Continuous Distributions
Part 2

DASC 512

Overview

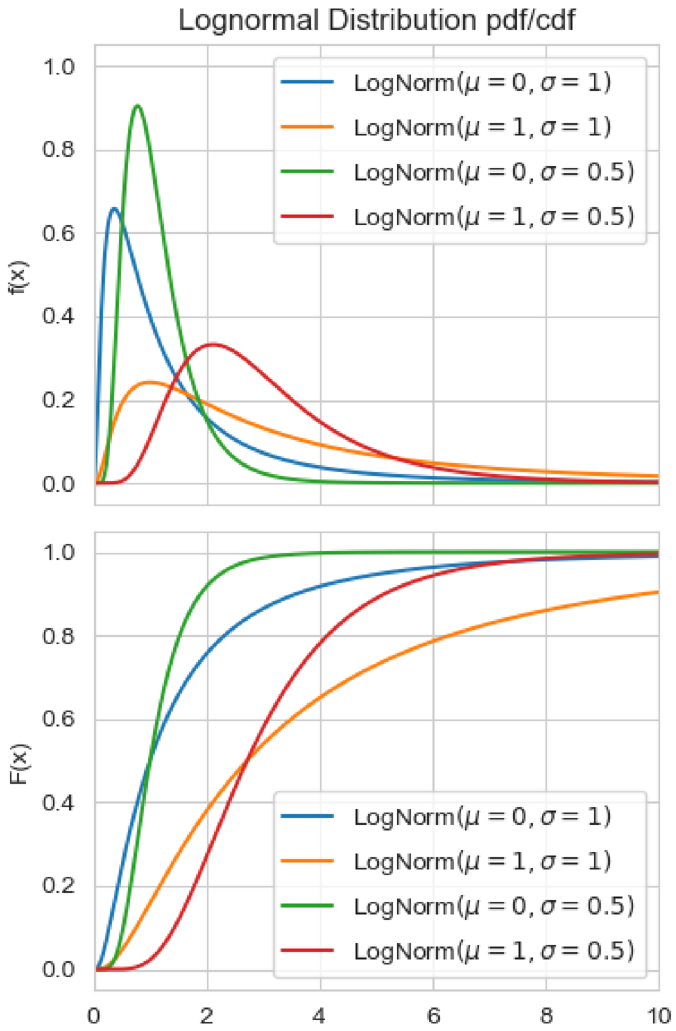
- Normal distribution
- Student's *t* distribution
- Chi Squared distribution
- F distribution
- Lognormal distribution
- Exponential distribution
- Beta distribution
- Uniform distribution
- Other distributions

Lognormal Distribution

Lognormal is a common distribution when something is approximately normally distributed but can only take positive values.

Like the χ^2 distribution, it is derived from the Standard Normal (Z)

Note that μ and σ are not the mean and standard deviation for this distribution.



