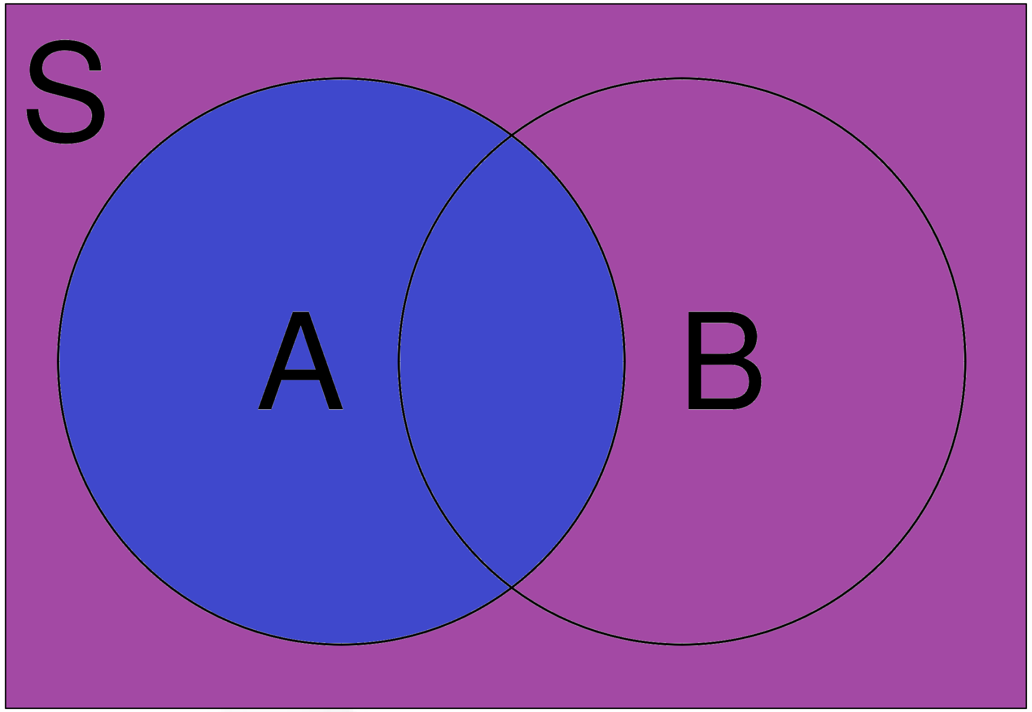


Probability Rules
Complement

Let A and B be events in the sample space S

$$P(A) + P(A^C) = 1$$

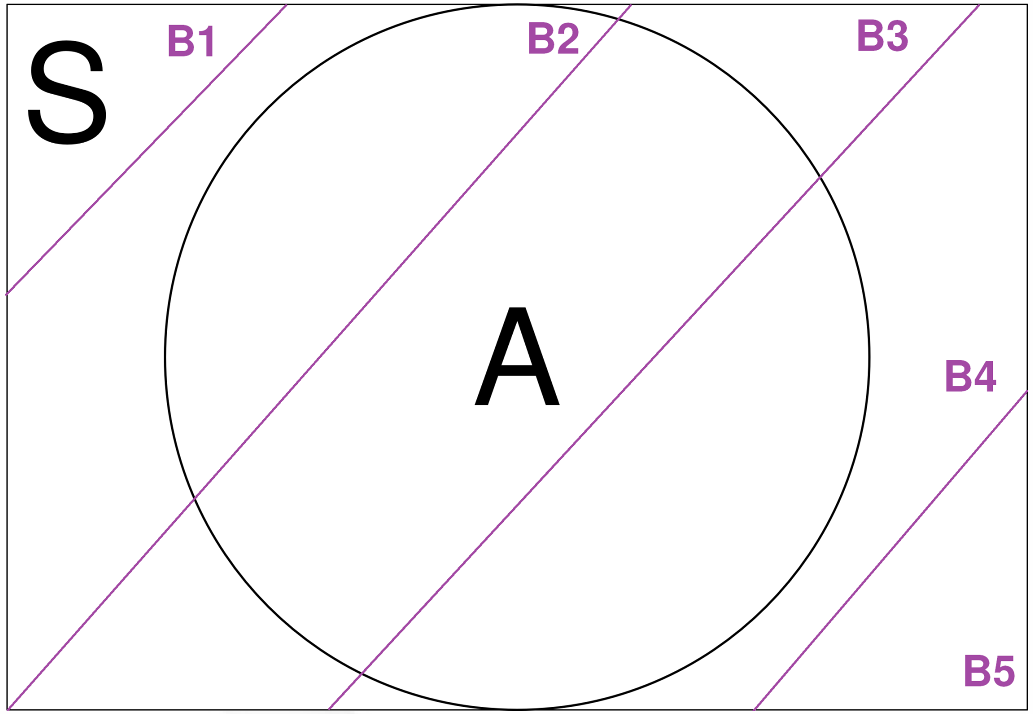
Any event and its complement form a partition.



