

Problem 1: 30 points

Each day a hospital records the number of people who come to the emergency room for treatment.

(a)

In a particular week, the arrivals to the ER are:

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|--------|--------|---------|-----------|----------|--------|----------|
| 10 | 8 | 14 | 7 | 21 | 44 | 60 |

Do you think that the Poisson distribution might describe the distribution of the number of daily arrivals (i.e., using an interval of one day)? Why or why not? Use graphical and/or analytical probability results to justify your answer.

(b)

Building upon your answer the previous question, would you expect the Poisson distribution to better describe, or more poorly describe, the total number of *weekly* admissions to the ER (i.e., using an interval of one week)? Why?

Problem 2: 35 points

Lake Wobegon Junior College admits students only if they score above 400 on a standardized achievement test. Applicants from Group A have a mean of 500 and a standard deviation of 100 on this test, and applicants from Group B have a mean of 450 and a standard deviation of 100. Both distributions are approximately normal, and both groups have the same size.

Note: This problem builds on examples from the Progress Checks. If you haven't done those, this will be far more difficult.

(a)

About 45% of accepted students come from Group B with the remainder coming from Group A (See Progress Check 14).

Assume that students with a score above 600 will decline their admission to Lake Wobegon Junior College in favor of another school. What is the expected proportion of those students who both (1) are accepted and (2) choose to attend who come from Group B?

(b)

Expanding on the last part, create a plot of the proportion of students from Group B as a function of the upper score cutoff (i.e., the score above which students will choose another school). At what upper cutoff score do you expect the proportion of students from each group to be equal?

Problem 3: 35 points

Demonstrate that the Central Limit Theorem works on a Poisson distributed variable with $\lambda = 2$. Use this as the population distribution for all sub-parts.

(a)

Simulate taking the mean of 2 observations of the variable 1000 times. Make a histogram and boxplot of the means. Comment on the approximate ‘normality’ of the sampling distribution.

(b)

Simulate taking the mean of 10 observations of the variable 1000 times. Make a histogram and boxplot of the means. Comment on the approximate ‘normality’ of the sampling distribution.

(c)

How large does n (the number of observations used to calculate the mean) need to be for the sampling distribution to **approximate** normality? Justify your answer. Use logic to argue your point augmented by math as needed.