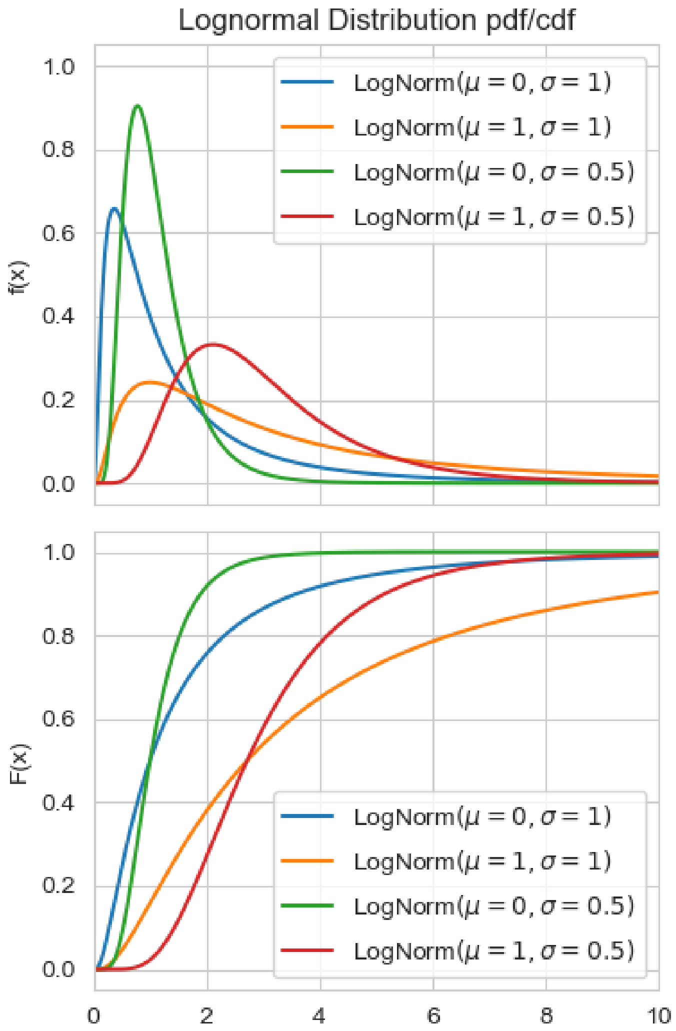
Lognormal Distribution

$$X \sim \text{LogNorm}(\mu, \sigma)$$

$$X = e^{\mu + \sigma Z} \sim \text{LogNorm}(\mu, \sigma),$$
where $Z \sim N(0,1)$

Mean: $e^{\mu + \frac{\sigma^2}{2}}$

Variance: $\left(e^{\sigma^2} - 1\right)e^{2\mu + \sigma^2}$

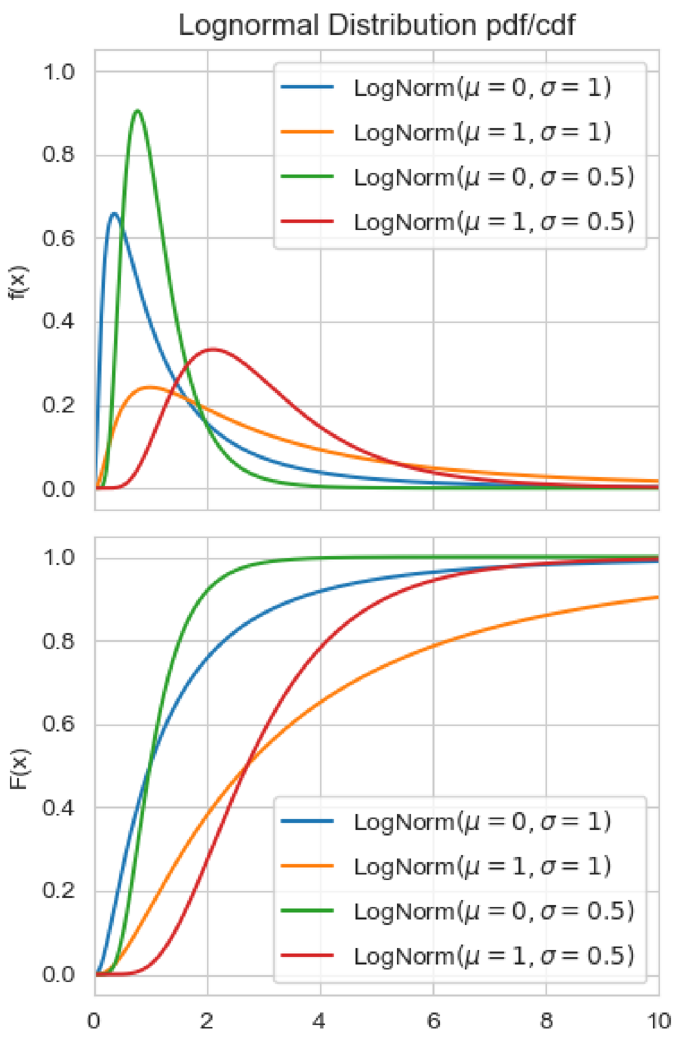


Lognormal Distribution

In SciPy:

scipy.stats.lognorm(x, s=σ, loc, scale)

- μ is not an input: use scale=e^{μ} instead
- loc and scale here refer to the location and scale family of distributions