Probability Rules
Law of Total Probability

Let A be an event in the sample space S and let $B = \{B_1, \ldots, B_n\}$ form a partition of S.

$$P(A) = \sum_{n} P(A \cap B_n)$$

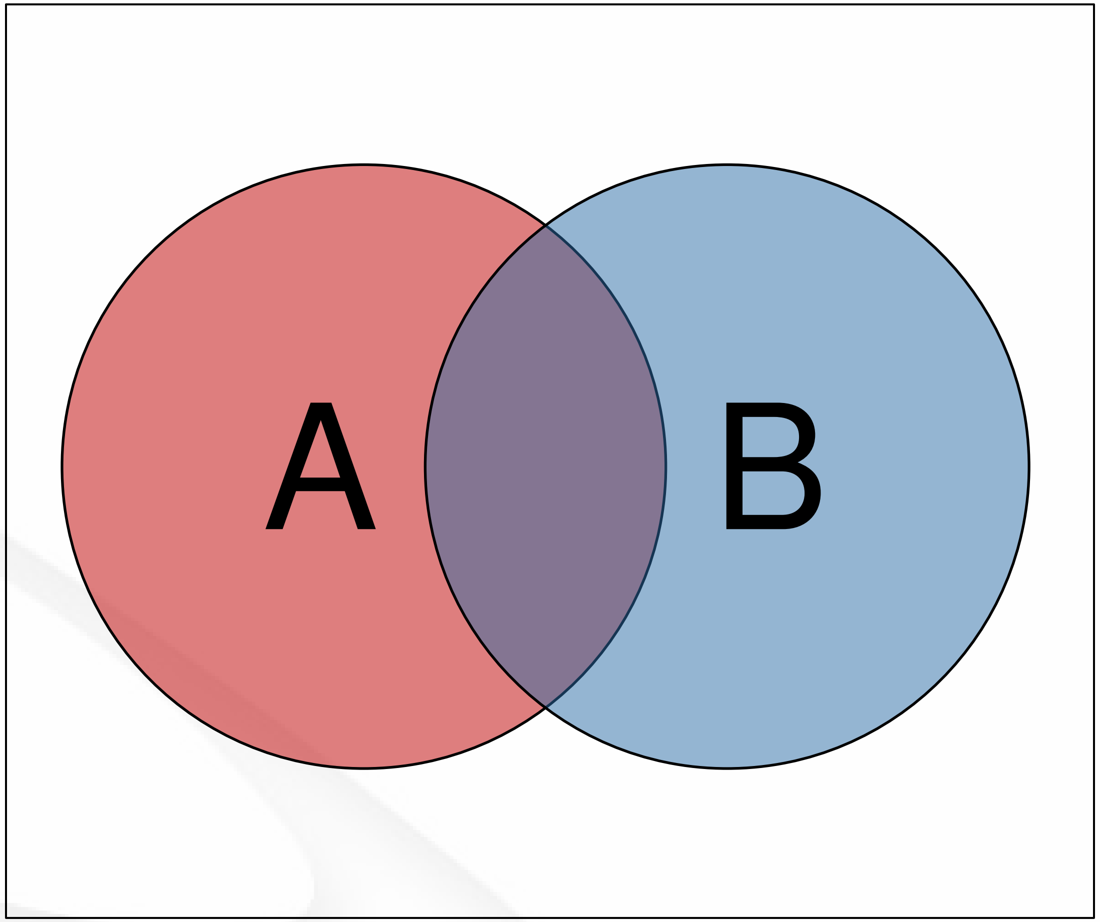


Probability Rules

Additive Rule

Let A and B be events in the sample space S

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

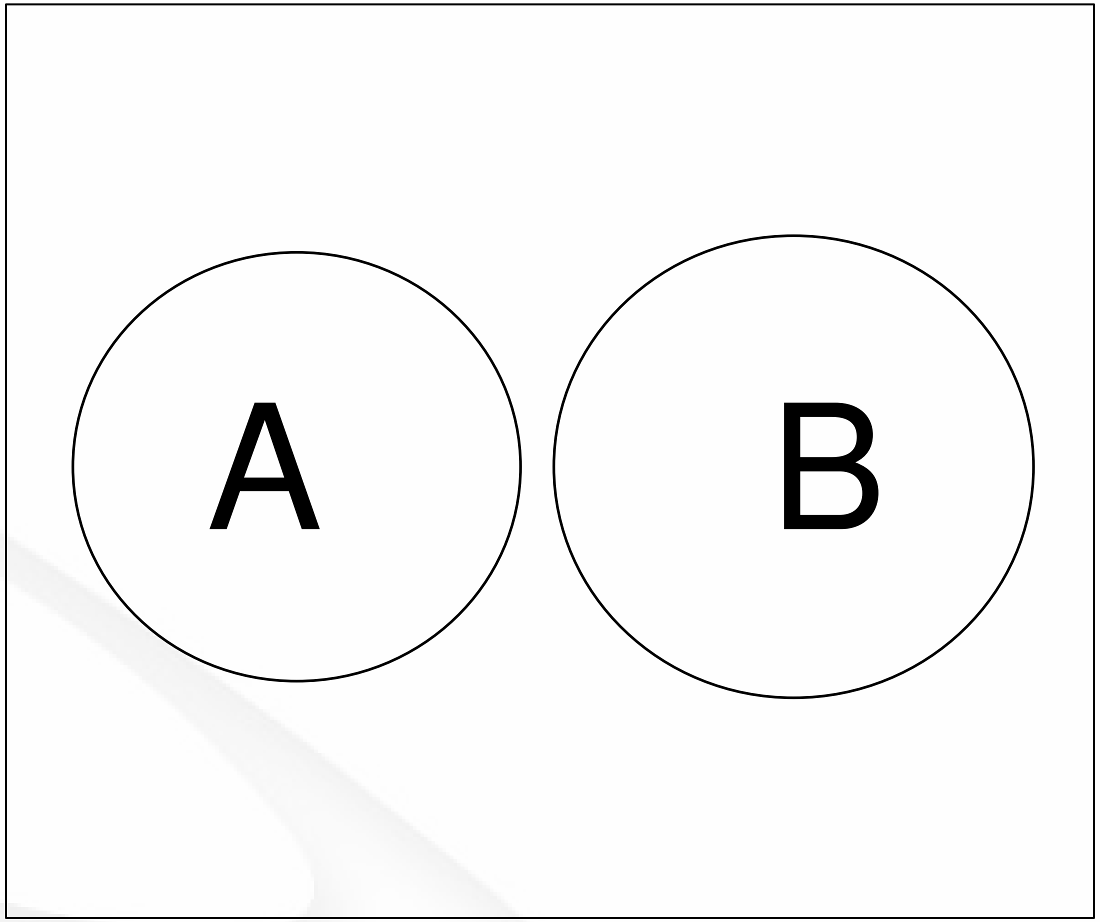


Probability Rules
Disjoint Events

Let A and B be events in sample space S

If, and only if, A and B are disjoint,

$$P(A \cap B) = 0$$

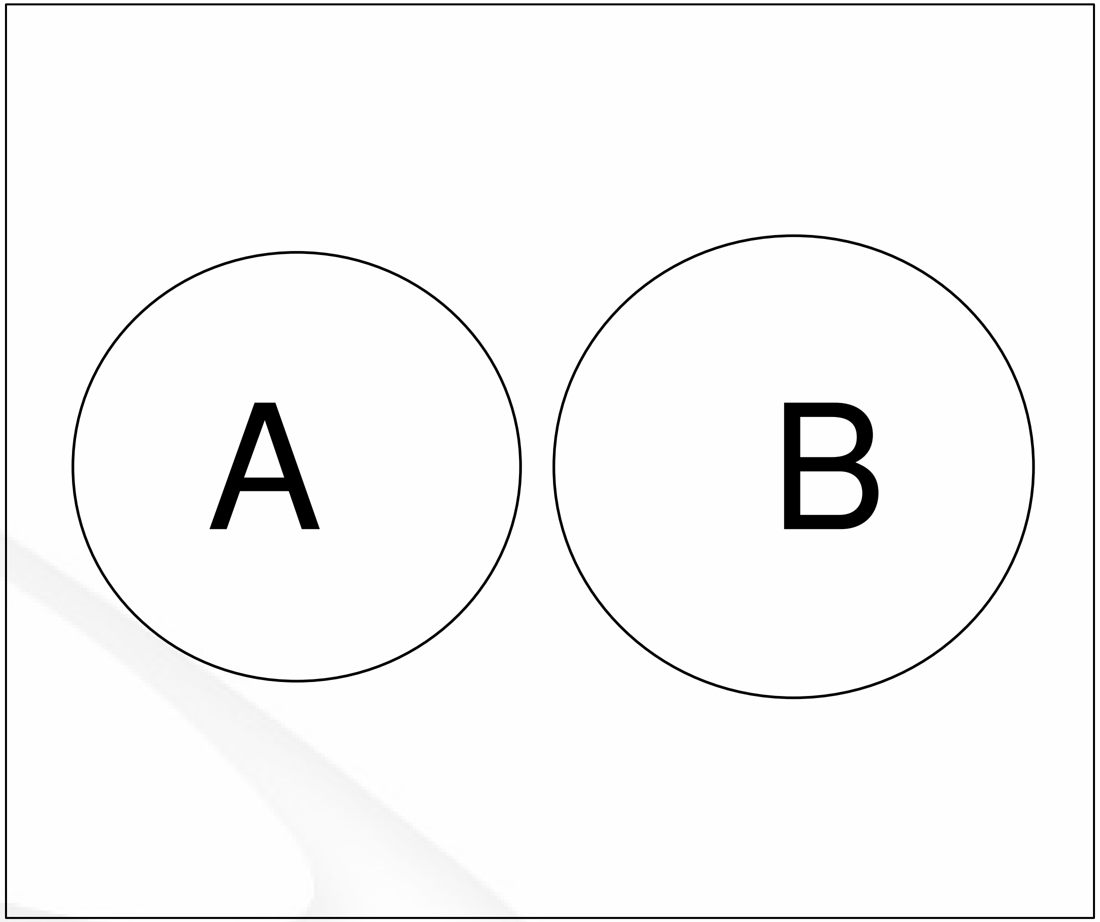


Probability Rules
Additive Rule – Special Case for Disjoint Events

Let A and B be events in sample space S

If, and only if, A and B are disjoint,

$$P(A \cup B) = P(A) + P(B)$$



Example

- A sample of Google Play Store apps in 2019 had the following attributes:

	Everyone	Ages 10+	Teen 13+ (A)	Mature 17+	Total
Free	8019	380	1156	479	**10034**
Paid (B)	695	33	52	20	**800**
Total	**8714**	**413**	**1208**	**499**	**10834**

- Let us define the following events, if we randomly select an app.
  - *A*: The app is rated Teen 13+.
  - *B*: The app is not free.
- Convert this contingency table into a relative frequency table.

https://www.kaggle.com/lava18/google-play-store-apps

Example

- A sample of Google Play Store apps in 2019 had the following attributes:

	Everyone	Ages 10+	Teen 13+ (A)	Mature 17+	Total
Free	0.7402	0.0351	0.1067	0.0442	**0.9262**
Paid (B)	0.0641	0.0030	0.0048	0.0018	**0.0738**
Total	**0.8043**	**0.0381**	**0.1115**	**0.0461**	**1**

- Calculate the following:
  - P(A)
  - P(B)

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- Calculate the following:
  - $P(A) = 0.1115$
  - $P(B) = 0.0738$

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- What is the probability that the app is both free and rated Teen 13+?

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- What is the probability that the app is both free and rated Teen 13+?

- $P(A \cap B^c) = 0.1067$

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- What is the probability that the app is either paid or rated Teen 13+?

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- What is the probability that the app is either paid or rated Teen 13+?

- $$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.1115 + 0.0738 - 0.0048$$
$$= 0.1805$$

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Recap

- Probability and Statistics
- Events
- Sets
- Venn Diagrams
- Rules of Probability