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Uses of Infinity: Avicenna and Descartes

Infinity, today, is a very intuitive concept that not all people fully understand. Because of this lack of understanding, it can be used in ways that seem intuitive but perhaps not reasonable. In philosophical arguments that show the existence of a "turtle with a jetpack," such as an atom or uncaused cause, bundling an infinite set of regular turtles to find the turtle with a jetpack seems intuitive and a legitimate step in an argument, even though it may not be reasonable. Perhaps foreseeing this problem, Descartes uses our (humans') lack of true understanding (grasp) of infinity to his advantage in a proof of an infinite being. However, an infinite being in some forms may be absurd, like a square circle. In this essay, I will first explore Avicenna's proof of a necessary being which uses the bundling step and make an argument attacking the bundling step based on Levey's *Paradox of the PSR*. Then, I will describe Descartes' argument for an infinite being and explain why Levey's proof creates problems for Descartes.

Avicenna argues for an existent that is necessary through itself. He starts by defining a necessary existent as an existent when considered as not existing, we would arrive at an absurdity or contradiction. More importantly for this proof, we consider two kinds of necessary existences. First, there are those that are necessary through themselves (NTT), which are necessary existences that each could, without raising a contradiction, exist by itself. Contrast this with necessary existences that must exist only because another existence exists. These are necessary through another and possible through themselves (PTT). Any necessary existence must exist either because of itself or because of another.

Separate from the necessary existences discussed above, there are also possible and impossible existences. Impossible existences are existences that would cause a contradiction if they were to exist. Everything that is neither impossibly existent nor necessarily existent is possibly existent. For anything existing, before it existed, it was a possible existence, because it did not exist but could exist. Therefore, if an existing thing is merely possibly existent, there is no difference between the thing before it existed and while it exists, since it is a possibly existent thing during both times. On the other hand, if the existing thing is necessary, then its emergence into existence is necessary and distinguishable from when it was merely possible. These necessary things must have a cause that brought it into existence, so they are called PTT. Thus, whatever exists must be necessary and can be described as either NTT or PTT.

Avicenna begins the argument by safely assuming there is something that exists. Let's call it existence X. If existence X is an NTT, then this proof is finished. Otherwise, it must be a PTT, and it must exist through another existence. But, "there cannot be for anything that is possible [in] itself a cause that is itself possible ad infinitum." To see this, we can take the set of that existence, any PTT existences that it exists through, and again any PTT existences that that existence exists through, and so on, and call it Y. Therefore, Y is a set of PTT existences, such that for any y in Y, every PTT existence that y exists through is also in Y. Y may be a finite or infinite set. This is the bundling step, and it is possible that Avicenna needs to bundle an infinite number of existences into set Y.

Continuing the proof, set Y is either NTT or PTT. If it is NTT, then we have reached a contradiction. If we have understood everything correctly, then set Y is composed of parts (the PTT inside Y), so the set's existence is reliant on the existence of each of those parts. Thus, set Y exists through its parts and cannot be considered an NTT.

If the set were composed of a circle of PTT existences, one might argue that the set can still be considered NTT. For example, if **a** causes **b** and **b** causes **a**, then the set of two PTT, **a** and **b**, can be considered an NTT. Avicenna argues that this is an absurdity, since it essentially posits that the cause of **a** exists after **a** started existing. Through the possibility of time travel or a non-temporal and non-transitive interpretation of causality, this absurdity may not seem so absurd.

However, if an opponent to Avicenna were to push on time travel or some other solution, we can think deeper about a set with parts that cause each other to find an NTT. Any existent in the set cannot exist without another existent in the set, yet because of its own existence, it (through the other existences) must exist. Thus, either the entire set exists or none of it exists, so the existence of each existent in the set relies on the whole to exist. Since a set with parts conjures the image of a set relying on the existence of its elements, we can think of this set as an object with attributes that rely on the object for its existence. This object is the NTT that this proof has been looking for.

On the other hand, if we do not consider set Y as an NTT, it must be a PTT and exist through another existence, call it existence Z. Since every PTT cause of anything in Y is already included in set Y, Z cannot be a PTT, so it must be an NTT existence. Since we have found an NTT existence, this proof is done.

Avicenna's argument uses a bundling step, where he "bundles" all the causes of an existence X of a certain type into a set Y. However, he does not consider that it may be impossible to bundle all causes of X into a set. To be clear, the word set does not need to have any other meanings associated with it, other than just a conjunction or grouping together of

multiple things under one name. However, as we will see later, it may help to see an argument mathematically.

Our argument will exploit this, and it will primarily be an extension of Samuel Levey's argument against van Inwagen-Bennett's argument that the PSR leads to no contingent things. First, I will give a high-level overview of Levey's argument. Then, I will describe why it cannot be used as is against Avicenna, and finally, I will make the argumentative steps necessary to use Levey's argument to explain why Avicenna cannot bundle an infinite number of PTT existences.