Exponential Distribution

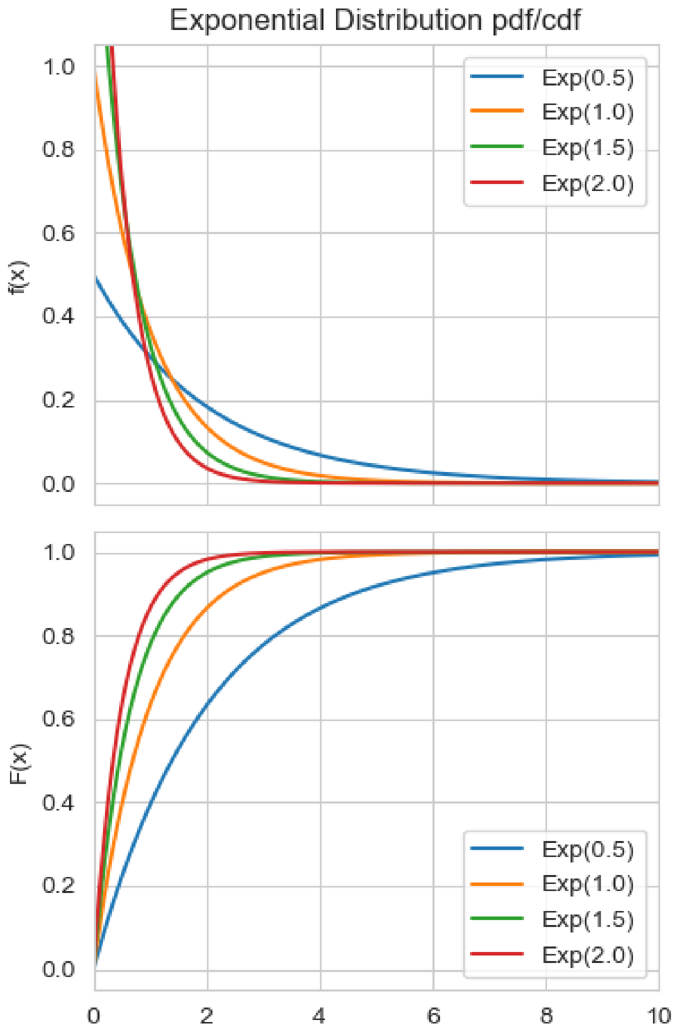
The exponential distribution is often used to model time to failure in reliability modeling.

It is notable for having the memorylessness feature:

$$P(X > x_1 + x_2) = P(X > x_2 | X > x_1)$$

What does this mean? It means the probability of failure in the next 2 minutes is the same whether the component has just been replaced or it has been operating for 1000 hours.

In other words, it represents time between events that occur with a constant rate – a "homogeneous Poisson process"



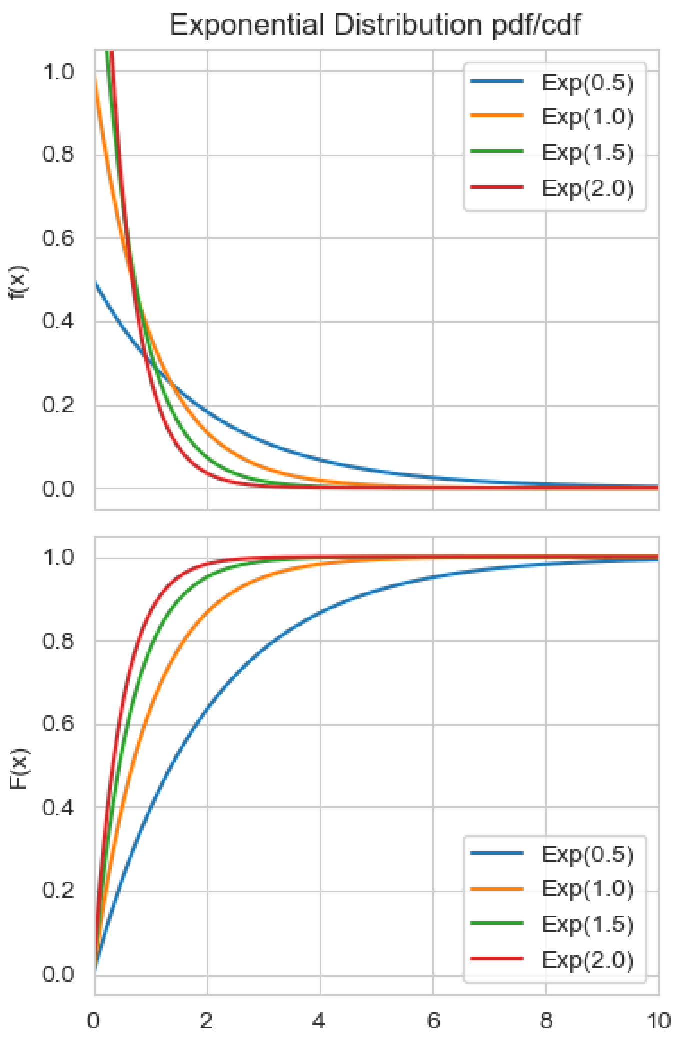
Exponential Distribution

$$X \sim \mathrm{Exp}(\lambda)$$

$$f(x) = \lambda e^{-\lambda x}, \qquad x \geq 0$$
$$F(x) = 1 - e^{-\lambda x}, \qquad x \geq 0$$

Mean: $\mu = \frac{1}{\lambda}$

Variance: $\sigma^2 = \frac{1}{\lambda^2}$

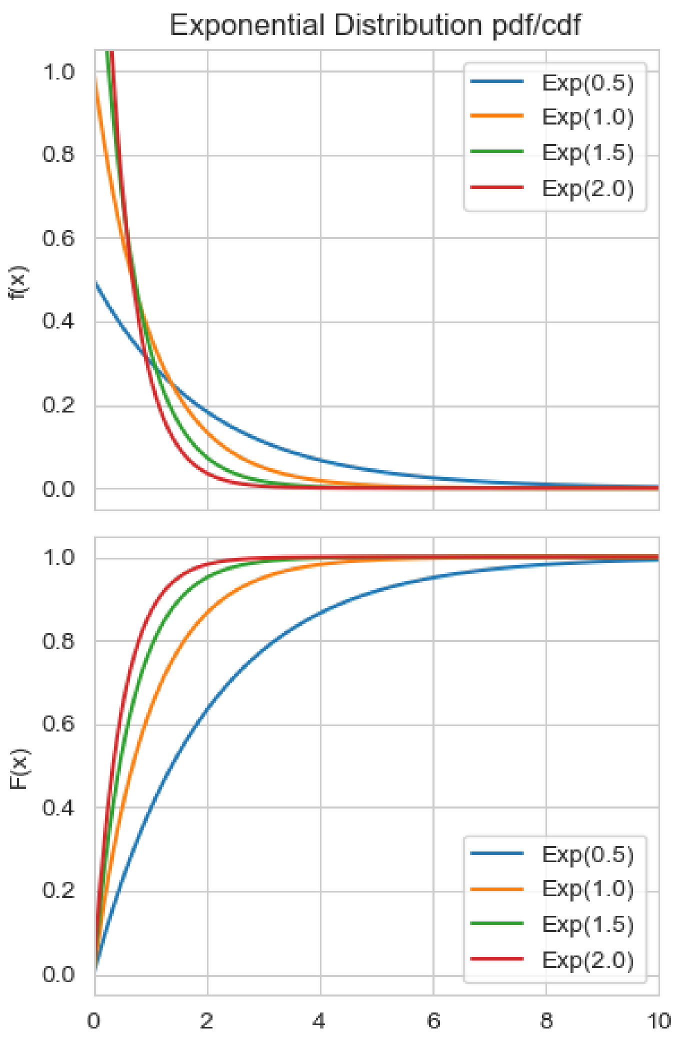


Exponential Distribution

In SciPy:

scipy.stats.expon(x, loc, scale)

- λ is not an input: use scale=$\frac{1}{\lambda}$ instead
- loc and scale here refer to the location and scale family of distributions