Van Inwagen-Bennett argues that under the PSR, there are no contingent truths. His argument, briefly, is that given the PSR and a conjunction of all contingent truths C, there must be a non-contingent reason. Then, the reason for C is necessary, and thus all of C is necessary. Levey believes that this creates a contradiction, which he calls the paradox of the explanatory ground: "Let C be the conjunction of all contingent truths. Suppose there is an explanatory ground G for C that explains and entails C. Is G itself in C? Answer: G is in C if and only if it is not. Contradiction." To clarify the contradiction, if G were contingent, then G must be in C and is self-explaining, which contradicts the definition of a contingent truth. Otherwise, if G were necessary, all of C must also be necessary, contradicting how we defined C. These contradictions tend to lead to a rejection of the PSR.

However, Levey believes that these contradictions instead point to the impossibility of creating C, a set or conjunction of all contingent truths. He sees that implicit in van Inwagen-Bennett's argument is the auxiliary: "If there are any contingent truths, there is a conjunction of all contingent truths." I will use the word set and conjunction interchangeably. Without the auxiliary, we could not even consider a set C of all contingent truths. There are two assumptions for this auxiliary that Levey attacks to support his argument that we cannot consider a

conjunction of all contingent truths. We will only consider one of these assumptions, called Completeness, because Levey makes a stronger argument rejecting this one. Completeness states that if there are any contingent truths, there is such a thing as all contingent truths. It is straightforward to see that if Completeness fails, then the auxiliary fails, and it would be impossible to create a set of all contingent truths.

To reject Completeness, he takes what he calls an Extensibilist stance: contingent truths are indefinitely extensible. Levey defines indefinitely extensible: "A concept F is indefinitely extensible iff for any totality T of things all of which satisfy F, one can, by reference to T, identify a further object x satisfying F but not belonging to T." To be clear, this is not as simple as saying for any set T, in which each element is described by F, we can add another element that is described by concept F to T. Rather, to be indefinitely extensible, a concept F must be such that any set T, in which each element is described by F, can be used to create another element that is described by F. In summary, if we can take a set T, which we claim is all of the elements under concept F, and use T to make another element under concept F, then it is impossible to create a set that is truly "all" of the elements defined under concept F.

Levey shows that contingent truths, according to the PSR, are indefinitely extensible in a simple proof. To make the result more obvious, we should first assume that we can have a set of all contingent truths (the concept contingent truth is not indefinitely extensible).

Let P be a totality of contingent truths. By PSR, there is an explanatory ground G for P. Now, G cannot be necessary, for then P would be necessary. Nor can G be included in P, for then G would be self-explaining. So, G is a contingent truth not in P. The argument is general, so it holds for any totality of contingent truths. Therefore, the concept contingent truth is indefinitely extensible. (Levey)

Notice that this is incredibly similar to the proof of the paradox of the explanatory ground. Instead of arriving at a contradiction and rejecting PSR, Levey keeps PSR and concludes that there is always a contingent truth outside of any set of contingent truths. Thus, we cannot create a set of all contingent truths.

At first glance, we would like to equate contingent truths with Avicenna's PTT in order to find it is indefinitely extensible and reject the bundling step in Avicenna's proof for an NTT existence. While the concepts are very similar, it is not so easy to apply Levey's proof directly. First, PTT are necessary existences that exist through another existence. In Levey's proof, contingent truths cannot be necessary, so the outside grounding, G, cannot be necessary. In the Avicenna parallel, there is no problem letting G be an NTT and still have a set P of PTT.

However, if PTT were indefinitely extensible, Avicenna would not be able to bundle all the PTT, ask for a cause, and claim the cause is NTT. Already, this is gives opponents a way out of Avicenna's argument – he must show that his argument still works even if the world is composed of indefinitely extensible PTT. In other words, he must be able to find an NTT even without bundling all the PTT of a certain kind. Continuing my argument, I will show that Avicenna cannot get out of an indefinitely extensible PTT without raising more questions about his argument.

First, I will show that for any PTT, it must have a PTT that temporally caused it. As we noted earlier, Avicenna's PTT must have a cause that changed its state from nonexistence to existence. We claim that this cause cannot be an NTT and must be a PTT. Assume for reductio, that the cause was an NTT. Then, we can think of the NTT before it caused the PTT to exist and the NTT after it caused the PTT to exist as two parts of the NTT. Such an NTT would have parts and be dependent on those parts. For example, the NTT after causing the PTT would depend on

the NTT before causing the PTT, the NTT's causing of the PTT, and even the PTT. Since this NTT depends on others, it is no longer necessary through itself, which is a contradiction. Therefore, the temporal cause of each PTT cannot be an NTT and must be a PTT. Since each PTT must have a temporal cause, every PTT must be temporally caused by another PTT.