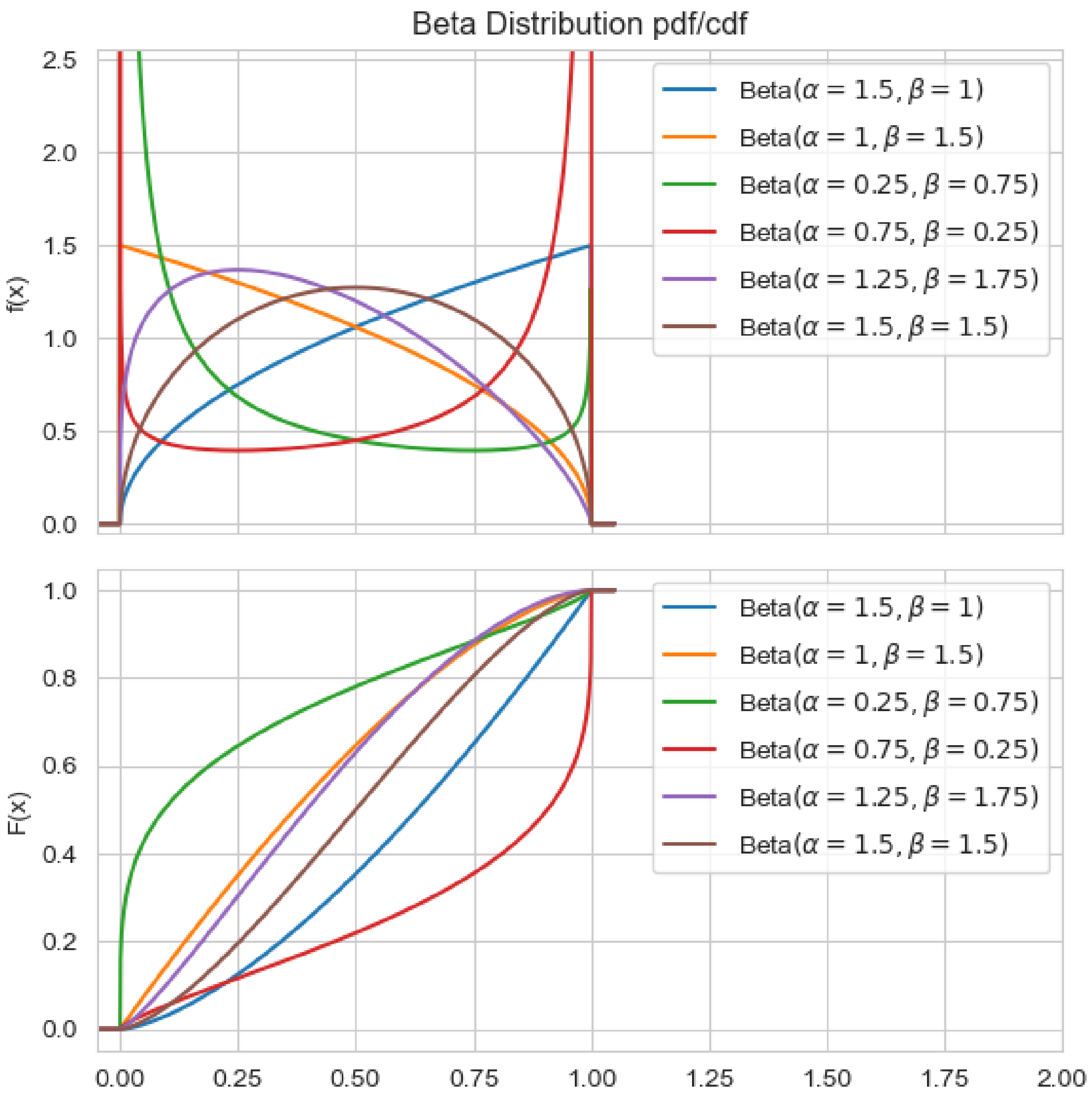


Beta Distribution

Beta is a flexible distribution for values between 0 and 1 – very useful for applications where the RV is a proportion

Varying parameters α>0 and β>0 can result in:

- Increasing distributions $(\alpha > 1, \beta = 1)$
- Decreasing distributions $(\alpha = 1, \beta > 1)$
- U-shaped distributions $(\alpha < 1, \beta < 1)$
- Unimodal distributions $(\alpha > 1, \beta > 1)$
- Symmetric distributions $(\alpha = \beta)$

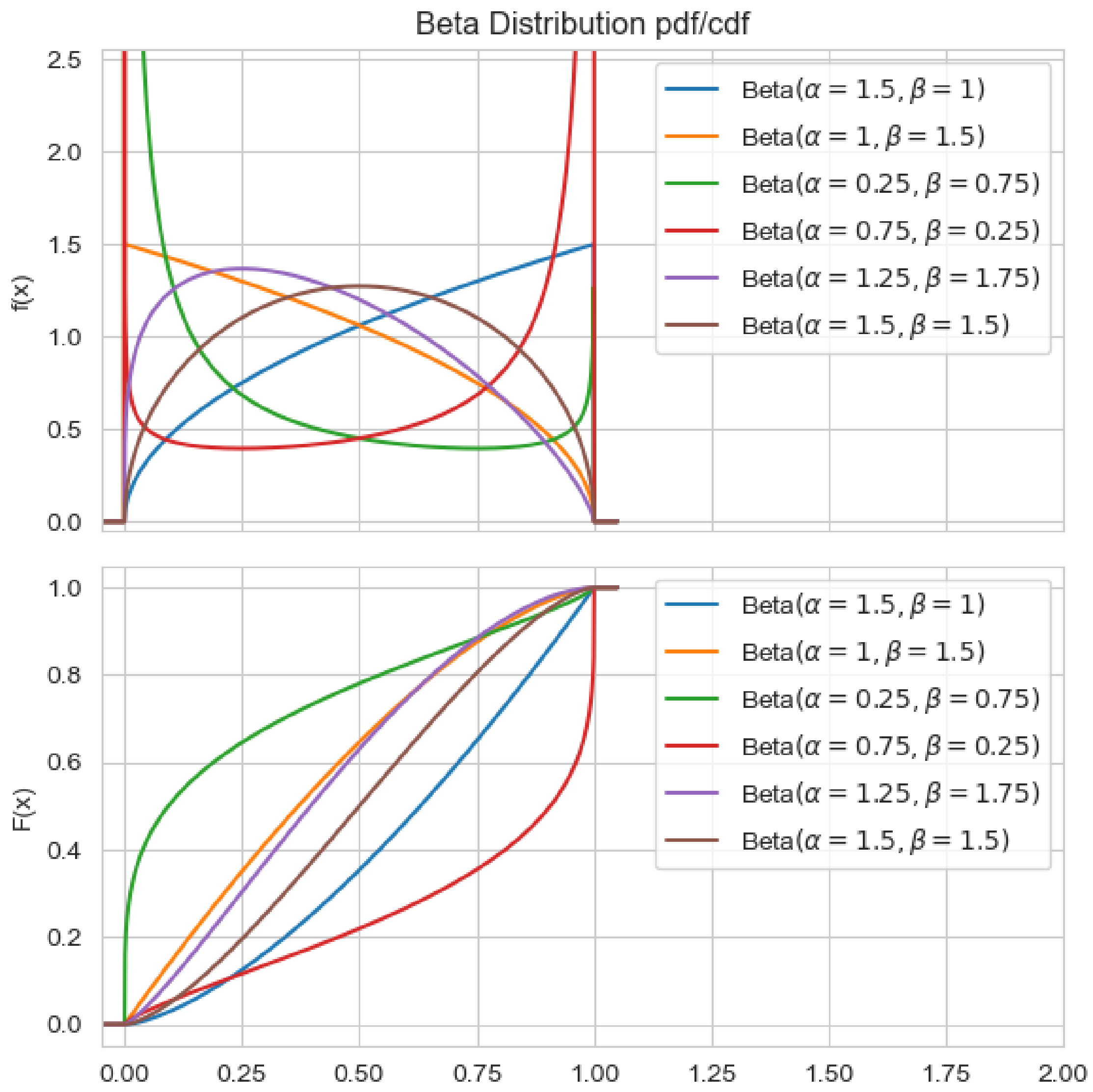


Beta Distribution

$$X \sim Beta(\alpha, \beta)$$

Mean: $\dfrac{\alpha}{\alpha + \beta}$

Variance: $\dfrac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$

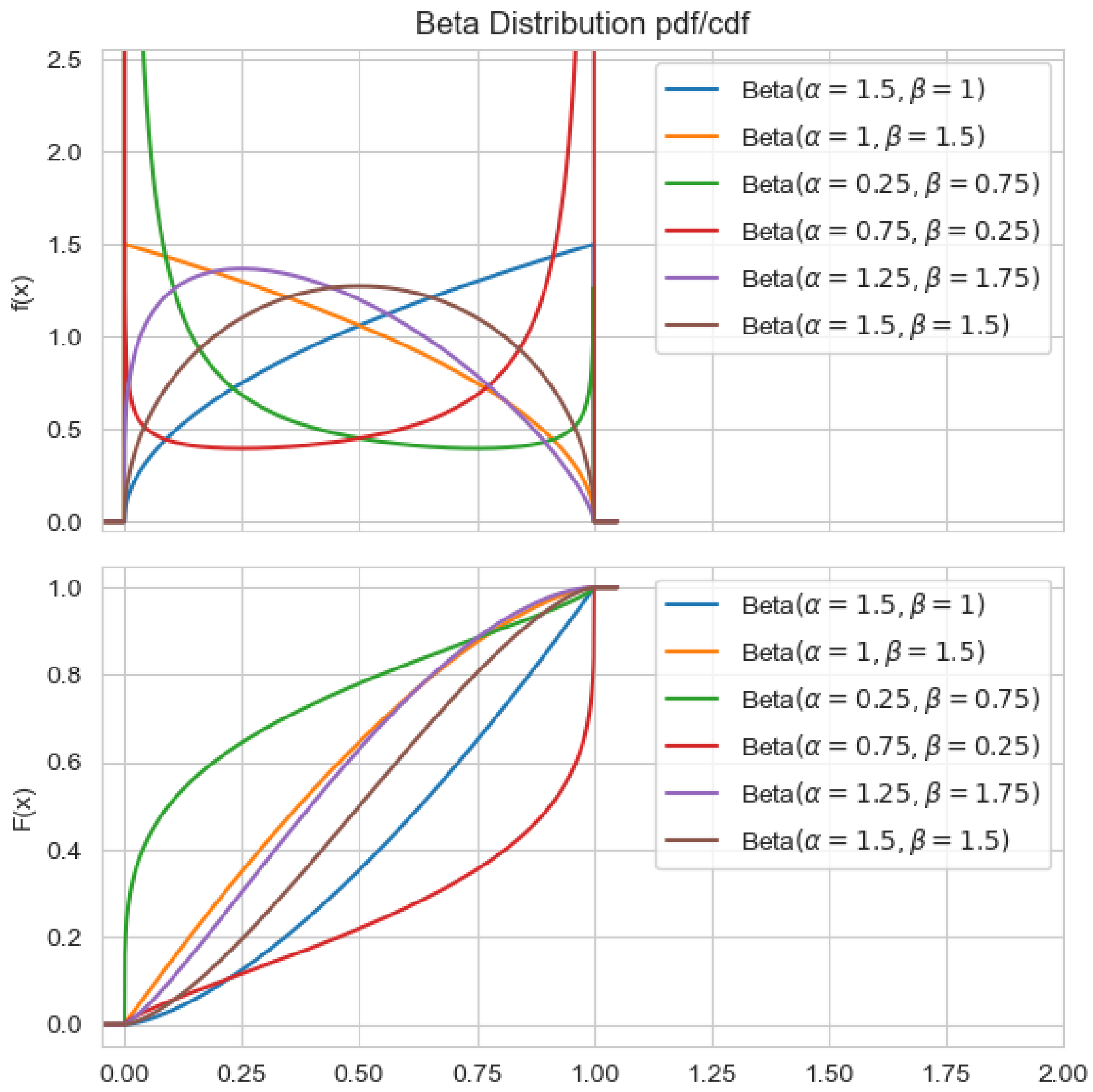


Beta Distribution