Some may argue that the NTT may not need to change after it creates the PTT if it were "programmed" to create the PTT at a certain time. I have three responses to this. First, I believe there is still a difference between the NTT before it caused the PTT and after it did. Second, that may not be convincing enough, so we also note that a "program" would need to specify a time, which presupposes the objectivity of time, which is hard to see in an infinite timeline. Finally, a "program" would specify the creation and existence of a PTT, which means the program and the PTT depend on each other (the program could not exist if the PTT did not exist, and vice versa). Since the "program" is an attribute or characteristic of the NTT, that would mean the NTT is dependent on the PTT as well as vice versa. That would also result in a contradiction, so the NTT could not be the temporal cause of the PTT.

We can now repeat Levey's argument in this way: Let P be a totality of all PTT. If P is not an NTT, it must be a PTT, and according to the above paragraph, it must have a PTT cause. According to Avicenna, PTT cannot be temporally self-causing, since that would mean it is not necessary through another. Therefore, the PTT cause of P must be outside of P. This argument is general, so it holds for any totality of PTT. Therefore, the concept PTT is indefinitely extensible.

Now, we can return to Avicenna's argument and try to reject each of the steps where he claims to find an NTT. First, Avicenna considers any particular thing X in order to look for an NTT. If X is NTT, his proof is done. My argument cannot fight him here, but if this were the entire proof, it would be an unjustified claim that no one would typically accept. Then, if X is a

PTT, Avicenna considers what caused it. Since I showed that any PTT must have a PTT cause, we can satisfactorily say that it is possible that when Avicenna considers causes of X, he continues to find more PTT. Note that it is possible Avicenna finds an NTT, too, but claiming that he does is as arbitrary and unjustified as claiming X is an NTT.

Finally, Avicenna tries to skip past all the PTT causes by bundling all of them and asking for a cause of the entire bundle. My proof that PTT are indefinitely extensible prevents Avicenna from bundling all PTT causes of X, so he cannot ask for a cause of a bundle. Therefore, Avicenna is unable to prove the existence of an NTT, ignoring the path of just claiming the existence.

A potential criticism of my argument stems from the fact that I show that any PTT must have a temporal cause which must be PTT. However, when we bundle all the PTT causes of X into a set Y, the set may potentially cover all of time until X. Then, asking for a temporal cause of this set no longer seems possible, and I would not be able to show that PTT are indefinitely extensible. Indeed, at this point, it seems reasonable to look for a non-temporal cause, which must be the NTT Avicenna is looking for, since a non-temporal existent was never non-existent thus never merely possible.

However, going down this route breaks down Avicenna's definition of PTT. For his argument to work, he needs PTT to require a cause that changes the PTT's state from non-existence to existence. Since the PTT no longer needs to be caused to change from non-existence to existence (there was no time before its existence), it no longer needs a cause. So, even if an Avicenna proponent argues that a PTT set Y could cover all of time, s/he would still be unable to find a NTT.

Therefore, Avicenna cannot guarantee the existence of an NTT. In Avicenna's argument for an NTT, the bundling step was what guaranteed an NTT. If that step fails, Avicenna can no longer do that. In my argument, I showed that we cannot think or define a set (even mathematically) that describes the bundle of all causes of a certain thing. Perhaps foreseeing these difficulties, Descartes uses the idea that we humans cannot fully understand or describe infinity to prove his version of God, an infinite being. After describing his argument, I will show that nonetheless, Descartes may not get the God he wants.

In *Meditations*, Descartes mentally withdraws from the physical world around from him to think about reality. One idea that is in our minds is that of God, an infinite being. Even for people who deny they have any idea of such a being, they undoubtedly have thoughts of potential, growth, or imperfection - they believe they could improve in some way. Descartes argues that we can only have such ideas of imperfection because we already have an understanding of perfection, which we will call God. God is maximally perfect, such that he could not be *more* in any way, i.e. He is infinitely perfect. Note that Descartes only needs us to have a vague understanding of the infinite, not necessarily a true understanding or grasp of the concept.

Next, Descartes considers where this idea of God comes from. It could not have come from nothingness. For a rationalist, that is absurd,

> "For where, I ask, could the effect get its reality from, if not from the cause? It follows ... that something cannot arise from nothing, and also that what is more perfect—that is, contains in itself more reality—cannot arise from what is less perfect."