In SciPy:

scipy.stats.beta(x, a=α, b=β, loc, scale)

- loc and scale here refer to the location and scale family of distributions



Uniform Distribution

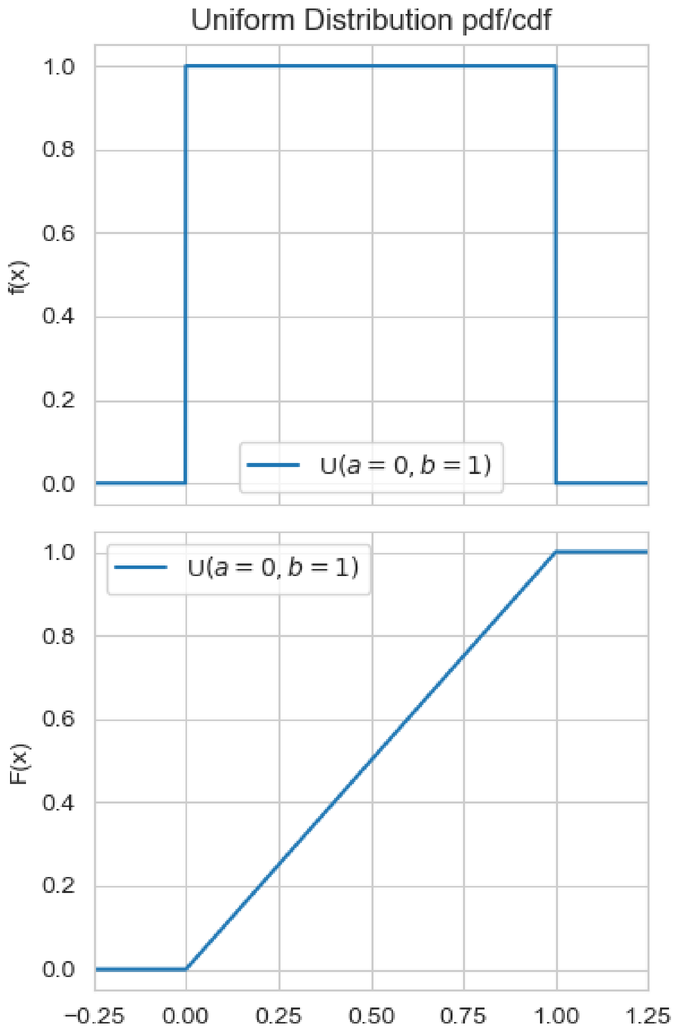
Of course, there is also a continuous uniform distribution

$$X \sim U(a, b)$$

$$f(x) = \frac{1}{b-a}, a \leq x < b$$

$$F(x) = \frac{x-a}{b-a}, a \leq x < b$$

Mean: $\frac{a+b}{2}$ Variance: $\frac{(b-a)^2}{12}$

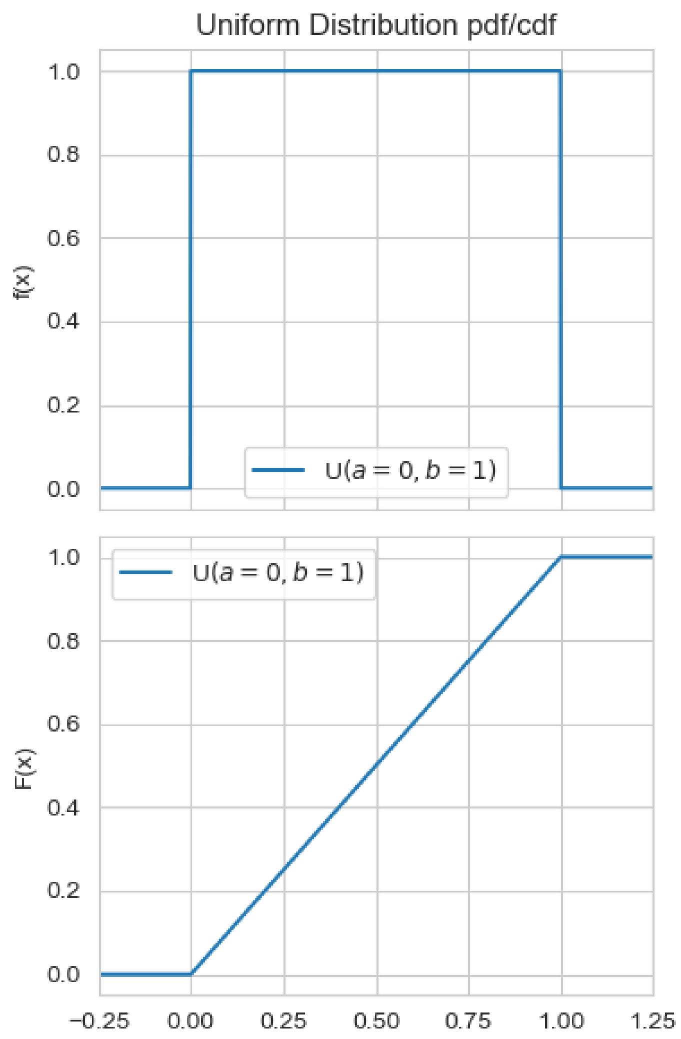


Uniform Distribution

In SciPy:

scipy.stats.uniform(x, loc, scale)

- Set loc=a, scale=$b - a$



Other Distributions

- Weibull distribution
  - A more flexible form of the exponential distribution used in reliability modeling
  - Exponential is a special case of Weibull
- Gamma distribution
  - Very flexible distribution used to approximate odd distributions
  - Exponential and χ^2 are special cases of Gamma
- Cauchy distribution
  - This distribution breaks everything. It has no finite mean or variance.
  - Equivalent to the $t(\nu = 1)$
- Double exponential (Laplace) distribution
  - This is a symmetric version of the exponential distribution defined for all $x \neq 0$

Resources

Wikipedia

- https://en.Wikipedia.org

SciPy.Stats Reference

- https://docs.scipy.org/doc/scipy/reference/stats.html

For deep theory, the STAT 601/602 textbook

- Casella, G., & Berger, R. L. (2002). *Statistical inference*. Cengage Learning.

Recap

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