Also, an idea of the infinite could not have come from the finite. Descartes claims that there is more reality in the infinite than the finite, so the finite could not have created an idea of the infinite. Without going into the specific definition of what reality is or how to judge the amount of reality, we can think of God as a being infinite in many attributes, including intelligence. We have an understanding of an infinitely intelligent being, because we understand the imperfect intelligence in ourselves and even in the most intelligent of our peers. We have an idea of an infinitely intelligent being, so there is some part of that infinite intelligence that could not have been created from our finite intelligence, "For if we suppose that an idea contains something which was not in its cause, it must have got this from nothing." Since the infinite intelligence is clearly not contained within our finite minds, it must have come from something infinite.

Opponents to Descartes may try to argue that we could build up to an infinite using the finite. We can think of this more mathematically. First, think of adding one to a number continuously without end. Mathematically, we would think this gives us infinity using only finite pieces. However, to do this indefinite series of summations, we first need to understand that numbers can be summed up until infinity. Thus, we would first need to understand a concept of infinity before doing the summation. Furthermore, we could think of any two points in a plane. (With regards to the earlier infinitely intelligent God, we can think of one of the axes as quantifying level of intelligence) We can then draw a line through the two points and extend it indefinitely. This way, we have built a concept of the infinite (and even a concept of the infinitely intelligent being). In the same way as Descartes would refute the first case (of the summations), we would need to understand an infinitely large plane before extending the line. To build anything finite into the infinite, we would first need to understand that there is "space" or possibility of the infinite, which presumes an idea of the infinite. Thus, we already have an

understanding of the infinite, and it could not have come from anything finite, so it must have come from an infinite being.

Now, we can consider how Levey's argument works against Descartes. In the extensions of his conclusion, Levey considers that a religious God could not know everything or have caused everything, but rather for any single thing that exists, God knows it and causes it. The effect is very similar, but this puts a limitation on what God can do. A religious opponent may not be happy with this. S/he would most likely say that his/her God is so much greater than us and our logic that God can know everything and must have caused everything. Does this same defense work for Descartes?

First, what kind of infinity would Descartes want for his proof of God? Most likely, the biggest infinity there could be, because otherwise, God is no longer necessarily the most powerful or the most intelligent or the most anything. Then, it must be an infinite being whose infinity is at least as big as the indefinitely extensible one. Earlier, we showed that any concept that is indefinitely extensible could not possibly be all fit into a single set. A set is merely a mathematical descriptor of things. Each thing could be represented by a set of all its attributes/properties. Therefore, it is a contradiction to say that there exists an indefinitely extensible infinity all in one singular thing, no matter how easy we think it is to imagine such a thing. I use thing in the broadest sense – whatever you can think of or point to or even have a faintest idea of is a thing.

So, an indefinitely extensible infinite being is a contradiction. An opponent might take the view similar to the religious opponent above - that an infinite being is so powerful that it surpasses these contradictions. This is one way to go, but it opens doors for square circles and

other illogical existences to exist in our world. After all, if an illogical being could exist, there's no reason square circles could not. This is too unreasonable.

Another opponent might attack my claim that Descartes' infinite being must be infinite in the sense of indefinitely extensible. It is hard already to prove that Descartes' infinite being is good or has any specific attribute, other than being infinite. Why not downgrade its infinity, too? Descartes' proof still works even if the infinite being only has the cardinality of, say, the counting numbers. However, we can use Descartes' own proof to show that his infinite being must be indefinitely extensible.

We have an idea of the indefinitely extensible infinite. We showed our at least vague understanding of the concept in the proof against Avicenna. Since such an infinity is clearly greater than the infinity in counting numbers or any other set that can be considered a singular set, our idea of the indefinitely extensible could not have come from any infinite being that is only infinite in the sense of the counting numbers. Therefore, by Descartes' own logic, there must exist an indefinitely extensible infinite being, which is a contradiction as we have shown. There must be something wrong in Descartes' proof if the same logic can be used to prove a contradiction.

Dismissing the possibility that contradictions should be allowed to exist, there is one more route a Descartes supporter might take. An indefinitely extensible being is only a contradiction if it is singular, as we have only shown that an indefinitely extensible set (singular) cannot exist. It is possible that Descartes' God is not one being, but multiple (perhaps an infinite number of) that together hold attributes (power, intelligence, etc.) that are indefinitely extensible. Since this broadens the definition of what God could be (we could all be gods), I will not try to defend against this route of argument.

Levey's argument effectively against bundling shows that our intuition of infinity is not necessarily correct. As easy as it is to think that we can bundle an infinity of any object or that our idea of infinity must have come from some infinite being, it does not always make logical sense. For Avicenna, we are left with a need to show that in a universe of indefinitely extensible PTT, we can still find an NTT. For Descartes, we must accept infinite number of beings, such that all beings together have indefinitely extensible infinite attributes or reject infinite beings altogether.

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